Demodulation of Interferograms of Closed Fringes by Zernike Polynomials using a technique of Soft Computing

Luis Ernesto Mancilla Espinosa, Juan Martín Carpio Valadez, F.J. Cuevas

***Abstract*—We present the results about to recover the phase of interferograms of closed fringes by Zernike polynomials using a technique of soft computing, applying genetic algorithms (AG) and using an optimization fitness based with Zernike polynomials, with results very satisfactory.**

***Index Terms*— Genetic Algorithms, Interferometry, Phase recover, Zernike polynomials.**

I. Introduction

The interferometry is an optical technique does not destructive; it used to measure physics variables (stress, temperature, acceleration, curvature, and so on) with high degree of resolution, because it follows from the magnitude of wavelength used of the light [1].

In optical metrology, the mathematical model to state the phenomena of interference is modeling across a fingers pattern that modulates a signal with cosenoidal profile as [2]:

$$I(x,y)=a(x,y)+b(x,y)\cos(w_x x+w_y y+\Theta(x,y)+\eta(x,y))$$

Where $a(x,y)$ is the background lighting, $b(x,y)$ is the amplitude modulation,

w_x and w_y are the carrier frequencies on the axis x and y, $\Theta(x,y)$ is the phase term related to the physical quantity being measured, $\eta(x,y)$ is an additive uniform noise.

The goal of genetics algorithm is recover $\Theta(x,y)$ from the fringe pattern, which it is directly relationship with the physics quantity that we want to measure. By other part, the experimental array to provoke the interference phenomenon requires recording the interference image that to generate, and it processed digitally in the computer to obtain the information of the phase.

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One way to calculate the phase term $\Theta(x,y)$ is the phase shift technical (PST) [10-14], which needs unless of three phase interferograms. The shift of phase between the interferograms must to be known and controlled experimentally. This technical can be used when the mechanical conditions are satisfied in the interferometry experiment.

In other conditions, when the stability condition is not cover, there are a lot of techniques to obtain the term of phase from the fingers pattern, as: the Fourier Method [15, 16], Synchronous Method [17] and the Phase Lock Loop (PLL) [18], between others. However, these techniques work well only if the interferogram analyzed has a carrier frequency, a narrow bandwidth and the signal has low noise. Even more, these methods fail when we calculate the phase of an interferogram with closed fringers, as the figure 1. Besides, the Fourier and Synchronous methods estimate the phase wrapped because of the arctangent function used in the phase calculation, so an additional unwrappong process is required. The unwrapping process is difficult when the fringers pattern includes high amplitude, which causes differences greater than 2π radians between adjacent pixels [19-20]. In PLL technique, the phase is estimated by following the phase changes of the input signal by varying the phase of a computer simulated oscillator (VCO) such that the phase error between the fringe pattern and VCO's signal vanishes.

Recently, the regularization [21-24] and neural network techniques [25,26] have been used to work with fringers pattern, which contain a narrow bandwidth and noise. The regularization technique established a cost function that constrains the estimated phase using two considerations: (a) the fidelity between the estimated function and the observed fringe pattern and (b) smoothness of the modulated phase field. In the neural network technique, a multilayer neural network is trained with a set of fringers pattern and a set of phase gradients provided from calibrated objects. After that the network has been trained, the phase gradients is estimated in the network output when the fringers pattern (interferograms) are presented in the network input. The defect of this technique is the requirement of a set of training fringer images and theirs related phase measurements.

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Therefore to the case of interferograms as the figure 1, we require different techniques, as the proposals in the articles [2], [3], [4], [8], [9] .

Our proposal consist to use Zernike polynomials to optimistic a quadratic criterion. The Zernike polynomials are used to obtain the interferogram phase that phase is showed as a surface and due to the orthogonal characterictics of the Zernike polynomials, these are very suitables to carry out the fitting of that surface. By other way, the parameters of the Zernike polynomials have direct relation with the physics properties as: aberration spherical coefficient, come coefficient, astigmatism coefficient, coefficient of focus shift, tilt in y, tilt in x, so on. [1].

Therefore, we use the Zernike polynomials to recover the closed interferograms phase using AG.

The paper is organized as follows, in the section 2 we establish the mathematics models that used to carry out the optimization by AG. In the section 3 we show the experimental results obtained. And finally in the section 4 conclusions and comments.

II. Mathematics Models

The model used to represent the phenomena of interference is:

$$I(x,y) = a(x,y) + b(x,y)\cos(w_x x + w_y y + \Theta(x,y))$$

The optimization criterion that we use, it was:

$$f_{opt}(z^k) = \alpha - \sum_{y=1} \sum_{x=1} \{(I(x,y) - \cos(w_x x + w_y y + f_{ajuste}(z^k,x,y))^2 + \lambda\{(f_{ajuste}(z^k,x,y) - f_{ajuste}(z^k,x-1,y))^2 + (f_{ajuste}(z^k,x,y) - f_{ajuste}(z^k,x,y-1))^2\}\}$$

Where:

x , y are integer values representing indexes of the pixel location in the fringe image, α is used to convert the proposal from minimal to maximal optimization, w_x is the frequency in the axe x , w_y is frequency in the axe y , λ parameter which control the smoothness level of the image to obtain, z^k is the k eth chromosome of the population, k is the index into the whole population of chromosomes.

The fitness function that we use is the Zernike polynomial which cartesian model is [2, pag.465], [7]:

$$f_{ajuste}(z^k,x,y) = \sum_{j=1}^{j=L} z^k{}_j U_j(x,y)$$

Where:

j is the order of Zernike polynomial, U is the result polynomial of mapping the plane (ρ,θ) to plane (x,y).

The AG that we use to have the following characteristic:

Select Operator, Boltzman type.

Cross Operator with two points of cross.

Mutation Operator with stepped degraded.

The Zernike polynomial that we use is the four order.

The similarity criterion between the inteferograms that we used was the calculus of the Euclidian distance between the images [8].

The search intervals to the Zernike variables polynomial were obtained across tests, although actuality we are working over the find the global upper limits of the variables, for posteriori characteristic the individual limits of the parameters, we are working in this research process.

III. Experimetal Results

The computer equipment that we use to do the tests is a PC-Pentium Centrino 1.6GHz, 256 MB of RAM.

Case 1.-Image of 40x40 pixels shows figure 1, resolved with: population=1000, cross probability=90%, mutation probability=10%, number of generations=50, time used to obtain the figure 2 is of: 540 seconds.

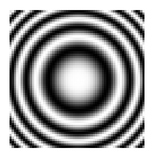

Figure 1

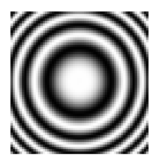

Figure 2

The figures 3,5,7,9 y 11; were obtained [5] with a fitness function:

$$f_{ajuste}(z,x,y) = z_0 + z_1 x + z_2 y + z_3 x^2 + z_4 xy + z_5 y^2 + z_6 x^3 + z_7 x^2 y + z_8 xy^2 + z_9 y^3 + z_{10} x^3 y + z_{11} x^2 y^2 + z_{12} xy^3 + z_{13} x^4 + z_{14} y^4$$

And with search interval:

$z_1, z_2 \in [-2,2]$; $z_3, z_4, z_5 \in [-0.05,0.05]$;

$z_6, z_7, z_8, z_9 \in [-0.003,0.003]$;

$z_{10}, z_{11}, z_{12}, z_{13}, z_{14} \in [-0.0001, 0.0001]$

The figures 4,6,8,10 and 12; were obtained with a fitness function:

$$f_{ajuste}(z,x,y) = z_1U_1 + z_2U_2 + z_3U_3 + z_4U_4 + z_5U_5 + z_6U_6 + z_7U_7 + z_8U_8 + z_9U_9 + z_{10}U_{10} + z_{11}U_{11} + z_{12}U_{12} + z_{13}U_{13} + z_{14}U_{14} + z_{15}U_{15}$$

Case 2.-Image of 40x40 pixels show figure 3, resolved with: population=1500, cross probability=90%, mutation probability=1%, number of generations=40, time used to obtain the figure 4 is of: 360 seconds.

With a search interval:

$z_1 \in [0.99, 1.01], z_2 \in [-1, 0], z_3 \in [0, 0.5],$
$z_4, z_8 \in [0, 0.001],$
$z_5 \in [-0.01, 0], z_6 \in [-0.1, 0], z_7 \in [-0.001, 0],$
$z_9, z_{12} \in [-0.000001, 0], z_{10}, z_{11} \in [-0.00001, 0],$
$z_{13}, z_{15} \in [0, 0.00001], z_{14} \in [0, 0.0001]$

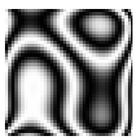

Figure 3

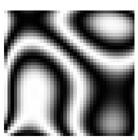

Figure 4

Case 3.-Image of 40x40 pixels show figure 5, resolved with: population=500, cross probability=100%, mutation probability=4%, number of generations=170, time used to obtain the figure 6 is of: 560 seconds.

With a search interval:

$z_1 \in [0.99, 1.01], z_2 \in [-1, 0],$
$z_3, z_9, z_{12} \in [-0.000001, 0], z_4, z_8 \in [0, 0.001],$
$z_5, z_7 \in [-0.01, 0], z_6 \in [-0.1, 0],$
$z_{10}, z_{11} \in [-0.00001, 0], z_{13} \in [0, 0.00001],$
$z_{14} \in [0, 0.0001], z_{15} \in [0, 0.000001]$

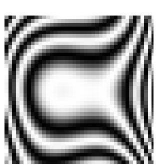

Figure 5

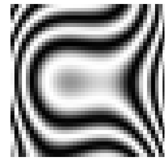

Figure 6

Case 4.-Image of 40x40 pixels show figure 7, resolved with: population=500, cross probability=100%, mutation probability=4%, number of generations=170, time used to obtain the figure 8 is of: 560 seconds.

With a search interval:

$z_1 \in [0.99, 1.01], z_2, z_3 \in [-1, 0], z_4, z_5, z_8 \in [0, 0.001],$
$z_9, z_{12} \in [-0.000001, 0],$
$z_{10}, z_{11} \in [-0.00001, 0], z_{13}, z_{14} \in [0, 0.00001],$
$z_6 \in [-0.01, 0], z_7 \in [-0.001, 0],$
$z_{15} \in [0, 0.0001]$

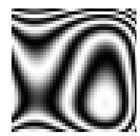

Figure 7

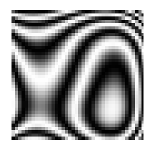

Figure 8

Case 5.-Image of 40x40 pixels show figure 9, resolved with: population=500, cross probability=100%, mutation probability=4%, number of generations=170, time used to obtain the figure 10 is of: 360 seconds.

With a search interval:

$z_1 \in [0.99, 1.01], z_2, z_3 \in [0, 1], z_4 z_5 \in [-0.01, 0],$
$z_8, z_{11} \in [-0.0001, 0],$
$z_6 \in [0, 0.01], z_7 \in [-0.001, 0], z_9 \in [0, 0.0001],$
$z_{12}, z_{14} \in [-0.00001, 0], z_{13}, z_{15} \in [0, 0.00001],$
$z_{10} \in [0, 0.001]$

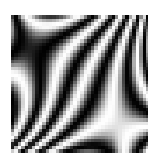

Figure 9

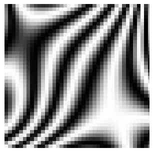

Figure 10

Case 6.-Image of 40x40 pixels show figure 11, resolved with: population=500, cross probability=100%, mutation probability=4%, number of generations=170, time used to obtain the figure 12 is of: 360 seconds.

With a search interval:

$$z_1 \in [0.99,1.01], z_2, z_6 \in [-0.1,0], z_3 \in [-1,0],$$
$$z_9, z_{12} \in [-0.000001,0], z_8 \in [0,0.001]$$
$$z_5 \in [-0.01,0], z_6 \in [-0.1,0], z_7 \in [-0.001,0],$$
$$z_{10}, z_{11} \in [-0.00001,0], z_{13}, z_{14} \in [0,0.00001],$$
$$z_{15} \in [0,0.00001], z_4 \in [0,0.01]$$

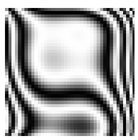

Figure 11

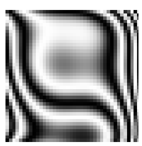

Figure 12

The case 1 is selected from reference [2] and the cases 2 to 6 is selected from reference [5].

Case 7.-Image of 40x40 pixels shows figure 13, resolved with: population=500, cross probability=100%, mutation probability=1%, number of generations=30, time used to obtain the figure 14 is of: 60 seconds.

With a search interval:

$$z_1 \in [5,7], z_2 \in [6,8], z_3 \in [4,6], z_4 \in [3,5],$$
$$z_5 \in [2,4], z_6 \in [1,3], z_7 \in [0,-2], z_8 \in [-1,-3],$$
$$z_9 \in [-2,-4], z_{10} \in [3,5], z_{11} \in [5,7], z_{12} \in [7,9],$$
$$z_{13} \in [4,6], z_{14} \in [6,8], z_{15} \in [0,2]$$

Figure 13

Figure 14

Case 8.-Image of 40x40 pixels shows figure 15, resolved with: population=500, cross probability=100%, mutation probability=1%, number of generations=20, time used to obtain the figure 16 is of: 60 seconds.

With search interval:

$$z_1 \in [0.05,0.15], z_2 \in [0.15,0.25], z_3 \in [0.25,0.35],$$
$$z_4 \in [0.35,0.45], z_5 \in [0.45,0.55], z_6 \in [0.55,0.65],$$
$$z_7 \in [0.65,0.75], z_8 \in [0.75,0.85], z_9 \in [0.15,0.05],$$
$$z_{10} \in [-0.25,-0.15], z_{11} \in [-0.35,-0.25],$$
$$z_{12} \in [-0.45,-0.35], z_{13} \in [-0.55,-0.45],$$
$$z_{14} \in [-0.65,-0.55], z_{15} \in [-0.75,-0.65]$$

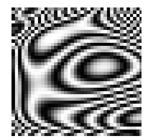

Figure 15

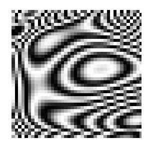

Figure 16

The cases 7 and 8 is selected from the reference [6]

Case 9.-Image of 64x64 pixels shows figure 17, resolved with: population=500, cross probability=90%, mutation probability=1%, number of generations=30, time used to obtain the figure 18 is of: 180 seconds.

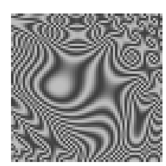

Figure 17

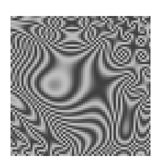

Figure 18

The case 9, we were selected from the reference [2].

IV. Conclusions

We have presented results selected the phase recover, in basis to its recover time, for several closed interferograms applying a fitness function by Zernike polynomial and using AG.
We searched to prove the advantages of our technique with respect to others and under criterions as:
a).-Recover the interferogram and consequently its phase.
b).-Direct relationship of physics parameters.
c).-Recover Time.

We can to declare:
1.-In the figures 3,5,7,9,11 that were generated with not Zernike polynomials and using AG, in accordance with reference [5], and similar conditions of search range, we get similar results, with the difference that we can correlation the parameters of Zernike polynomials with optics values and the reference [5] did not. Moreover, we were reported the recover time and the reference [5] did not, in that point we could not contrast the advantages.
2.-For the figures 13, 15 of the reference [6] that were generated with Zernike polynomial and recovered the interferogram parameters with technique of quadratic minimal using Zernike polynomial, also were recovered satisfactorily and acceptable time. Newly, we can not to compare recover times because the reference [6] did not report.

3.-For the figure 17 of the reference [2] that use AG but not Zernike polynomial, it report a recover time 330 sec., we recover it in 180 sec., under similar conditions of computer equipment.

4.-About to reference [8],
a).-They used real interferograms, we did not by the moment, we are working in this part.
b).-Use an evolutionary algorithm and not a genetics algorithm.
c).-Use a Seidel polynomial and not a Zernike polynomial.
d).-Establish the problem as an optimization problem in the real space, similar to ours.
e).-Use an IV Pentium, but did not mention the speed computer.
f).-The experimentals (2) take between 3 to 6 minutes, we carry out several experimentals with results of better time.
g).-Use MatLab, we use “C”.
h).-Have serious problems for Seidel Polynomial of order bigger than 3, we have not problem with order Zernike polynomial.
i).-Assert that the evolutionary strategy is generally recommended to the genetics algorithms to resolve problems that deal with the optimization function of real numbers, but has not reference to this assert.
j).-Use a factor c=0.01 for the experimentals but does not say how to obtain it.
k).-Assert that if the success of mutation happen rarely, the search say us that we are near to a minimal, by recommend to decrease the size of neighborhood when we search. If the mutations are successful and they happen very frequently, it means that the converse can acceleration across the increase of the step, however did not show any references about theses assertions.
l).-They mention that after a lot of attempt found the cross and mutation configuration in terms following: a 10% for the equation (12), a 30% for the equation (13) and a 70% for the equation (14). Similar to ours, they carry out several attempts for fit the ranks.

5.-Respect to the reference [9],
a).-Use a polynomial equation not Zernike to obtain the phase.
b).-Use simulate interferograms with noise.
c).-Use an evolution algorithm and not a genetics algorithm.
d).-Drive one experimental, we report several.
e).-Do not report recover time, we do.
f).-Do not report the equipment type that they were used, we do.
g).-Use Matlab, we use “C”.

We conclusion that our propose show better results respect to other techniques.

V. Future works

We are working as mention in the limit of search interval of the parameters of genetics algorithms, first in the global form and after in individual form, we seek to carry AG to embedded software using FPGA technology in way does not parallel, after in way parallel to recover the phase in real time, after parallel with same technology with same goal, reduce the recover time.

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