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Constructive Analysis of Intensional Phenomena in Natural Language

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Abstract— **Chierchia [2, 3, 4], pointed out the inadequacy of Montague's approach in the analysis of certain natural language constructions, such as nominalization and propositional attitude reports. The problem seems to be related to the strong typing of Montague's Intensional Logic, and its interpretation of propositions as sets of possible worlds. Turner in [9], following Bealer's intuitions [1], offers an interesting solution by building an intensional logic in which the intension of propositions and propositional functions are treated as individuals of a different kind, Turner's framework is called *property theory.* For a classic approach to intensionality see [5, 6]. In this paper, we show that is possible to recast Turner's proposals in the *constructive type theory* providing a simple analysis of intentionality and an elegant solution to the puzzle sentences.**

Keywords: formal semantics, natural language processing, intensionality, constructive mathematics, type theory

1 Introduction

There are some natural language constructions that seem to require a type shifting to be properly analysed. An example of these are the so called 'That'-clauses, as shown in the following sentences (taken from [9]):

John is daft.
That John has arrived is daft.

A semantic account of these two sentences seems to require that propositions be taken to be individuals of a certain kind, which can be subjects for predication.

Another class of intensional phenomena is apparent in the representation of propositional attitude operators.

Ranta [7], offers an interesting characterisation of *belief* in terms of hypothetical judgements, his treatment allows for an analysis of Geach's Hob, Bob, Nob and Cob sentences.

There are other constructions, which seem to require quantification over propositions, as the following example shows (taken from [9]):

$$\frac{\begin{array}{l}\textit{Peter believes everything that John does}\\ \textit{John believes that Henry is daft}\end{array}}{\textit{Peter believes that Henry is daft}}$$

Ranta would analyse the previous example by defining Peter's belief context as an extension of John's belief context: introducing an anchoring (injective function) from John's context to Peter's context. This would provide a solution to the problem, aboiding the high order quantification.

Another problem concerns the nominalization of predicative expressions. There are many examples in natural language of constructions like:

running is fun.
red is a color.
To arrive late is strange.

These constructions suggest the need to treat predicative expressions as individuals as well. Turner, in his intensional framework, defines a nominalization operator that packs the predicative expressions into individuals. Such individuals correspond to functions which form propositions when given an individual from the domain.

On this document we provide a different solution which follows Bealer intuitions [1]. He suggests that the simplest way to analyse these natural language constructions is by considering propositions as individuals, hence we may provide an interpretation of them with the technical apparatus of first order logic. In the following section we introduce the Universe of Small Sets which provides a constructive framework for recasting the multi-sorted domain proposed by Turner in [9].

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2 Intensionality and the Universe of Small Sets

One of the appealling features of Martin-Löf's Type Theory for natural language semantics is that it provides a highly intensional notion of proposition. One important approach consists in defining the universe of small sets U, this provides a level of representation in which sets become objects of the universe. This is accomplished by introducing a representation of every 'small' set in the universe[1] (not including the universe itself, as it would render the theory inconsistent), and then by providing a transformation function which obtains, for each representation, the actual set it stands for. As the constructive type theory is built on the Curry-Howard's *propositions as types* correspondence (which we interpret as 'propositions as sets'), we have a representation of every proposition (but not the proposition itself) as an object in the universe of small sets. We propose that these representations can be interpreted as the intensions (sense) of the propositions. On this view, the elimination rule for the transfomation function (that we will call from now on the *extensional* function, as it provides the extension of the proposition) has an interesting interpretation. The rule is stated as follows:

$$\frac{a = b : \mathsf{U}}{\mathsf{T}(a) = \mathsf{T}(b)}$$

This rule justifies the application of Leibniz's *substitution rule* provided that the intensions of both propositions are the same. It asserts that equal intensions have equal extensions, but not the other way round. That is, if the sense of two propositions is the same, then we can safely substitute the occurrence of one for another in an expression without risk of altering the meaning of that expression.

3 Propositions as Individuals

Using the universe of small sets to analyse intensionality means that the intensions of propositions become objects in the universe. In this sense they are like the objects of any other sets. This means that they may become arguments for predicative expressions[2]. Some constructions of natural language, such as the 'That'-clauses, seem to require such a treatment of propositions, as the following example shows (taken from [9]):

John is daft.
That John has arrived is daft.

[1] We may even decide which sets are to be included.

[2] In this section, we will assume that predicative expressions (such as adjectives and verbs), take as arguments not just an inividual(s), but also the type of the individual(s). This modification does not have any effect in the meaning of the predicative expression.

A semantic account of these two sentences seem to require that propositions be taken to be individuals of a certain kind, which can be subjects for predication. We may provide a categorisation of 'That' in the type theory, which allows for nominalization of sentences, in the following way:

$$\begin{array}{c}\text{That} : (\text{A}{:}\mathsf{U};\ \text{P}{:}(\text{X}{:}\ \textit{set};\ \text{X})\ \ \textit{prop})\ \ \textit{prop} \\ \text{That}(\text{A},\text{P}) = \text{P}(\mathsf{U},\text{A}) :\ \ \textit{prop}\end{array}$$

This provides for a uniform treatment of predicative expressions. On this categorisation, 'That' takes as arguments an object from the universe (the intension of a proposition), and a propositional function (such as *is_daft*), and builds a new propostion. The two sentences in the example will be analysed, respectively, as:

is_daft(Man, John)
is_daft(U, has_arrived'(Man',John))

where the predicative expression *is_daft* has been applied to individuals of different kinds. In the first sentence, it takes as an argument an individual *John* of type *Man*, and in the second sentence it takes an individual *has_arrived'(Man',John)* of type U. We may obtain the extension of the proposition from its intension by applying the extensional function T. If we apply it to the intension of *John has arrived*, we obtain the following result:

T(has_arrived'(Man',John)) = has_arrived(Man,John)

Another class of intensional phenomena is apparent in the representation of propositional attitude operators. Ranta in [7], offers an interesting characterisation of *belief* in terms of hypothetical judgements. His treatment allows for an analysis of Geach's Hob, Bob, Nob and Cob sentences. There are other constructions, which seem to require quantification over propositions, as the following example shows (taken from [9]):

$$\frac{\begin{array}{l}\textit{Peter believes everything that John does} \\ \textit{John believes that Henry is daft}\end{array}}{\textit{Peter believes that Henry is daft}}$$

Ranta would analyse the previous example by defining Peter's belief context as an extension of John's belief context: introducing an anchoring (injective function) from John's context to Peter's context. This would provide a solution to the problem, aboiding the high order quantification.

On this document we provide a different solution which follows Bealer intuitions [1]. He suggests that the simplest way to analyse these sentences is by considering propositions as individuals, hence we may provide an interpretation of them with the technical apparatus of first order logic. With the constructive type theory, it is possible to provide for such a characterisation by categorising the verb *believe* as taking the intension of propositions as one of its arguments, as shown below:

$$\text{believe} : (\text{X}: \ \textit{set}; \text{X}; \mathsf{U}) \ \ \textit{prop}$$

Then it is possible to achieve high order quantification using the standard constructive counterparts of the logical connectors.

$$(\forall x \in \mathsf{U}) \ (\textit{believes}(Man, John, x) \supset \textit{believes}(Man, Peter, x))$$

It is important, to observe that the universe of small sets contains a representation for every *existing* set. It is not possible to analyse non-existing objects, like *unicorns* for example, if the set of *unicorns* does not have an extensional counterpart. To analyse expressions dealing with non-existing objects, one possibility is to follow Ranta's approach building a representation using hypothetical judgements.

4 The Extensional Universe

A formal definition of the extensional universe XU. This universe contains elements which are all the objects of the sets whose representations appear in the universe of small sets. In that sense, the extensional universe represents a *domain of individuals* as is used in model-theoretic semantics. The justification for the introduction of this universe will be provided in Section 5, below. To define the extensional universe we provide its formation rule:

XU-Formation

$$\mathsf{XU} \ \textit{set}$$

The cannonical elements of the extensional universe, are defined by the introduction rule shown below:

XU-Introduction

$$\frac{A : \mathsf{U} \qquad a : \mathsf{T}(A)}{\mathsf{obj}(A, a) : \mathsf{XU}}$$

As can be observed, every cannonical element consists of two components, the actual individual, in this case a, and the representation of its type A in the universe of small sets. Hence, any element belonging to a set which is not represented in the universe of small sets, does not belong to the extensional universe. This means that the elements of the universe of small sets are not part of it. Further, as the extensional universe can not be represented in the universe of small sets, the extensional universe's elements are not represented in it.

It is possible to provide a general elimination rule for objects in the extensional universe, which introduces the constant XE. An equality rule explains the meaning of this constant:

XU-Elimination

$$\frac{c : \mathsf{XU} \ \ d(X, x) : C(\mathsf{obj}(X, x)) \ [X : \mathsf{U}, x : \mathsf{T}(X)]}{\mathsf{XE}(c, (X, x)d(X, x)) : C(c)}$$

XU-Equality

$$\frac{A : \mathsf{U} \ \ a : \mathsf{T}(A) \ \ d(X, x) : C(\mathsf{obj}(X, x)) \ [X : \mathsf{U}, x : \mathsf{T}(X)]}{\mathsf{XE}(\mathsf{obj}(A, a), (X, x)d(X, x)) = d(A, a) : C(\mathsf{obj}(A, a))}$$

Using the constant XE, is possible to define operators for taking the corresponding elements from a given object of the extensional universe, as follows:

$$\begin{aligned} \mathsf{xuleft}(c) \ &\equiv \ \mathsf{XE}(c, (x, y).x) : \mathsf{U} \ [c : \mathsf{XU}, x : \mathsf{U}, y : \mathsf{T}(X)] \\ \mathsf{xuright}(c) \ &\equiv \ \mathsf{XE}(c, (x, y).y) : \mathsf{T}(x) \ [c : \mathsf{XU}, x : \mathsf{U}, y : \mathsf{T}(X)] \end{aligned}$$

Another possibility is to define the left and right injections independently of the constant XE. This can be done by providing independent elimination and equality rules for each injection function:

XU-Elimination 1

$$\frac{c : \mathsf{XU}}{\mathsf{xuleft}(c) : \mathsf{U}}$$

XU-Elimination 2

$$\frac{c : \mathsf{XU}}{\mathsf{xuright}(c) : \mathsf{T}(\mathsf{xuleft}(c))}$$

XU-Equality 1

$$\frac{A : \mathsf{U} \qquad a : \mathsf{T}(a) \qquad \mathsf{obj}(X, x) : \mathsf{XU} \ [X : \mathsf{U}, x : \mathsf{T}(X)]}{\mathsf{xuleft}(\mathsf{obj}(A, a)) = A : \mathsf{U}}$$

XU-Equality 2

$$\frac{A : \mathsf{U} \qquad a : \mathsf{T}(a) \qquad \mathsf{obj}(X, x) : \mathsf{XU} \ [X : \mathsf{U}, x : \mathsf{T}(X)]}{\mathsf{xuright}(\mathsf{obj}(A, a)) = a : \mathsf{T}(\mathsf{xuleft}(\mathsf{obj}(A, a)))}$$

In the following section the introduction of the *extensional universe* will be justified by showing its application to solve puzzle natural language constructions.

5 Predicative Expressions as Individuals

Up to this point, a constructive framework has been provided for some natural language constructions, such as 'That'-Clauses and attitude reports, which require that propositions be treated as individuals of a certain kind. Another problem concerns the nominalization of predicative expressions. There are many examples in natural language of constructions like:

running is fun.
red is a color.
To arrive late is strange.

These constructions suggest the need to treat predicative expressions as individuals. Turner, in his intensional framework, defines a nominalization operator that packs the predicative expressions into individuals. Such individuals correspond to functions which form propositions when given an individual from the domain. To recast Turner's treatment in the type theory is a challenging problem, as there is nothing corresponding to a domain of individuals in the theory. Every set contains its own individuals, and it seems that the most we can do is to define a nominalization operator corresponding to each set. This seems a rather awkward and impractical solution. Instead, we propose to explore the possibility of making use of the new set XUjust defined (see Section 4), which contains all the individuals of the other sets. Using the set XU, is it possible to define the type:

$$\text{PFunction} \equiv \Pi(\mathsf{XU}, (x)\mathsf{U}) : \ set$$

whose elements are functions that map individuals of the domain into intensions of propositions, that is, into representations in the universe of small sets. Armed with this technical apparatus, it is possible to provide a characterisation of 'predicative-expressions as individuals'. For this purpose we introduce the nominalization operator nom, which takes the intension of a predicative expression to produce an individual of type PFunction. We also need an unpack operator, which takes nominalized expressions and returns the original function, these operators can be formalized as follows:

nom-definition

$$\frac{P(X,x) : \mathsf{U}\ [X : \mathsf{U}, x : \mathsf{T}(X)]}{\mathsf{nom}((X,x)P) : \Pi(\mathsf{XU}, (y)\mathsf{U})}$$

with the equality rule:

$$\mathsf{nom}((X,x)P) = (\lambda z)P(\mathsf{xuleft}(z), \mathsf{xuright}(z)) : \Pi(\mathsf{XU}, (y)\mathsf{U})$$

unpack-definition

$$\frac{A : \mathsf{U}\quad a : \mathsf{T}(A)\quad Q : \Pi(\mathsf{XU}, (y)\mathsf{U})}{\mathsf{unpack}(Q, A, a) : \mathsf{U}}$$

with the equality rule:

$$\mathsf{unpack}(Q, A, a) = \mathsf{ap}(Q, \mathsf{obj}(A, a)) : \mathsf{U})$$

The nominalization and unpack operators are related in the following way:

$$\mathsf{unpack}(\mathsf{nom}(P)) = P : (A : \mathsf{U}; \mathsf{T}(A))\mathsf{U}$$

We can also define the constant 'To', for infinitivals, in terms of the nominalization constant. It takes the intension of a propositional function and a propositional function as arguments, and renders a proposition. It is defined as follows:

$$\text{To} : \ ((A : \mathsf{U}; \mathsf{T}(A))\mathsf{U}; (X : \ set; X)\ prop)\ prop$$
$$\text{To(P,Q)} = Q(PFunction, \mathsf{nom}(P)) : \ prop$$

We can now analyse the previous examples within the constructive framework:

running is fun $\Longrightarrow$
is_fun(PFunction,nom(running'))
= is_fun(PFunction, (λz) running'(xuleft(z),$\mathsf{xuright}$(z)))

red is a color $\Longrightarrow$
is_a_color(PFunction,nom(red'))
= is_a_color(PFunction, (λz) red'(xuleft(z),$\mathsf{xuright}$(z)))

To arrive late is strange $\Longrightarrow$
To(arrive_late',is_strange)
= is_strange(PFunction,nom(arrive_late'))
= is_strange(PFunction,
(λz) arrive_late'(xuleft(z),$\mathsf{xuright}$(z)))

so, the desired interpretation for the predicative expressions are obtained.

6 Conclusions

In the present document a constructive framework has been proposed to undertake the analysis of intensional phenomena in natural language, such as nominalization and attitude reports. The proposal is based in Turner's intensional framework [8] which formalises Bealer's intuitions [1] in a classical model. Turner's approach consists in allowing the treatment of propositions and predicative expressions as individuals of a certain kind, which seems to be required in the analysis of infinitivals and nominalized expressions. For a more classic approach to intensional phenomena see [5, 6].

In the analysis presented, it is claimed that the Universe of Small Sets provides a good level of description in which the intensions of propositions can be treated as individuals of a certain kind and then used as arguments of predicative expressions and infinitivals. Two operators ('That' and 'To'), have been defined to provide an interpretation to those constructions. The problem of nominalization of predicative expressions is more difficult, as it requires the introduction of an extensional universe. The notion of the extensional universe XU, has been formalized in the type theory and two operators, nom and unpack were defined to provide for the manipulation of propositional functions as individuals. The former takes a predicative expression and packs it as an element of the type PFunctions (propositional functions) defined in terms of the extensional universe. The later unpacks a nominalized expression allowing its application to arguments of the right type.

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