Averaged Model of a Buck Converter for Efficiency Analysis

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Abstract—In this work a buck converter model for multi-domain simulations is proposed and compared with a state-of-the-art buck converter model. In the proposed model no switching events are calculated. By avoiding the computation of the switching events in power electronic models the processing time of multi-domain simulations can be decreased significantly. The proposed model calculates any operation point of the buck converter in continuous inductor current conduction mode (CICM) while considering the conduction losses and switching losses. It is possible to utilize the proposed modeling approach also for other dc-to-dc converter topologies. Laboratory test results for the validation of the proposed model are included.

Index Terms—simulation, DC-DC power conversion, losses

I. INTRODUCTION

For the efficient utilization of multi-domain simulation software it is of high importance to have fast simulation models of power electronic components on hand. Especially in simulations of vast and complex electromechanical systems (e.g. power trains of hybrid electric vehicles [1] or drive systems in processing plants [2]) it is crucial to limit the processing effort to a minimum. Many times such electromechanical systems contain power electronic subsystems such as rectifiers, inverters, dc-to-dc converters, balancing systems (for energy sources), etc. When simulating these power electronic devices together with the other electrical and mechanical components of the application, computing the quantities of the power electronic models requires a large share of the available processing power if switching events are calculated in the power electronic models. Simulation models including power electronic devices with switching frequencies around 100 kHz require at least four calculation points within simulation times of around 10 µs for calculating the switching events. However, if the energy flow in an electromechanical system has to be investigated by simulation it is not necessary to calculate the switching events in the power electronic model as long as the relevant losses are considered.

In this work two different buck converter models are described. The first model, model A, which is state-of-the-art describes the behavior of a conventional buck converter, as shown in fig. 1, including the calculation of switching events. This means that in model A the switching of the semiconductors in the circuit is implemented with if-clauses. Therefore, model A directly calculates the ripple of the current through the storage inductor, as shown in the upper diagram of fig. 2 and the ripple of the voltage across the buffer capacitor. (Because of the large capacitance the ripple of the voltage across the buffer capacitor is too small to be noticed in the lower diagram of fig. 2.) Due to the if-clauses in model A the duration of the computing time is very high.

The second model in this work, indicated as model B, describes the behavior of the buck converter without calculating the switching events with if-clauses. Only the mean and RMS values of the voltages and currents are calculated. Therefore, the computation times of model B are significantly shorter than the computation times of model A.

In both models the conduction losses are considered by an ohmic resistance of the storage inductor, the knee voltage and the on-resistance of the diode, and the on-resistance of the MOSFET. Linear temperature dependence is implemented for the ohmic resistances of the storage inductor, the knee voltage and the on-resistance of the diode and the on-resistance of the MOSFET in both buck converter models.

The switching losses are calculated assuming a linear dependency on the switching frequency, the blocking voltage and the commutating current between the MOSFET and the diode. A controlled current source connected to the positive...
and the negative pin of the supply side of the buck converter is used to model the switching losses. This current source assures that the energy balance between the supply side and the load side of the buck converter is guaranteed.

II. THE MODEL FOR CALCULATING SWITCHING EVENTS

If the buck converter circuit in fig.1 is operated in continuous inductor current conduction mode (CICM) the circuit can be in two different states. As long as the MOSFET S is on and the diode D blocks the current, the buck converter is in state 1. The corresponding equivalent circuit of the buck converter in state 1 is shown in fig.3. \( v_{in} \) is the input voltage and \( v_{out} \) is the output voltage of the converter. \( R_S \) indicates the on-resistance of the MOSFET and \( i_D \) denotes the current through the MOSFET. \( R_L \) represents the ohmic contribution of the storage inductor \( L \) and \( i_L \) is the current through \( L \). \( C \) stands for the buffer capacitor, \( i_{load} \) indicates the output current of the converter and \( i_D \) represents the current through the diode (in state 1, \( i_D = 0 \)).

After \( S \) switched from on to off, the diode begins to conduct. If \( S \) is off and \( D \) conducts the circuit is in state 2. The corresponding equivalent circuit of the buck converter in state 2 is shown in fig.4. \( R_D \) is the on-resistance and \( V_D \) represents the knee voltage of the diode. In fig.2 the waveforms of the buck converter operating in CICM with a duty cycle of \( \frac{1}{3} \) are shown. In the inductor current and the inductor voltage waveforms the switching between the two different states of the buck converter can be seen. If the inductor voltage is positive and the inductor current increases, the buck converter is in state 1 and if the inductor voltage is negative and the inductor current decreases, the buck converter is in state 2.

Discontinuous conduction mode could be considered in a third state where \( S \) and \( D \) are open at the same time. The buck converter is in discontinuous conduction mode if \( S \) is open and the current passing through the diode becomes zero.

A buck converter model for calculating switching events can be implemented according to the pseudo code given in alg.1 where \( d \) stands for the duty cycle, \( f_s \) represents the switching frequency, and \( t \) indicates the time. \( s_{control} \) is the Boolean control signal of the MOSFET, is true during

\[
t_{on} = d T_s \tag{1}
\]

and false during

\[
t_{off} = (1 - d) T_s \tag{2}
\]
in a switching period \( T_s = \frac{1}{f_s} \). In alg.1 only CICM is considered. However, it is easy to modify the model so that discontinuous conduction mode can be simulated as well.

The basic principle of the modeling approach described in alg.1 is used in many state-of-the-art simulation tools. A disadvantage of such a model is the processing effort that is caused by the if-clauses. Strictly speaking, the whole set of equations describing the circuit changes whenever the converter switches from state 1 to state 2 and vice versa. In such a model the relevant conduction losses are considered inherently. For the consideration of the switching losses a model expansion as described in section V is necessary.

III. THE AVERAGED MODEL

If the dynamic behavior of the buck converter is not of interest but the energy flow needs to be investigated it is possible to model the buck converter without calculating the switching events. Assuming the buck converter is in steady state the integral of the inductor voltage \( v_L \) over one switching period \( T_s \) equals zero [3]. Hence,

\[
\int_0^{T_s} v_L \, dt = \int_0^{t_{on}} v_L \, dt + \int_{t_{on}}^{t_{off}} v_L \, dt = 0. \tag{3}
\]

During the time \( t_{on} \) the equivalent circuit of state 1 describes the behavior of the buck converter. In the circuit in fig.3 the inductor voltage is given by

\[
\overline{v}_{L, state \ 1} = v_{in} - v_{out} - \overline{v}_{RL} - \overline{v}_{RS, state \ 1}, \tag{4}
\]

where the voltage across \( R_L \)

\[
\overline{v}_{RL} = \overline{i}_L R_L \tag{5}
\]

and the voltage across \( R_S \)

\[
\overline{v}_{RS, state \ 1} = \overline{i}_L R_S, \tag{6}
\]

Algorithm 1 Pseudo code of a buck converter model for calculating switching events in CICM

**Model:**

BuckConverter

**Parameter:**

\( L, C, R_S, R_L, R_D, V_D, f_s \)

**Real variables:**

\( v_{in}, v_{out}, L, i_L, i_D, i_{load}, t, d \)

**Boolean variables:**

\( s_{control} \)

**Equations:**

if \( (s_{control} = \text{true}) \),

consider equations corresponding to the equivalent circuit of state 1 (fig.3)

else

consider equations corresponding to the equivalent circuit of state 2 (fig.4)

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with the mean value of the inductor current
\[ \bar{I}_L = \bar{I}_{\text{load}}. \]  
(7)

The equivalent circuit of state 2 (shown in fig. 4) describes the behavior of the buck converter during the time \( t_{\text{off}} = T_s - t_{\text{on}}. \) In state 2 the inductor voltage
\[ \tau_{\text{L, state 2}} = -\tau_{\text{out}} - \tau_{\text{RL}} - \tau_{\text{RD, state 2}} - V_D, \]  
(8)
where \( \tau_{\text{RL}} \) is given by (5) and the voltage across \( R_D \)
\[ \tau_{\text{RD, state 2}} = \bar{I}_L R_D. \]  
(9)
Combining (3) with (4) and (8) one can derive
\[ dT_s \{ \tau_{\text{in}} - \tau_{\text{RL}} - \tau_{\text{out}} - \tau_{\text{RD, state 1}} \} + \]  
\[ + (1 - d)T_s \{ -\tau_{\text{out}} - \tau_{\text{RL}} - \tau_{\text{RD, state 2}} - V_D \} = 0. \]  
(10)
From (10) it is possible to find the mean value of the output voltage by
\[ d(\tau_{\text{in}} - \tau_{\text{RD, state 1}} + \tau_{\text{RD, state 2}} + V_D) = \]  
\[ - (\tau_{\text{RL}} + \tau_{\text{RD, state 2}} + V_D) = \tau_{\text{out}}. \]  
(11)
\( \tau_{\text{out}} \) is a function of the duty cycle \( d, \) the input voltage \( \tau_{\text{in}}, \) and the mean value of the load current \( \bar{I}_{\text{load}}. \) Consequently, it is possible to calculate the average output voltage with considering the conduction losses if there are relations for \( d, \tau_{\text{in}}, \) and \( \bar{I}_{\text{load}} \) available in other models, which is usually the case. The result of (11) can be used as the input of a voltage source that is linked to the connectors of the load side of the buck converter model. Please note that with (11) only the influence of the conduction losses on the average output voltage is considered. In order to calculate the influence of the conduction losses on the supply current the conduction losses of the individual elements (MOSFET, diode, and inductor) need to be known. From the equivalent circuits in fig. 3 and 4 it appears that by approximation (with the assumption that \( \tau_{\text{out}} \) only changes insignificantly in one switching period) in state 1 the inductor current rises with a time constant
\[ \tau_{\text{state 1}} = \frac{L}{R_D + R_L}. \]  
(12)
and in state 2 the inductor current decays exponentially with
\[ \tau_{\text{state 2}} = \frac{L}{R_D + R_L}. \]  
(13)
Provided that the time constants \( \tau_{\text{state 1}} \) and \( \tau_{\text{state 2}} \) are much larger than the switching period \( T_s \) (which applies practically to all buck converters with proper design), the instantaneous current through the inductor can be assumed to have a triangular waveform as
\[ i_L = \begin{cases} 
\bar{I}_{\text{L, state 1}} & \text{if } nT_s < t \leq (n + d)T_s \\
\bar{I}_{\text{L, state 2}} & \text{if } (n + d)T_s < t \leq (n + 1)T_s 
\end{cases} \]  
(14)
with \( n = 0, 1, 2, 3, \ldots \) and
\[ i_{\text{L, state 1}} = \bar{I}_{\text{load}} - \frac{\Delta I_L}{2} + \frac{\Delta I_L}{dT_s} t \]  
(15)
\[ i_{\text{L, state 2}} = \bar{I}_{\text{load}} + \frac{\Delta I_L}{2} - \frac{\Delta I_L}{(1 - d)T_s} t, \]  
(16)
where the current ripple
\[ \Delta I_L = \frac{\tau_{\text{out}} + V_D}{L} (1 - d)T_s. \]  
(17)
Considering the two states of the buck converter circuit and using (14) - (17) the waveform of the current through the MOSFET
\[ i_s = \begin{cases} 
i_{\text{L, state 1}} & \text{if } nT_s < t \leq (n + d)T_s \\
0 & \text{if } (n + d)T_s < t \leq (n + 1)T_s 
\end{cases} \]  
(18)
and the waveform of the current through the diode
\[ i_D = \begin{cases} 
0 & \text{if } nT_s < t \leq (n + d)T_s \\
i_{\text{L, state 2}} & \text{if } (n + d)T_s < t \leq (n + 1)T_s. 
\end{cases} \]  
(19)
For calculating the conduction losses of the individual elements in the converter, the RMS values of the current through the MOSFET \( I_{\text{rms}, \text{L}, \text{MOSFET}} \), the current through the diode \( I_{\text{D}, \text{rms}} \) and the inductor current \( I_{\text{L}, \text{rms}} \) have to be available. Applying the general relation
\[ I_{\text{rms}} = \sqrt{\frac{1}{T_s} \int_{t_0}^{t_0 + T_s} i(t)^2 dt} \]  
(20)
to (18) and (19) results in
\[ I_{\text{rms}, \text{L}, \text{MOSFET}} = \sqrt{d \left[ I_{\text{L, min}}^2 + I_{\text{L, max}} \Delta I_L + \frac{\Delta I_L^2}{3} \right]}, \]  
(21)
with
\[ I_{\text{L, min}} = \bar{I}_{\text{load}} - \frac{\Delta I_L}{2}. \]  
(22)
and
\[ I_{\text{D}, \text{rms}} = \sqrt{1 - d} \left[ I_{\text{L, max}}^2 - I_{\text{L, max}} \Delta I_L + \frac{\Delta I_L^2}{3} \right], \]  
(23)
with
\[ I_{\text{L, max}} = \bar{I}_{\text{load}} + \frac{\Delta I_L}{2}. \]  
(24)
Using (21) - (24) and considering
\[ i_L = i_s + i_D \]  
(25)
the RMS value of the inductor current can be written as
\[ I_{\text{L, rms}} = \sqrt{I_{\text{rms}, \text{L}, \text{MOSFET}}^2 + I_{\text{D, rms}}^2}. \]  
(26)
The conduction losses of the MOSFET \( P_{\text{L, MOSFET}} \) and the storage inductor \( P_{\text{L, \text{con}} \text{L}} \) can be calculated by
\[ P_{\text{L, MOSFET}} = R_D I_{\text{rms}, \text{MOSFET}}^2 \]  
(27)
and
\[ P_{\text{L, \text{con}} \text{L}} = R_L I_{\text{L, rms}}^2. \]  
(28)
When calculating the conduction losses of the diode also the portion of the power emission contributed by the knee voltage has to be taken into account. Since the knee voltage is modeled as a constant voltage source the mean value of the current through the diode
\[ \bar{i}_D = (1 - d)\bar{I}_{\text{load}} \]  
(29)
has to be used to calculate the respective contribution to the conduction losses. The total conduction losses in the diode can be written as
\[ P_{\text{D, con}} = R_D I_{\text{D, rms}}^2 + V_D \bar{i}_D. \]  
(30)
Using (27), (28), and (30) the total amount of conduction losses can be calculated by
\[ P_{\text{tot, con}} = P_{\text{L, con}} + P_{\text{D, con}}. \]  
(31)
IV. TEMPERATURE DEPENDENCE OF CONDUCTION LOSSES

To assure good accuracy of the models in section II and III also the temperature dependences of the conduction losses need to be considered. In many cases a linear approximation of the temperature dependence is sufficient. Ideally, the reference values (e.g. resistances, voltages, etc.) and the linear temperature coefficients \( \alpha \) are generated from measurement results. When using a linear temperature coefficient it is crucial to take into account that it only applies to one specific reference temperature. A widely used relation for a resistance with linear temperature dependence is

\[
R_1 = R_0 [1 + \alpha_0 (\vartheta_1 - \vartheta_0)],
\]

(32)

where \( R_1 \) is the resistance at the temperature \( \vartheta_1 \), \( R_0 \) is the resistance at the temperature \( \vartheta_0 \) and \( \alpha_0 \) is the linear temperature coefficient at the temperature \( \vartheta_0 \). If only a resistance measurement point \( R_2 \) for a different temperature \( \vartheta_2 \) than \( \vartheta_0 \), to which the linear temperature coefficient \( \alpha_0 \) applies, is available the temperature coefficient needs to be recalculated to

\[
\alpha_2 = \frac{\alpha_0}{1 + \alpha_0 (\vartheta_2 - \vartheta_0)}.
\]

(33)

With (33) it is possible to calculate the linear temperature dependent resistance \( R_1 \) at any temperature \( \vartheta_1 \) according to

\[
R_1 = R_2 \left[ 1 + \frac{\alpha_0}{1 + \alpha_0 (\vartheta_2 - \vartheta_0)} (\vartheta_1 - \vartheta_2) \right]
\]

(34)

if \( \alpha_0 \) (not \( \alpha_2 \)), \( R_2 \) (not \( R_0 \)), and \( \vartheta_2 \) are given. The approach that results in (32) - (34) can also be applied to model linear temperature dependence of the knee voltage in the diode. In the buck converter models described in section II and III the on-resistance of the MOSFET, the on-resistance and the knee voltage of the diode as well as the ohmic contribution of the inductance are modeled with linear temperature dependence according to the approach used in (32) - (34).

V. CONSIDERATION OF SWITCHING LOSSES

Models on different levels of detail for switching loss calculation have been published. In many MOSFET models the parasitic capacitances are considered and in some also the parasitic inductances at the drain and at the source of the MOSFET are taken into account. In [4] a model considering the voltage dependence of the parasitic capacitances is proposed. A model in which constant parasitic capacities as well as parasitic inductances are considered is suggested in [5] and in [6] voltage dependent parasitic capacities together with the parasitic inductances are used for the calculation.

In data sheets such as [7] an equation combining two terms is used. In the first term constant slopes of the drain current and the drain source voltage are assumed and in the second the influence of the output capacitance is taken into account. Also for this approach the parasitic capacities as well as the switching times (or at least the gate switch charge and the gate current) have to be known. In [8] is stated that the approach in [7] leads to an overestimation of the switching losses in the MOSFET.

A general approach for switching loss calculation in power semiconductors using measurement results with linearization and polynomial fitting is presented in [9]. In [10] the switching losses are considered to be linear dependent on the blocking voltage, the current through the switch, and the switching frequency. This approach was initially developed for modeling switching losses in IGBTs but it can also be applied to the calculation of MOSFET switching losses. In [11] a modified version of the model proposed in [10] is presented. The difference is that in [11] the switching losses are dependent on the blocking voltage, the current through the switch, and the switching frequency with higher order.

In the presented work the approach described in [10] is used to model the switching losses. The two buck converter models in section II and III can be expanded with switching loss models using

\[
P_{\text{switch}} = P_{\text{ref.switch}} \frac{f_s}{f_{\text{ref.s}}} i_{\text{load}} \frac{v_{\text{in}}}{v_{\text{in.ref}}},
\]

(35)

where \( P_{\text{switch}} \) represents the sum of the actual switching losses in the MOSFET and the diode of the buck converter, \( f_s \) denotes the actual switching frequency, \( i_{\text{load}} \) is the actual commutating current between the diode and the MOSFET, and \( v_{\text{in}} \) is the actual blocking voltage of the diode and the MOSFET. \( P_{\text{ref.switch}} \) represents a measured value of the switching losses at a reference operation point defined by \( f_{\text{ref.s}}, i_{\text{ref.load}}, v_{\text{ref.in}} \).

With this approach no knowledge of the parasitic capacitances and inductances is needed. Neither the switching energy nor the switching times need to be known. For the accuracy of the model given in (35) the precision of the measurement results at the reference operation point is very important.

VI. IMPLEMENTATION OF THE SIMULATION MODELS

The buck converter models described in section II and III got implemented with Modelica modeling language [12]
using the Dymola programming and simulation environment. *Modelica* is an open and object oriented modeling language that allows the user to create models of any kind of physical object or device which can be described by algebraic equations and ordinary differential equations. Elementary models (e.g. resistors, diodes, energy sources, etc.) get connected via their respective potential quantities (e.g. electric potential, temperature, etc.) and flow quantities (e.g. electric current, thermal power, etc.) to form more complex models such as a power electronic circuit. Graphical as well as textual programming are facilitated by *Modelica*.

In both models the conduction losses including temperature dependence according to section IV and the switching losses according to section V are considered.

In fig. 5 the scheme of model A, the model calculating the switching events (as explained in section II) is shown.

The conduction losses are inherently considered in alg. 1 and the switching losses are considered by means of a controlled current source with

$$i_{\text{model A}}^* = \frac{P_{\text{switch}}}{v_{\text{in}}}$$

whereas $P_{\text{switch}}$ is calculated by (35).

Fig. 6 illustrates the scheme of the averaged buck converter model (as explained in section III) with consideration of the switching and conduction losses. The basic components of model B are the current source controlled with $i_{\text{model B}}^*$, the voltage source controlled with $v^* = \overline{v_{\text{out}}}$, and the power meter measuring the averaged output power $P_{\text{out}} = \overline{v_{\text{out}} i_{\text{load}}}$. In model B the control signal of the voltage source $v^*$ is computed according to (11) and the control signal of the current source $i_{\text{model B}}^*$ is calculated by

$$i_{\text{model B}}^* = \frac{P_{\text{switch}} + P_{\text{tot,con}} + P_{\text{out}}}{v_{\text{in}}},$$

In (37) $P_{\text{switch}}$ is given by (35), $P_{\text{tot,con}}$ is calculated from (31), and $P_{\text{out}}$ is the output signal of the power meter in fig. 6.

VII. SIMULATION AND LABORATORY TEST RESULTS

The approach applied in model A is well established. Therefore the results of model A are used as a reference for the verification of model B. For the comparison of the two models a buck converter ($f_s = 100\text{ kHz}$) supplied with a constant voltage of 30 V and loaded with a constant load current of 40 A was simulated using model A and model B. In the two simulations the duty cycle was decreased step by step from 0.8 to 0.2. Fig. 7 shows the duty cycle signal of the two simulations. The purpose of model B is to calculate the efficiency and the electric quantities in steady state. The supply current signals and the load voltage signals in fig. 8 and 9 show that after the transients decay both models reach the same operation point. Please note that in fig. 8 the instantaneous supply current signal computed with model A is averaged over a switching period.

Both simulations were computed on a state-of-the-art PC with 3 GHz dual core and 3 GB RAM. It took only 2.8 s to process the results of the simulation with model B whereas the CPU time for the simulation with model A was 36 s. The large difference between the CPU times indicates that it is much more efficient to use model B if the energy flow through a converter is the focus of the simulation.
For the validation of the two simulation models several laboratory tests have been conducted. The measurement setup is illustrated in fig. 10. In order to avoid core losses an air-cored coil was implemented as the storage inductor. As the passive and the active switch two IRFPS3810 power MOSFETs were chosen whereas the body diode of one of the MOSFETs was used as the freewheeling diode. The temperatures of the two MOSFETs and the air-cored coil were measured with type-K thermocouples.

The circuit in fig. 10 was tested with a similar duty cycle reference signal as shown in fig. 7. However, the step time of the duty cycle signal in the laboratory test was significantly longer compared to the signal in fig. 7. Because of this, the temperatures of the semiconductors increased significantly. Fig. 11 shows the measured efficiency of the circuit under test and the respective (steady state) simulation results of model A and B. The measured and simulated results show satisfactory coherence.

In fig. 12, 13 and 14 the measured losses of the buck converter operated with $d = 0.2$, $d = 0.5$ and $d = 0.8$ during a warm-up test are compared with the results of a simulation carried out with model B. In fig. 12 it can be seen that the conduction losses decrease with increasing time and temperature. This is because the knee voltage of the freewheeling diode has a negative temperature coefficient and at $d = 0.2$ the freewheeling diode conducts 80% of the time in a switching period. In fig. 13 and 14 the conduction losses raise with increasing time and temperature. The reason for this is the positive linear temperature coefficient of the on-resistance of the MOSFET and the longer duration in which the MOSFET conducts during a switching period. It is also important to consider that if the buck converter is operated with higher duty cycles the MOSFET dissipates more energy and reaches higher temperatures than operated with lower ones.

**VIII. CONCLUSION**

An analytical approach to calculate the steady state behavior of a buck converter including the consideration of conduction losses is described. The presented model B is generated from the derived equations and expanded so that switching losses and temperature dependence of the conduction losses are considered. For the verification of the described modeling approach two simulation models (model A and B) are programmed with Modelica language. In steady state model A, the model calculating switching events, matches the behavior of model B, the model based on the approach of system averaging. When comparing the CPU times of model A and model B it appears that model B can be computed more
than 10 times faster than model A. Consequently, it is recommended to preferably use model B in simulations whenever only the steady state values of the electrical quantities in the buck converter are of interest. This is for instance the case in energy flow analyzes and in simulations for core component dimensioning of electromechanical systems. The simulation results of model A and B show satisfying conformity with the laboratory test results. Further work will include the application of the modeling approach used for model B to other dc-to-dc converter types.

**REFERENCES**


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