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Two Dimensional Object Recognition Using Rajan Transform

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Abstract—This transform was introduced in the year 1997 by Rajan [2], [4] and [5] on the lines of Hadamard Transform. This paper presents, in addition to its formulation, the algebraic properties of the transform and uses in pattern recognition, the technology related to object recognition, that is, about the use of Rajan Transform (RT) in recognizing regular and irregular objects. Rajan Transform is a homomorphism that maps a set consisting of a number sequence, its graphical inverse and their cyclic and dyadic permutations, to a set consisting of a unique number sequence ensuring the invariance property under such permutations. This paper describes in detail the techniques of using RT for recognizing regular and irregular shape objects.

Index Terms— **Image processing, Object Recognition, Pattern classification, Homomorphic Transform**

I. Introduction

Pattern recognition is essentially a classification process. It is after a prolonged research and study of related techniques like Permutation Invariant Systems and Number Theory, Rajan Transform was introduced in the year 1997 as a novel algorithm for classification purposes, which was earlier known as Rapid Transform. Subsequent research on the algebraic properties of this transform has exposed its richness. The purpose of this paper is to present a comprehensive introduction to RT and its algebraic properties, and its role in developing high-speed algorithms for recognizing regular and irregular shape objects. Basically any shape could be described in terms of certain regular shapes defined in a 3X3 neighborhood structure in the manner how an arbitrary signal is expressed in terms of the orthogonal functions of sinusoidal and co-sinusoidal functions as a power series (Fourier series) consisting of weighted sinusoids and co-sinusoids.

Signal processing is either carried out in time domain or frequency domain depending on the needs and requirement. Similarly shape processing (recognition) is carried out on the representative polygons (similar to frequency components) using Rajan Transform. This paper is intended to present comprehensive details about the state-of-the-art technique of recognizing shapes of objects by analyzing representative polygons.

II. Rajan Transform

Rajan Transform is essentially a fast algorithm developed on the lines of Decimation-In-Frequency (DIF) Fast Fourier Transform algorithm, but it is different from the DIF-FFT algorithm. Given a number sequence x(n) of length N, which is a power of 2, first it is divided into the first half and the second half each consisting of (N/2) points so that the following hold good.

$g(j) = x(i)+x(i+(N/2)) \quad ; \quad 0 \le j \le N/2 \; ; \; 0 \le i \le N/2$

$h(j) = |x(i) - x(i-N/2)| \quad ; \quad 0 \le j \le N/2 \; ; \; (N/2) \le i \le N$

Now each (N/2)–point segment is further divided into two halves each consisting of (N/4) points so that the following hold good.

$g_1(k) = g(j) + g(j + (N/4))$; $0\le k\le (N/4)$; $0\le j\le (N/4)$

$g_2(k) = |g(j) - g(j -(N/4))|$;$0\le k\le(N/4)$;$(N/4)\le j\le (N/2)$

$h_1(k) = h(j) + h(j + (N/4))$; $0\le k\le (N/4)$; $0\le j\le (N/4)$

$h_2(k) = |h(j) -h(j - (N/4))|$;$0\le k\le (N/4)$;$(N/4)\le j\le (N/2)$

This process is continued till no more division is possible. The total number of stages thus turns out to be $\log_2 N$. Let us denote the sum and difference operators respectively as + and ~. If x(n) is a number sequence of length $N = 2^k$; K>0, then its Rajan Transform is denoted as X(k). RT is applicable to any number sequence and it induces an isomorphism in a class of sequences, that is, it maps a domain set consisting of the cyclic and dyadic permutations of a sequence on to a range set consisting of sequences of the form X(k)E(r) where x(k) denotes the permutation invariant RT and E(r) an encryption code corresponding to an element in the domain set. This map is a one-to-one and on to correspondence and an inverse map also exists. Hence it is viewed as a transform. Consider a sequence x(n) = 3, 8, 5, 6, 0, 2, 9, 6. Then X(k) = 39, 5, 13, 9,

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13, 1, 7, 5. The signal flow graph for this transform together with the encryption key (number 0 or 1 inside the brackets) is given in figure 2.1.

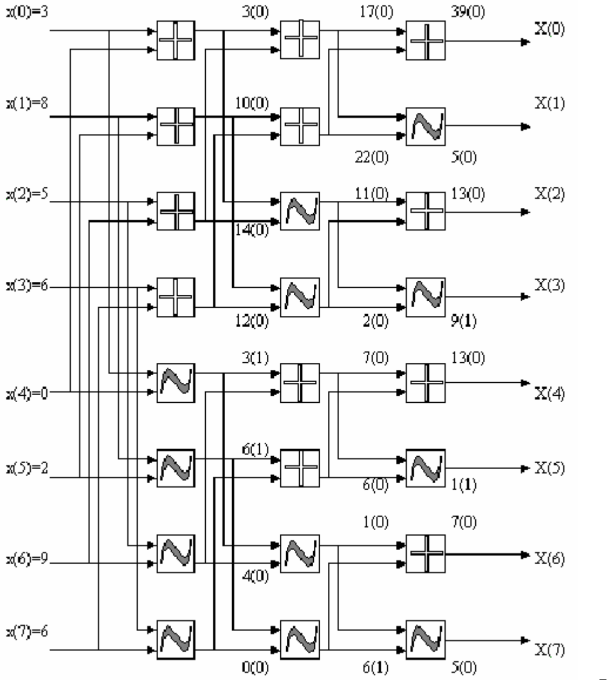


Fig. 2.1: Signal flow diagram of one dimensional 8-point RT

Observe from the diagram that E(r) is a union of three sequences $E_1(r)$ = 0, 0, 0, 0, 1, 1, 0, 0 , $E_2(r)$ = 0, 0, 0, 0, 0, 0, 0, 1 and $E_3(r)$ = 0, 0, 0, 1, 0, 1, 0, 0. That is, $E(r) = E_1(r)E_2(r)E_3(r)$. Now, the sequence X(k)E(r) = 39, 5, 13, 9, 13, 1, 7, 5, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0 is the RT of the input sequence x(n) = 3, 8, 5, 6, 0, 2, 9, 6. In general, the first point in a RT is called 'Cumulative point Index' (CPI).

III. INVERSE RAJAN TRANSFORM

Retrieval of the information or the signal x(n) can be done by Inverse Rajan Transform (IRT) [2], [4] and [5]. The basic requirements for the IRT computation are the RT coefficients associated with the encryption values (k values) that are generated by encryption function while computing forward RT. The strategy adopted here is to retrace the forward transform signal flow graph. It was observed that the point-wise addition of a constant value, say K to the sample sequence x(n) while computing forward RT, changes only the DC component X_0, that is, the CPI to X_0+NK, which is the first coefficient of the newly computed RT, and the remaining spectral values remain the same of the original spectrum. For example, the RT spectrum X(k) of the sequence x(n) = 3, 8, 5, 6, 0, 2, 9, 6 is 39, 5, 13, 9, 13, 1, 7, 5. Now let us build a new sequence $x_1(n)$ = 4, 9, 6, 7, 1, 3, 10, 7 by adding K=1 to the sequence x(n) = 3, 8, 5, 6, 0, 2, 9, 6. The RT spectrum of the new sequence is $X_1(k)$ = 47, 5, 13, 9, 13, 1, 7, 5. Now, in order to work with sequences containing negative sample values, we proceed as usual in the case of forward transform. But, the inverse transform is calculated just by adding a constant value $N(2^M-1)$ to the CPI value of the spectrum. M is the bit length required to represent the maximum quantization level of the samples and N is the length of the sequence. This constant factor $K = (2^M-1)$ is chosen such that all the maximum possible negative values of the sequence x(n) are level shifted to 0 or above. This DC shift is required, because we hide the sign of the negative values that are generated while computing the forward RT. As mentioned earlier, RT induces an isomorphism between the domain set consisting of the inverse, cyclic, dyadic and dual class permutations of a sequence on to a range set consisting of sequences of the form X(k)E(r) where X(k) denotes the permutation invariant RT and E(r) an encryption code corresponding to an element in the domain set. This map is a one-to-one and on-to correspondence and an inverse map also exists. Thus RT is viewed as a transform. Now we provide a technique for obtaining the inverse of Rajan Transform. Inverse Rajan transform (IRT) is a recursive algorithm and it transforms a RT code X(k)E(r) of length N(1+m) where $N = 2^m$ and m is the number of stages of computation, into one of its original sequences belonging to its permutation class depending on the encryption code E(r). The computation of IRT is carried out in the following manner. First the input sequence is divided into segments each consisting of two points so that either

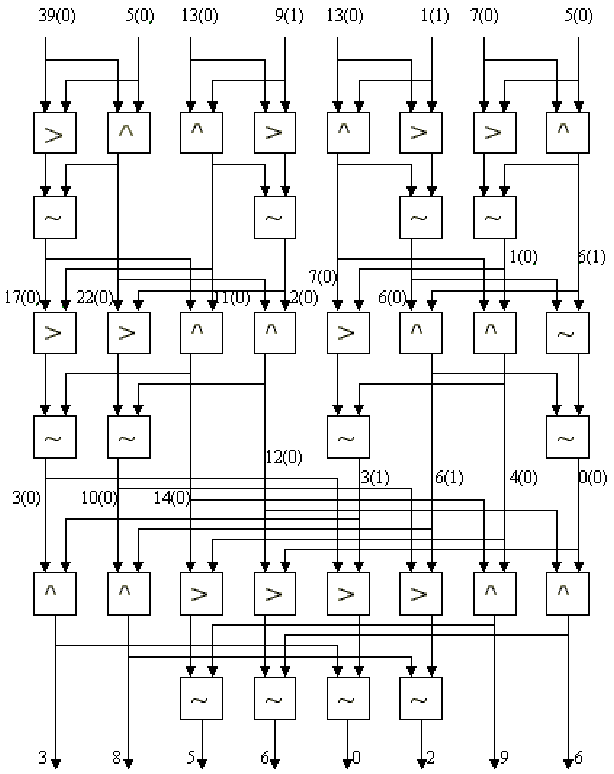
$g(2j+1) = (X(2k)+X(2k+1))/2$
$g(2j) = \max(X(2k), X(2k+1))-g(2j+1);$
if $E_1(2r)= 0$ and $E_1(2r+1) = 0$; $0 \le j<N$; $0 \le k \le N$; $0 \le r \le N$,

or

$g(2j) = (X(2k)+X(2k+1))/2$
$g(2j+1) = \max(X(2k), X(2k+1) - g(2j)$
if $E_1(2r) = 1$ or $E_1(2r+1) = 1$; $0 \le j \le N$; $0 \le k \le N$; $0 \le r \le N$.

The resulting sequence is divided into segments each consisting of four points. Each 4-point segment is synthesized as per the above procedure. The resulting sequence is further divided into segments each consisting of eight points and the same procedure is carried out. This process is continued till no more division is possible. Consider X(k) = 39,5,13,9,13,1,7,5. Then its IRT is x(n) = 3, 8, 5, 6, 0, 2, 9, 6. The inverse x(n) is obtained from the given X(k)E(r) as shown in figure 3.1.

The symbols ^ > and ~ respectively denote the operators average of two, maximum of two and difference of two. Note that IRT will work only in the presence of encryption sequence E(r) and for every member of the permutation class there would be a unique encryption sequence. Study of the class of encryption sequences corresponding input sequences itself is a field of active research. A specific example would make things easy. Let us consider a sequence x(n) = 3, 8, 5, 6, 0, 2, 9, 6 and its RT, X(k) = 39, 5, 13, 9, 13, 1, 7, 5. Now addition of 15 to each sample point in x(n) would yield a sequence $x_1(n)$ = 18, 23, 20, 21, 15, 17, 24, 21 and its RT would be $X_1(k)$ = 159, 5, 13, 9, 13, 1, 7, 5. Observe that $X_1(0) = X(0) + 120 = 39 + 120 = 159$. As outlined earlier, one can add $N(2^M-1)$ to X(0) of a spectral sequence and compute the IRT as usual. Then it is important to subtract 2^M-1 from every sample point so as to obtain the actual sample domain sequence. Let us take the case of X(k) = 39, 5, 13, 9, 13, 1, 7, 5. Now let us add 120 only to X(0) so that the spectral sequence becomes 159, 5, 13, 9, 13, 1, 7, 5. The IRT of this sequence is 18, 23, 20, 21, 15, 17, 24, 21. By subtracting 15 from each sample value we obtain the actual sequence 3, 8, 5, 6, 0, 2, 9, 6.



IV. ALGEBRAIC PROPERTIES OF RAJAN TRANSFORM

RT has interesting algebraic properties like permutation invariance, scalar property and linear pair forming property. All such properties are explained in short in the following.

Cyclic shift invariance property

Let us consider the same sequence x(n) = 3,8,5,6,0,2,9,6. Using this sequence one can generate seven more cyclic shifted versions such as $x_{c1}(n)$ = 6, 3, 8, 5, 6, 0, 2, 9 , $x_{c2}(n)$ = 9, 6, 3, 8, 5, 6, 0, 2 , $x_{c3}(n)$ = 2, 9, 6, 3, 8, 5, 6, 0 , $x_{c4}(n)$ = 0, 2, 9, 6, 3, 8, 5, 6 , $x_{c5}(n)$ = 6, 0, 2, 9, 6, 3, 8, 5 , $x_{c6}(n)$ = 5, 6, 0, 2, 9, 6, 3, 8 and $x_{c7}(n)$ = 8, 5, 6, 0, 2, 9, 6, 3. It is obvious that the cyclic shifted version of $x_{c7}(n)$ is x(n) itself. One can easily verify that all these eight sequences have the same X(k), that is, the sequence 39, 5, 13, 9, 13, 1, 7, 5 but different E(r).

Graphical inverse invariance property

The sequence x(n) = 3, 8, 5, 6, 0, 2, 9, 6 has the graphical inverse $x^{-1}(n)$ = 6, 9, 2, 0, 6, 5, 8, 3. Using this sequence one can generate seven more cyclic shifted versions such as $x_{c1}^{-1}(n)$ = 3, 6, 9, 2, 0, 6, 5, 8 , $x_{c2}^{-1}(n)$ = 8, 3, 6, 9, 2, 0, 6, 5 , $x_{c3}^{-1}(n)$ = 5, 8, 3, 6, 9, 2, 0, 6 , $x_{c4}^{-1}(n)$ = 6, 5, 8, 3, 6, 9, 2, 0 , $x_{c5}^{-1}(n)$ = 0, 6, 5, 8, 3, 6, 9, 2 , $x_{c6}^{-1}(n)$ = 2, 0, 6, 5, 8, 3, 6, 9 and $x_{c7}^{-1}(n)$ = 9, 2, 0, 6, 5, 8, 3, 6. It is obvious that the cyclic shifted version of $x_{c7}^{-1}(n)$ is $x^{-1}(n)$ itself. One can easily verify that all these eight sequences have the same X(k), that is, the sequence 39, 5, 13, 9, 13, 1, 7, 5 but different E(r)

Dyadic shift invariance property

The term 'dyad' refers to a group of two, and the term 'dyadic shift' to the operation of transposition of two blocks of elements in a sequence. For instance, let us take x(n) = 3, 8, 5, 6, 0, 2, 9, 6 and transpose its first half with the second half. The resulting sequence $T_d^{(2)}[x(n)]$ = 0, 2, 9, 6, 3, 8, 5, 6 is the 2-block dyadic shifted version of x (n). The symbol $T_d^{(2)}$ denotes the 2-block dyadic shift operator. In the same manner, we obtain $T_d^{(4)}[T_d^{(2)}[x(n)]]$ = 9, 6, 0, 2, 5, 6, 3, 8 and $T_d^{(8)}[T_d^{(4)}[T_d^{(2)}[x(n)]]]$=6, 9, 2, 0, 6, 5, 8, 3. One can easily verify that all these dyadic shifted sequences have the same X(k), that is, the sequence 39, 5, 13, 9, 13, 7, 5 but different E(r). There is yet another way of dyadic shifting the input sequence x(n) to $T_d^{(2)}[T_d^{(4)}[T_d^{(8)}[x(n)]]]$. Let us take x(n) = 3, 8, 5, 6, 0, 2, 9, 6 and obtain the following dyadic shifts: $T_d^{(8)}[x(n)]$ = 8, 3, 6, 5, 2, 0, 6, 9 , $T_d^{(4)}[T_d^{(8)}[x(n)]]$ = 6, 5, 8, 3, 6, 9, 2, 0 and $T_d^{(2)}[T_d^{(4)}[T_d^{(8)}[x(n)]]]$ = 6, 9, 2, 0, 6, 5, 8, 3. Note that $T_d^{(2)}[T_d^{(4)}[T_d^{(8)}[x(n)]]] = T_d^{(8)}[T_d^{(4)}[T_d^{(2)}[x(n)]]]$. One can easily verify from the above that other than $T_d^{(4)}[T_d^{(2)}[x(n)]]$ and $T_d^{(8)}[x(n)]$, all other dyadically permuted sequences fall under the category of the cyclic permutation class of x(n) and $x^{-1}(n)$. This amounts to saying that the cyclic permutation class of x(n) has eight non-repeating independent sequences, that of $x^{-1}(n)$ has eight non-repeating independent sequences and the dyadic permutation classes of x(n) has two non-repeating independent sequences. To conclude, all these 18 sequences could be seen to have the same X(k). Each of these 18 sequences has an independent encryption key E(r).

Dual class invariance

Given a sequence x(n), one can construct another sequence y(n) consisting of at least one number which is not present in x(n) such that X(k) = Y(k). In such a case, y(n) is called the dual of x(n). An arbitrary sequence x(n) of length $N=2^n$ is said to have a dual y(n) if and only if its CPI is an even number and is divisible by N/2. In other words, x(n) is said to form a dual pair with y(n). Consider two sequences x(n) = 2, 4, 2, 2 and y(n) = 3, 1, 3, 3. Note that X(k) = Y(k) = 10, 2, 2, 2 and the point-wise mean of the two sequences x(n) and y(n) is 2.5. From a dual pair one can generate another sequence (child sequence) consisting of numbers which are point-wise differences of the pair. Duality is a hereditary property that is transferred to child sequences.

Regenerative property

The sequence |x(0)-y(0)|, |x(1)-y(1)|, |x(2)-y(2)|, ….., |x(N-2)-y(N-2)|, |x(N-1)-y(N-1)| of a dual pair x(n) and y(n) of length N is eligible to form a dual pair with yet another sequence. For example let us consider a sequence x(n) = 3, 1, 3, 3. Its CPI is 10 and it is divisible by N/2, that is, 2 yielding the value 5. Now one can obtain the dual sequence y(n) = 2, 4, 2, 2 by subtracting each element of x(n) from 5. Now x(n) and y(n) form a pair and yield their 'first generation child sequence', say $x_1(n)$ = 1, 3, 1, 1 , which is obtained by finding the point wise difference between the parent sequences x(n) and y(n). One can easily verify that $x_1(n)$ also is eligible to form a dual pair with $y_1(n)$ = 2, 0, 2, 2. This hereditary property is termed here as 'regenerative property'. Another important observation follows. $x_1(n)$ forms a dual pair with $y_1(n)$ but this pair does not produce a child sequence as it was done by the pair <x(n), y(n)>.This could be easily verified by the fact that the point wise difference between $x_1(n)$ and $y_1(n)$ yields $x_1(n)$ only. Hence for brevity we call the sequences $x_1(n)$ and $y_1(n)$ as 'sterile sequences' and the pair <$x_1(n)$, $y_1(n)$> as a 'sterile pair'. This property opens up new avenues for further research on generative and sterile pairs which have potential applications to areas related to encryption, and cryptography..

Scalar property

Let x(n) be a number sequence and ℓ be a scalar. Then the RT of ℓx(n) will be ℓX(k), where X(k) is the RT of x(n). For example, let us consider a sequence x(n) = 1, 3, 1, 2 and a scalar ℓ of value 2. Now the RT X(k) of x(n) is 7, 3, 1, 1. The RT of ℓx(n) = 2, 6, 2, 4 is 14, 6, 2, 2 which is nothing but ℓX(k).

Linearity property

In general, RT does not satisfy the linearity property. However, it was observed that for a pair x(n) and y(n) which are number sequences either in the increasing order or in the decreasing order, the linearity property holds. That is, for ℓx(n)+my(n) where ℓ and m are scalars and x(n) and y(n) are two number sequences either in the increasing or decreasing order, the RT will be ℓX(k)+mY(k), where X(k) and Y(k) are respectively the RTs of x(n) and y(n). A characterization theorem is yet to be established for categorizing pairs of sequences which would satisfy linearity property.

Linear pair forming property

Two sequences x(n) and y(n) are said to form a linear pair when X(k)+Y(k) is the RT of x(n)+y(n), where X(k) and Y(k) are the RTs of x(n) and y(n) respectively. The symbol + denotes the point wise addition of two sequences. As outlined earlier, pairs of sequences consisting of increasing numbers of decreasing numbers only form linear pair. In general arbitrary sequences do not form linear pairs. Consequently RT could be viewed as a nonlinear transform. However, higher order RT spectra do form linear pairs, and this has been identified as a very useful property for pattern recognition purposes.

For instance let us consider two arbitrary sequences x(n) and y(n) given below:

x(n) = 2,2,2,1,6,2,6,1,2,3,0,0,2,5,5,4; (16-point sequence)

y(n) = 4,5,3,1,0,1,4,6,6,8,0,0,7,9,0,5; (16-point sequence)

Now the RT of x(n) denoted as $X_1(k)$ is computed as $X_1(k)$ = 43, 7, 7, 5, 19, 7, 7, 3, 15, 1, 1, 1, 9, 1, 3, 3 and the RT of y(n) denoted as $Y_1(k)$ is computed as $Y_1(k)$ = 59, 11, 21, 1, 17, 9, 9, 5, 29, 3, 11, 7, 11, 1, 9, 1. Now, z(n) = x(n) + y(n) is given by the sequence 6, 7, 5, 2, 6, 3, 10, 7, 8, 11, 0, 0, 9, 14, 16, 6, 8, 6, 8, 6, where as $X_1(k)$ + $Y_1(k)$ = 102, 18, 28, 6, 36, 16, 16, 8, 34, 4, 12, 8, 20, 2, 12, 4. Note that Z(k) is not equal to $X_1(k)$ + $Y_1(k)$. Now, the second order RT spectra of $X_1(k)$ and $Y_1(k)$ are respectively computed as $X_2(k)$ = 132, 76, 72, 64, 32, 32, 28, 28, 64, 32, 36, 20, 24, 16, 20, 12 and $Y_2(k)$ = 204, 128, 76, 56, 80, 68, 48, 44, 72, 20, 32, 20, 36, 32, 16, 12. Let $z_1(n)$ be the sequence $X_1(k)$ + $Y_1(k)$. Then $Z_1(k)$ is the RT of and $z_1(n)$ and is computed to be 326, 194, 138, 110, 98, 86, 70, 66, 138, 70, 86, 42, 66, 62, 42, 38. Note that $Z_1(k)$ is not equal to $X_2(k)$ + $Y_2(k)$ = 336, 204, 148, 120, 112, 100, 76, 72, 136, 52, 68, 40, 60, 48, 36, 24. This procedure is repeated for the third order spectra as follows: $X_3(k)$ = 688, 128, 128, 64, 304, 96, 96, 64, 240, 0, 32, 32, 144, 32, 32, 32 and $Y_3(k)$ = 944, 184, 336, 104, 272, 136, 144, 88, 464, 40, 176, 24, 176, 24, 144, 8 are the RTs of $X_2(k)$ and $Y_2(k)$ respectively. The RT of $X_2(k)$ + $Y_2(k)$ is computed to be 1632, 312, 464, 168, 576, 232, 240, 152, 704, 40, 208, 56, 320, 56, 176, 40 which is nothing but $X_3(k)$ + $Y_3(k)$.

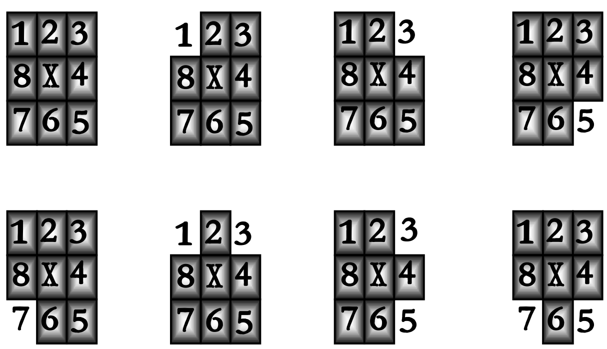
Thus it is clear from the above example that higher order RT spectra of arbitrary sequences of equal length do form linear pairs, and so the name 'linear pair forming property'. Linear pair forming property is also called self-organizing property. To be more precise, given any arbitrary random sequence of appropriate length, the successive RTs would introduce order in the sequences and exhibit linearity property.

V. OBJECT RECOGNITION USING RAJAN TRANSFORM

Shape representation

Any regular or irregular object image (two dimensional) could be represented by a set of basis convex polygons that are shown in figure 5.1. The convex polygons are labeled based on the vertices whose cells are dropped. The term `convex polygon' refers to the polygon which is drawn by connecting the boundary cells and the central cell is not touched by the lines drawn. For example the convex polygon $E_{1,3,5,7}$ is the polygon obtained by dropping the cells 1, 3, 5 and 7. Let us consider an irregular shape shown in figure 5.2. The image shown in figure 5.2 is quantized as shown in figure 5.3.

The quantized image is further scanned by the 3X3 neighborhood window and on each move the polygon that could be fitted inside the image is observed. For example the first scan position of the image is shown in figure 5.4. The polygon B_1 is found to be contained in the image. The window is moved to the right by one cell. Figure 5.5 shows the second scan position in the image. In this position polygon A is found to be contained in the image.



A B$_1$ B$_3$ B$_5$

B$_7$ C$_{1,3}$ C$_{3,5}$ C$_{5,7}$

C$_{7,1}$ C$_{5,1}$ C$_{7,3}$ D$_{1,3,5}$

D$_{3,5,7}$ D$_{1,5,7}$ D$_{1,3,7}$ E$_{1,3,5,7}$

Fig 5.1: Sixteen basis convex polygons in a 3X3 neighbourhood structure

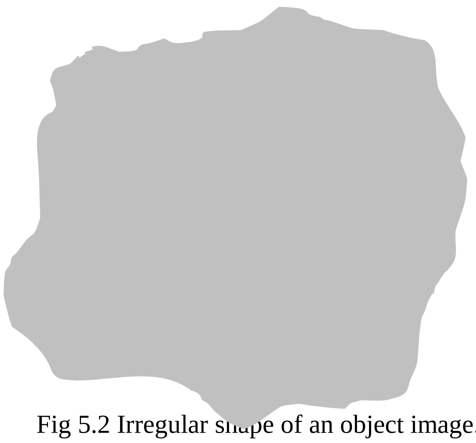


Fig 5.2 Irregular shape of an object image.

Any irregular image could also be represented in the same manner using 16 convex polygons. Now one can apply the Rajan Transform to all these 16 convex polygons as described below.

After scanning the image once by the 3X3 window the scanning window is brought down by one row and the image scanned from left to right. This procedure of raster scanning is continued till the entire image is scanned by the window.

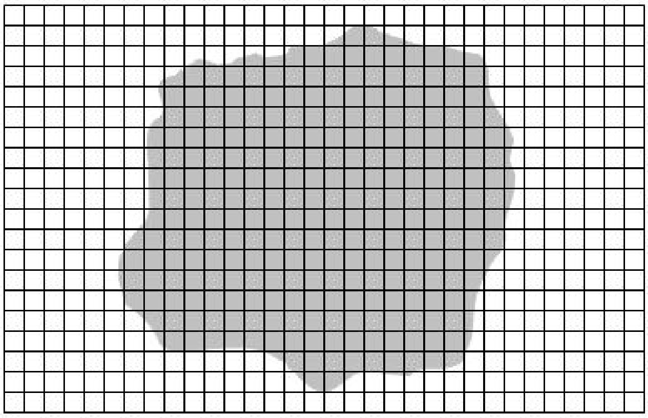


Fig 5.3 Quantized version of the irregular shape of an object image

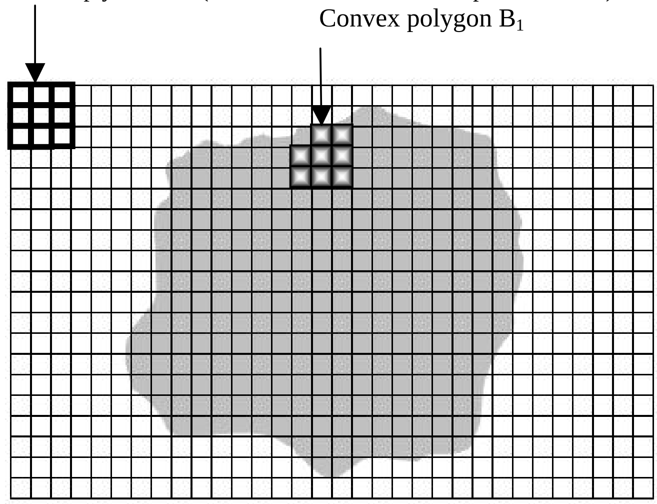


Fig 5.4 First scan position of the window on the image

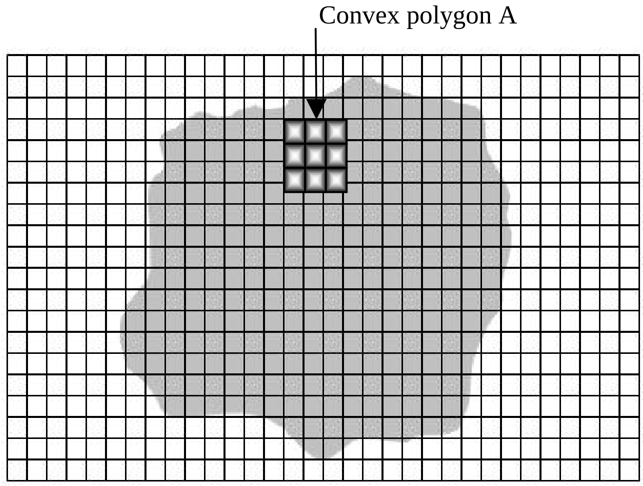
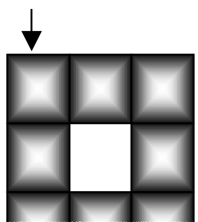


Fig 5.5 Second scan position of the window on the image

The overall effect of scanning the image yields a string of symbols denoting such convex polygons. This string is called pextral representation of the image just like the notion of spectral representation of a signal. For example, the pextral representation of the image given in figure 5.3 is computed as $\mathbf{0^{73}B_1A^7B_3 0^{16}B_1A^{12}0^{16}A^{13}0^{16}A^{13}B_5 0^{15}A^{14}0^{15} A^{14}0^{15}A^{14}0^{15}A^{13}0^{15}B_1A^{13}B_5 0^{12}B_1A^{15}0^{14}B_7A^{13}B_5 0^{15}B_7A^{12}0^{17}AB_7A^{10}0^{22}B_7A^4B_7 0^{70}}$ where the symbol $\mathbf{0^{73}}$ represents a string of 73 convex nulls.

RT spectrum of the pextra of 16 convex polygons

The central idea on which this concept has been developed is the fact that it is sufficient to find the pextrum of a shape by considering the contour of the given shape as a string of 0s and 1s and finding out the RT spectrum of it. For example, let us consider the contour of the pattern represented by the symbol A which is shown in figure 5.6



The string corresponding to this

Fig 5.6:Binary string representing the boundary of the pattern A

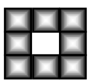
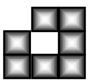
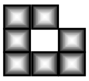
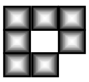
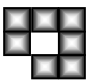
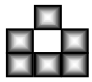
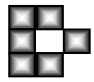
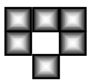
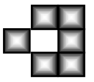
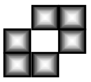
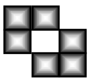
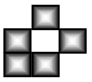
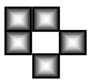
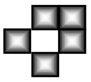
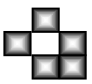
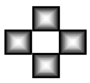
The RT spectrum of this binary string is 8,0,0,0,0,0,0,0. So, the pextrum of the shape A is 8,0,0,0,0,0,0,0. Table 5.1 provides the pextra of all 16 convex patterns.

Now, the RT spectrum of the pextral code $0^{73}B_1A^7B_30^{16}B_1A^{12}0^{16}A^{13}0^{16}A^{13}B_50^{15}A^{14}0^{15}A^{14}0^{15}A^{14}\ 0^{15}A^{13}0^{15}B_1A^{13}B_50^{12}B_1A^{15}0^{14}B_7A^{13}B_50^{15}B_7A^{12}0^{17}AB_7A^{10}0^{22}B_7A^4B_70^{70}$ is $0^{73},7,1,1,1,1,1,1,1, (8,1,1,1,1,1,1,1)^7,7,1,1,1,1,1,1,1,0^{16}, 7,1,1,1,1,1,1,1,(8,0,0,0,0,0,0,0)^{12},0^{16},(8.0.0.0.0.0.0.0)^{13},0^{16},(8,0,0,0,0,0,0,0)^{13},7,1,1,1,1,1,1,1,0^{15},(8,0,0,0,0,0,0,0)^{14},0^{15},(8,0,0,0,0,0,0,0)^{14},0^{15},(8,0,0,0,0,0,0,0)^{14},0^{15},(8,0,0,0,0,0,0,0)^{13},0^{15},7,1,1,1,1,1,1,1,(8,0,0,0,0,0,0,0)^{13},7,1,1,1,1,1,1,1,0^{12},7,1,1,1,1,1,1,1,(8,0,0,0,0,0,0,0)^{15},0^{14},7,1,1,1,1,1,1,1,(8,0,0,0,0,0,0,0)^{13},7,1,1,1,1,1,1,1,0^{15},7,1,1,1,1,1,1,1,(8,0,0,0,0,0,0,0)^{12},0^{17},8,0,0,0,0,0,0,0,7,1,1,1,1,1,1,1,(8,0,0,0,0,0,0,0)^{10},0^{22},7,1,1,1,1,1,1,1,(8,0,0,0,0,0,0,0)^4,7,1,1,1,1,1,1,1,0^{70}$

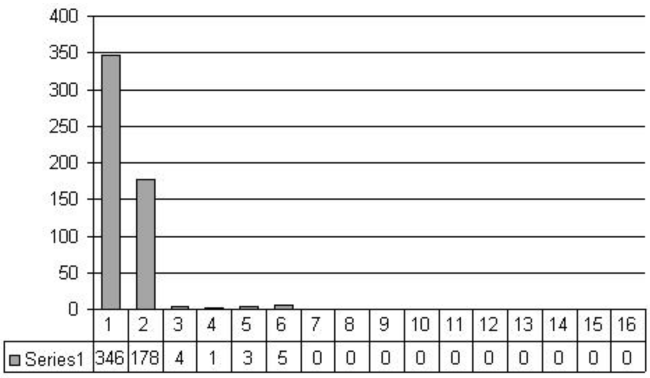
Object Recognition from RT spectrum of the pextra of 16 convex polygons

The technique of recognizing an object in a two dimanesional digital plane is as follows: The given digital image is represented as a string of the 16 basis convex polygons. Then each of the poygon is numerically coded in terms of 8 bits of 0s and 1s and RT applied to each block of 8 bits. From RT spectrum, one can characterize the shape of a given image. One can as well obtain the histogram of the pextral code and form an idea of the shape of a given image.

Table 5.1: Pextra of all 16 convex patterns

Pattern	Representative String	Corresponding Pextrum
	**1, 1, 1, 1, 1, 1, 1, 1**	**8, 0, 0, 0, 0, 0, 0, 0**
	**0, 1, 1, 1, 1, 1, 1, 1**	**7, 1, 1, 1, 1, 1, 1, 1**
	**1, 1, 0, 1, 1, 1, 1, 1**	**7, 1, 1, 1, 1, 1, 1, 1**
	**1, 1, 1, 1, 0, 1, 1, 1**	**7, 1, 1, 1, 1, 1, 1, 1**
	**1, 1, 1, 1, 1, 1, 0, 1**	**7, 1, 1, 1, 1, 1, 1, 1**
	**0, 1, 0, 1, 1, 1, 1, 1**	**6, 2, 0, 0, 2, 2, 0, 0**
	**1, 1, 0, 1, 0, 1, 1, 1**	**6, 2, 0, 0, 2, 2, 0, 0**
	**1, 1, 1, 1, 0, 1, 0, 1**	**6, 2, 0, 0, 2, 2, 0, 0**
	**0, 1, 1, 1, 1, 1, 0, 1**	**6, 2, 0, 0, 2, 2, 0, 0**
	0, 1, 1, 1, 0, 1, 1, 1	6, 2, 2, 2, 0, 0, 0, 0
	1, 1, 0, 1, 1, 1, 0, 1	6, 2, 2, 2, 0, 0, 0, 0
	**0, 1, 0, 1, 0, 1, 1, 1**	**5, 3, 1, 1, 1, 1, 1, 1**
	**1, 1, 0, 1, 0, 1, 0, 1**	**5, 3, 1, 1, 1, 1, 1, 1**
	**0, 1, 1, 1, 0, 1, 0, 1**	**5, 3, 1, 1, 1, 1, 1, 1**
	**0, 1, 0, 1, 1, 1, 0, 1**	**5, 3, 1, 1, 1, 1, 1, 1**
	**0, 1, 0, 1, 0, 1, 0, 1**	**4, 4, 0, 0, 0, 0, 0, 0**

For example, let us consider the example given above. The histogram of the pextral code is shown in figure 5.7.



Legend:

Index	1	2	3	4	5	6	7	8	9
Polygon	0	A	B_1	B_3	B_5	B_7	$C_{1,3}$	$C_{3,5}$	$C_{5,7}$
Index	10	11	12	13	14	15	16	17	
Polygon	$C_{7,1}$	$C_{1,5}$	$C_{3,7}$	$D_{1,3,5}$	$D_{3,5,7}$	$D_{5,7,1}$	$D_{7,1,3}$	$E_{1,3,5,7}$	

Fig 5.7: Histogram of the pextral code.

Observations:

The number of 0s, that is, convex polygons in an image indicates a measure of the background of the image, whereas the number of As in an image indicates the area spanned by the image. The presence of other polygons and their actual number gives an approximate nature of the boundary of the image.

Inference

Hence, it is proposed in this paper to study the spectra of the polygons rather than the spectra of the entire image.

VI. CONCLUSIONS

Rajan Transform has been used as a high-speed spectral domain tool to carry out object recognition and digital image processing operations. This paper mainly focuses on algebraic properties of the transform and uses in object recognition from RT spectrum of the pextra of 16 convex polygons. Research carry out so for clearly indicate that RT could be one of the best tools that could be used in cryptography. RT based watermarking of text and images and character recognition would yield a repertoire of tools of the future. Efforts are being made to abstract the notion of RT to Symbolic Processing of Signals and Images in the DNA computing paradigm.

VII. REFERENCES

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