Product Design and Process Planning in Using the Process Capability Index

Angus Jeang and Chien-Ping Chung

Abstract—**In the past, the process capability index (PCI) was the only one used in on-line quality management; there were no researches on off-line applications, such as product design or process planning. In using conventional PCI for off-line application, designers normally established the process mean close to the design target, and minimized the process variance so that the PCI value would be maximized. The process variance is determined by the process tolerance, which affects the production cost. This factor must be considered because cost will increase as process tolerance is minimized. Simply averting small tolerance for the sake of cost reduction is not always a rational choice, because a great tolerance value generally results in poor quality. Hence, the conventional PCI value does not truly represent the measurement score for off-line applications during product design or process planning. In this regard, the off-line PCI expression is developed with consideration of the balance between product quality and production cost. The product quality is represented by quality loss function, and production cost is expressed by tolerance cost function. These two functions are simultaneously related to process mean and process tolerance. Thus, the new PCI expression can be used to determine appropriate process mean and process tolerance, as well as a measurement score for comparison and selection among candidates. Consequently, an economical and quality of product design and process planning can be achieved during off-line applications.**

Index Terms—**Process capability index (PCI); Process Mean; Process tolerance; Off-line application.**

I. Introduction

Today's manufacturing industry is facing intensive competition, so both the cost and quality aspects have become important issues among management concerns. Thus producers thrive on providing economical processes which are also capable of meeting the customer's quality requirement. In recent years, as the concept of concurrent engineering has become widely accepted, design engineers hope to achieve simultaneous product design and process planning, side by side, at an early stage of product development [4]. The goals are to shorten the time span required for introducing the new product onto the market and to attain the lowest production cost and premium product quality. Hence, what is needed is a way to measure the degree of the producer's process capability, in satisfying the customer's quality requirement. More importantly, a growing number of producers include this measurement value in their purchase contracts with customers, as a documentation requirement. One such measurement is the process capability index (PCI).

The process capability index (PCI) is a value which reflects real-time quality status. The PCI acts as the reference for real-time monitoring that enables process controllers to acquire a better grasp of the quality of them on site processes [6,7]. Although the PCI is considered as one of the quality measurements employed during on-line quality management, several authors have pointed out that the PCI should be addressed at the beginning of the design stage rather than at the production stage, where process capability analysis is typically done [12]. For the sake of convenience, let us call the PCI for the former one, off-line PCI, and the latter one, on-line PCI. The on-line PCI has process mean and process variance that are obtained from the existing process. Conversely, the off-line PCI has the process mean and process variance as two unknown variables, which the process planner would have to determine. When cost is not considered as a factor for off-line PCI analysis; normally the process planners would do their best to set the process mean close to the design target, and minimize the process variance. Because the additional cost incurred for tightening the variance is not considered, obviously, the establishment of mean and variance values will result in a high PCI scale [9]. Thus, this research intends to develop an Off-line PCI that expression which contains cost factors.

The PCI value is typically defined as the ability to carry out a task or achieve a goal. In process capability analysis, the lower and upper limits are initially assumed to be firm and non-negotiable, unless it can be proven that the quality of the final product will not be lessened by changing these limits. The controllable factors are the process mean and process variance [8]. The deviation between process mean and design target can be reduced by locating the process mean close to the design target without additional cost being incurred. The process variance can be lowered by tightening the process tolerance, with extra cost incurred. In case the conventional on-line PCI is used for process capability analysis during the product development, designer engineers naturally intend to raise the PCI value by locating the process mean near the target value, and by reducing the tolerance value to ensure a better product quality. However, simply increasing the PCI value can easily create additional and unnecessary production costs that result from extra efforts and expensive devices for ensuring tolerance control. Hence, there is a need to balance customer demands for quality and production costs. In this regard, the off-line PCI value is introduced, in consideration of quality loss and production cost, simultaneously in this research. The quality loss is expressed by quality loss function, and the production cost is represented by tolerance cost function. Then, this new PCI expression can be used as linkage for concurrent product design and process planning, prior to actual production. The rationale will be discussed in the latter sections.

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II. BACKGROUND REVIEW

A. Population mean and population standard deviation of produced quality values

The process design requires determining the best quality value of process setting and process tolerance so that unit-to-unit variation is reduced to a minimum. Process tolerance is defined as the maximum range of variation for the quality value of interest in a particular process. By following the selected process setting and process tolerance, the quality value of production output is randomly distributed, with the average and variability being process mean and process variance. To reduce the variation of production output so that the functionality of a produced product can be accomplished, we usually have a process specification as maximum value of production output, which allows for deviation from the target value.

The process mean may or may not be same as the quality value of the process setting. By the same token, the process variance may or may not be the same as the quality value established by the process tolerance. However, having the process setting and process tolerance established at process design, with the possibility of process drifting or systematic error being excluded in the production process, we believe that the process mean will be close to the selected process setting, and that the process variance will be less than the process tolerance after production process. Suppose that $X_1, X_2, \ldots, X_n$ are the produced quality values. The sample average is:

$$\overline{X} = \sum_{i=1}^{n} \frac{X_i}{n} \tag{1}$$

The sample standard deviation is:

$$S = [\sum_{i=1}^{n} \frac{(X_i - \overline{X})^2}{n}]^{\frac{1}{2}} \tag{2}$$

The parameter U is the established process setting (process mean), and parameter σ is the process standard deviation estimated from determined process tolerance t. Obviously, U and σ are population mean and population standard deviation of the produced quality values. Thus, assume that the probability distribution of population representing the produced quality values, forms a normal distribution with $X \sim N(U, \sigma^2)$ from which the sample was taken. Both the mean U and tolerance t are two controllable variables which can be decided at the very beginning of the process or product design, namely, pre-production stage. It is believed that $\overline{X}$ and S observed from the samples will be close to U and σ as the sample size becomes great. For the sake of convenience, we will call these two statistics the process mean and process tolerance, in the following discussion.

B. Quality loss and tolerance cost

In the product life cycle, quality values will vary under different circumstances. Fig. 1 is a typical quality loss function. Let $X_1, X_2, \ldots, X_n$ be the quality values appearing in different situations. The average quality loss in its symmetric quality loss function is the following [11,14]:

$$E[L(X)] = K[(U-T)^2 + \sigma^2] \tag{3}$$

U represents the average value of the quality characteristic, while σ^2 is the variance of the quality characteristic. Equation (3) tells us that there are two sources of quality loss. (a) $K(U-T)^2$ is the deviation between the process mean and the design target. Examples are blade damage, machine breakdown, problems with raw materials and worker carelessness, which are usually easy to handle without cost becoming a factor. (b) $K\sigma^2$ is the loss resulting from process variances. To eliminate this source, better equipment, materials, and processes are usually required. There is cost containment in this effort. Hence, we usually improve (a) first, and then (b), for economic considerations.

Usually, a high tolerance cost is associated with a tight process tolerance, while a low tolerance cost results from a loose process tolerance. Fig. 2 is a typical tolerance cost function. The tolerance cost can be formulated in various function expressions. To evaluate the tolerance cost, this paper adopts the tolerance cost function as developed in the literature [3].

$$C_M(t) = a + b \cdot \exp(-c \cdot t) \tag{4}$$

where a, b, and c are the coefficients for the tolerance cost function, and t is the process tolerance.

From the above cost expression, it can be noted that a tight process tolerance results in a higher tolerance cost, due to additional manufacturing operations, more expensive equipment needed and slower production rates, while a loose process tolerance results in a lower tolerance cost [8-10].

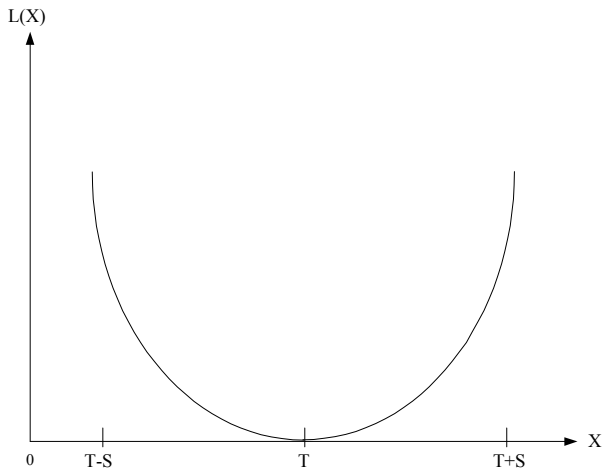

Fig. 1. Quality loss function

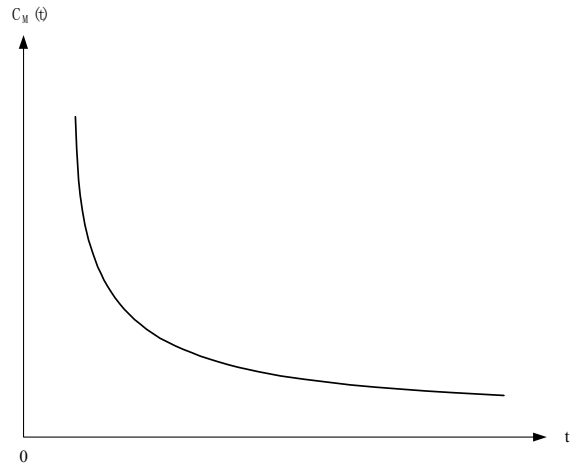

Fig. 2. Tolerance cost function

C. Process Capability Indices (PCI)

The frequently seen PCI includes C_p, C_{pk}, and C_{pm} expressions. C_p can be defined as follows [1,2,5,6,7].:

$$C_p = \frac{USL - LSL}{6\sigma} \tag{5}$$

The expression (USL-LSL) refers to the difference between the upper and lower limits which are specified by the customer's quality requirement; σ is the standard deviation which is actually incurred in the production process. However, during the production process, the process means U can be located at positions other than design target. If the process variance σ^2 did not change, the above C_p value would also remain unchanged; this was the major defect owing to the facts that only the spread of the process is reflected, and the deviation of process mean can not be reflected in the measurement. These are the main reasons why C_{pk} was developed; the C_{pk} expression is defined as below:

$$C_{pk} = Min(\frac{USL - U}{3\sigma}, \frac{U - LSL}{3\sigma}) \tag{6}$$

There is still a deficiency for C_{pk} expression: the same C_{pk} values may be constituted with different process means and variances. This situation has created a great deal of confusion, and uncertainty as to which would be the best process capability among the alternatives. To cope with the above arguments, another form of PCI, C_{pm}, was developed. C_{pm} is defined as follows:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (U - T)^2}} \tag{7}$$

When the process mean is equal to design target, the C_{pm} can be simplified as C_p. For the purpose of comparison, three processes: A, B, and C are depicted in Fig. 3. The C_p, C_{pk}, and C_{pm} values from processes A, B, and C are shown in Table 1. Because process C has the greatest C_p value, it is might be mistakenly concluded that process C had the best process capability among processes A, B, and C, when C_p is considered as a reference value. However, this erroneous conclusion originates from the fact that the C_p value is solely based on the magnitude of variance, and disregards the negative impact from the process deviation. Similarly, when C_{pk} was used in representing the levels of process capability, the process C_{pk} values for processes A, B, and C, are all just equal to one. Thus again, quite obviously, there is difficulty in ordering the superiority of process capability of the three processes. To overcome the defects appearing in C_p and C_{pk} expressions, another PCI expression, $C_{pm,}$ is introduced. Unlike the previous two expressions, C_{pm} can simultaneously reflect the impact from process deviation and process variance. This feature is particularly important because process mean and process variance are generally changed at the same time in most production process. Unfortunately, with C_{pm}, processes A and B are the best two choices. The non-different outcomes between processes A and B result from the fact that the contribution of C_{pm} value, from process mean deviation and process variance magnitude, is identical. Hence, there must be a way of measurement being provided to make mean deviation and process variance magnitude distinguishable in PCI expression.

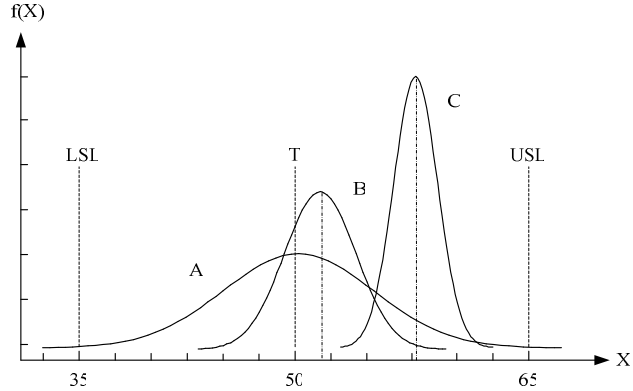

Fig. 3. The distribution for process A, B and C

Note: Process A: U_A= 50.0, σ_A =5.00, $C_M(t)$=\$2000,

Process B: U_B= 53.0, σ_B =3.83, $C_M(t)$=\$3500,

Process C: U_C= 57.5, σ_C =2.50, $C_M(t)$=\$6000

Table 1 PCI values for process A, B, and C

Process	C_p	C_{pk}	C_{pm}	C_{pmc}
A	1	1	1	0.051298
B	1.25	1	1	0.047673
C	2	1	0.632456	0.031782

D. Estimation of process mean and variance for an assembly chain

An assembled product consists of several components created by different processes; hence, determining which combination of component tolerances is best, is an important issue. In addition to tolerance design for a single component of a product, most design problems should allocate component tolerances so that the assembly dimensions of a completed product fall into acceptable quality and functionality limits. The assembly dimension of a completed product is a combination of the dimensions of several components from assembly chains; therefore, the overall assembly dimensions vary within a distributed range. As a result, most tolerance analyses with assembly dimensions relate the variation of assembly tolerances (resultant tolerances) to the variation of the component tolerances, whereby appropriate component tolerances are determined.

Thus, to extend tolerance design for a completed dimension, tolerance stack-up and variance build-up models will be applied. Tolerance stack-up and variance build-up are estimated by using the assembly chain, where the formation of tolerance stack-up is accumulated by the product/process tolerances appearing in the related components in the assembly chain. Similarly, the variance build-up possesses the same features, that is, worse cases of tolerance stack-up and variance build-up can be represented as:

$$t_Y \approx \sum_{j=1}^{M_0} t_j \tag{8}$$

$$\sigma_Y^2 \approx \sum_{j=1}^{M_0} \sigma_j^2 \tag{9}$$

where t_j is the process tolerance of the jth component/process, t_Y is the process tolerance stack-up of the product quality value (also called resultant process tolerance), and M_0 is the

total number of the components in a product. The term σ_j^2 is the process variance of the jth component/process, which can also be estimated as: $(\frac{t_j}{P})^2$, where P is a given value [10], while σ_Y^2 is the process variance build-up of the product quality value (also called resultant process variance).

Let X_Y represent the resultant product quality value that is summed up from several single component product quality values:

$$\sum_{j=1}^{M_0} A_j \times X_j = X_Y \tag{10}$$

A_j is the assembly chain vector for the jth component. The example in this study discusses an assembly chain vector that involves linear equations with coefficients being 1 or -1; X_j is the single product quality value. By taking the expectation on both sides of (10), the expected value of the resultant product quality value is:

$$\sum_{j=1}^{M_0} A_j \times U_j = U_Y \tag{11}$$

U_j is the process mean of a single product quality value and U_Y is the resultant process mean.

III. Simultaneous Determination of Parameter and Tolerance Values

Product functionality is achieved through the assignment of appropriate design target and design tolerance, at the stage of product design and process planning. Generally, when the manufacturer attempts to attain a high process capability with a small process tolerance, a higher manufacturing cost will usually result. But when the design tolerance exceeds the process tolerance, additional space or process distribution is provided for a possible shift or drift. Consequently, the parameter value may be set at various positions within a feasible range, which in turn, can lead to quality improvement and cost reduction. The mathematical relationship of the design target *T*, design tolerance *S*, parameter value *U* and process tolerance *t*, can be represented as [8]:

$$|T - U| \leqq S - t \tag{12}$$

The tolerance value *t* is constrained by t_L and t_U which refer to upper and lower process capability limits, respectively. However, the parameter value is also contained within an acceptable range, between U_U and U_L. That is,

$$t_L \leq t \leq t_U \tag{13}$$

$$U_L \leq U \leq U_U \tag{14}$$

The values of t_L, t_U, U_U, U_L, *S*, and *T* are known in advance. Equation (12) states that the deviation between design target and parameter value should be less than the distance between design tolerance and process tolerance. By satisfying (12), manufacturability for all possible combinations will be ensured. As a result, the feasible combinations increase the flexibility of both product design and process planning, to a great amount. In addition, the effort of obtaining the precision required for the control of the process is reduced. Quality improvement and cost reduction are thus achieved. In general, there are no costs associated with the parameter design which changes the nominal value of a parameter value; conversely, there are costs involved in the tolerance design. Hence, the best setting of the nominal values is often determined prior to the tolerance design, for the sake of economics. However, based on the discussion regarding (12), in some situations it is important to simultaneously determine parameter and tolerance values, due to the dependency between these two values on the quality characteristics of interest. The dependency phenomena discussed in (12) mainly focus on one single quality characteristic. However, the dependency phenomena are still true in a design with multiple quality characteristics. This is due to the fact that (20) and (23) or (26) and (29), shown in Appendix, are mutually dependent when design function *f(x)* is a nonlinear function. The mutual dependency can be explained by the fact that U_Y and t_Y are in a functional relationship with component parameter value U_i, and tolerance value t_i concurrently. Because U_Y and t_Y still need to be constrained by (12), and will be substituted into the PCI expression eventually, the degree of dependency will be even greater than in the former situation. Although, most of the time design function *f(x)* is unknown, we believe that it should inherently exist in a nonlinear function form, for all product design and process planning. Based on the above discussion, both the parameter and tolerance values have to be simultaneously determined in order to optimize the design.

IV. Process Capability Index for Off-Line Application

A. Single quality characteristic

The conventional PCI values are mainly applied to measure the on-line spread resulting from actual process deviation and process variance. The reality of this spread is a consequence of the production process, in which there are no more controllable elements. Hence, the conventional PCI expression is valid only in the on-line application. In this regard, there is an attempt to explore the process capability analysis, at the pre-production stage, to enable designers to integrate the aspects of product design and process planning at an early time. According to the preceding discussion, unlike the other two PCI expressions, C_{pm} is able to simultaneously reflect the influences of process deviation and process variance. However, this is only legitimate at the post-production stage, due to the fact that U and t are realized values which are not controllable for design. However, when C_{pm} is used as a measurement scale in the pre-production stage, U and t become controllable variables. Then, it is possible that various combinations of U and t will result in the same C_{pm} value. Thus, it is difficult to make a distinction among alternatives, in order to make a correct choice from among them. As is known, the process mean U can be adjusted at no additional cost. The designers would most likely establish the process mean U as close as possible to the design target T, within the process feasible range, and attempt to decrease the process variance as much as possible within the process capability limits in order to attain a higher PCI value. In other words, with the exclusive use of the process mean and process tolerance as the determinants of conventional PCI, regardless of the cost impact on customer and production, there is a tendency for designers to position the process mean as close to the target value as possible, and solely cut down the process tolerance to lower capability limit in order to increase the PCI value. Apparently, the found PCI value is erroneous.

The degree of proximity reflects the different quality loss according to the customer's view. Reducing the process variance is normally completed by tightening the tolerance

value through tolerance design which usually involves additional cost. Therefore, in addition to the constraints from feasible ranges and capability limits, the influence exerted by the relevant costs representing the selected process mean and process tolerance, should be considered as well. This brings us to the next section, a discussion on the requirement that costs related to process mean and process tolerance must be contained in PCI expression, when referred to as off-line process capability analysis, during product design and process planning.

As indicated in Fig. 4, various combinations of U and t will result in the same C_{pm} value, 1.2. This unhelpful facet will prevent the conventional C_{pm} from being a suitable index for differentiating possible alternatives during product design or process planning. To overcome the above weakness, the lack of consideration of the cost influence from various U and t values, should be resolved. As is known, all costs incurred within a product life cycle, include the material and production costs which are incurred before the product reaches the consumer, and quality loss, which occurs after a sale. In these regards, let the denominator of C_{pm} be replaced with the sum of quality and production related cost, which includes quality loss $K[\sigma^2+(U-T)^2]$ and tolerance cost $C_M(t)$. Have both U and t be controllable variables so that a maximum C_{pmc} can be achieved. This cost effectiveness PCI expression C_{pmc} is shown as follows.

$$C_{pmc}=\frac{USL-LSL}{6\sqrt{K[\sigma^2+(U-T)^2]+C_M(t)}} \tag{15}$$

where σ is $\frac{t}{P}$ [10].

Fig. 4 depicts that there are infinite combinations of U and t which have C_{pm} as 1.2. Table 2 shows that different C_{pmc} values can be obtained under various combinations of U and t, when C_{pm} is fixed as 1.2. Apparently, the conventional C_{pm} is not capable of representing all possible alternatives for off-line application during design and planning stage. Table 1 shows that processes A and B are identical choices based on C_{pm}; however, process A is the only selection based on C_{pmc} value. Different selections are being made because C_{pm} lacks consideration of the combined influence from quality loss for customers and tolerance costs for production.

Let design target value T = 30 mm, design tolerance S = 0.05mm, quality loss coefficient K = 1200, coefficients of tolerance function a = 50.11345, b = 119.3737, c= 31.5877, P = 3, t_L = 0.024 mm, t_U = 0.086 mm. Substitute these values into (12), (13), (14), and (15), to proceed the following discussion.

The optimal C_{pmc}^* is 0.0019. The optimal process mean U^* is 30.0000 mm and the process tolerance t^* is 0.05 mm. Figs. 5 and 6 show that when the process mean is located at the target with fixed tolerance value, the maximum C_{pm} and C_{pmc} can be reached. The explanation is discernable by looking into the common expression, $(U-T)^2$, in the denominator of (7) and (15). On the other hand, in Fig. 7, with fixed process mean at the target value, the maximum value C_{pm}, which is infinite, is reached when t is near to zero and the maximum C_{pmc} , which is finite, arrives when t is 0.04. The same conclusions are derived by examining Figs. 7 and 8. The fact behind different optimal t values being found when the PCI is at its maximum, is comprehensible because the variance in C_{pmc} is cost related, while the variance in C_{pm} is cost unrelated. The t value in C_{pm} can be any small value regardless of the cost impact resulting from tolerance cost.

Table 2 Various C_{pmc} when C_{pm} is 1.2

(U,t)	(50.00,12.50)	(50.05,12.49)	(51.00,12.13)	(51.05,12.09)	(52.00,10.96)	(52.05,10.88)
C_{pm}	1.2000000	1.2000000	1.2000000	1.2000000	1.2000000	1.2000000
C_{pmc}	0.0345994	0.0346019	0.0356385	0.0357504	0.0394258	0.039727

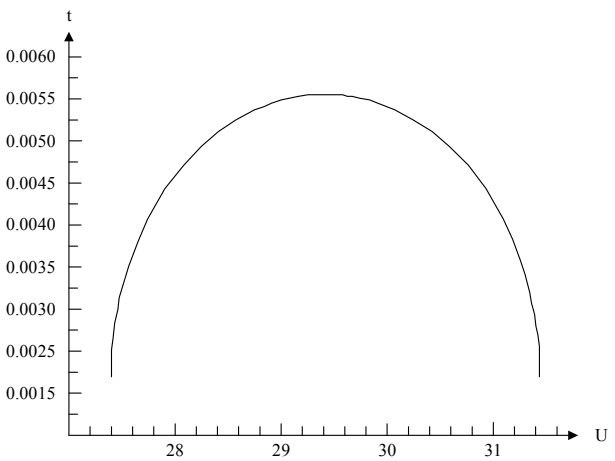

Fig. 4. Feasible range for various combination of U and t with T=30, S=0.05, and P =3 when C_{pm}=1.2

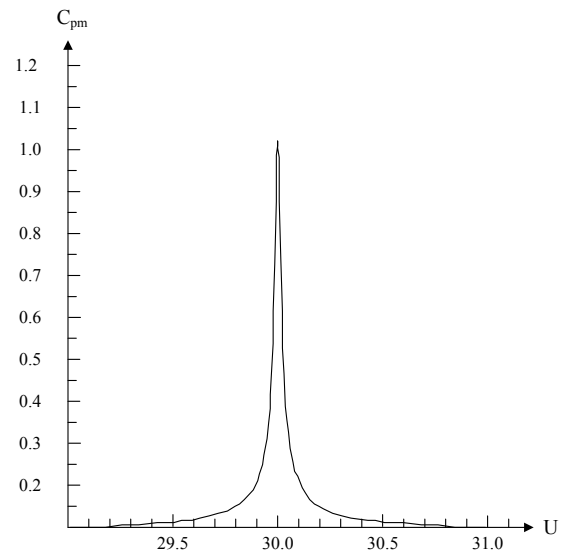

Fig. 5. C_{pm} vs U with T=30, S=0.05, t=0.399863, and P=3

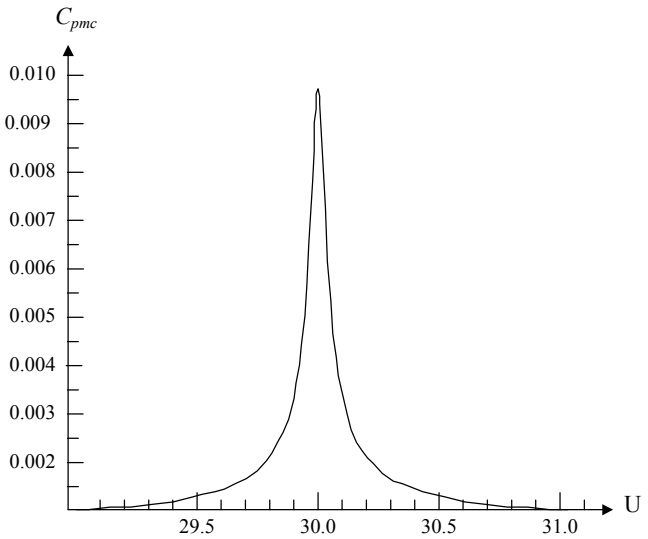

Fig. 6. C_{pmc} vs U with T=30, K=1200, S=0.05, t=0.0399863, a=50.1134, b=119.3737 and c=31.58775

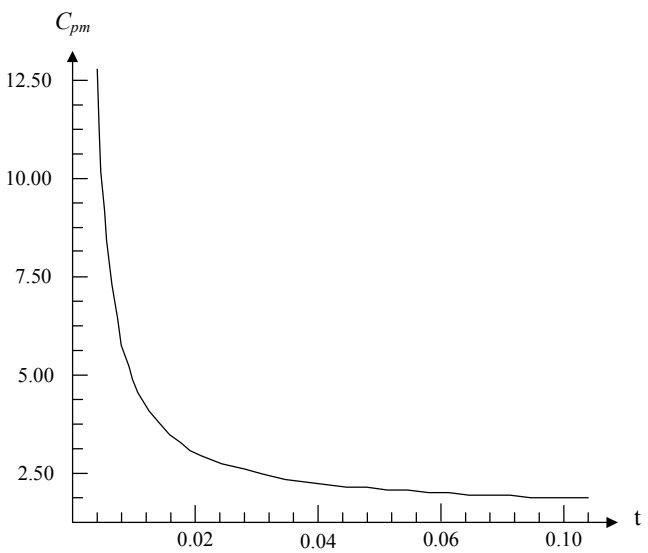

Fig. 7. C_{pm} vs t with T=30, U=30, S=0.05, and P=3

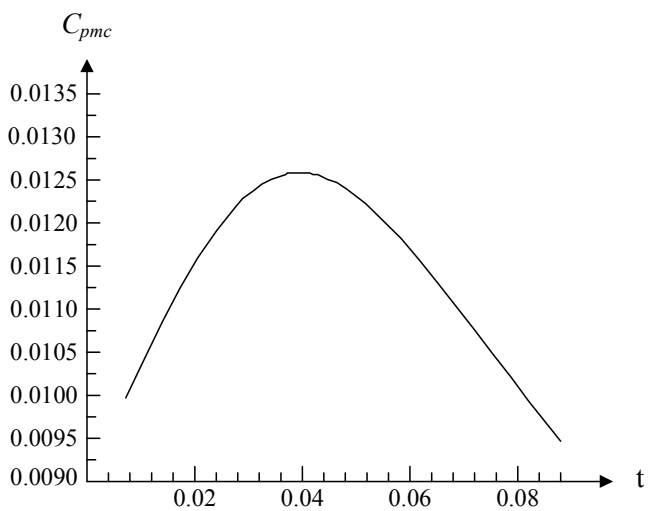

Fig. 8. C_{pmc} vs t with T=30, K=1200, S=0.05, a=50.11345, b=119.3737, and c=31.58775

A. Multiple quality characteristics

As discussed in the preceding section, a lower quality loss (better quality) implies a higher tolerance cost, and a higher quality loss (poorer quality) indicates lower tolerance cost. Hence, the design parameters must be adjusted to reach an economic balance between reduction in quality loss and tolerance cost, so that the cost effectiveness PCI expression, C_{pmc}, is maximized. The model developed in the following illustration attempts to achieve this objective in the case of multiple quality characteristics.

Before model development, a functional relationship between the dependent variable Y and independent variable X should be identified and thoroughly understood. Based on this relationship, the resultant overall quality characteristic such as σ_Y and U_Y can be estimated from the set of individual quality characteristics in both product and process. The proposed process capability index, C_{pmc} for the multiple quality characteristics is:

$$C_{pmc} = \frac{USL - LSL}{6\sqrt{K[(U_Y - T_Y)^2 + \sigma_Y{}^2] + \sum_{i=1}^{M_0} C_M(t_i)}} \tag{16}$$

where M_0 is the total number of quality characteristics in a product or process.

Assembly is the process by which the various components and sub-assemblies are brought together to form a completed product which is designed to fulfill a certain product function. Tolerance design as well as parameter design should be considered simultaneously, particularly when the aspects of quality loss, tolerance cost, minor maintenance cost, and major maintenance cost are included in the objective function of interest. Therefore, to ensure that the required functionality and quality of a product are met, a proper determination of process mean and process tolerance among the assembly components is of critical importance.

The assembly application shown in Fig. 9 is a classic gearbox assembly which consists of five components: X_1, X_2, X_3, X_4, and X_5. In this example M_0 is 5 and the assembly chain vector is: $A_j = [1,1, -1, -1, -1]$; the assembly chain describing the product quality value X_Y of interest is:

$$X_Y = X_1 + X_2 - X_3 - X_4 - X_5 \tag{17}$$

Thus, the assembly chain in describing the resultant process mean of the product quality value X_Y of interest is:

$$U_Y = U_1 + U_2 - U_3 - U_4 - U_5 \tag{18}$$

Associated process means that U_1, U_2, U_3, U_4, and U_5, as well as process tolerances t_1, t_2, t_3, t_4, and t_5 must be determined so that the gap X_Y between the bushing and gearbox falls within the specification limits, USL and LSL, where T_1 is 0.9 mm, USL is 0.985 mm, and LSL is 0.825 mm. Let K be $1500. The P value for estimating the process variance is assumed to be 3. Table 3 provides the upper and lower process capability limits for each component. Table 4 provides the coefficients a, b, and c for the tolerance cost functions, which can be found based on actual data collected in factories and analyzed through a statistical regression method. Based on given conditions from these tables, the problem is formulated via one of the optimization techniques such as mathematical programming. The objective intends to find optimal parameter and tolerance values so that the C_{pmc} is maximized under a set of process constraints. In our example, these optimal values are:

$U_1^* = 16.0121$ mm, $U_2^* = 18.0150$ mm, $U_3^* = 29.0056$ mm, $U_4^* = 1.8078$ mm, $U_5^* = 2.3087$ mm, $t_1^* = 0.0140$ mm, $t_2^* = 0.0196$ mm, $t_3^* = 0.0240$ mm, $t_4^* = 0.0115$ mm and $t_5^* = 0.0107$ mm, and $C_{pmc}^* = 0.003844$.

For the purpose of comparison, a further study is carried with the same formulation except that objective function C_{pmc} is replaced with the expression of C_p, C_{pk}, or C_{pm}. The results along with the above outcomes are summarized in Table 5. Clearly, C_{pmc} mirrors relevant quality and cost sensitively and is most applicable during the stage of product design. In other words, C_{pmc} is an appropriate expression for application in the process capability analysis during off-line application; particularly, for product design and process planning at the stage of blueprint.

Table 3 Upper and lower process capability limits for component j

j	t_{Lj} [mm]	t_{Uj} [mm]
1	0.014	0.042
2	0.018	0.052
3	0.024	0.072
4	0.009	0.027
5	0.010	0.030

Table 4 Tolerance cost function coefficients

j	a_j	b_j	c_j
1	10.0045	1.0036	4.0773
2	12.0127	1.0189	6.0921
3	10.0045	1.0036	4.0773
4	5.0981	0.9871	7.6381
5	6.5690	0.7621	5.9331

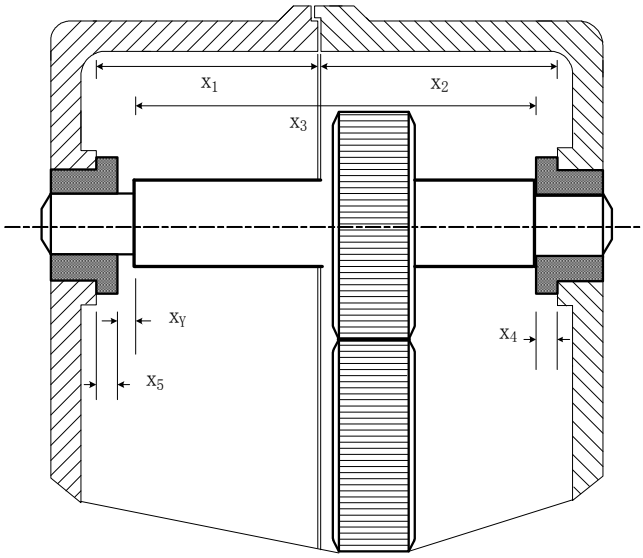

Fig. 9. A classic gearbox

Table 5 PCI values under various quality loss coefficient K

PCI \ K	15	150	1500	15000	150000
C_{pmc}	0.003846	0.003844	0.003832	0.003728	0.003005
C_{pm}	1.865577	1.865577	1.865577	1.865577	1.865577
C_{pk}	0.874489	0.874489	0.874489	0.874489	0.874489
C_p	1.865577	1.865577	1.865577	1.865577	1.865577

V. Conclusion

In the past, in using conventional PCI for measuring the process consequence, designers normally establish the process mean close to the design target, and minimize the process variance to small process capability limits so that the PCI value will be maximized. This one-way attitude in establishment will drive the PCI value to be unusually high since conventional PCI expression lacks consideration of quality loss and production cost involved during variance reduction and mean adjustment. Moreover, these are some of the reasons that concurrent engineers suggesting that possible issues occurring in the production stages should first be considered at the time that the new product is developed. That can reduce the time span for introducing a new product onto market, and increase the chance for obtaining a superior edge among competitors. Thus, the present research introduces a PCI measurement, C_{pmc}, for the process capability analysis, to ensure that lower production cost and high product quality can be achieved at the earlier time of the blue print stage. As a result, an effective PCI for product life cycle becomes actualized. In this regard, a new PCI expression C_{pmc} is developed, which not only considers the impact from process mean and process variance as done conventionally, but also considers relevant quality cost and production cost incurred in a product's life cycle.

APPENDIX. PROOF THE DEPENDENCE BETWEEN PROCESS MEAN AND PROCESS TOLERANCE

Case 1 – Single quality characteristic

The expansion of $Y = f(X)$ about the point U_X by Taylor's series up to the first three terms is:

$$Y = f(X) = f(U_X) + (X - U_X)\left(\frac{\partial f(X)}{\partial X}\bigg|_{Ux}\right) + \frac{1}{2!}(X - U_X)^2\left(\frac{\partial f(X)}{\partial X}\bigg|_{Ux}\right) + \Re \quad (19)$$

where $\Re$ is the remainder. Taking the expectation of (19), the outcome is

$$U_Y = E(Y) = f(U_X) + \frac{1}{2}\frac{\partial^2 f(X)}{\partial X^2}\bigg|_{U_X} V(X) + E(\Re)$$

$$\approx f(U_X) + \frac{1}{2}\frac{\partial^2 f(X)}{\partial X^2}\bigg|_{U_X} V(X)$$

$$\approx f(U_X) + \frac{1}{2}\frac{\partial^2 f(X)}{\partial X^2}\bigg|_{U_X} \sigma_X^{\,2}$$

where $\sigma_X = \dfrac{t_X}{P}$

$$\approx f(U_X) + \frac{1}{2}\frac{\partial^2 f(X)}{\partial X^2}\bigg|_{U_X} \left(\frac{t_X}{P}\right)^2 \quad (20)$$

To have an approximate value of V(Y), consider the Taylor's series expansion up to the first two terms. That is:

$Y = f(X)$

$$= f(U_X) + (X - U_X)\left(\frac{\partial f(X)}{\partial X}\bigg|_{Ux}\right) \quad (21)$$

Then

$$\sigma_Y^{\,2} \approx \mathrm{V(Y)} = \mathrm{V}\,(f(U)) + \mathrm{V}\left((X - U_X)\left(\frac{\partial f(X)}{\partial X}\bigg|_{Ux}\right)\right])$$

$$= \sum_i^n \left[\left(\frac{\partial f(X)}{\partial X}\bigg|_{Ux}\right)^2 \sigma_X^{\,2}\right] \quad (22)$$

Because $\sigma_X = \dfrac{t_X}{P}$, (22) can be further arranged as:

$$\sigma_Y^{\,2} = \left(\frac{\partial f(X)}{\partial X}\bigg|_{Ux}\right)^2 \left(\frac{t_X}{P}\right)^2 \quad (23)$$

Case 2 - Mutiple quality characteristics

Extending the above derivation to the n design factors; assuming $X_1, X_2, ..., X_n$ are independent; such as $Y = f(X_1, X_2, ..., X_n)$; that is:

$Y = f(X_1, X_2, ..., X_n)$

$$= f(U_1, U_2, ..., U_n) + \sum_i^n \left[(X_i - U_i)\left(\frac{\partial f(X_1, X_2, X_3, ..., X_n)}{\partial X_i}\bigg|_{U_1, U_2, U_3, ..., U_n}\right)\right] + \frac{1}{2!}\sum_j^n\sum_i^n \left[(X_i - U_i)(X_j - U_j)\left(\frac{\partial f^2(X_1, X_2, X_3, ..., X_n)}{\partial X_i \partial X_j}\bigg|_{U_1, U_2, U_3, ..., U_n}\right)\right] + \Re \quad (24)$$

$U_Y = E(Y)$

$$= f(U_1, U_2, ..., U_n) + \frac{1}{2}\sum_i^n \left[\frac{\partial^2 f(X_1, X_2, X_3, ..., X_n)}{\partial X_i^{\,2}}\bigg|_{U_1, U_2, U_3, ..., U_n} V(X_i)\right] + E(\Re)$$

$$\approx f(U_1, U_2, ..., U_n) + \frac{1}{2}\sum_i^n \left[\frac{\partial^2 f(X_1, X_2, X_3, ..., X_n)}{\partial X_i^{\,2}}\bigg|_{U_1, U_2, U_3, ..., U_n} V(X_i)\right] \quad (25)$$

Assuming the relationship between tolerance value and standard deviation is $t = P\sigma$, (25) can be further expressed as:

$$U_Y \approx f(U_1, U_2, ..., U_n) + \frac{1}{2}\sum_i^n \left[\frac{\partial^2 f(X_1, X_2, X_3, ..., X_n)}{\partial X_i^{\,2}}\bigg|_{U_1, U_2, U_3, ..., U_n} \left(\frac{t_i}{P}\right)^2\right] \quad (26)$$

To have an approximate value of V(Y), consider the Taylor's series expansion up to the first two terms. That is:

$Y = f(X_1, X_2, ..., X_n)$

$$= f(U_1, U_2, ..., U_n) + \sum_i^n \left[(X_i - U_i)\left(\frac{\partial f(X_1, X_2, X_3, ..., X_n)}{\partial X_i}\bigg|_{U_1, U_2, U_3, ..., U_n}\right)\right] \quad (27)$$

Then

$$\sigma_Y^{\,2} \approx \mathrm{V(Y)} = \mathrm{V}\,(f(X_1, X_2, ..., X_n)) + \mathrm{V}\left(\sum_i^n \left[(X_i - U_i)\left(\frac{\partial f(X_1, X_2, X_3, ..., X_n)}{\partial X_i}\bigg|_{U_1, U_2, U_3, ..., U_n}\right)\right]\right)$$

$$= \sum_i^n \left[\left(\frac{\partial f(X_1, X_2, X_3, ..., X_n)}{\partial X_i}\bigg|_{U_1, U_2, U_3, ..., U_n}\right)^2 \sigma_{Xi}^{\,2}\right] \quad (28)$$

Because $\sigma = \dfrac{t}{P}$, (28) can be further arranged as:

$$\sigma_Y^{\,2} = \sum_i^n \left[\left(\frac{\partial f(X_1, X_2, X_3, ..., X_n)}{\partial X_i}\bigg|_{U_1, U_2, U_3, ..., U_n}\right)^2 \left(\frac{t_i}{P}\right)^2\right] \quad (29)$$

$$t_Y^{\,2} = \sum_i^n \left[\left(\frac{\partial f(X_1, X_2, X_3, ..., X_n)}{\partial X_i}\bigg|_{U_1, U_2, U_3, ..., U_n}\right)^2 t_i^{\,2}\right] \quad (30)$$

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