Adaptive Sliding Mode Control with PID Tuning for Uncertain Systems

T. C. Kuo, *Member, IAENG,* Y. J. Huang, *Member, IAENG,* C. Y. Chen, and C. H. Chang

Abstract—**This paper proposes a novel adaptive sliding mode control with PID tuning method for a class of uncertain systems. The goal is to achieve system robustness against parameter variations and external disturbances. Suitable PID control gain parameters can be systematically on-line computed according to the developed adaptive law. To reduce the high frequency chattering in the switching part of the controller, the boundary layer technique is utilized. The proposed method controller is applied to a brushless DC motor control system. Simulation results demonstrate that satisfactory trajectory tracking is achieved effectively and the input chattering is eliminated completely.**

Index Terms—**PID controller, adaptive control, sliding mode control, robustness.**

I. Introduction

The proportional-integral-derivative (PID) controller is widely used in many control applications because of its simplicity and effectiveness. Though the use of PID control has been a long history in the field of control engineering, the three controller gain parameters, proportional gain K_P, integral gain K_I, and derivative gain K_D, are usually fixed. The disadvantage of PID controller is poor capability of dealing with system uncertainty, i.e., parameter variations and external disturbance. Robustness has gained more and more attention.

In recent years, there has been extensive interest in self-tuning these three controller gains. For examples, the PID self-tuning methods based on the relay feedback technique were presented for a class of systems [1, 2]. An adaptive PID control PID control tuning was proposed to cope with the control problem for a class of uncertain chaotic systems with external disturbance [3]. A genetic algorithm was used to find the optimum tuning parameters of the PID controller by taking integral absolute error as fitting function [4]. Sliding mode control (SMC) is one of the popular strategies to deal with uncertain control systems [5-7]. The main feature of SMC is the robustness against parameter variations and external disturbances. Various applications of SMC have been conducted, such as robotic manipulators, aircrafts, DC motors, chaotic systems, and so on [8-13].

Because there was no powerful microprocessor systems in the past and there existed considerable chattering in SMC systems, SMC did not receive wide attention among engineering professionals back to 1970s. A rise in the interest in SMC occurred in the 1980s because the property of robustness of SMC was gradually recognized Generally speaking, in most cases the system model parameters are not precisely known and environmental disturbance is quite difficult to determine. Good rejection to external disturbances and low sensitivity to parameter variations have become the hallmark of SMC systems.

In this paper, the adaptive PID with SMC is proposed for second-order uncertain systems. The goal is to achieve system robustness against parameter variations and external disturbances. Suitable PID control gain parameters can be systematically obtained according to the developed adaptive law. To reduce the high frequency chattering in the controller, the boundary layer technique is applied [14]. The proposed method controller is applied to the brushless DC motor control system. Simulation results demonstrate that the chattering is eliminated and satisfactory trajectory tracking is achieved. In addition, suitable PID control gain parameters are on-line obtained.

II. Definition of The Systems

Consider a second-order uncertain system which can be described by

$$\dot{x}_1(t) = \dot{x}_2(t)\,, \tag{1}$$

$$\dot{x}_2(t) = f(x_1, x_2, t) + \Delta f(x_1, x_2, t) + d(t) + bu\,, \tag{2}$$

$$y(t) = x_1(t)\,, \tag{3}$$

where $x_1(t)$ and $x_2(t)$ are measurable states, u is the input, y is the output, b is the input gain, $f(\cdot)$ is nominal parameter of plant, $\Delta f(\cdot)$ is the plant uncertainty applied to the system, and $d(t)$ denotes the external disturbance. It is assumed that there exist two positive upper bounds, g and α, satisfying $|\Delta f(\cdot)| \le g$ and $|d(t)| \le \alpha$.

Let e be the error between the desired trajectory y_d and the output y, i.e.,

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$$e = y_d - y \,. \tag{4}$$

III. Controller Design

In order to have a second-order error dynamics, we define a signal x_r as

$$\dot{x}_r = \ddot{y}_d + K_1\dot{e} + K_0 e\,, \tag{5}$$

where K_1 and K_0 are chosen by designers such that roots of $s^2 + K_1 s + K_0 = 0$ are in the open left-half complex plane. In general, we can choose $K_1 = 2\zeta\omega_n$ and $K_0 = \omega_n^2$, where ζ is the damping ratio and ω_n is the natural frequency.

In the following, the proposed controller is described step by step. The design procedure of the proposed sliding mode control is divided into two steps. The first step is to define a sliding surface function such that in the sliding mode the system behaves equivalently as a linear system. The second step is to determine a control law such that the system will reach and stay on the sliding surface $\sigma = 0$.

First, define the sliding surface function as

$$\sigma = x_2 - x_r \,. \tag{6}$$

If sliding mode occurs, i.e. $\sigma = 0$, then

$$x_r = x_2 \,. \tag{7}$$

Substituting (7) into (5) yields

$$\ddot{e} + K_1\dot{e} + K_0 e = 0\,. \tag{8}$$

This implies that tracking error will tend to zero ($e \to 0$) as time goes to infinity ($t \to \infty$).

Next, let the control input u be

$$u = u_{PID} + u_s\,, \tag{9}$$

where

$$u_{PID} = \frac{1}{b}\left[K_P e + K_I \int e dt + K_D \dot{e}\right], \tag{10}$$

$$u_S = -\frac{1}{b}\left(|f| + g + \alpha + |\dot{x}_r| + b|u_{PID}| + K_2\right)\mathrm{sgn}(\sigma)\,. \tag{11}$$

In (11), the gain K_2 is a positive scalar and $\mathrm{sgn}(\cdot)$ is the sign function, i.e.,

$$\mathrm{sgn}(\sigma) = \begin{cases} +1, & \sigma > 0, \\ -1, & \sigma < 0. \end{cases} \tag{12}$$

The three PID controller gains, K_P, K_I, and K_D, are on-line computed, not fixed at all time, by the following adaptive laws,

$$\dot{K}_P = -\eta_1 \sigma e\,, \tag{13}$$

$$\dot{K}_I = -\eta_2 \sigma \int e dt\,, \tag{14}$$

$$\dot{K}_D = -\eta_3 \sigma \dot{e}\,, \tag{15}$$

where $\eta_i > 0$ is defined as the learning rate, $i = 1, 2, 3$.

In order to prove the stability, let the Lyapunov function candidate be

$$V = \frac{1}{2}\sigma^2\,. \tag{16}$$

Taking derivative of (16) yields,

$$\begin{aligned}
\dot{V} &= \sigma\dot{\sigma}\,, \\
&= \sigma\left[f + \Delta f + d + b\left(u_{PID} + u_S\right) - \dot{x}_r\right] \\
&= \left[\sigma f - |\sigma||f|\right] + \left[\sigma\Delta f - |\sigma|g\right] + \left[\sigma d - |\sigma|\alpha\right] \\
&\quad + \left[b\sigma u_{PID} - b|\sigma||u_{PID}|\right] - \left[|\sigma||\dot{x}_r| + \sigma\dot{x}_r\right] - |\sigma|K_2 \\
&\le \left[|\sigma||f| - |\sigma||f|\right] + \left[|\sigma||\Delta f| - |\sigma|g\right] + \left[|\sigma||d| - |\sigma|\alpha\right] \\
&\quad + \left[b|\sigma||u_{PID}| - b|\sigma||u_{PID}|\right] - \left[|\sigma||\dot{x}_r| + |\sigma||\dot{x}_r|\right] - |\sigma|K_2 \\
&\le 0
\end{aligned} \tag{17}$$

Thus, the control law given by (9)-(12) and adaptive laws (13)-(15) guarantee the reaching and sustaining of the sliding mode.

In general, the inherent high-frequency chattering of the control input may limit the practical application of developed method. We further replace $\mathrm{sgn}(\sigma)$ in (11) by the saturation function, $\mathrm{sat}(\frac{\sigma}{\delta})$, i.e.,

$$\mathrm{sat}(\frac{\sigma}{\delta}) = \begin{cases} 1, & \frac{\sigma}{\delta} \ge 1, \\ \frac{\sigma}{\delta}, & -1 < \frac{\sigma}{\delta} < 1, \\ -1, & \frac{\sigma}{\delta} \le -1, \end{cases} \tag{18}$$

where δ is the width of the boundary layer. With this replacement, the sliding surface function σ with an arbitrary initial value will reach and stay within the boundary layer $|\sigma| \le \delta$.

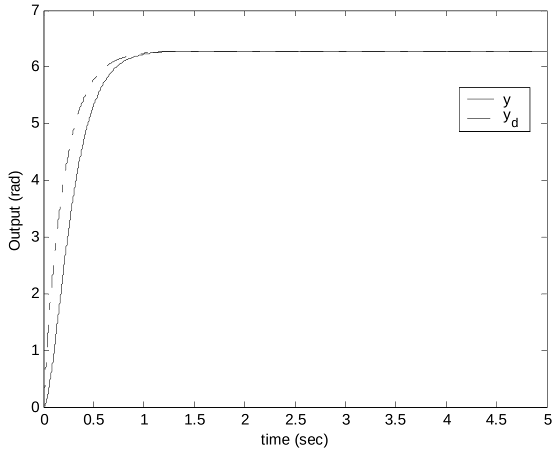

Fig. 1. Output performance.

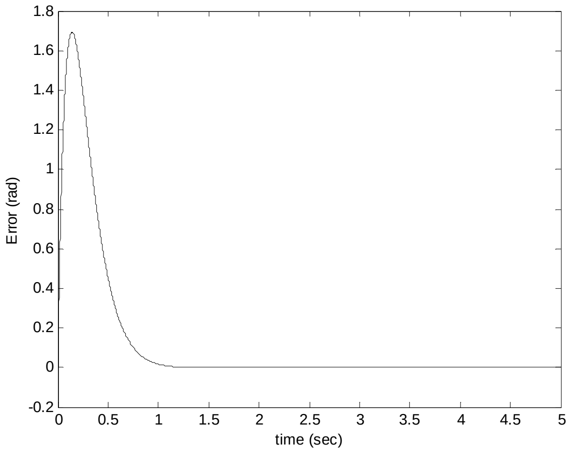

Fig. 2. Tracking error.

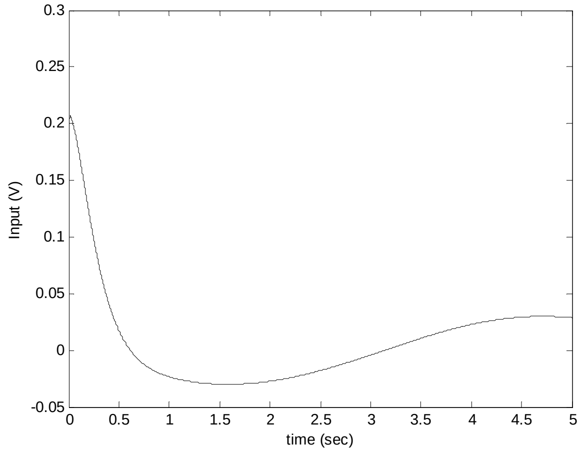

Fig. 3. Control input.

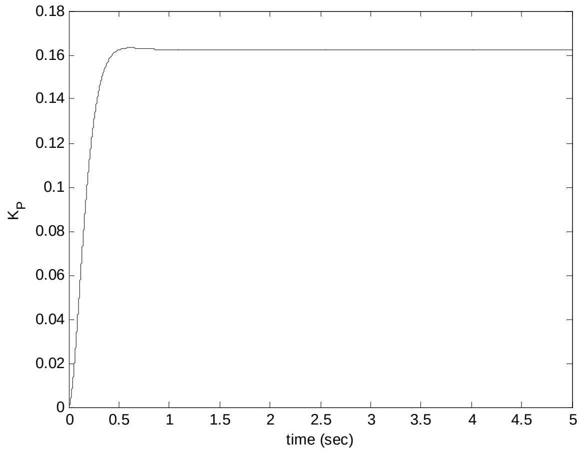

Fig. 4 Controller gain K_P.

IV. Simulation Results

In the computer simulation, we apply the proposed controller to the permanent magnet brushless DC motor with unknown but bounded parameter variations and external disturbance. The dynamics of the brushless DC motor system is described as [13]

$$\ddot{\theta} + a_1\dot{\theta} = b(u + d)\,, \tag{19}$$

where θ is the position angle, $\dot{\theta}$ is the angular velocity, $a_1 = \frac{B}{J}$ is composed of the viscous-friction coefficient B and an unknown but bounded rotor inertia and load J, $b = \frac{K_t K_c}{J}$ factors in the motor torque coefficient K_t and the PWM inverter current coefficient K_c. The control input u is the voltage input. The input uncertainty and disturbance is noted as d.

Let $x_1 = \theta$ and $x_2 = \dot{\theta}$. Therefore, the state-space equation of the brushless DC motor can be rewritten as

$$\dot{x}_1 = x_2\,, \tag{20}$$

$$\dot{x}_2 = -a_1 x_2 + b(u + d)\,, \tag{21}$$

$$y = x_1\,. \tag{22}$$

The bounds on the uncertain parameters and disturbances of the brushless DC motor were given in [13]. The desired trajectory is

$$y_d = 2\pi - 2\pi \exp(-5t)\,. \tag{23}$$

The control input can be implemented by using the formulation (9)-(11), adaptive law (13)-(15), and the saturation function (18). We choose damping ratio $\zeta = 1$ and natural frequency $\omega_n = 7$ so that the roots of $s^2 + K_1 s + K_0 = 0$ are in the open left-half complex plane with $K_1 = 14$ and $K_0 = 49$. The PID controller gains, K_P, K_I, and K_V, are initiated at $K_P(0) = 0$, $K_I(0) = 0$, and $K_D(0) = 0$. The learning rate η_i is set to be 1, $i = 1, 2, 3$. The boundary layer is set to be $\delta = 0.1$.

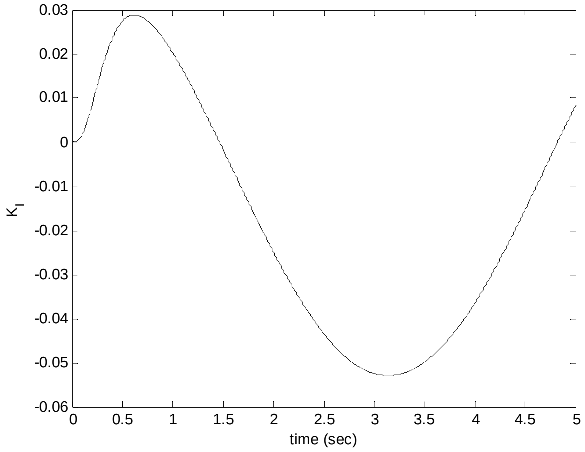

Fig. 5 Controller gain K_I.

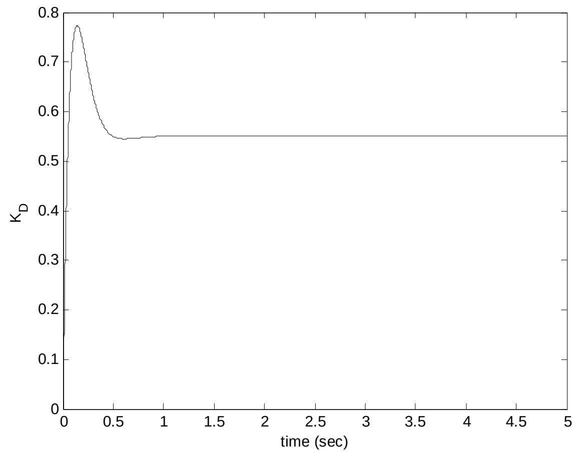

Fig. 6 Controller gain K_D.

The simulation results are shown in Figs. 1 to 6. The sampling time is equal to 0.001 sec. The initial condition is $x_1(0) = x_2(0) = 0$. As shown in Figs. 1 and 2, trajectory tracking results show that the output y converges to the desired trajectory y_d rapidly. From Fig. 3, it is obvious that chattering of the control input is eliminated by using boundary layer technology. The parameters of PID controller, i.e., proportional gain, K_P , integral gain, K_I , and derivative gain, K_D , are obtained with on-line adaptive law as shown in Figs. 4-6, respectively. K_P and K_D are smooth and reach optimal values. K_I reaches a small limit cycle smoothly.

V. Conclusions

In this paper, a novel robust controller design for uncertain systems is proposed. The control law consists of a continuous adaptive PID control part and a discontinuous switching control input. PID controller gains used to be fixed. Robustness could not be guaranteed. Here the three PID controller gains can be systematically on-line computed using adaptive laws. Therefore, the system stability as well as the robustness can be guaranteed. The high frequency chattering in the control input is eliminated by using the boundary layer technology. The proposed method is successfully applied to a brushless DC motor control system. The computer simulation results demonstrate that satisfactory trajectory tracking can be achieved effectively and the chattering is eliminated completely.

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