Numerical Determination of Crack Opening and Closure Stress Intensity Factors

L.C.H. Ricardo*

Abstract—**The present work shows the numerical determination of fatigue crack opening and closure stress intensity factors of a C(T) specimen under variable amplitude loading using a finite element method. A half compact tension C(T) specimen, assuming plane stress constraint was used by finite element method covering the effects in two-dimensional (2D) small scale yielding models of fatigue crack growth under modified wind turbine standard spectrum loading WISPER. The crack propagation of the finite element model was based on release nodes in the minimum loads to minimize convergence problems. To understand the crack propagation processes under variable amplitude loading, retardations models are observed.**

Index Terms— **Fatigue, Finite Element Method, Crack Propagation Process, Variable Amplitude Loading**

I. INTRODUCTION

The most common technique for predicting the fatigue life of automotive, aircraft and wind turbine structures is Miner's rule [1]. Despite the known deviations, inaccuracies and proven conservatism of Miner's cumulative damage law, it is at present being used in the design of many advanced structures. Fracture mechanics techniques for fatigue life predictions remain as a back up in design procedures.

The most important and difficult problem in using fracture mechanics concepts in designs seems to be the use of crack growth data in predicting fatigue life. This experimentally obtained data is used to derive a relationship between stress intensity range (ΔK) and crack growth per cycle (da/dN). In cases of fatigue loaded parts containing a flaw under constant stress amplitude fatigue, the crack growth can be calculated by simply integrating the relation between da/dN and ΔK. However, for complex spectrum loadings, simple addition of the crack growth occurring in each portion of the loading sequence produces results that very often are more erroneous than the results obtained using Miner's rule with an *S-N* curve.

Retardation tends to cause conservative Miner's-rule life predictions where the fatigue life is dominated by the crack growth. However, the opposite effect generally occurs where the life is dominated by the initiation and growth of small cracks.

In these cases, large cyclic strains, which might occur locally at stress raisers due to overload, may pre-damage the material and lower its resistance to fatigue.

This effect is generally handled by basing the crack initiation life prediction on a modified (lowered) strain-life or stress-life curve that includes the effect. In 1960 Schijve [2] observed that experimentally derived crack growth equations were independent of the loading sequence and depended only on the stress intensity range and number of cycles for a given portion of loading sequence. The central problem in the successful utilization of fracture mechanics techniques applied in a fatigue spectrum is to obtain a clear understanding of the influence of loading sequences on fatigue crack growth. Of particular interest in the study of crack growth under variable-amplitude loading is the decrease in growth rate called crack growth retardation that usually follows a high overload.

Most of the reported theoretical descriptions of retardation are based on data fitting techniques, which tend to hide the behavior of the phenomenon. If the retarding effect of a peak overload on the crack growth is neglected, the prediction of the material lifetime is usually very conservative [3]. The small scale yield model employs the Dugdale [4] theory of crack tip plasticity, modified to leave a wedge of plastically stretched material on fatigue crack surfaces. Fatigue crack growth was simulated by Skorupa and Skorupa [5] using the strip model over a distance corresponding to the fatigue crack growth increment as shown in Fig. 1.

In order to satisfy the compatibility between the elastic plate and the plastically deformed strip material, tensile stress must be applied on the fictitious crack surfaces. Tensile stresses are also needed over some distance ahead of the crack tip, in the crack wake region, Fig. 1 ($a_{\text{open}} \le x < a$), where a_{open} indicate crack open, where the plastic elongations of the strip *L(x)* exceed the fictitious crack opening displacements, *V(x)*, in the plastic zone ($a \le x < a_{\text{fict}}$),where a_{fict} indicate ficticious crack extension, as in the original Dugdale model.

Crack propagation simulation by finite element method is a well consolidated technique to simulate the crack propagation process under constant amplitude loading. Newman [6] was the first to simulate the crack propagation process using a finite element method with the difference of the released node being in the maximum load whereas in the current work the release node is in the minimum load. Ricardo [7] presents results that show that the releasing node at minimum load does not affect the quality of the results. McGlung [8] in 1989 shows results have no relevant difference in node release in the maximum or in the minimum load.

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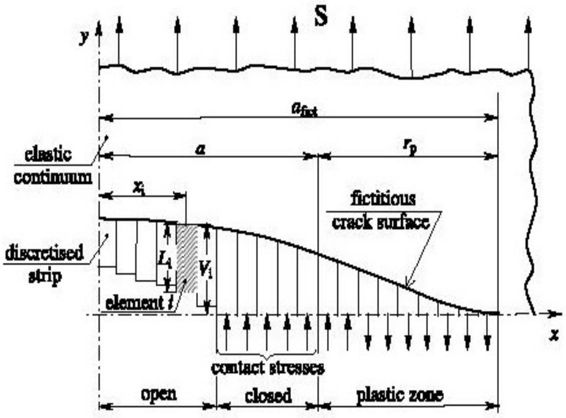

Figure 1: Schematic Small Scale Yield Model [5]

II. RETARDATION PHENOMENON

Corbly & Packman [9] describe some aspects of the retardation phenomenon. Despite the recent increase in research into retardation effects in crack propagation, there are many aspects of load interaction phenomena that lack adequate explanations. Aspects of the retardation phenomena that are generally agreed upon are presented below.

1. Retardation increases with higher values of peak loading σ_{peak} for constant values of lower stress levels [10,11].

2. The number of cycles at the lower stress level required to return to the non-retarded crack growth rate is a function of ΔK_{peak}, ΔK_{lower}, R_{peak}, R_{lower}, and number of peak cycles [12].

3. If the ratio of the peak stress to lower stress intensity factors is greater than about 1.5, complete retardation (arrest) at the lower stress intensity range is observed. However, some tests of this may not have continued long enough to see if the crack ever propagated again [12].

4. With a constant ratio of peak to lower stress intensity, the number of cycles to return to non-retarded growth rates increases with increasing peak stress intensity [11,12].

5. Given a ratio of peak stress to secondary stress, the number of cycles required to return to non-retarded growth rates decreases with increased time at zero load before cycling at the lower level [12].

6. Increased percentage delay effects of peak loading, given a percent overload, are greater at higher baseline stress intensity factors [13].

III. DESCRIPTION OF MODEL

A compact tension specimen was modeled using a commercial finite element code, MSC/Patran, r1 [14] and ABAQUS Version 68 [15] used as solver. Half of the specimen was modeled and symmetry conditions applied. Fig. 2 shows the compact tension specimen from ASTM 647-E95a, and Fig. 3 shows the model used in the present work. A plane stress constraint is modeled by the finite element method covering the effects in two-dimensional (2D) small scale yielding models of fatigue crack growth variable spectrum loading. The boundary conditions are presented in Fig. 4. The finite element model has triangle elements, S8R, with quadratic formulation and spring elements, SPRING1.

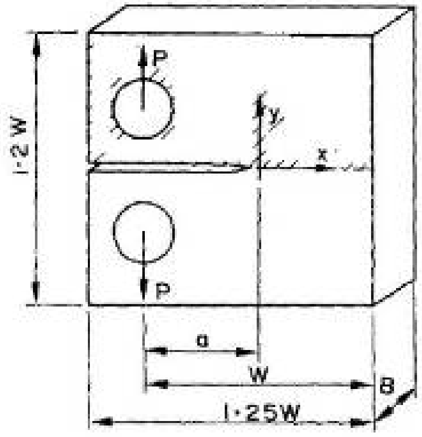

Figure 2: Compact Tension Specimen

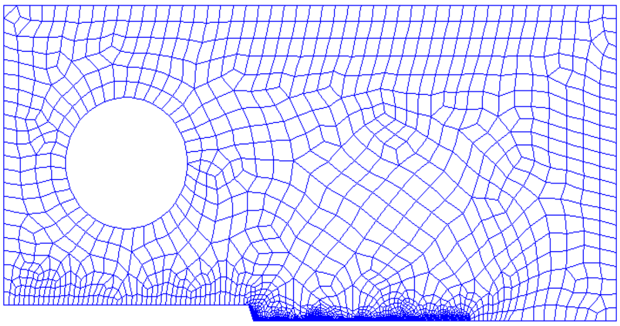

Figure 3 Half Compact Tension Modeled by Finite Element Method

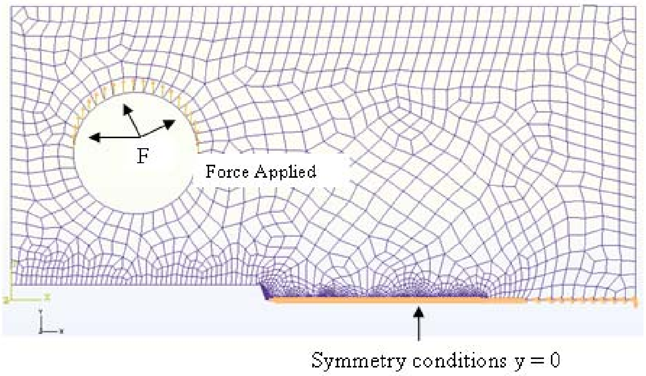

Figure 4 Boundary Conditions used in the FEM Crack Propagation Model

WISPER is a standard variable-amplitude test loading histories for use in the fatigue design of horizontal axis wind turbine blades. This load history was develop by ten Have [16] and is based on flap load service measurements on 9 different horizontal axis wind turbines, covering a wide range of materials, rotor diameters and geographical locations. In this work was used MINI-WISPER spectrum loading generated by Genesis [17]. Genesis edits the WISPER load history. The spectrum loading used to simulate the crack propagation is a modified wind turbine standard loading MINI-WISPER, having only positive loads, as shown in Fig. 5.

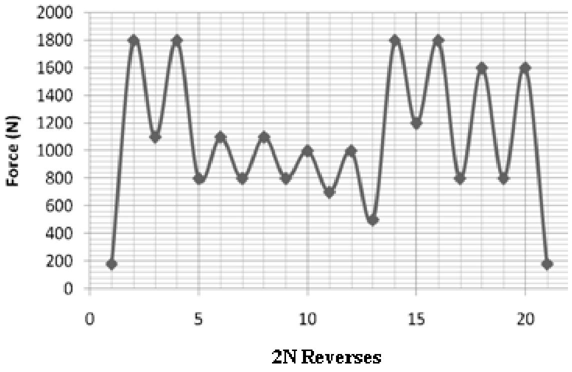

Figure 5 Wind Turbine Standard Loading Modified (WISPER)

To convert force in stress intensity factor, the equation (1) is used, Fatigue crack growth is simulated by releasing crack tip node at K_{min}, followed by a single loading cycle $K_{min} \rightarrow K_{max} \rightarrow K_{min}$, Fig. 6.The force is divided into steps of loads P_{min}- P_{max} and nine steps of loads P_{max}-P_{min}, in each cycle. To evaluate the crack propagation, a nonlinear analysis is used to compute the deformation history, cycle by cycle, using the Newton-Rapson method. The procedure to estimate if the crack is opened or closed is based on the work of Wei & James [18]. These authors considered that the crack closure occurs at the first contact behind the crack tip; a second criterion is that the surface at the crack tip must be in compression. This can be observed when the displacements of nodes in the crack tip area are negatives in (y) direction.

$$K_{\min} = \frac{P_{\min}}{BW^{1/2}} f\left(\frac{a}{W}\right);\ K_{\max} = \frac{P_{\max}}{BW^{1/2}} f\left(\frac{a}{W}\right) \qquad (1)$$

Where: $\boldsymbol{K_{min}}$ = minimum stress intensity factor; $\boldsymbol{K_{max}}$ = maximum stress intensity factor; $\boldsymbol{P_{min}}$ = minimum applied load; $\boldsymbol{P_{max}}$= maximum applied load ; ***B*** = specimen thickness; ***a*** = crack length; ***W*** = width of the specimen; ***a/W*** = ratio of the crack length to the specimen width; ***f(a/W)*** = characteristic function of the specimen geometry. Table I shows the materials properties for compact tension specimen, C(T).

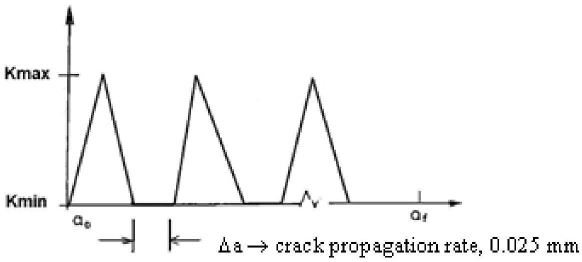

Figure 6: Numerical Crack Propagation Rate

Table I: Material Properties of a Low Alloy Steel

σ_{ys}	σ_u	E	δ	ν
MPa	MPa	MPa		
230	410	210000	0.21	0.3

Where: σ_{ys} = yield stress; σ_u = ultimate tensile stress; E = elastic modulus; δ = elongation; ν = Poisson's coefficient

The dimensions of the compact tension specimen were: B=3.8 mm; W= 50.0 mm; a/W= 0.26. Table II shows the estimated and used values of lower element size and effective plastic zone. The smaller element 0.025 mm, was estimated based on the plastic zone size ahead of the crack tip and computed by Irwin equation (2),

$$w = \frac{1}{\pi}\left(\frac{K_{max}}{\sigma_0}\right)^2 \text{, (plane stress)} \qquad (2)$$

Table II: Smaller Element of Model

	Plastic Zone (mm)	Smaller Element (mm)
Estimated	0.48	0.048
Used	0.10	0.025

The stress level in the crack tip, Fig. 7, must to be positive to characterize the crack opening and negative to characterize the crack closure. Antunes & Rodrigues [19] consider two basic points as criteria to determine the crack opening or closure:
– the first contact of the crack flank, which corresponds to the contact of the first node behind the current crack tip. This is the conventional definition proposed by Elber [20] and has been widely used by Jiang et al. [21]. However, results are mesh-dependent, since the proximity of the first node to the crack tip increases the opening load;
– the first contact of other nodes behind the crack tip. Pommier [22] and Roychowdhury and Dodds [23] considered the second node behind the crack tip. In this paper the released nodes in the crack tips were located at the minimum load of a load cycle to simulate crack growth and will be considered the first contact of other nodes behind the crack tip, positive stress (+Syy) to characterize the crack opening and negative stress (-Syy) to characterize the crack closure.

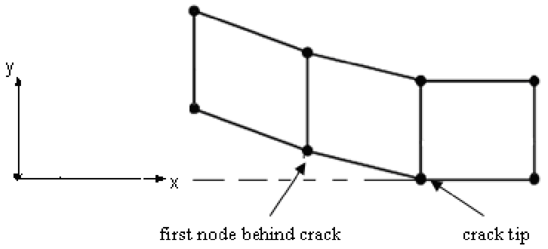

Figure 7: Criterion to Crack Opening and Closure

IV. RESULTS

Table III presents the numerical results to be compared with experimental data, where; K_{min} = Lower Stress Intensity Factor; K_{max} = Maximum Stress Intensity Factor; K_{op} = Crack Opening Stress Intensity Factor; K_{cl} = Crack Closure Stress Intensity Factor.

Table III: Numerical Crack Opening and Closure Data

Cycle N.o	K_{min} $MPa\sqrt{mm}$	K_{max} $MPa\sqrt{mm}$	K_{op} $MPa\sqrt{mm}$	K_{cl} $MPa\sqrt{mm}$
1	78	255	58	144
2	152	260	88	147
3	112	159	119	90
4	104	162	91	91
5	107	241	62	136
6	144	345	138	195
7	140	226	128	128
8	126	230	130	130
9	149	210	99	118
10	140	213	87	120

V. DISCUSSION OF RESULTS

In the present work it was very difficult to determine with good precision the crack opening or closure. It was necessary to use the iteration process in the crack surface step by step during loading and unloading to find the as Ricardo [24] crack opening or closing.

The retard effect is present in some cycles in special where there are overloads. In constant amplitude loading, the effective plastic zone increases with the extension of the crack length; the crack propagation rate has no influence in the quality of results, assuming that it is in respect to the Newman [25] recommendation to have four elements yielded in the reverse plastic zone. In variable amplitude loading the crack length can not progress until a new overload occurs or the energy spent during cyclic process creates a new plastic zone and the driving force increases the crack length. The researchers normally work with simple overloads or specific load blocks; this approach can induce some mistakes in terms of results that can be conservative or nonrealistic.

The methodology used in this work to simulate the crack propagation is the same as Ricardo [24] under constant amplitude loading, as shown in Fig. 8, providing good correlation between numerical and experimental data.

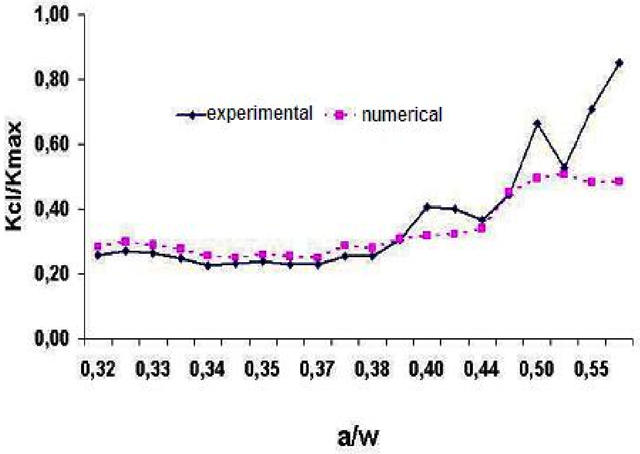

Figure 8: Correlation Numerical x Experimental Data

It is expected that the same level of correlation in the present work using the same crack propagation mode under variable amplitude loading approach, than Ricardo [24]. It will be necessary to test real crack propagation in specimens to validate the numerical results from the crack propagation model.

Of course there are many factors related with the crack propagation rate like reverse and effective plastic zones; that can calculated using Irwin expression as well as the numbers of the elements that must have yielded inside of the reverse plastic zone.

VI. CONCLUSION

In this work it was possible to identify the crack opening and closure using the finite element method. In the literature there are few works covering crack propagation simulation with random load histories like WISPER.

Normally just few load blocks are used to reduce the complexity; this should provide conservative answers when used to develop wind turbine components. The next step in this work in progress will be to perform the same model and load history with different crack propagation rates to identify whether or not the retard effect can be observed. These data will be compared with experimental test and, if necessary, adjustment of the crack propagation model will done to improve the crack propagation model.

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REFERENCES

[1] Miner, M. A., Cumulative Damage in Fatigue, Journal of Applied Mechanics, *ASME,* Vol.12, , USA, 1945 p. A159-A164.

[2] Schijve, J., Fatigue Crack Propagation in Light Alloy Sheet Material and Structures, *NLR*, Report MP195, Amsterdam, 1960

[3] Ditlevsen, O. & Sobczyk, K. ;Random Fatigue CrackGrowth with Retardation; *Eng. Fracture Mech.;*Vol.24; N.o 6, pp. 861-878, 1986

[4] Dugdale, D.S., Yielding of Steel Sheets Containing Slits, *Journal Mech. Phys. Solids*, pp. 100-104, n.0 8, USA, 1960

[5] Skorupa, M & Skorupa, A., Experimental Results and Predictions on Fatigue Crack Growth in Structural Steel, *Int. Journal of Fatigue*, 27, pp. 1016-1028, 2005

[6] Newman, J .C. *A;* Finite Element Analysis of Fatigue Crack Closure, *NASA TM X-72005*, National Aeronautics and Space Administration, Hampton,VA, 1975.

[7] Ricardo, L.C.H.; Pimenta, P.M. & Spinelli, D., “Crack Closure Simulation by Finite Element” In, Blom, A.F.; *Fatigue 2002*, Vol. 4/5, West Midlands, Emas, pp. 2293 – 2300, 2002

[8] McClung, R.C & Sehitoglu, H. On The Finite Element Analysis of Fatigue Crack Closure-2 Numerical Results, *Eng. Frac. Mechanics***,** Vol. 33, N.o 2, pp. 253-272, 1989

[9] Corbly, D.M. & Pacmasn, P.F. “ On The Influence of Single and Multiple Peak Overloads on Fatigue Crack Propagation in 7075-T6511 Aluminum” , *Eng. Fracture Mech.* 5, pp. 479-497, 1973

[10] Schijve, J. ; Brock, D. & de Rigle, P., Report M2094, Amsterdam, 1962

[11] Hardrarth, H.F. & McEvily, A . T., Proc. *Crack Propagation Symposium,* Vol. 1 Cranfield, 1961

[12] Jonds, D. & Wei, R.P. , An Exploratory Study of Dealy in Fatigue Crack Growth, *Eng. Fracture Mech.* 7,1971

[13] von Ewu, E. ; Hertzberg, R. & Roberts, R., Delay Defects in Fatigue Crack Propagation, *Nat. Symposium F.M.*; 7; 1971

[14] MSC/Patran r1, USA, 2008

[15] Abaqus version 68, USA, 2008

[16] ten Have, A.A. WISPER and WISPERX - Final definition of two standard fatigue loading sequences for wind turbine blades, *NLR TP 91476 U***,** 1991

[17] Genesis, Version 1.0, *NRL*, National Research Laboratory, Amsterdam, 2001

[18] Wei, L.W. & James, M.N., A Study of Fatigue Crack Closure in Polycarbonate CT Specimens, *Eng. Fracture Mechanics***,** Vol. 66, pp. 223-242, 2000

[19] F.V. Antunes, & Rodrigues, D.M, Numerical Simulation of Plasticity Induced Crack Closure: Identification and Discussion of Parameters, *Eng. Fracture Mech.*, Vol. 75, pp. 3101-3120, 2008

[20] Elber W. Fatigue crack closure under cyclic tension, *Eng Fracture Mech*., Vol.2, pp.37–45,1970

[21] Jiang Y, Feng M, Ding F., A re-examination of plasticity-induced crack closure in fatigue crack propagation, *Int. J Plasticity*,21, pp. 1720–40, 2005

[22] Pommier S. A study of the relationship between variable level fatigue crack growth and the cyclic constitutive behavior of steel. *Int J. Fatigue*,pp.111–118, 2001

[23] Roychowdhury S, Dodds Jr RH. A numerical investigation of 3D small-scale yielding fatigue crack growth. *Eng. Fracture Mech.*;Vol. 70, pp.2363–83, 2003

[24] Ricardo, L.C. H, Andrade, A.H.P.; Goldefroid, L.B. & Topper, T.H.; (2004) “Finite Element Method to Simulate Crack Closure Stress Intensity Factor” SAE Paper Number 2004- 01-2229, *II International SAE Fatigue Conference*, June 22-23, Sao Paulo, Brazil

[25] Newman , J.C. Jr., “The Merging of Fatigue and Fracture Mechanics Concepts, A Historical Perspective”, *Progress in Aerospace Science***s**, N.o 34, pp. 347-390, 1998