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Statistical Models for Nonstationary and Non-Gaussian Road Vehicle Vibrations

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***Abstract*—This paper presents some of the outcomes of a research project concerned with the development of a method for synthesizing, under controlled conditions in the laboratory, the random vibrations generated by road transport vehicles. Firstly, the paper deals with the development of a technique for decomposing non-stationary random vibration signals into constituent Gaussian elements. The hypothesis that random non-stationary vehicle vibrations are essentially composed of a sequence of zero-mean random Gaussian processes of varying standard deviations is tested and the paper reveals that the variations in the magnitude of the vibrations are the cause of the leptokurtic, non-Gaussian nature of the process. It is shown how non-stationary vibration signals can be systematically decomposed into these independent random Gaussian elements by means of a numerical curve-fitting procedure. The paper describes the development of the algorithm which is designed to automatically extract the parameters of each constituent Gaussian process namely the RMS level and the Vibration Dose. The validity of the Random Gaussian Sequence Decomposition (RGSD) method was tested using a set of road vehicle vibration records and was found to be capable of successfully extract the Gaussian estimates as well as the corresponding Vibration Doses. Validation was achieved by comparing the sum of these Gaussian estimates against the PDF of the original vibration record. All validation cases studied show that the RGSD algorithm is very successful in breaking-down non-stationary random vibration records into their constituent Gaussian processes. Secondly, the paper describes the development of a statistical model for characterising the nonstationarity of road vehicle vibrations. It shows that the Rayleigh and two-parameter Weibull distributions cannot be used to model the magnitude distribution. An alternative model, which is a modified form based on the Rayleigh and Weibull distributions, is introduced and is shown to offer good agreement with the statistical distribution of a broad range of experimental data.**

***Index Terms*—Gaussian vibrations, non-stationary vibrations, random vibrations, vehicle vibrations. Rayleigh, Weibull.**

I. Introduction

It is self-evident that the primary source of vertical vibrations generated by road vehicles can be attributed to the unevenness of pavement surfaces. When wheeled vehicles traverse irregular surfaces, the interaction between the vehicle and the terrain give rise to a dynamic process that produces complex forces and motions within the vehicle. Because pavement surface irregularities are generally random in nature, the resulting vehicle vibrations are also random. Furthermore, the levels of vibrations are not solely dependent on the pavement roughness but are also a function of vehicle type, payload and vehicle speed. The effect of these parameters tend make the complex mechanical interactions between the vehicle and pavement surface difficult to characterise and predict. It is therefore widely acknowledged that the analysis and synthesis of road-related vehicle vibrations demand some level of sophistication. As the importance and significance of optimising protective packaging designs intensifies, the need for closer and more accurate monitoring and understanding of hazards in the distribution environment increases.

Although vibrations generated by road vehicles have been thoroughly studied on numerous occasions, because of their inherent complexity, variability and unpredictability, there does not exist a definitive method to predict, analyse or synthesize them. There have been, however, a number of attempts in characterising some aspects of the process. By far the most common approach is to compute the average Power Spectral Density (PSD) of the vibrations. The technique is useful in many ways, such as identifying prevalent frequencies and the overall (RMS) vibration level, and is still widely used today to characterise ride quality. One major drawback of the average PSD is that it effectively describes the average energy level (in this case acceleration) for each frequency band within the spectrum. It does not contain information on time-variant parameters such as possible variations in amplitude or frequency or the time at which these variations occur. Furthermore, the temporal averaging process inherent to the PSD cannot separate the effects of transients within the signal. This is of no consequence if the process is both Gaussian and stationary. In such cases the nature of the signal is well defined by the normal distribution and its higher-order moments. However, as it has repeatedly been shown, road vehicle vibrations can often be significantly non-stationary and non Gaussian mainly due to variations in pavement roughness and vehicle speed [1][2]. One such example is illustrated in Fig. 1.

The main consequence of ignoring the non-Gaussian nature of vehicle vibrations becomes critical when the average PSD of the vibration sample is used to synthesize these vibrations using laboratory vibration generators. In such cases, the resulting synthesized vibrations are unavoidably Gaussian and,

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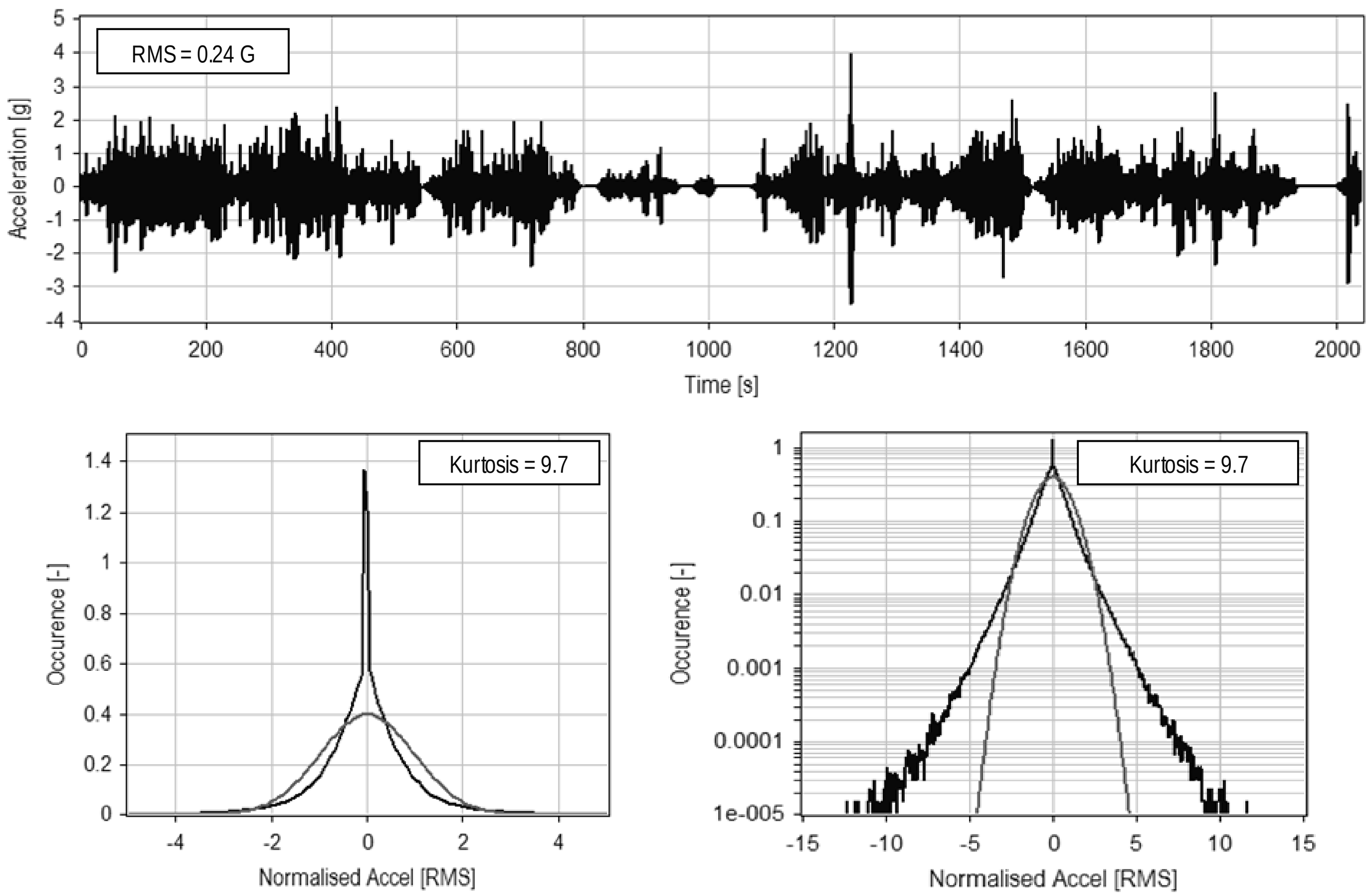


Figure 1. Example of the non-stationary, leptokurtic nature of road vehicle vibrations.

consequently, fail to reproduce the fluctuations in amplitudes that are inherent to the process.

One approach that is sometimes used to (partially) compensate for the amplitude non-stationary of vehicle vibrations is the peak-hold spectrum. In effect, instead of averaging the signal amplitude for each narrow frequency band, the peak-hold spectrum uses the largest amplitude within each frequency band. Typically, given a suitably large sample, the peak-hold spectrum is an amplified version of the average PSD and is often used to reveal the relationship between the mean and peak spectral values which are related by the crest factor. In reality, the interpretation of the peak-hold spectrum is difficult especially when the vibrations are time dependent (non-stationary) and contain transients. Further uncertainty arises due to the fact that statistical uncertainties between the average and peak-hold spectral density estimates are not consistent. In general, the use of peak-hold spectra for establishing the severity of vibration tests can lead to conservative results [1]. This is especially so if the process is highly non-stationary and the peak-hold spectral values are the results of severe but short-lived excursions in vibration levels. While such statistically unlikely events can dramatically distort the peak-hold spectrum, they have little or no effect on the average PSD.

A variant on the peak-hold spectrum is the method developed by the US Army at its Aberdeen Proving Ground for inclusion in its Mil Std 810D [3]. The analysis was based on vibration data collected from a range of road surface types and vehicle speeds. The data is analysed in 1 Hz frequency bands where both the mean RMS vibration value and one standard deviation are calculated. This method is advantageous over the peak-hold spectrum in that the statistical confidence of the mean and standard deviation is consistent [1]. The Aberdeen Proving Ground method, like the peak-hold method, is significantly affected when the vibrations are non-stationary [1].

An alternative approach, described by Murphy [4], involves the use of the rainflow count algorithm to determine the frequency of occurrence (amplitude density in cycles per mile) for a predetermined set of acceleration ranges (rainflow amplitude). Data, collected from a typical tractor-trailer travelling over a wide range of pavements including freeways, secondary roads and urban routes, were used to propose an exponential relationship between the amplitude density and acceleration range:

$$N = a\, e^{bx} \tag{1}$$

where N is the amplitude density in cycles per unit length, x is the acceleration range while *a* and *b* are empirical constants. This method does provide some information on the amplitude non-stationary of the vibrations and may be useful when used in conjunction with the PSD.

The non-stationary nature of road vehicle vibrations was discussed by Richards [1] who attributed it to variations in vehicle speed. He produced data showing the variations in RMS acceleration levels as a function of vehicle speed for a typical 40-minute journey. Richards also identified that the dynamic response of road vehicles contain both continuous (steady-state) and transient components. Richards [1] recognised the difficulties in identifying transients given that they occur at random intervals with large variations in amplitudes. It was also acknowledged that, although desirable for establishing test requirements, the separation of transients from the underlying vibrations is, in reality, arbitrary and almost always difficult to achieve.

Charles [5] was one among many to recognize that there existed problems relating to the interpretation of vertical vibration data from road vehicles for use to generate laboratory test specifications. He also acknowledged that wheeled vehicle vibrations are "unlikely to be stationary" due to variations in road surface quality and vehicle speed. He also showed that the statistical distribution of vehicle vibrations is more likely to contain larger extrema than that a true Gaussian process as illustrated in Fig. 2.

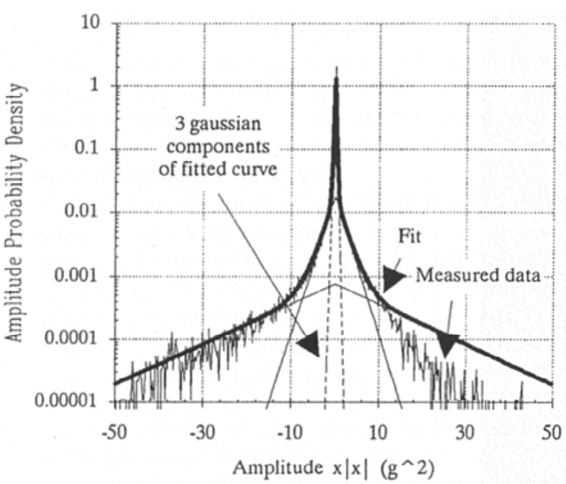


Figure 2. Illustration of the non-Gaussian nature of vehicle vibrations (after Charles [5]).

Charles [5], who studied a variety of road types, stated: "even for a good classified road, a whole range of surface irregularities may be encountered". He acknowledged that there exist difficulties associated with distinguishing shocks from vehicle vibrations. Charles [5] suggested that the analysis method for characterising non-stationary vehicle vibrations should include the identification of stationary sections using the "RMS time histogram (sic)" (presumably meaning time history), the examination of vibration severity in terms of RMS and peak amplitude as a function of vehicle speed and verification of the normality of the data by computing the amplitude probability analysis.

Despite the manifest non-stationarity and non Gaussian nature of road vehicle shocks and vibrations, they are not taken into account by most analysis methods in use today. However, more recently, attempts have been made to account for the non Gaussian characteristics of vehicle vibrations by applying a non-linear transformation to a Gaussian function by means of a Hermite polynomial thus enabling control of the skewness and kurtosis parameters [6,7,8]. The main limitation of this technique is that it fails to recognise that the primary cause of the leptokurtic nature of road vehicles vibrations is the result of the non-stationarity of the process rather than an inherent non-Gaussian character. Consequently, it does not succeed in reproducing the variations in the processes' amplitude that are considered essential if realistic simulations are to be achieved.

This paper builds on Charles' proposition that non-stationary random vehicle vibrations consist of Gaussian segments and introduces a method by which measured and numerically-simulated road vehicle vibration data can be decomposed into its constituent Gaussian components.

II. RANDOM GAUSSIAN SEQUENCE DECOMPOSITION

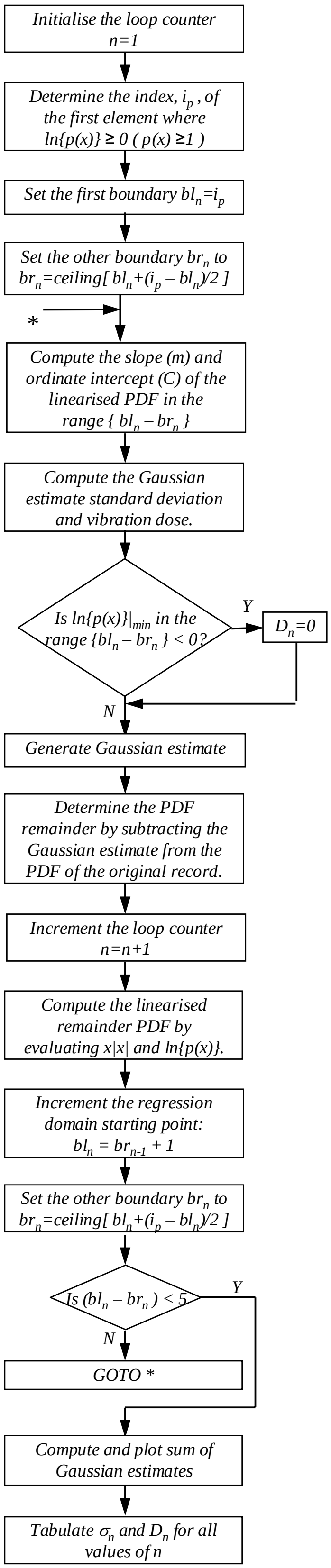
The Probability Distribution Function of a signal composed of a sequence of random Gaussian processes can be expressed as the sum of the individual distribution functions each weighted by, what will here be termed, the Vibration Dose. The Vibration Dose effectively describes the time fraction for which a Gaussian process of a particular standard deviation exists. The decomposition method described here relies on the fact that the distribution function of a sequence of zero-mean Gaussian processes can be described intrinsically as a function of two parameters, namely the vibration dose, D_i, and the standard deviation, σ_i, as follows:

$$\sum ln\left\{p\left(x\right)\right\}=\sum_{i=1}^{n} ln\left\{\frac{D_i}{\sqrt{2\pi}\sigma_i}\right\}-\frac{1}{2\sigma_i^2}x\left|x\right| \tag{2}$$

In this form the function produces a linear relationship between $x|x|$ and $ln\{p(x)\}$ represented by the slope $-1/2\sigma^2$ and the ordinate intercept $ln\{D/\sqrt{2\pi}\,\sigma\}$. This shows that the distribution parameters of a Gaussian process can be determined by fitting a straight line through one half (or side) the distribution estimates to obtain the Vibration Dose and standard deviation as follows:

$$\sigma_i=\sqrt{\frac{1}{2m_i}} \quad and \quad D_i=\sqrt{2\pi}\cdot\left(-\frac{1}{2m_i}\right)\cdot exp(C_i) \tag{3}$$

Where m_i and C_i are, respectively, the ith slope and ordinate intercept of the linear regression fit.



Algorithm commentary

The PDF of the record is computed between limits determined from the absolute minimum and maximum of the entire vibration record. This section of the algorithm deals with determining the optimum region at the 'tail' of the distribution function to determine the first Gaussian estimate which, by design, will represent the Gaussian element with the largest standard deviation.

First the element containing the maximum, $p(x)_{max}$ is identified. Then, elements within the distribution function which contain '-inf' values are detected. These represent elements where $p(x) \to 0$. The very adjacent element toward the centre of the distribution (the mean) is identified and represents the ultimate boundary of the distribution function, bl_n. The other boundary of the region, br_n, is defined as half way between bln and the distribution peak element, i_p. This coefficient of ½ was arrived at by experimentation and was found to yield the most consistent and accurate results. The coordinates ln{p(x)} and x|x| within the domain { bl_n - br_n } are used as the first set of values to determine the parameters of the first Gaussian estimate. This is best illustrated in Fig. 4.

Linear regression by the method of least squares is used to determine the line of best fit through the data and estimates of the slope and ordinate intercept. The Gaussian estimate's standard deviation (σ_n) (or RMS for a zero-mean process) and vibration dose (D_n) are computed from estimates of the slope and ordinate intercept. A numerical vector for the Gaussian estimate is generated for the entire range (shown in Fig. 5).

The difference between the original PDF and the Gaussian estimate is computed and used to fit the next Gaussian estimate. The PDF remainder is linearised and any negative value is truncated to zero as a necessity for computing the natural log.

The next range for regression is established by moving the starting point (left boundary, bl_n) by one and computing the end point (the right boundary, br_n) as half way between bln and the PDF peak i_p. If the number of elements in the range {bl_n - br_n} is less than 5, it is deemed that there is no longer a sufficient number of points to accurately extract a line of best fit by regression. The programme is terminated and the results displayed as shown in Fig. 6.

Figure 3. Random Gaussian Sequence Decomposition algorithm flow chart.

The challenge in developing an automated algorithm to extract a number of Gaussian parameters from a non-Gaussian distribution are related to the data range (or boundary) for each Gaussian element, determining a suitable number of Gaussian elements in the sequence and the effect of fluctuation in the distribution estimates, especially in the high standard deviation, low count region. A description of the algorithm developed in this study is given in Fig. 3 along with illustrations of its operation and the results it generates.

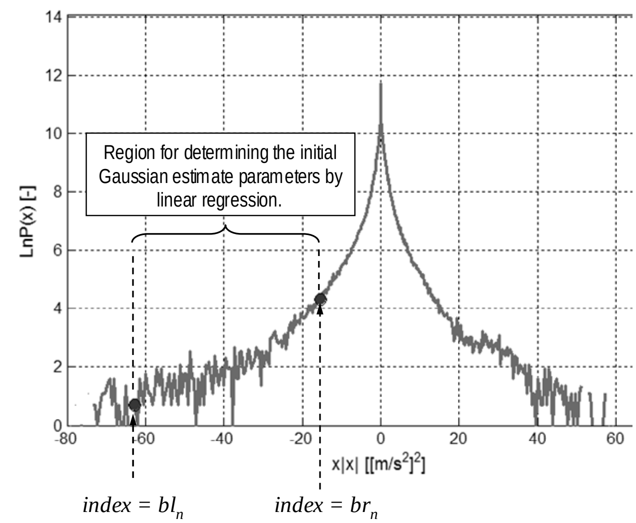


Figure 4. Identification of the region to determine the initial Gaussian estimate parameters by linear regression

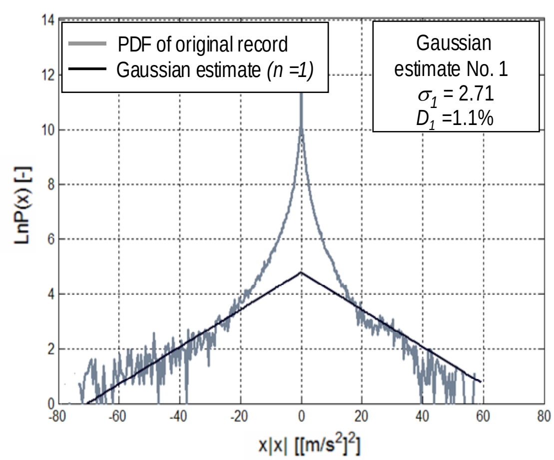


Figure 5. First Gaussian estimate along with PDF of original record. Note the relatively small dose that makes the estimate difficult to distinguish in linear scales.

The validity of the Random Gaussian Sequence Decomposition (RGSD) method was tested using a broad set of typical of vibration records collected from a variety of vehicle types, routes and payload conditions (Table I). In addition, the vertical acceleration responses of various linear quarter-car numerical models, made available in the literature, were computed for a range of pavement profiles a (Table II) to supplement and complement the collection of measured vibration records. Although it is acknowledged that the rudimentary nature of the simulation models (only linear elements were used) produces vibration estimates that are not necessarily accurate, the simulation is sufficiently realistic to reproduce the random non-stationarities that occur in reality and are, therefore, deemed adequate to the purpose of this study. The simulation was carried out with a purposed-design program coded in Matlab® and Simulink®. The boundary conditions were accounted for by introducing a vehicle velocity ramp at a constant forward acceleration until the target cruise speed was reached. The vertical vibrations of the quarter-car model were then computed at constant vehicle velocity for the entire pavement profile.

The Random Gaussian Sequence Decomposition method was found to be capable of successfully extracting the Gaussian estimates as well as the corresponding vibration doses for every single test record. Validation was achieved by comparing the sum of these Gaussian estimates against the PDF of the original vibration record. A number of typical results are presented the appendix.

Table I. Summary of measured vibration record parameters.

ID	Vehicle type & load	Route Type
MA	Utility vehicle (1 Tonne capacity). Load: < 5% cap.	S'urban streets
MB	Prime mover + Semi trailer (Air ride susp.). Load: 90% cap.	Country roads
MC	Transport van (700 kg capacity). Load: 60% cap.	Suburban streets
MD	Transport van (700 kg capacity). Load: 60% cap.	Suburban hwy.
ME	Transport van (700 kg capacity). Load: 60% cap.	Motorway
MF	Prime mover + Semi trailer (leaf spring susp.). Load: < 5% cap.	Country roads
MG	Tipper truck (16 Tonnes capacity, Air ride susp.). Load: 25% capacity.	Country roads
MH	Small flat bet truck (1 Tonne capacity, leaf spring susp.). Load <5% cap.	Suburban streets
MJ	Flat bed truck (5 Tonnes capacity, leaf spring susp.). Load >95% cap.	Country roads
MK	Sedan car. Load: 1 passenger	Suburban

Table II. Summary of routes used for numerically-generated vibration records.

ID	Route (Victoria, Australia)
SA	Murray Valley Highway (Major county road)
SB	Bendigo – Maryborough road (Major county road)
SC	Princess Highway (Freeway)
SD	Timboon Road, Victoria, Australia (Major county road)
SE	Road sequence: Timboon Road – Princess Hwy and Murray Valley Hwy, Victoria, Aust.

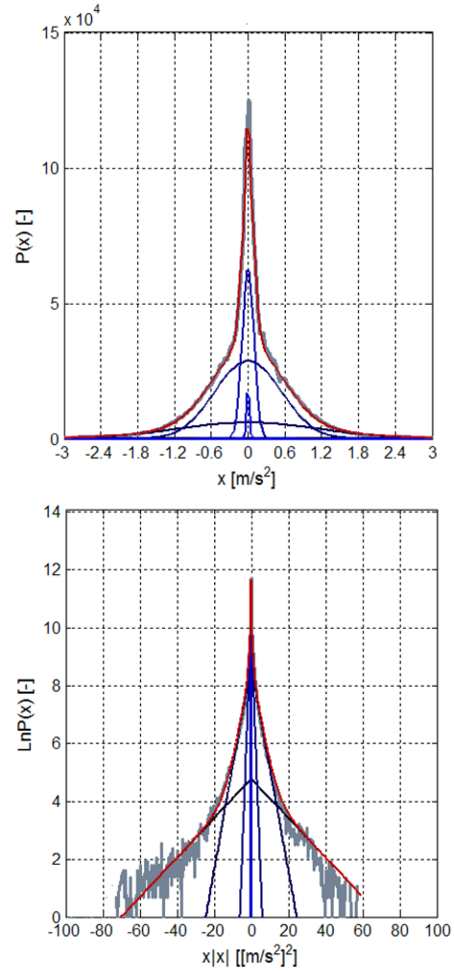


Figure 6. Typical plot of the decomposed Gaussian estimates (blue lines) along with the sum of the Gaussian estimates (red line) and the PDF of the original record.

III. Statistical distribution of magnitude

One useful approach for revealing the overall level-type nonstationary character of random vibrations is to compute the probability density function (PDF) of the instantaneous magnitude of the random vibrations obtained by means of the Hilbert Transform. Analysis of vehicle vibration samples, representing a wide range of pavement conditions and vehicle types, have revealed a similarity between the statistical distribution of the vibration magnitude and the Rayleigh distribution. However, there exists a consistent, and not altogether unexpected, deviation from the Rayleigh distribution as illustrated by the examples in Fig. 7 which shows the distribution of the vibration magnitude for a number of typical vehicles and pavement conditions. The non-conformance with the Rayleigh distribution is clearly evident and has been found to be representative of the process in general.

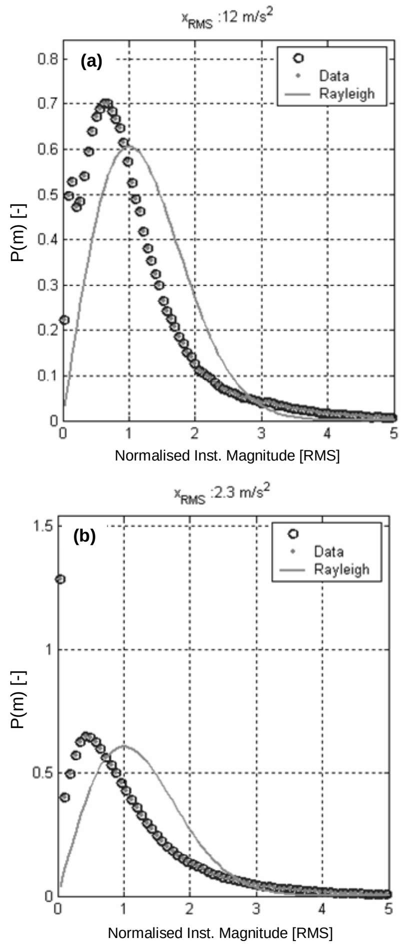


Figure 7. Typical normalised vibration magnitude probability density estimates along with the best-fitting Rayleigh distribution Data source: a) MA, b) MB.

These deviations from the Rayleigh distribution can be explained since, fundamentally, the characterisation of peaks or magnitude with the Rayleigh Distribution (Eqn. 4) relies on the signal being Gaussian as well as narrow-banded which is defined as peaks occurring above zero and troughs below zero (spectral width parameter, $\varepsilon = 0$) [6]. While it is reasonable to assume that, in the main, the vertical vibrations of road vehicle are very nearly narrow banded, it is often the case that the statistical distributions of road vehicle vibrations are non-Gaussian and are characterised by high kurtosis values.

$$P(m) = \frac{m}{\sigma^2} \cdot exp\left\{ -\frac{1}{2} \cdot \left(\frac{m}{\sigma} \right)^2 \right\} \quad 0 \le m \le \infty \qquad (4)$$

Where m is the instantaneous vibration magnitude and σ is the standard deviation.

Due to the strong prevalence of the Gaussian (hence stationary) assumption with respect to vehicle vibrations, little information is available regarding better-suited probability density functions for vibration magnitudes on non-stationary processes. It has been suggested by Nigam [7] and Newland [6] that the two-parameter Weibull distribution, given in Eqn. 5, (of which the Rayleigh distribution is a special case) may be suited to characterizing non-stationary random processes. Two examples given by Newland [6] are for the circulation of wave-induced bending moments in ships, proposed by Mansour [8] and for the wind loading of buildings proposed by Melbourne [9, 10].

$$P(m) = \alpha\, \beta^{-\alpha}\, m^{(\alpha - 1)} \cdot exp\left\{ -\left(\frac{m}{\beta} \right)^{\alpha} \right\} \quad 0 \le m \le \infty \qquad (5)$$

Where m is the instantaneous vibration magnitude, α is referred to as the shape factor and β scale parameter.

Numerous validation tests against the entire set of vibration record samples collected in this study have shown that, although the Weibull distribution offers a significant improvement on the Rayleigh distribution, there are too may instances in which adequate correlation is not achieved across the entire vibration magnitude range. Further analysis has led toward the development of three alternative forms of the Rayleigh distribution (Eqns. 6 (a), (b) and (c)) which afford some control on the shape of the distribution. The validity of all three models was thoroughly tested against the available data and was found to offer various level of improvement on the Weibull distribution.

$$P(m) = \frac{m}{(\beta\sigma)^2} \cdot exp\left\{ -\frac{1}{2} \cdot \left(\frac{m}{\sigma} \right)^{\alpha} \right\} \quad 0 \le m \le \infty \qquad (6(a))$$

$$P(m) = \frac{m}{(\beta\sigma)^2} \cdot exp\left\{ -\frac{1}{2} \cdot \left(\frac{m}{\beta\sigma} \right)^{\alpha} \right\} \quad 0 \le m \le \infty \qquad (6(b))$$

$$P(m) = \frac{m}{(\beta\sigma)^2} \cdot exp\left\{ -\frac{1}{2} \cdot \left(\frac{m}{\gamma\sigma} \right)^{\alpha} \right\} \quad 0 \le m \le \infty \qquad (6(c))$$

Where α is the exponent parameter and β and γ are the scale parameters. Note that the two-parameter Modified Rayleigh distribution (Eqn. 6(b)) is a special case of the three-parameter function (Eqn.6(c)) when the scale parameters are equal ($\beta = \gamma$). Fig. 8 shows how the parameters α, β and γ influence the shape of the two and three-parameter Modified Rayleigh distribution functions (Eqn. 8(b) and (c)). The exponent α alters the slope of the right-hand (tail) of the curve while having little effect on the left-hand portion of the curve. The scale parameter β alters the overall width of the function by effectively scaling the standard deviation. Consequently, the value of β also determines the location of the peak of the distribution function. The second scale parameter γ does not alter the shape of the distribution function but merely changes the overall scale of the curve.

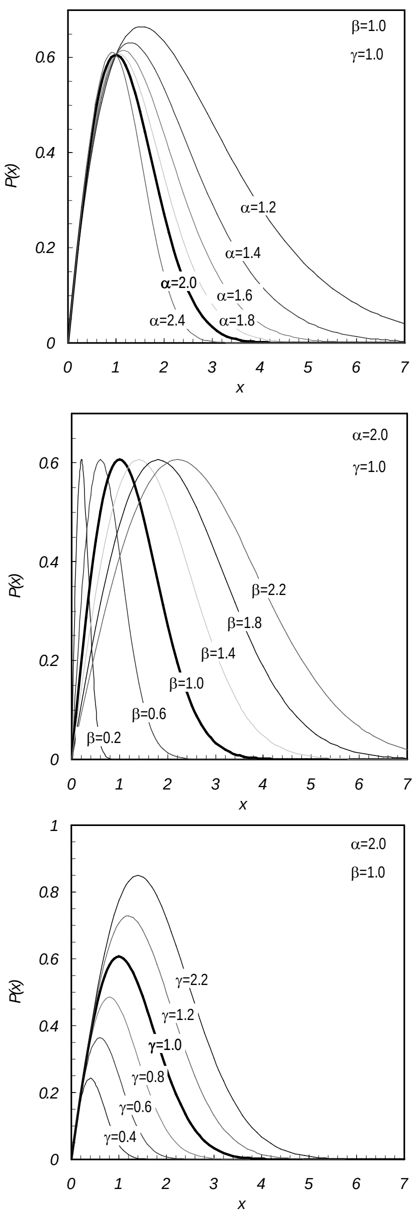


Figure 8. Effect of the exponent parameter α (top) and the scale parameters β (centre) and γ (bottom) on the overall shape of the two and three-parameter Modified Rayleigh distribution functions. (Thick dark line represents the Rayleigh distribution).

These various statistical models were compared with the probability density of the instantaneous magnitude of recorded vibration data by means by means of nonlinear least-squares regression using the Gauss-Newton method. Fig. 9 shows a typical example of the suitability of the various statistical models when compared to the probability density distribution of measured vibration records. Further cases are included in the appendix. These results present strong evidence that, in all cases presented here, the Weibull distribution fails to adequately compensate for the discrepancies between the probability density of actual non-stationary vibration realizations and the Rayleigh distribution. The same applied for the single-parameter Modified Rayleigh distribution (Eqn. 6(a)). In most cases - all but four of those analysed - (see table III) the two-parameter Modified Rayleigh distribution offers a very acceptable alternative.

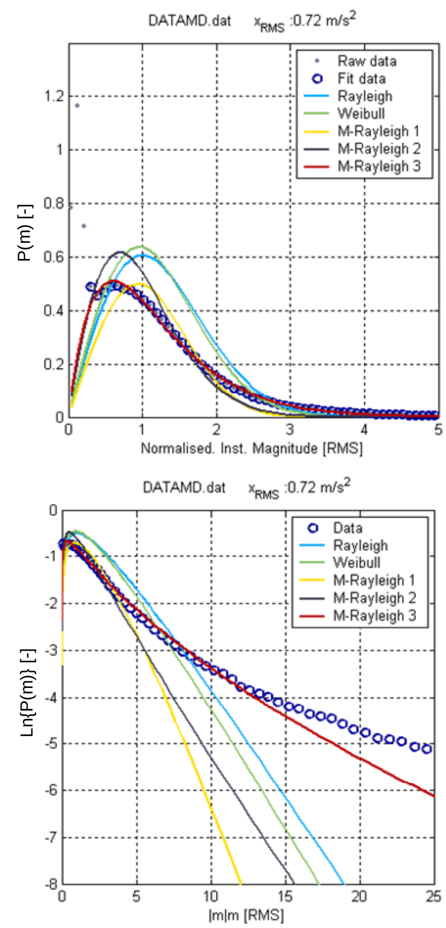


Figure 9. Typical normalised vibration magnitude distribution along with various statistical models.

It can be seen from Fig. 9 that the three-parameter modified Rayleigh distribution generally and consistently offers the best agreement with the instantaneous magnitude PDF of a wide range of vibration realizations. The main discrepancies generally occur at high levels of vibration magnitude as illustrated in the logarithmic plot. Ultimately, the three-parameter model (Eqn. 6(c)) was found to offer the best agreement over the widest range of vibration realizations. This was mainly attributed to the fact that three independent parameters are used. It must be noted that, at this stage, the exponent and scale parameters used in both the two-parameter and three-parameter Modified Rayleigh models (Eqn. 6(b) and (c)) have no direct physical significance to the process itself. Despite this, these modified Rayleigh distributions allow the formation of models that better represent the statistical nature of the non-Gaussian random vehicle vibrations for a wide range of cases. The main relevance of these models is that it enables the characterisation of the overall variation in the instantaneous vibration magnitude of a large variety of vibration records with a small number (no more than three) of statistical parameters.

Table III. Results of regression analysis for two and three-parameter Modified Rayleigh model.

	2-parameter model		Three-parameter model					2-P vs 3-P
Vibration record	β_2	α_2	β_3	γ_3	α_3	$\beta_3{:}\gamma_3$	Mean $(\beta_3+\gamma_3)/2$	Visual correlation
Data MA	0.362	1.167	0.386	0.403	1.246	4.4	0.39	Good
Data MB	0.295	1.113	0.198	0.144	0.816	27.3	0.17	Poor
Data MC	0.631	1.570	0.521	0.400	1.125	23.2	0.46	Poor
Data MD	0.593	1.573	0.447	0.305	1.026	31.8	0.38	Poor
Data ME	0.502	1.358	0.418	0.362	1.108	13.4	0.39	Good
Data MF	0.321	1.097	0.311	0.303	1.064	2.6	0.31	Good
Data MG	0.411	1.201	0.404	0.398	1.178	1.5	0.40	Good
Data MH	0.454	1.330	0.307	0.206	0.877	32.9	0.26	Poor
Data MK	0.268	1.034	0.237	0.212	0.943	10.5	0.22	Good
Data SA 1 100	0.227	0.982	0.205	0.188	0.905	8.3	0.20	Good
Data SA 2 100	0.406	1.203	0.508	0.612	1.663	20.5	0.56	Poor
Data SA 3 100	0.217	0.965	0.240	0.261	1.054	8.8	0.25	Good
Data SA 4 100	0.283	1.049	0.310	0.333	1.144	7.4	0.32	Good
Data SB 1 120	0.252	1.008	0.283	0.309	1.118	9.2	0.30	Good
Data SB 2 120	0.791	1.728	0.807	0.874	1.975	8.3	0.84	Good
Data SB 3 120	0.295	1.062	0.367	0.429	1.326	16.9	0.40	Good
Data SB 4 120	0.354	1.133	0.422	0.485	1.390	14.9	0.45	Good
Data SC 1 80	0.345	1.127	0.402	0.457	1.352	13.7	0.43	Good
Data SC 2 80	0.295	1.068	0.325	0.354	1.187	8.9	0.34	Good
Data SC 3 80	0.226	0.981	0.289	0.348	1.236	20.4	0.32	Good
Data SC 4 80	0.240	0.996	0.291	0.338	1.193	16.2	0.31	Good
Rayleigh	1	2	1	1	2			

Table III lists the extracted exponent and scale parameters from the two and three-parameter regression analysis for all the analysed vibration records. An interesting outcome of these results is the relationship between the exponent parameter, α_2 and the scale parameter β_2 for the two-parameter Modified Rayleigh model as shown in Fig. 10.

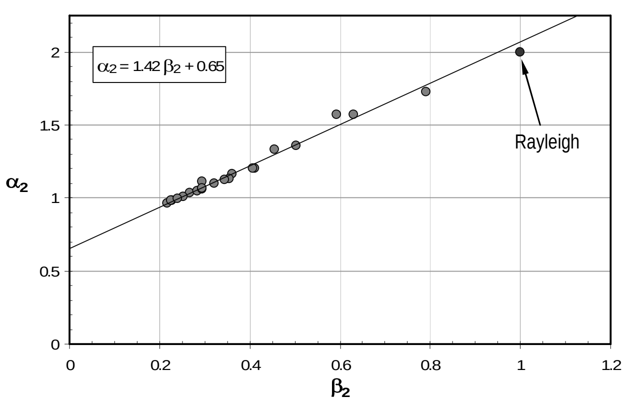


Figure 10. Relationship between exponent and scale parameter for the two-parameter Modified Rayleigh probability density distribution model.

The strong linear relationship revealed here is somewhat astonishing given that the vibration data collected consists of both measured and numerically simulated vibrations using a wide variety of vehicle and pavement surface types not to mention the uncontrolled (set by ambient traffic conditions) vehicle speed for measured records. This outcome indicates that, in many cases, there exists a generic single-parameter statistical model that accurately describes the probability density distribution of a broad range of vehicle vibration realizations. The obvious benefit of this outcome is that, for a significant portion of vibration events (16 out of the 21 used here) the non-stationarity of the random vibrations can be characterised by a single statistical parameter.

Further analysis of these two-parameter models that produce good agreement with the PDF of actual data shows that there does not appear to be any relation between the type of data (measured or simulated) and vehicle type. In other words, although α_2 and β_2 remain well correlated throughout, the values of α_2 and β_2 do not appear to represent a special kind or magnitude of non-stationarity as shown in Fig. 11. Establishing such a relationship may very possibly require many more independent and varied vibration records to be collected and analysed.

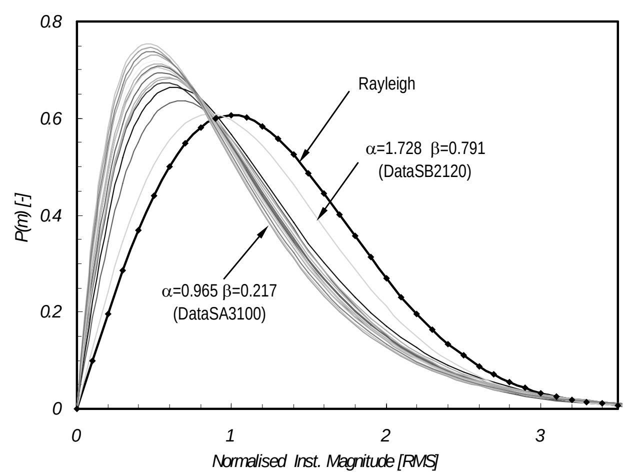


Figure 11. Two-parameter Modified Rayleigh PDF models for 16 vibration records where good agreement was achieved.

As for the two-parameter Modified Rayleigh model, some interesting relationships exist between the exponent parameter α_3 and the scale parameters β_3 and γ_3 for the three-parameter Modified Rayleigh model. Firstly, although the relationship between the scale parameters β_3, γ_3 and the exponent parameter α_3 is approximately linear, a significant amount of scatter is present as shown in Fig. 12. However, it is interesting to note that the arithmetic average of the scale parameters ($\overline{\beta_3 : \gamma_3}$) yields a stronger linear relationship with α_3 in a similar, albeit less well-defined, fashion as is the case with the two-parameter model. Nevertheless, this correlation remains remarkable given the variety of the data used.

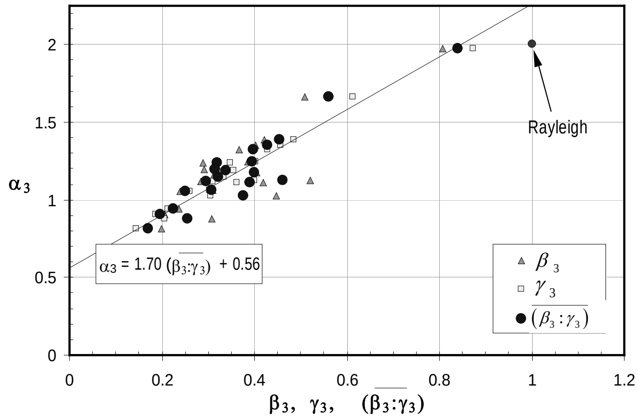


Figure 12. Relationship between exponent and scale parameters for the three-parameter Modified Rayleigh probability density distribution model.

Also of interest is the relationship between the scale parameters β_3 and γ_3 as shown in Figure 6.21. These results graphically reveal cases where the three-parameter model is well approximated by the two-parameter model represented by data points lying close to the unity ratio line (dashed blue line). Also shown here is the effect of computing the arithmetic mean of the scale parameters thus enabling the reduction of the three-parameter model to a two-parameter model by replacing β_3 and γ_3 with ($\overline{\beta_3 : \gamma_3}$). These relationships for the three-parameter model are clearly less robust than that of the two-parameter model. This is not entirely unexpected especially in those cases where there is a departure in agreement between the models.

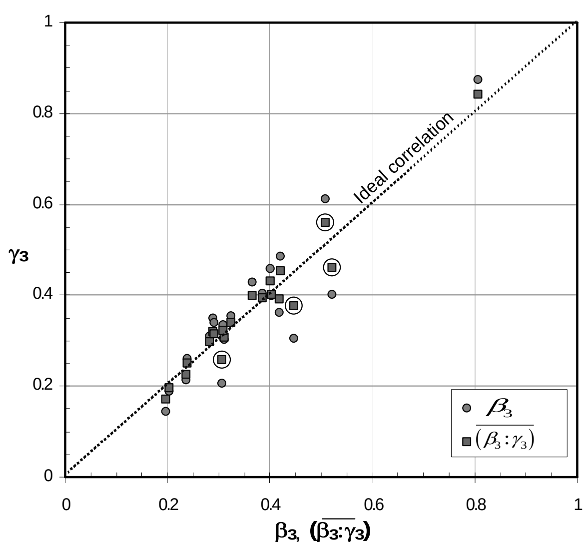


Figure 13. Relationship between scale parameters for the three-parameter Modified Rayleigh probability density distribution model. Circled squares indicate cases where the inequality is large.

These five cases where the PDF of actual vibrations records cannot be adequately represented by the two-parameter Modified Rayleigh model are shown in Fig. 14 which, when compared with Fig. 11, exhibits obvious differences. Again, there appears to be no specific reason for the fact that the PDF of these particular five vibration records can only be adequately represented by the three-parameter Modified Rayleigh model. As previously suggested, this may be the subject of further research aimed at correlating the statistical parameters with any particular feature in the type of non-stationarity within the vibration signal.

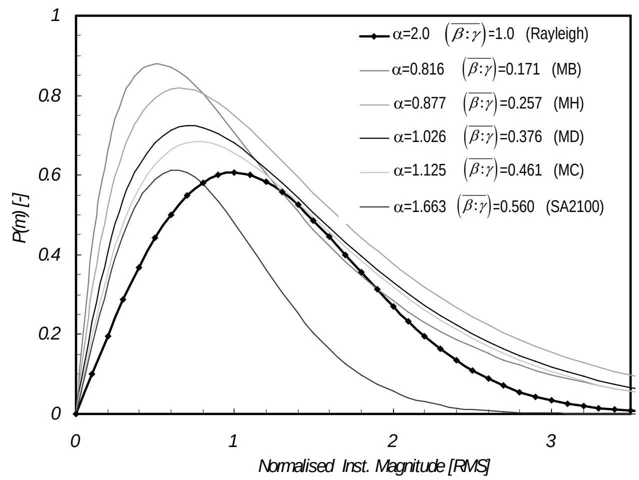


Figure 14. Three-parameter Modified Rayleigh PDF models for 5 vibration records where poor agreement was achieved with the two-parameter model.

IV. Conclusions

The paper has introduced a novel approach for the characterisation of nonstationary, non-Gaussian random vibrations generated by road vehicles. A Random Gaussian Sequence Decomposition algorithm which automatically extracts the parameters of each constituent Gaussian process, namely the RMS level and the Vibration Dose, was successfully developed and was found to be very effective in characterising non-stationary random vehicle vibrations in such a way as to facilitate laboratory synthesis. The development of a method for characterising the non-stationarity of random vibrations by means of the statistical distribution of the instantaneous magnitude produced positive and useful results. A two-parameter distribution function based on a combination of Rayleigh and Weibull models was shown to offer acceptable agreement with a wide range of experimental data and can be used quite effectively to provide some characterisation of the nature of non-stationary wheeled vehicle vibrations.

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APPENDIX

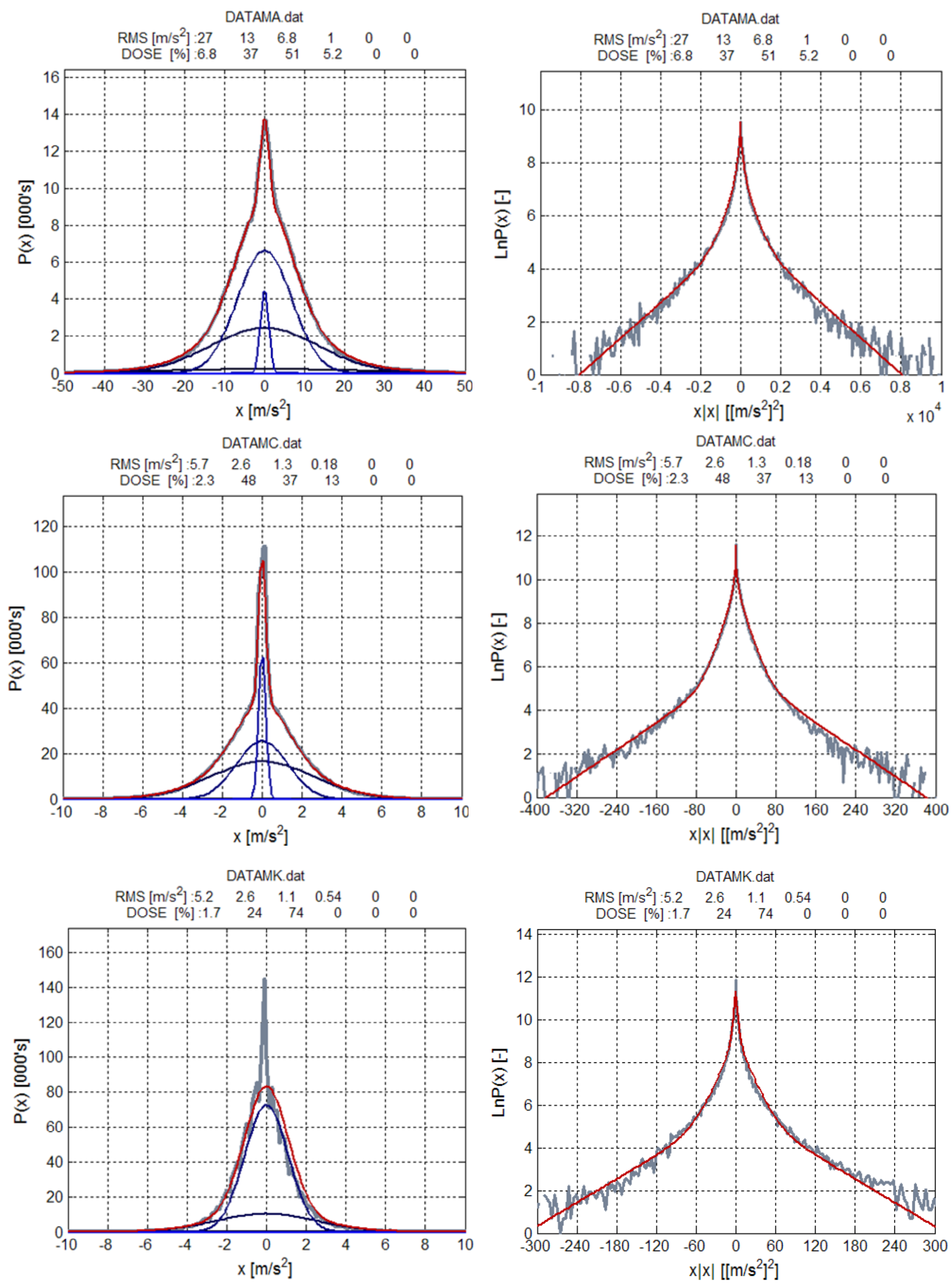

Figure A1. Validation results for the Random Gaussian Sequence Decomposition method for a range of typical vibration records. (Red line: sum of Gaussian segments, Grey: PDF of actual record)

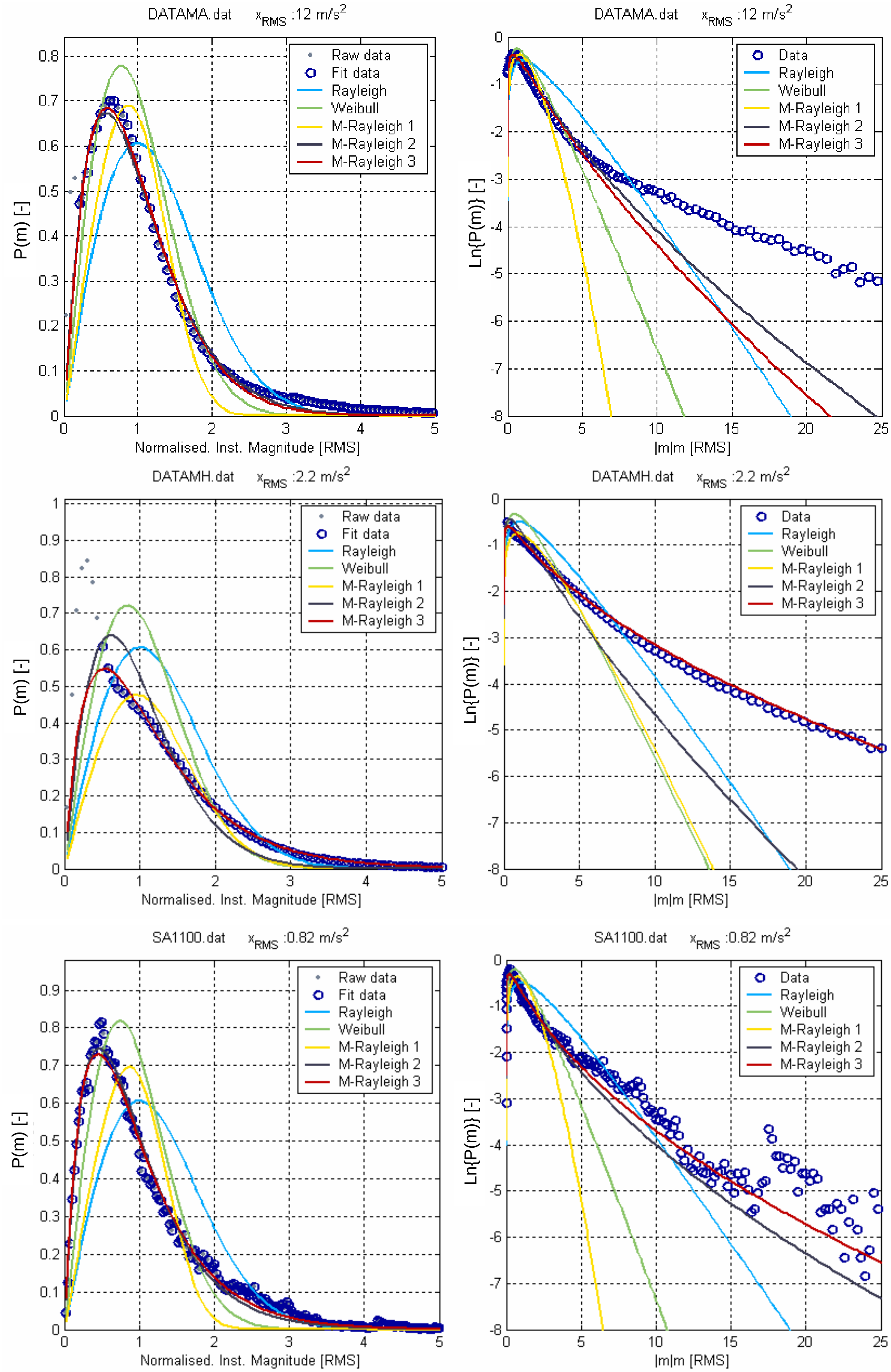

Figure A2. Validation results for the PDF models for a range of typical vibration records.