Generation of Crowned Parabolic Novikov gears

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***Abstract* - The Wildhaber-Novikov gear is one of the circular arc gears, which has the large contact area between the convex and concave profiled mating teeth. In (June 28, 1999), a new geometry of W-N gear with parabolic profile in normal section has been developed. This paper studies the generation of rack-cutters for parabolic crowned profiles with its generation in order to select the requirements of W-N gears.**

Index Terms- crowned parabolic profile, generation of gears, Novikov gears

I. INTRODUCTION

Circular arc helical gears were proposed by Wildhaber and Novikov. However, there is a significant difference between the ideas proposed by the previously mentioned inventors. Wildhaber's idea [1] is based on generation of the pinion and gear by the same imaginary rack-cutter that provides conjugate gear tooth surfaces that are in *line* contact at every instant. Novikov [2] proposed the application of two mismatched imaginary rack-cutters that provide conjugated gear tooth surfaces that are in *point* contact at every instant. Point contact of Novikov gears has been achieved by application of two mismatched rack-cutters for generation of the pinion and gear, respectively.

There are two versions of Novikov gears (with circular-arc profile), the first having one zone of meshing, and the other having two zones of meshing. The design of gears with two zones of meshing was an attempt to reduce high bending stresses caused by point contact.

The proposed new version of helical gears is based on the following ideas [3]:

1. The bearing contact is localized and the contact stresses are reduced because of the tangency of concave-convex tooth surfaces of the mating gears.
2. The normal section of each rack-cutter is a parabola as shown in Fig.1. A current point of the parabola is determined in an auxiliary coordinate system S_i by the equations

$$x_i = u_i \qquad y_i = a_i u_i^2 \tag{1}$$

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where u_i is a variable parameter that determines the location of the current point in the normal section and a_i is the parabolic coefficient.

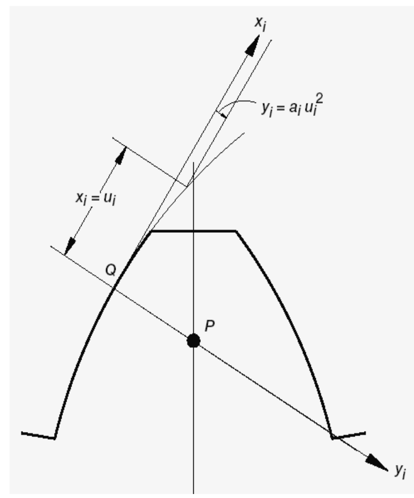

Fig. 1- Parabolic profile of rack-cutter in normal section.

II. DERIVATION OF PINION TOOTH SURFACE

A. Pinion Rack-Cutter Surface $\sum_c$

The derivation of rack-cutter surface $\sum_c$ is based on the following procedure:

1. The normal profile of $\sum_c$ is a parabola and is represented in coordinate system S_a, Fig. 2-b, by equations that are similar to (1):

$$\mathbf{r}_a(u_c) = \begin{bmatrix} u_c & a_c u_c^2 & 0 & 1 \end{bmatrix}^T \tag{2}$$

where a_c is the parabolic coefficient; u_c is the variable parameter.

2. The normal profile is represented in S_b by matrix equation

$$\mathbf{r}_b(u_c) = \mathbf{M}_{ba}\,\mathbf{r}_a(u_c) \tag{3}$$

$\mathbf{M}_{ba}$ indicates the 4×4 matrix used for the coordinate transformation from coordinate system S_a to S_b [4]:

3. Consider that rack-cutter surface $\sum_c$ is formed in S_c while coordinate system S_b with the normal profile performs a translational motion in the direction *a-a* of the skew teeth of the rack-cutter, Fig. 3. Surface $\sum_c$ is

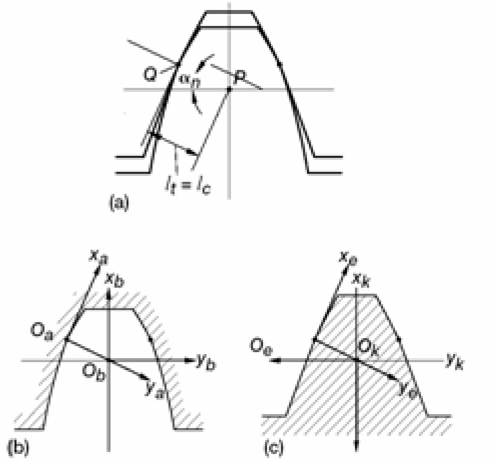

Fig. 2- Normal sections of rack-cutters
(a) Profiles of pinion and gear rack-cutters.
(b) Pinion rack-cutter. (c) Gear rack-cutter.

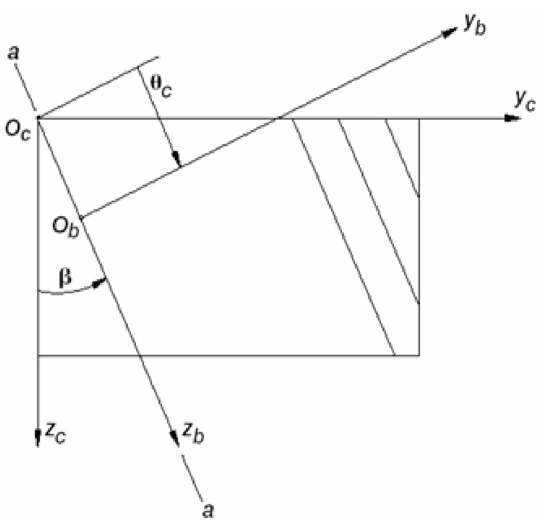

Fig. 3- For derivation of pinion rack-cutter surface $\sum_c$

determined in coordinate system S_c in two-parameter form by the following matrix equation

$$\mathbf{r}_c(u_c,\theta_c) = \mathbf{M}_{cb}(\theta_c)\ \mathbf{r}_b(u_c) \quad (4)$$

4. The normal $\mathbf{N}_c$ to rack-cutter surface $\sum_c$ is determined by matrix equation, [4]

$$\mathbf{N}_c(u_c) = L_{cb}\ L_{ba}\ \mathbf{N}_a(u_c) \quad (5)$$

Here

$$\mathbf{N}_a(u_c) = \mathbf{k}_a \times \frac{\partial \mathbf{r}_a}{\partial u_c} \quad (6)$$

and the unit normal to the surface is

$$\mathbf{n}_c(u_c) = \frac{\mathbf{N}_c}{|\mathbf{N}_c|} = \frac{\mathbf{N}_c(u_c)}{\sqrt{1+4a_c^2\,u_c^2}} \quad (7)$$

where L_{cb} indicates the 3×3 matrix that is the sub-matrix of $\mathbf{M}_{cb}$ and is used for the transformation of vector components; $\mathbf{k}_a$ is the unit vector of axis z_a. The transverse section of rack-cutter $\sum_c$ is shown in Figs. 4-a and b.

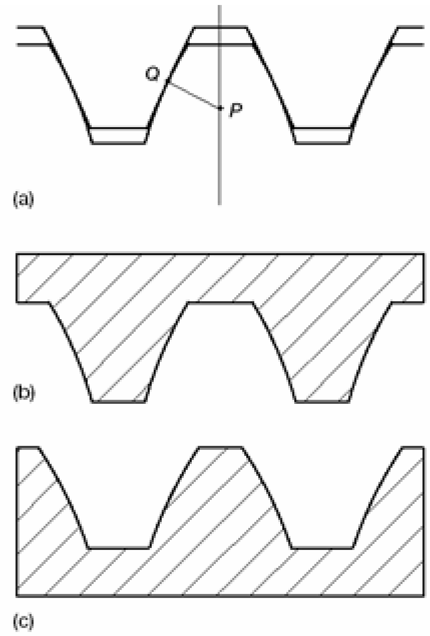

Fig. 4- Rack-cutter transverse profiles.
(a) Mating profiles. (b) Pinion rack-cutter profile.
(c) Gear rack-cutter profile.

B. Determination Of Pinion Tooth Surface $\sum_1$

The determination of $\sum_1$ is based on the following considerations:

1. Movable coordinate systems S_c and S_1, Fig. 5, are rigidly connected to the pinion rack-cutter and the pinion, respectively. The fixed coordinate system S_m is rigidly connected to the cutting machine.
2. The rack-cutter and the pinion perform related motions, as shown in Fig. 5, where $s_c = rp_1\,\psi_1$ is the displacement of the rack-cutter in its translational motion, and ψ_1 is the angle of rotation of the pinion.
3. A family of rack-cutter surfaces is generated in coordinate system S_1 and is determined by the matrix equation

$$\mathbf{r}_1(u_c,\theta_c,\psi_1) = \mathbf{M}_{1c}(\psi_1)\ \mathbf{r}_c(u_c,\theta_c) \quad (8)$$

Here

$$\mathbf{M}_{1c}(\psi_1) = \mathbf{M}_{1m}\ \mathbf{M}_{mc} \quad (9)$$

The pinion tooth surface $\sum_1$ is generated as the envelope of the family of surface $\mathbf{r}_1(u_c,\theta_c,\psi_1)$ .Surface $\sum_1$ is determined by

$$f_{1p}(u_c,\theta_c,\psi_1) = 0 \quad (10)$$

simultaneous consideration of vector function $\mathbf{r}_1(u_c,\theta_c,\psi_1)$ and the so-called equation of meshing .

4. To derive the equation of meshing (10), apply the theorem of [5] and [4] to obtain,

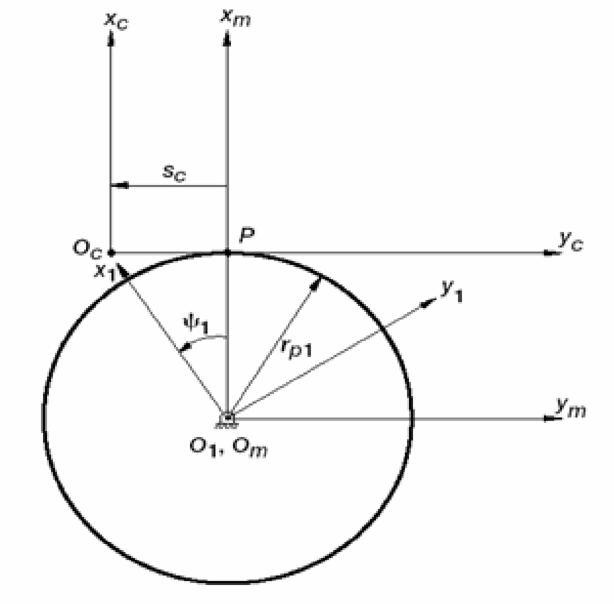

Fig. 5- Generation of pinion by rack-cutter $\sum_c$

$$\left|\overline{O_c O_1}\right| = -rp_1\,\mathbf{i} + rp_1\psi_1\,\mathbf{j} \qquad (11)$$

$$\mathbf{v}_c = \omega^{(1)}\ rp_1\,\mathbf{j} \quad where \quad \omega^{(1)} = \omega\ \mathbf{k} \qquad (12)$$

$$\mathbf{v}_1 = \omega^{(1)} \times \mathbf{r}_c + \left|\overline{O_c O_1}\right| \times \omega^{(1)} \qquad (13)$$

The relative velocity is

$$\mathbf{v}_{c1} = \mathbf{v}_c - \mathbf{v}_1 = -\omega\,[(rp_1\psi_1 - y_c)\,\mathbf{i} + x_c] \qquad (14)$$

Thus, the equation of meshing is

$$\mathbf{N}_c \bullet \mathbf{v}_{c1} = 0 \qquad (15)$$

That yields

$$f_{1c}(u_c,\theta_c,\psi_1) = (rp_1\psi_1 - y_c)N_{xc} + x_c N_{yc} = 0 \qquad (16)$$

where (x_c, y_c, z_c) are the coordinates of a current point of $\sum_c$; $(\mathbf{N}_c)$ is the normal to the surface $\sum_c$; ω is the angular velocity; $\mathbf{v}_c$ *and* $\mathbf{v}_1$ are the velocities of the rack-cutter c and pinion respectively; $\mathbf{v}_{c1}$ represent the relative velocity (sliding velocity) between the rack-cutter and pinion, Fig. 5.

Equations (8) and (16) represent the pinion tooth surface by three related parameters. Taking into account that the equations above are linear with respect to θ_c, hence θ_c may be eliminated and represent the pinion tooth surface by vector function $\mathbf{r}_1(u_c,\psi_1)$.

III. DERIVATION OF GEAR TOOTH SURFACE

A. Gear Rack-Cutter Surface $\sum_t$

The derivation of rack-cutter surface $\sum_t$ is based on the procedure similar to that applied for derivation of $\sum_c$. The normal profile of $\sum_t$ is a parabola represented in S_e, referring to Fig. 2-c,

$$\mathbf{r}_e(u_t) = \begin{bmatrix} u_t & a_t u_t^2 & 0 & 1 \end{bmatrix}^T \qquad (17)$$

which is similar to (2). Use coordinate systems S_k, Fig. 2-c, and S_t that are similar to S_b and S_c, Fig. 3, to represent surface S_t by matrix equation,

$$\mathbf{r}_t(u_t,\theta_t) = \mathbf{M}_{tk}(\theta_t)\mathbf{M}_{ke}\mathbf{r}_e(u_t) \qquad (18)$$

The normal to the surface $\sum_t$ is determined by equations similar to (5) to (7). The difference in the representation of $\sum_t$ is the change in the subscript *c* to *t*.

B.Determination Of Gear Tooth Surface $\sum_2$

The generation of $\sum_2$ by rack-cutter surface $\sum_t$ is represented schematically in Fig. 6. The rack-cutter and the gear perform related translational and rotational motions designated as $s_t = rp_2\psi_2$ *and* ψ_2.

The gear tooth is represented

$$\mathbf{r}_2 = \mathbf{r}_2(u_t,\theta_t,\psi_2)$$

(19)

$$f_{2t}(u_t,\theta_t,\psi_2) = 0$$

(20)

Equation (20) represents in S_2 the family of rack-cutter surfaces $\sum_t$ determined as,

$$\mathbf{r}_2(u_t,\theta_t,\psi_2) = \mathbf{M}_{2t}(\psi_2)\mathbf{r}_t(u_t,\theta_t) \qquad (21)$$

Here

$$\mathbf{M}_{2t}(\psi_2) = \mathbf{M}_{2m}(\psi_2)\mathbf{M}_{mt}(\psi_2) \qquad (22)$$

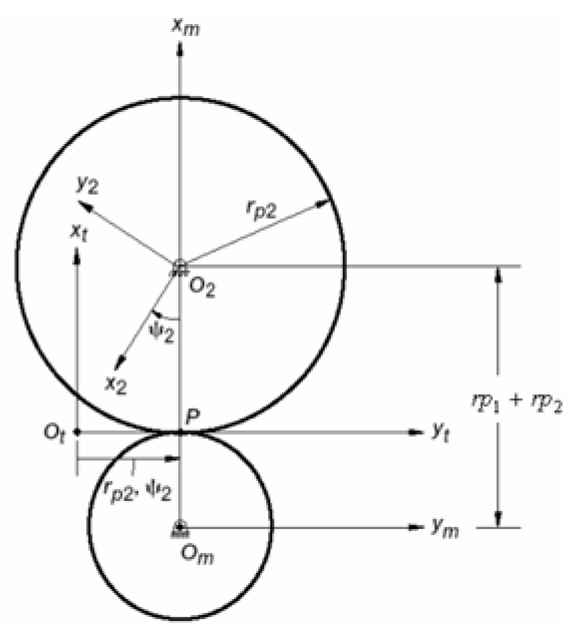

Fig. 6- Generation of gear by rack-cutter $\sum_t$

The derivation of the equation of meshing (20) may be accomplished similarly to that of (16),

$$f_{2t}(u_t,\theta_t,\psi_2) = (rp_2\,\psi_2 - y_t)N_{xt} + x_t\,N_{xt} = 0 \quad (23)$$

Equations (21) and (23) represent the gear tooth surface by three related parameters. The linear parameter θ_t can be eliminated and the gear tooth surface represented in two-parameter form by vector function $\mathbf{r}_2(u_t,\psi_2)$.

IV. MATHEMATICAL SIMULATION OF RACK FILLET

A fillet part is a circular arc which has coordinates *x* and *y*, *k* and *h* –center coordinates and a radius r_f.

This arc lies between points *A* and *B*, where point *A* represent the mating point, which satisfy smoothing contact, between the circular-arc or parabolic curve and the fillet curve, point *B* represent the meeting point, which satisfy smoothing contact, between the fillet curve and the horizontal straight line, as shown in Fig. 7. Therefore, to find the *x and y*-coordinates of points *A* and *B*, angles θ_A *and* θ_B which satisfy smoothing contact must be found.

To satisfy smoothing contact at point *B*, angle θ_B may equal to 90° because the radius of fillet may be perpendicular on the tangent, which is the horizontal straight line at this point, also to satisfy smoothing contact at point *A* the fillet radius may be perpendicular on the tangent at this point, or in other words the slope of circular-arc or parabolic curve must be equal to the slope of the fillet curve at this point, [6].

Thus, to find the slope of parabolic curve at any point, using (3), to get,

$$\frac{dx}{dy} = \frac{dx}{du} * \frac{du}{dy} = \frac{\cos(\alpha_n) - 2a_c u_c \sin(\alpha_n)}{\sin(\alpha_n) + 2a_c u_c \cos(\alpha_n)} \quad (24)$$

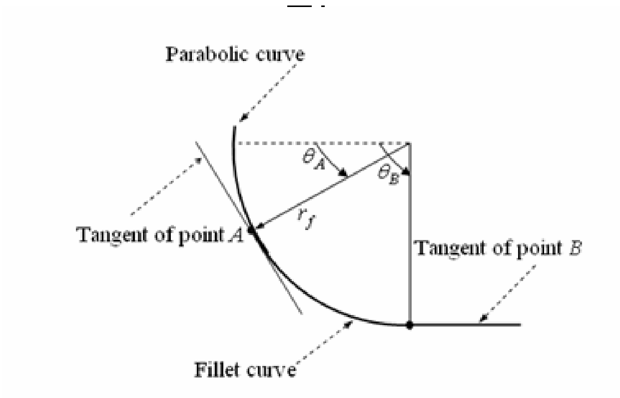

Fig. 7- Fillet part of rack-cutter.

Also to find the slope of fillet curve at any point, the circle equation is:-

$$\begin{aligned}&(y-h)^2 + (x-k)^2 = r_f^2\\ &y^2 - 2h\,y + h^2 + x^2 - 2k\,x + k^2 = r_f^2\end{aligned} \quad (25)$$

differentiating (25) with respect to *y* as follows:

$$y - h + x\frac{dx}{dy} - k\frac{dx}{dy} = 0 \quad \Rightarrow \quad h = y + x\frac{dx}{dy} - k\frac{dx}{dy} \quad (26)$$

by substituting (13) , (24) and (26) in (25), thus obtaining a non-linear equation which is solved numerically using ***Secant Method*** to get the coordinates of smoothing point (*A*).

For the crowned parabolic profile, the fillet curve can be represented as follows:

$$\mathbf{r}_{b_f} = \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = \begin{bmatrix} r_f \sin(\theta_f) + x_{of} \\ r_f \cos(\theta_f) + y_{of} \\ 0 \end{bmatrix} \quad (27)$$

Here, r_f is the fillet radius; (x_{of}, y_{of}) are the arc center coordinates; θ_f is the variable parameter.

Using coordinates systems similar to S_b and S_c, Fig. 3, to represent surface S_{c_f}, the subscript *f* means the fillet surface, thus

$$\mathbf{r}_{c_f}(\theta_f,\theta_c) = \mathbf{M}_{c_f\,b_f}(\theta_c)\,\mathbf{r}_{b_f}(\theta_f) \quad (28)$$

The unit normal to the surface can be found as

$$\mathbf{N}_{c_f} = \frac{\partial \mathbf{r}_{c_f}}{\partial \theta_f} \times \frac{\partial \mathbf{r}_{c_f}}{\partial \theta_c} \quad \text{and} \quad \mathbf{n}_{c_f} = \frac{\mathbf{N}_{c_f}}{\left|\mathbf{N}_{c_f}\right|} \quad (29)$$

The representation of the pinion tooth fillet surface by equations similar to (8) and (16), and also the representation of the gear tooth fillet surface by equations similar to (21) and (23). Then the generation of pinion and gear for parabolic profiles with fillet radius can be obtained as shown in Fig. 8.

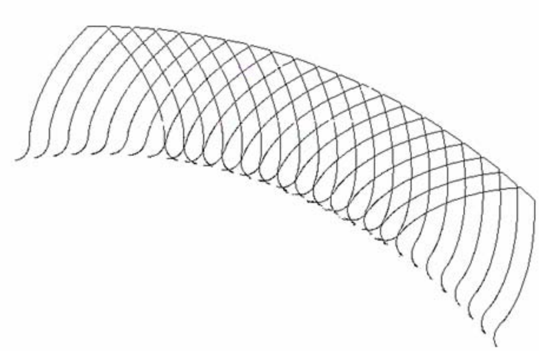

Fig. 8- Generation of parabolic tooth with fillet radius.

V. CONCLUSIONS

The developed approach of design and generation of the crowned parabolic Novikov gear drives has successfully been applied. The conjugation of gear tooth surfaces with profile crowning is achieved by applying two rack-cutters with crowned profile in normal section.

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Nomenclature

a_i ***Parabolic coefficients of profiles of pinion rack cutter (i=c) and gear rack cutter (i=t).***

f_{ij} ***Equation of meshing between tooth surface (i) and rack-cutter (j).***

l_i ***Parameter of location of point tangency Q for pinion (i=c) or gear (i=t).***

$\mathbf{M}_{ij}, \mathbf{L}_{ij}$ ***Matrices of coordinate transformation from coordinate system*** S_i *to* S_j **.**

$\mathbf{n}_i^{(j)}, \mathbf{N}_i^{(j)}$ ***Unit normal and normal to surface*** $\sum_i$ ***in coordinate system*** S_j .

$\mathbf{r}_i$ ***Position vector of a point in coordinate system*** S_i **.**

r_f ***Fillet radius*** .

rp_i ***Radius of cylinder of pinion (i=1) or for gear (i=2).***

S_i ***Displacement of rack-cutter for pinion (i=1) or for gear (i=2) .***

S_i (O_i,x_i,y_i,z_i) *Coordinate system (i=c,t,p,g,1,2,m,a,b,f,fs,cf,r,k,e)*

α_n ***Pressure angle in Normal section.***

β ***Helix angle .***

$\sum i$ ***Surfaces (i=c,t,p,g,1,2).***

ϕ_i ***Angle of rotation of the pinion (i=1) or the gear (i=2) in the process of generation for circular-arc profile .***

ψ_i ***Angle of rotation of profiled-crowned pinion (i=1), the double crowned- profile (i=p) or for gear (i=2) in the process of generation for circular-arc profile .***

(u_i, θ_i) ***Parameters of surface*** $\sum_i$ **.**

θ_A, θ_B ***Angles which satisfy smoothing contact curves .***

ρ_i ***Profile radii (i=p,g,c,t,1,2).***