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Estimation for the Weibull Power Law Parameters in the Step-up Voltage Test

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Abstract—**In assessing the insulation withstand level of the electric apparatus, the step-up test method is used. However, we have still many unknown matters regarding the treatment of the results. In this paper, we assume that the underlying probability distribution of failure time with a constant voltage level follows a Weibull distribution and that an inverse power law relationship between the mean lifetime and the imposed voltage holds; that is, the Weibull power law is assumed. Under such a condition, we first investigate whether we can estimate the unknown Weibull power law parameters using the breakdown voltage results obtained from the step-up test. We assume two models: one is the independence model, and the other is the cumulative exposure model. When we use the maximum likelihood estimation (MLE) method, the estimation is well performed in both the models. On the contrary, the method of least squares (LS), commonly used for electric engineers in obtaining the Weibull parameters for the breakdown voltage, performs badly. We compare the estimation results between those using the MLE and those using the LS both the models. The LS has a tendency to yield a bias for the Weibull shape parameter, and it generates a larger standard deviation. Consequently, the RMSE using the LS becomes larger than that using the MLE. We conclude that the MLE is superior to the LS. Regarding the model selection of which model between the independence model and the cumulative exposure model should be used, we recommend the cumulative exposure model from both viewpoints of the model derivation and the RMSE.**

Keywords: Weibull distribution, power law, step-up voltage test, maximum likelihood estimation, method of least squares

1 Introduction

To assess the insulation withstand level of the electric apparatus in electric power substations by using the voltage application test is crucial for insulation design. Although the test method is rather simple, we have still many unknown matters; e.g., how we deal with the test results. Assuming, e.g., that the voltage level imposed to the apparatus is continuously increased. Then, we can obtain the breakdown voltage results; using the basic statistics we may estimate the voltage blow which the breakdown probability is specified to be low, e.g., $\mu - 3\sigma$ is a simple guess to that (see [4]) where μ and σ are the sample mean and the sample standard deviation. However, we know that values μ and σ are affected by the voltage increasing rate per second; the faster the rate, the higher the value of μ. Using the step-up test method (Figure 1), a similar tendency will be observed. Unlike the breakdown time observation with same constant voltage stress imposing, we aware how difficult to assess the insulation withstand level by using the step-up voltage test [1, 6, 7, 8, 9]. This is because the step-up voltage test includes the random variable of time T and the explanation variable of voltage stress v together.

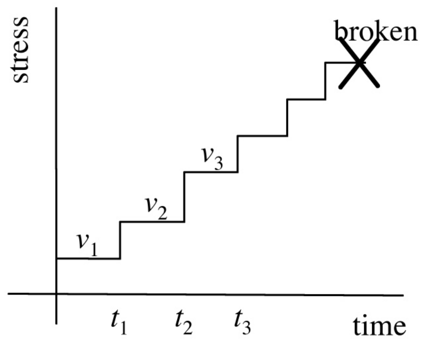

Figure 1: Step-up voltage test. *The test is performed as follows: Stress v_i is imposed to the insulation one minute; if the insulation is not broken, then the stress level is raised higher to v_{i+1} and v_{i+1} is imposed one minute; this continues until the insulation is broken, and the final stress v_f is used for the estimation.*

Here, we assume that the underlying probability distribution of failure time with a constant voltage level follows a Weibull distribution with shape parameter a and that there is a inverse power law between the mean lifetime

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t_{meanlife} and the imposed voltage v, that is

$$t_{\text{meanlife}} = kv^{-n}, \tag{1}$$

where n is the power law constant, and k is a constant. Therefore, we assume the Weibull power law model [2, 3],

$$F(t; v) = 1 - \exp[-K(v^n t)^a], \tag{2}$$

where, t expresses the random variable and v expresses the explanation variable. In this paper, we first investigate whether we can estimate the unknown values of a and n using the results obtained by the step-up test. We use the maximum likelihood estimation method (MLE).

Superficially, one sometimes uses the relationship of

$$m = na, \tag{3}$$

to evaluate the voltage stress endurance parameter, as if v works as the random variable and m behaves as the Weibull shape parameter. As seen in the literature, e.g., [7], it is common for electric engineers to use the method of least squares (LS) in obtaining the Weibull parameters for the breakdown voltage, i.e., the Weibull plot method is used. In such a case, the shape parameter m is often addressed.

We next compare the results between those using the MLE and those using the LS. We finally show the superiority of the MLE over the LS.

2 Step-up Test Method

The step-up test method is performed as follows: 1) stress v_1 is imposed to the insulation one minute, 2) if the insulation is not broken, then the stress level is raised higher to v_2 and v_2 is imposed one minute, 3) this continues until the insulation is broken, and the final stress v_f is used for the estimation. Here, the initial stress level is set such that the breakdown would not occur, and the step-up distance d is set such that too many stress levels are not imposed; in actual case, v_1 is 70% to 90% of the mean value of v_f, i.e., the sample mean μ, and d is 4% to 8% of the mean value of v_f. In the simulation study in this paper, we set $v_1 = 0.1$ and $d = 0.1$; the number of samples is 200; the number of trials is 100 each.

We have two cases of random number generation method according to the two proposed methods for insulation evaluation. One is the *independence model* and the other is the *cumulative exposure model*. In reliability fields, the latter is common.

2.1 Independence model

The independence method, called by [7, 8] and adopted by JEC-012 [4], is a probability model that each insulation event phenomenon such as TOV (temporary over voltages) independently affects the insulation failure regardless of its history such as aging or deterioration of the insulation. Then, the probability distribution model, when we observe the final breakdown voltage of v_f, is expressed as,

$$F(v) = 1 - \exp[-A\sum_{i=1}^{f}(v_i^{na} \cdot t_i^a)], \tag{4}$$

where A is a constant. If we use unit time for each t_i, then, t is hidden. Note that in this model the random variable is superficially expressed by v; then, the unknown shape parameter in this Weibull model is na, resulting that the parameters n and a are not obtained simultaneously. Therefore, the use of the parameter m is common.

By a simple calculation when time duration to each stress is small enough, the estimated shape parameter in Weibull analysis using this independence model has a bias for m even if the sample size is large enough; this is because, by approximating the summation part by integration, (4) tends to the ordinary Weibull distribution with shape parameter $m + 1$; thus, the bias of quantity 1 is always observed if the Weibull plot is used. See appendix.

2.2 Cumulative exposure model

The cumulative exposure model [3], also called the accumulation model by [7, 8], is a probability model that every step v_i, where stress v_i is imposed to the insulation, affects the next step v_{i+1} failure probability. This may be interpreted that we assume that the quantity $v^n t$ is accumulated to the insulation for future failure probability. Then, the probability distribution model, when we observe the final breakdown voltage v_f, is expressed as,

$$F(v) = 1 - \exp[-B\{\sum_{i=1}^{f}(v_i^n \cdot t_i)\}^a], \tag{5}$$

where B is a constant.

By a similar approximation to the independence model, we mention that we observe a bias of quantity a to the shape parameter m when we admit (3).

2.3 Bias for the Weibull shape parameter

As mentioned above, the bias will always be observed even if the sample size is large; an exception is seen when $a = 1$, the case of the exponential model. Otherwise, by looking at the functions in the exponential function in (4) and (5), we can see that the convexity varies by the value of a; see appendix. This is explained in [7] in a more concrete case when the actual step-up setting is assumed.

3 Parameter Estimation using the MLE

In both the models of 2.1 and 2.2, we use the likelihood function such that

$$L = \prod_{k=1}^{N} \{F(v_{f_k+1}) - F(v_{f_k})\}, \tag{6}$$

where N is the sample size. The maximum likelihood estimates are obtained by searching for the parameters such that the log-likelihood function becomes the maximum. We, here, use the simplex method [5] in optimization.

When we adopt the independence model, the unknown parameters are m and A. When we adopt the cumulative exposure model, the unknown parameters are a, n, and B.

4 Parameter Estimation using the LS

When we use the LS, the Weibull plot is required. To fit the straight line on the Weibull plot to the observed data, we transform the voltage values v_{f_k} $(k = 1, 2, \ldots, N)$ to its logarithmic values, and the probability of kth order statistics is set usually to $\frac{k}{N+1}$, where $v_{(f_1)} \leq \ldots \leq v_{(f_N)}$. The superficial estimate for the shape parameter in the Weibull model is then obtained.

When we adopt the independence model, the unknown parameters are m and A. When we adopt the cumulative exposure model, the unknown parameters are m and B.

5 Simulation

5.1 Simulation cases

For the independence model we consider the cases where $m = 6.67, 10, 15, 33.3$. The simulation cases for the cumulative exposure model are shown in Table 1. The value of m is set according to (3). The value of A is set to 0.053^m.

Table 1: Simulation cases for the cumulative exposure model

a	n	m
0.3	6.67	2
	10	3
	33.3	10
1	6.67	6.67
	10	10
	33.3	33.3
1.5	6.67	10
	10	15
	33.3	49.95

In all the cases, we set $v_1 = 0.1$ and $d = 0.1$; the number of samples is 200; the number of trials is 100 to each case.

5.2 Simulation results

The simulation results for the independence model are shown in Table 2. In the table, the RMSE is computed as

$$\text{RMSE} = \sqrt{\text{bias}^2 + \text{variance}},$$

where variance is s.d.^{2}.

Table 2: Simulation results for m in the independence model

true m	estimation method	estimate bias	s.d.	RMSE
6.67	MLE	−0.003	0.427	0.427
	LS	0.754	1.109	1.341
	MLEc	*0.951*	*0.448*	*1.052*
10	MLE	0.028	0.612	0.613
	LS	0.339	2.322	2.346
	MLEc	*0.900*	*0.706*	*1.144*
15	MLE	−0.065	0.896	0.898
	LS	0.418	2.452	2.487
	MLEc	*0.764*	*1.028*	*1.280*
33.3	MLE	−0.005	1.913	1.913
	LS	−0.079	5.075	5.076
	MLEc	*0.975*	*1.676*	*1.939*

From the table, we can see that the large bias is not observed in the MLE, but is in the LS as indicated in 2.1. The more important matter is that the standard deviation by the LS is markedly larger than that by the MLE. Consequently, the estimates by using the MLE are always far superior to those using the LS. In the table, we can see *MLEc* for reference; this value is the maximum likelihood estimate for m when we regard the data as the continuous data. We discuss this in the next section.

The simulation results for the cumulative exposure model are shown in Tables 3 and 4. In Table 3, the results by the MLE are shown, and in Table 4, the value for m using the MLE by (3) and using the LS are shown.

From the tables, we do not see the large bias in the MLE, but we do in the LS as indicated in 2.2. Similar to the results in the independence model, the standard deviation by the LS is larger than that by the MLE. Consequently, the estimates by using the MLE are always superior to those using the LS.

6 Discussion

As stated, using the independence model, we can estimate the superficial Weibull shape parameter induced from (4) by the LS, which is no more the same as m. On the contrary, the MLE will not yield the large bias. Similarly in the cumulative model, the MLE will not produce the large bias, while the LS will. In addition, the standard deviation is larger in the LS than in the MLE. As a consequence, the RMSE tends to larger in the LS than in the MLE. This is a first merit in using the MLE.

Table 3: Simulation results in the cumulative exposure model (1): a and n by the MLE

true a	n	estimate a bias	s.d.	RMSE
0.3	6.67	0.001	0.018	0.018
	10	0.005	0.023	0.023
	33.3	0.003	0.019	0.019
1.0	6.67	−0.003	0.046	0.046
	10	0.021	0.048	0.053
	33.3	0.000	0.038	0.038
1.5	6.67	−0.001	0.025	0.025
	10	−0.003	0.045	0.045
	33.3	−0.008	0.031	0.032
true a	**n**	**estimate n bias**	**s.d.**	**RMSE**
0.3	6.67	0.112	0.313	0.332
	10	0.157	0.480	0.505
	33.3	0.415	0.289	0.506
1.0	6.67	0.100	0.118	0.155
	10	0.332	0.203	0.389
	33.3	0.170	0.142	0.222
1.5	6.67	0.171	0.083	0.190
	10	0.146	0.120	0.189
	33.3	0.070	0.078	0.105

Which model between the independence model and the cumulative exposure model should be used? This is a next question. We can adopt the cumulative exposure model because of the natural derivation of the model. The independence model has long been used because the estimation method is simple and is easy to use. There seems no difference between the two models as long as we use the LS. However, if we use the MLE, a big difference between the two models is seen apparently. In the independence model (4), the parameter m can be defined by $m = na$. However, we cannot estimate the parameters a and n simultaneously. On the contrary, we can estimate both the parameters a and n simultaneously, if we use the MLE. Moreover, from Tables (2) and (4), the RMSE for m is smaller in the cumulative exposure model than in the independence model. We can regard that the cumulative exposure model can be recommended from both viewpoints of the model derivation and the RMSE.

We have shown the *MLEc* values for reference in Tables 2 and 4; these value are the maximum likelihood estimates for m when we regard the data as the continuous data. That is, we fit the continuous Weibull model,

$$f(v) = \frac{m}{u}(\frac{v}{u})^{m-1}\exp[-(\frac{v}{u})^m]. \tag{7}$$

The estimated values by the LS and by the $MLEc$ are, in principle, dealt with the data as continuous. On the contrary, the estimates by the MLE are dealt with the data as grouped. It would be recommended that we use the maximum likelihood method in the continuos model too. However, the difference of the RMSE value between the LS and the $MLEc$ is smaller than that between the MLE and the LS. The important point is that the MLE can deal with the data more accurately to each model than the LS can.

Table 4: Simulation results in the cumulative exposure model (2): m by the MLE and the LS

true m (a, n)	estimation method	estimate bias	s.d.	RMSE
2 (0.3, 6.67)	MLE	−0.027	0.125	0.128
	LS	0.224	0.180	0.288
	MLEc	*0.251*	*0.136*	*0.286*
3 (0.3, 1.0)	MLE	0.004	0.176	0.176
	LS	0.259	0.226	0.344
	MLEc	*0.308*	*0.200*	*0.368*
10 (0.3, 33.3)	MLE	-0.015	0.630	0.630
	LS	0.108	0.693	0.702
	MLEc	*0.331*	*0.655*	*0.733*
6.67 (1, 6.67)	MLE	−0.123	0.296	0.321
	LS	0.799	0.469	0.927
	MLEc	*0.902*	*0.435*	*1.002*
10 (1, 10)	MLE	−0.135	0.434	0.454
	LS	0.900	0.787	1.196
	MLEc	*0.881*	*0.591*	*1.060*
33.3 (1, 33.3)	MLE	−0.164	1.206	1.217
	LS	0.468	2.248	2.296
	MLEc	*1.160*	*1.822*	*2.160*
10 (1.5, 6.67)	MLE	−0.262	0.222	0.343
	LS	1.277	0.754	1.483
	MLEc	*1.376*	*0.623*	*1.510*
15 (1.5, 10)	MLE	−0.250	0.432	0.499
	LS	1.166	0.963	1.512
	MLEc	*1.298*	*0.844*	*1.548*
49.95 (1.5, 33.3)	MLE	−0.358	0.982	1.045
	LS	0.795	3.604	3.691
	MLEc	*1.168*	*2.679*	*2.923*

7 Conclusion

To assess the insulation withstand level of the electric apparatus, the step-up test method is often used. However, we have still many unknown matters regarding the treatment of the results. Assuming that the underlying probability distribution of failure time with a constant voltage level follows a Weibull distribution with shape parameter a and that an inverse power law $t_{\mathrm{meanlife}} = kv^{-n}$ between the mean lifetime t_{meanlife} and the imposed voltage v holds, then, we first investigate whether we can estimate the unknown values of n and a using the breakdown voltage results obtained by the step-up test. We have used the maximum likelihood estimation (MLE) method for this purpose. We have assumed two proposed models: 1) the independence model, 2) the cumulative exposure model. The maximum likelihood estimation is well performed in both the models.

It is common for electric engineers to use the method of least squares (LS) in obtaining the Weibull parameters for the breakdown voltage. In such a case, the shape parameter m is often addressed. We next compare the results between those using the MLE and those using the LS in both the models. The LS is inclined to yield a bias for parameter m, and it generates a larger standard deviation. Consequently, the RMSE using the LS becomes larger than that using the MLE. That is, the MLE is superior to the LS.

Regarding the model selection of which model between the independence model and the cumulative exposure model should be used, we recommend the cumulative exposure model from both viewpoints of the model derivation and the smaller RMSE.

Appendix

We consider the case that time duration t is 1 for all the stress levels. We use a simple approximation such that

$$\begin{aligned}\sum_{i=1}^{f} v_i^{na} &= (dv)^{na} + (2dv)^{na} + \cdots + (v_f)^{na} \\ &\approx C\int_0^{v_f} u^{na} du = C'(v_f)^{na+1}. \end{aligned} \tag{8}$$

In the independence model, from (4),

$$\begin{aligned} F(v) &= 1 - \exp[-D\sum_{i=1}^{f} v_i^{na}] \\ &\approx 1 - \exp[-D' v_f^{na+1}]. \end{aligned} \tag{9}$$

Transforming this into the Weibull plot structure,

$$\log\log\frac{1}{1-F(v)} = (na+1)\log v_f + \text{const.} \tag{10}$$

We can obtain the estimated mean of $na+1$ for true value of na; i.e., the bias is 1.

In the cumulative exposure model, transforming (5) into the Weibull plot structure,

$$\log\log\frac{1}{1-F(v)} = a\log[\sum_{i=1}^{f}(v_i^n)]. \tag{11}$$

By the approximation described above,

$$\log\log\frac{1}{1-F(v)} \approx a(n+1)\log v_f + \text{const,} \tag{12}$$

which deduces the bias of a.

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