A Decomposition Approach to Solve a General Delivery Problem

L. Lian and E. Castelain

Abstract—**This paper presents a new distribution and route planning problem, General Delivery Problem (GDP). Such problem is more general than the well-known Vehicle Routing Problem. To solve a GDP, a three-phase framework based on decomposition techniques is introduced. The decomposition techniques are employed to divide an original problem into a set of sub-problems, which can reduce the problem size. A kind of decomposition technique, Capacity Clustering Algorithm (CCA), is embedded into the three-phase framework with Simulated Annealing (SA) to solve a special GDP. The proposed three-phase framework with the above two algorithms is compared with five other decomposition methods in a distribution instance of the Regional Fire and Emergency Center in the north of France.**

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Index Terms—**Decomposition Technique, General Delivery Problem, Heuristic Method.**

I. INTRODUCTION

The vehicle dispatching and its route planning have a considerable economical impact on a distribution center. In practical aspect, this problem contributes directly to reduce costs of all logistic systems [1]. In the literature, such a problem is almost researched as a Vehicle Routing Problem (VRP). In general, the VRP can be defined as the problem of designing optimal delivery or collection routes from one or several depots to a number of geographically scattered cities or customers, subject to side constraints [2].

This paper focuses on a General Delivery Problem (GDP), which is more general than the well-known VRP and Traveling Salesman Problem (TSP). In GDP, there is no hypothesis on the way that origins and destinations will be linked to organize and to realize the whole set of deliveries. That is to say, when a company confronts a transportation problem and it just knows the locations of the customers and the quantities of the customers' demands, the company needs to decide the strategies to manage the transportation process and to organize the routing sequence. Generally speaking, TSP or VRP is kind of the routing sequence problem. As we know, TSP and VRP are NP-hard combinatorial optimization problems [3]. Therefore, GDP is difficult to solve as TSP and VRP are sub-problems of GDP.

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Gilbert Laporte [4] surveyed the main exact and approximate algorithms developed for the VRP, at a level appropriate for a first graduate course in combinatorial optimization. Furthermore, the approximate algorithm mainly includes two kinds of algorithms, constructive heuristic algorithm and meta-heuristic algorithm. Some research efforts were oriented towards the development and analysis of approximate heuristic techniques capable of solving real-size VRP problems. Bowerman et al. [5] classified the heuristic approaches to the VRP into five classes:

(1) cluster-first/route-second (CFRS),
(2) route-first/cluster-second (RFCS),
(3) savings/insertion,
(4) improvement/exchange,
(5) Simpler mathematical programming representations through relaxing some constraints. For the two clustering procedures, the cluster-first /route- second looks more effective.

Decomposition techniques have been applied to solve VRP. Such decomposition techniques can reduce the problem size and expand the choice of searching strategies. Several authors have previously proposed decomposition technique to solve VRP. Previous works are classified into three types: (1) using optimization model with Lagrangian relaxation as decomposition technique, i.e., Paolo Toth&Daniele Vigo[6], Chi-Bin Cheng&Keng-Pin Wang[7] and Byung-In Kim, Seongbae Kim&Surya Sahoo[8]; (2) heuristic approach to construct the groups. Rodolfo Dondo and Jaime Cerda[9] presented a new three-phase heuristic/algorithmic approach which embedded a heuristic-based clustering algorithm within a VRP with time windows(VRPTW) optimization framework. In other words, they used a preprocessing method to cluster nodes into groups, and then took the group as node to apply the optimization method; (3) cluster analysis is usually proposed. Sergio Bargio, Carlos Ferreira et al. [10] integrated several hierarchical and non-hierarchical clustering techniques into a sequential heuristic algorithm for the location-routing problem (LRP) model. Byung-In Kim, Seongbae Kim et al.[11] developed a capacitated clustering-based algorithm to deal with the real life waste collection problems. K.Ganesh and T.T.Narendran[12] provided an initial solution with k-means clustering methods and thereby accelerated convergence of the genetic algorithm to solve the vehicle routing problem with deliveries and pickups.

Cluster analysis [13] studies the division of entities (as objects or individuals) into groups based on one or several of their characteristics. According to Jain and Dubes : ''Cluster

(Advance online publication: 1 February 2010)

may be described as connected regions of a multi-dimensional space containing a relatively high density of points, separated from other such regions by a region containing a relatively low density of points''. Evidently, it is reasonable to regard cluster analysis as a decomposition technique. However, in all of the above papers, the authors use only one decomposition criterion to divide VRP. Instead, to resolve the large-scale GDP, this paper presents a heuristic framework with multi-decomposition criterions.

Section II describes the framework of the heuristic decomposition procedure and introduces the decomposition criterions. In Section III, a heuristic approach with Capacity Clustering Algorithm (CCA) and Simulated Annealing (SA) is embedded into the proposed framework to solve a special GDP. In Section IV, we provide a concrete instance of the decomposition procedure and illustrate the computational results. The paper is concluded in Section V.

II. The Decomposition Framework

Laporte, Mercure and Nobert (1986)[14] have provided the optimal solution to randomly generated asymmetrical CVRPs involving up to 260 vertices with branch-and-bound-algorithm. So far, it is one of the best solutions to VRP with the most nodes. But the GDP with a huge number of nodes is more difficult to solve. In order to overcome the limitation and to help the distribution centers solve their special vehicle dispatching and routing problem, we introduce a heuristic framework. The proposed heuristic framework includes a three-phase heuristic decomposition procedure which can be used to divide the large-scale GDP into some sub-problems (Transportation Problem, TSP, VRP or basic GDP). Through adding Simulated Annealing (SA) Algorithm into the framework, the delivery routes generated from the proposed framework can be improved, which is explained in Section III.

A. Decomposition procedure

The procedure for the proposed framework consists of three phases as follows:

Phase 1: Divide the huge number of nodes (including original nodes and destination nodes) in GDP into some groups with decomposition techniques.

Phase 2: Determine the distribution route for each group with the existing tools and/or heuristics algorithms.

Phase 3: Improve the routes between groups.

Chi-Bin Cheng and Keng-Pin Wang have used an iterative interaction between the original problem and many sub-problems to solve VRPTW. The decomposition procedure in this paper is similar to that provided by Chi-Bin Cheng and Keng-Pin Wang and it is illustrated in Fig.1.

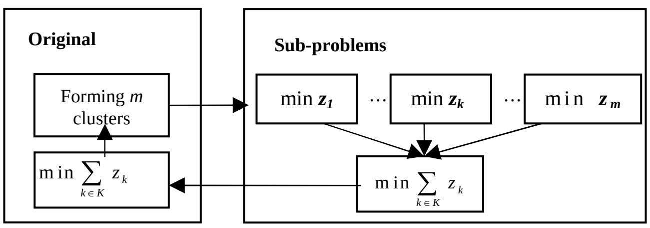


Fig 1. Interaction between original problem and sub-problem

In Phase 1, decomposition criterion in strategic view and in operational view should be decided at first. Noticeably, this is fundamental. The different decomposition criterion may lead to different resolution. The description of the division criterions will be presented in the Section II B. The large-scale problem is divided into some sub-problems by a decomposition technique. The decomposition process continues until the problem is divided into the smaller ones, TP, TSP, and VRP, that can be solved successfully.

In Phase 2, the distribution routes in each group are determined. For TP, it's easy to solve since it is a polynomial problem. We use ILOG CPLEX, high-performance optimization software, to generate the solutions. For TSP, we get the optimal solution by well-known and efficient TSP solvers like Concorde, which can solve large-scale TSP instance up to the 15,112 cities in Germany in 2001. For other kinds of VRP, genetic algorithm and other meta-heuristic algorithm can be used.

The decomposition procedure of GDP in Phase 1 and Phase 2 is an interaction between the original problem and its sub-problems. The original problem can be transformed into several sub-problems by dividing all nodes of original problems into several groups in terms of decomposition criterions. And then each sub-problem optimizes its own routing sequence. During the improvement of the routes (Phase 3), an improvement heuristics, i.e., 2-opt, 3-opt, Lin-Kernighan can be used to reform the current sequence. The sum of the objective values of all new sub-problems is returned to the original problem as a performance indicator for evaluating the current decomposition result. Fig.2 shows the two phases of the proposed framework.

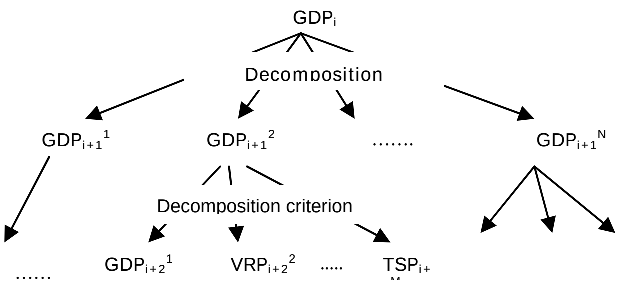


Fig. 2 Two Phases of Proposed Framework

B. Distribution network structure

It is necessary to present several general distribution network structures before the decomposition criterions are introduced. In a broad sense, a distribution network necessarily includes one primary layer of distribution facilities and sometimes includes a number of secondary layers. A primary facility represents either the origin or destination of a vehicle journey. Some examples of primary facilities are manufacturing plants, hospitals, waste collection centers, airports and landfills. On the other hand, a secondary facility represents an intermediate or a transshipment point such as a plant, a depot, a warehouse, a distribution center and an agency. There are mainly three classes of distribution network structures in the real world: 0-level distribution network, 1-level distribution network and 2-level distribution network.

1) 0-level distribution network

At first, we refer to a kind of distribution network structure without any secondary facility between the original nodes and destination nodes as 0-level distribution network (see Fig.3). TSP and VRP are two versions of the distribution problem with 0-level.

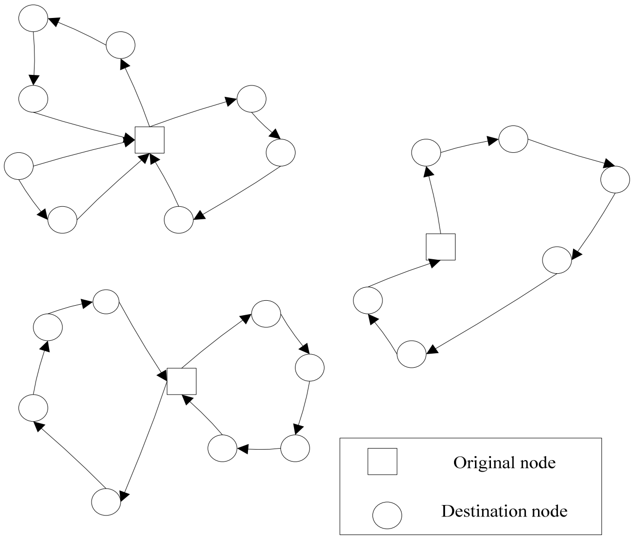


Fig. 3 0-level distribution network

2) 1-level distribution network

Another class is a 1-level distribution network which is concerned with the distribution network structure with just one layer of secondary facility. Many researches in the 1-level distribution network are typically assumed that primary facilities are located at known and fixed sites, but the secondary facilities should be located as shown in Fig. 4

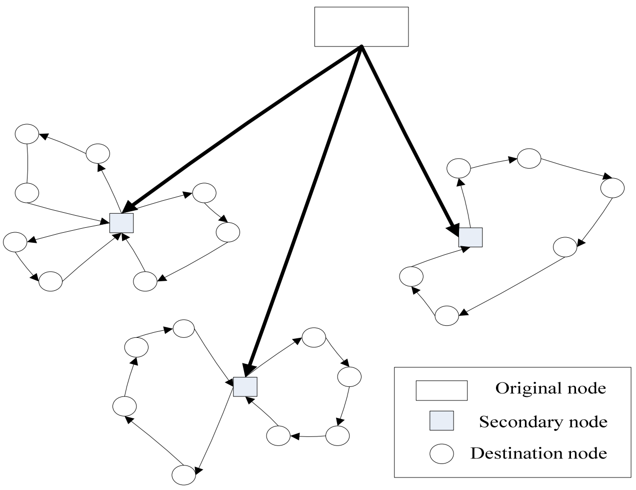


Fig. 4 1-level distribution network

3) 2-level distribution network

2-level distribution network occurs when the products are delivered from original nodes to destination nodes through two secondary layers. Fig 5 illustrates a typical 2-level distribution network system arising from an actual application in a mail-order company. A mail-order company offers several products (typically, packages containing various types of goods, such as clothes, electronic devices, appliances…) that must be delivered on time to the customers requesting them. To satisfy these requests, the firm operates the 2-level distribution network system as follows: starting the trips from the suppliers, then the products are delivered to the entrepots, where the products are transferred and shipped to agencies, where the products are sorted and delivered to the customers.

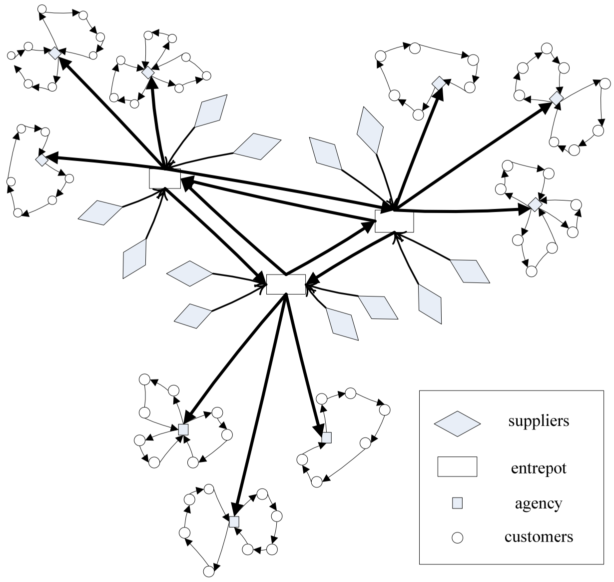


Fig. 5 2-level distribution network

C. Decomposition Criterion

In Phase 1, origin nodes and destination nodes are sorted and divided into several groups according to decomposition criterions. In this section, we describe these decomposition criterions in two aspects.

1) Decomposition Criterion in Strategic View

The strategy in the logistics activities of a company including the determination of the best configuration regarding location, size, technology content and product range is defined to achieve the company's long-term performance. In the strategic view, the company locates several layers of secondary facilities between the supplier nodes and the destination nodes to divide the GDP into smaller sub-problems. Each supplier and each customer is allocated to certain corresponding secondary facility at the same time.

In this situation, the distribution network structure is changed in consequence of the addition of the number of layers in the network. The distribution process of GDP is divided into several parts: distribution problem between original nodes and secondary facilities, distribution problems between secondary facilities and other secondary facilities, and finally distribution problem between secondary facilities and destination nodes. In the other word, a 1-niveau distribution network system is divided into a combination of two 0-level distribution networks. On the other hand, a 2-level distribution network system is divided into a combination of one 0-level distribution network and one 1-level distribution network or a combination of three 0-level distribution networks. In strategic view, the number and the locations of the secondary facilities should be decided.

Furthermore, each customer is assigned to a certain secondary facility. Fig 6 shows a simple example of the decomposition criterion in strategic view. In the Fig.6, the ring represents the origin and the other points are customers. Firstly, the four other depots are located, and then the origin delivers products to the four distribution centers. At last, the decomposition of the original problem has resulted in four sub-problems (TSP or VRP) from each of the 4 depots. In the real world, this approach can help the companies to improve the management of their transportation process. Hence, we propose the strategic criterion to locate the secondary facilities to optimize the distribution systems.

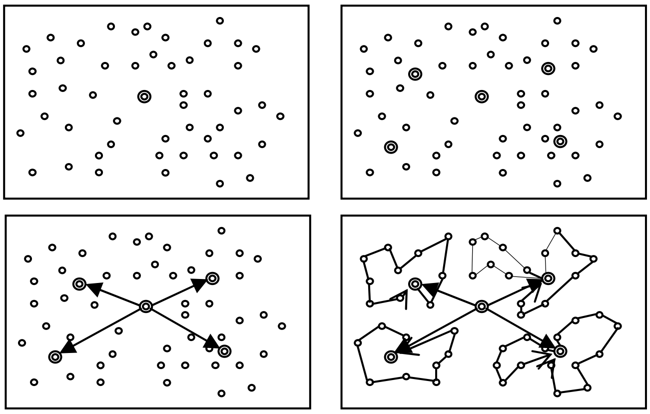


Fig. 6 An example of the division criterion in Strategic view

2) Decomposition criterion in operational view

Space criterion is to choose the adjacent nodes as the same group. There are several approaches to divide the large-scale problem with space criterions. Heung Suk Hwang[15] has developed a sector-clustering algorithm to convert a multi-supply centre problem into single supply center problems. Clarisse Dhaenens-Flipo[16] has investigated a spatial decomposition to divide a multi-facility production and distribution problem into some sub-problems and then developed a branch-and-bound algorithm to obtain the exact solutions of the sub-problems. Fig 7 illustrates an example of the space criterion. First of all, the nodes are divided into 4 groups with space criterion, and then we decide the itinerary for each group.

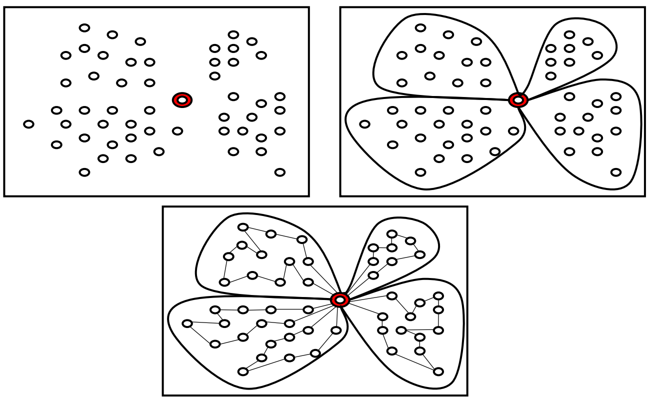


Fig. 7 An example of space criterion

Time criterion is used to divide the destination nodes into several priority-ranked groups according to customers' urgencies of the requirement deadline. Fig.8 presents an example of the time criterion. According to time criterion, the delivery destinations are divided into three subsets distinguished by different symbols (◊,□,O). In other words, the destinations defined by the same symbol must be served in the same day. The routing sequence for each day is finally presented.

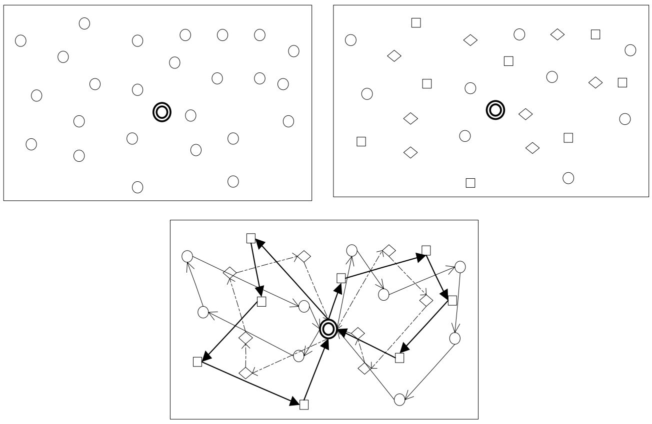


Fig. 8 An example of time criterion

III. Heuristic APPROACH to capacitated vehicle routing problem (CVRP) based on the Proposed Framework

A framework to decompose the large-scale problem and improve the solution has been designed in the section II. This paper aims to design a framework to assist the company to solve their GDP. Here, this study is applied to a special case, of GDP: the Capacitated Vehicle Routing Problem (CVRP).

In the last two decades branch-and-bound, including those that make use of the set partitioning formulation and column generation schemes, and branch-and-cut are two main exact approaches to the CVRP. Several branch-and-bound algorithms are available for the solution of CVRP. Toth and Vigo ([17], [18]) reviewed the structure of the branch-and-bound algorithm strategies and dominance rules. Branch-and-cut algorithms currently constitute the best available exact approach for solution of the CVRP. Research in this area has been strongly motivated but the approach is still quite limited and most of it is not yet published. Naddef and Rinaldi[19] have given a detailed presentation in this approach. In practice, the best CVRP exact algorithms can rarely tackle instances involving more than 100 vertices. In order to overcome the limitation of the vertices number and to solve larger CVRP, we apply our proposed approach to CVRP.

In this case, we take the CCA as decomposition technique, apply Concorde as the solver, and then improve the solution with SA. The approach is described in Section III A.

A. Heuristic approach based on CCA and SA

Heuristic approach based on CCA and SA

Phase 0: Estimate the number of vehicles, N, based on the total workload. Construct the distance matrix.

Phase 1: Decompose the large-scale problem into several TSPs with CCA.

Phase 2: Use Concorde to determine the routing sequence for each TSP.

Phase 3: Modify the routes between each TSP with SA.

Phase 4: Finish the heuristic approach.

The number of required vehicles (phase 0) is estimated by the total distribution workload divided by the daily workload capacity of each vehicle. Note that we assume that the vehicles are of the same type. The explanations of the other phases will be done in Section III B.

B. Capacitated clustering algorithm

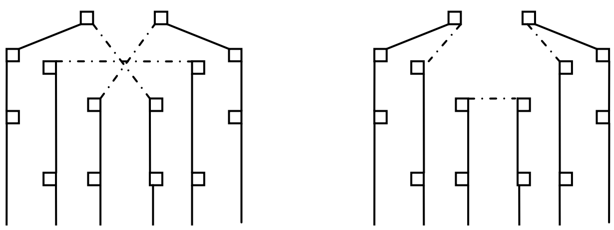
In phase 1, we decompose the original problem into several TSPs by CCA. In standard k-means algorithm, nodes are clustered according to the distances between the nodes and the centroids, i.e., a node is assigned to the cluster whose centroid is the closest to the node. But in the CCA, the centroid of the centroids is also considered, which would be called "the grand centroid". Additionally, in order to minimize the number of vehicles, we begin the algorithm with the number obtained in Phase 0 of the method in Section III A.

Capacitated clustering algorithm

Step 1: N initial centroid seed nodes are selected according to k-means cluster algorithm, the remaining nodes are assigned to the clusters.
Step 2: Let the cycle index m=0, a new centroid of each cluster and the grand centroid are calculated.
Step 3: Sort the nodes and assign nodes to new clusters with the grand centroid are repeated until there is no change
Step 4: If the capacity of any cluster is not satisfied the capacity of vehicle, the cycle index m=m+1, return to step 3; If m passes cycle index mmax, N=N+1, return to step 2;
Step 5: Finish the algorithm until the groups of nodes are found.

Simulated Annealing Improvement Heuristic

The optimal solution to each group is produced by Concorde in Phase 2. In Phase 3, we modify the routing sequence between the routings to approach the global optimal solution. SA with 3-opt improvement heuristic algorithm has been developed for this problem. In the basic iteration of SA, the neighbors of each state are obtained by 3-opt improvement heuristics. The basic idea of 3-opt local search algorithm is to start from the routing sequence, to choose three edges from the routing sequence, to remove them, and to combine the six parts to a routing sequence in the cheapest way. The idea is shown in the Fig. 9.



(a) Routing before 3-opt (b) Routing after 3-opt

Fig. 9 Idea of 3-opt Algorithm

3-opt improvement heuristic algorithm

Step 0 (Initialization) Let cycle index =1; Let the solution S from Phase 3 as the best solution S*, the objective f corresponding to *S** as the best objective *f**.
Step 1(Edges choosing) Apply 3-opt algorithm to three edges r_1, r_2, r_3, randomly choosing from edge set W. Then, at most, 15 new solutions S_1, S_2, …, S_{15} can be produced.
Step 2(Exchange evaluation) Evaluate all the candidates of 3-opt exchanges. Calculate the objective (the total distance) $f_1, f_2, \ldots, f_{15}$. Index f'=min $\{f_1, f_2, \ldots, f_{15}\}$, and S' corresponded to f'.
Step 3(Improvement) If $f' < f^*$, let $S^* = S'$, t =1; Else, t = t+1, go to Step 4.
Step 4(End standard) If f^* doesn't change within n_{max} loops, end; else, go back to Step 1.

IV. Computational Results

To evaluate the performance of the proposed approach, our approach with the decomposition technique is compared with the other three decomposition approaches, 1-level distribution network strategy with TSP and the Old Strategies used by a distribution instance of the Regional Fire and Emergency Center in the north of France.

The regional service centre needs to delivery the medicine to its firefighter centers in five regions each week. In the case of the Regional Fire and Emergency Center in the north of France, the distribution process is not associated with the time windows. From the performance point of view, the center just aims to minimize the number of the vehicles and the traveling distance. The centre has 10 vehicles with the same capacity. There are 110 firefighter centers in five different regions.

A. Performance of Old Strategy

Before, the regional service center firstly distributed the medicine to five distribution centers in each administrative region. Each distribution center organized the transportation sequence in the form of the round-trip vehicles between the distribution center and each firefighter centers in its region. Here, we regard the transportation strategy as Old Strategy. Obviously, the distribution network structure of the Old Strategy is as a 1-level distribution network in which the regional service center and all of the firefighters are primary facilities and the distribution centers in the five regions are the secondary facilities. The routing sequence of the Old Strategy is shown in Fig.10.

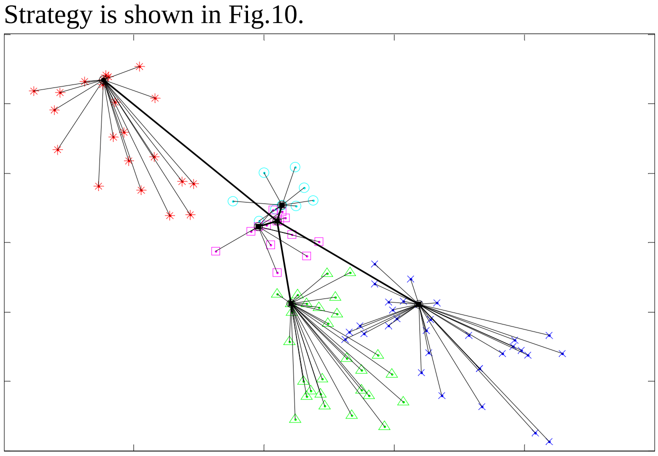


Fig. 10 Groups Produced by Capacitated Clustering

B. Performance of 1-level distribution strategy with TSP

In the Old Strategy, the routing sequence is not optimal because there are too many round-trips. Here we take another 1-level distribution network with TSP as distribution strategy to improve the Old Strategy. In this strategy, the distribution process is starting from the regional service center to five distribution centers in each administrative region where the medicines are sorted and distributed to the firefighter centers in its region. The distribution routing problem from the regional service center to the firefighter centers in each region is regarded as a TSP.

The Fig. 11 illustrates the routing of this strategy. In the Fig. 11, the firefighters in five regions are distinguished by five different colors and the regional service centers are represented by black points.

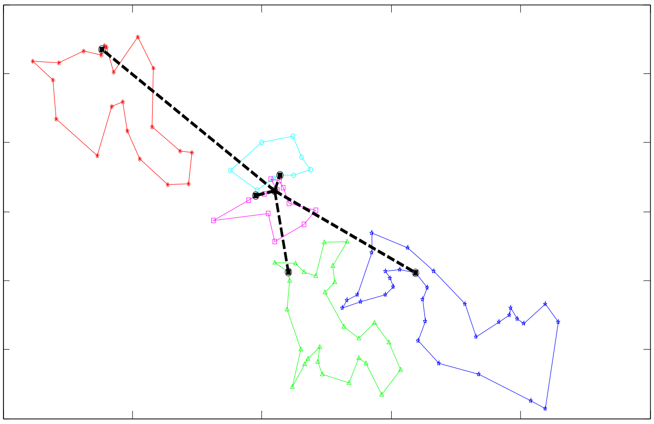


Fig.11 Performance of 1-level distribution strategy with TSP

C. Performance of the proposed heuristic approach

Fig.12 shows the results of each phase in the heuristic approach based on CCA and SA when the approach is used to solve the problem instance. Fig.12 (a) indicates the groups of firefighter centers produced by Phase 1, which represent by different symbols. After Phase 2, routing sequence for each group of the firefighter centers is shown in Fig.12 (b). At last, Fig. 12(c) shows the routing sequence improved by SA in Phase 3.

D. Performance comparison between different approaches

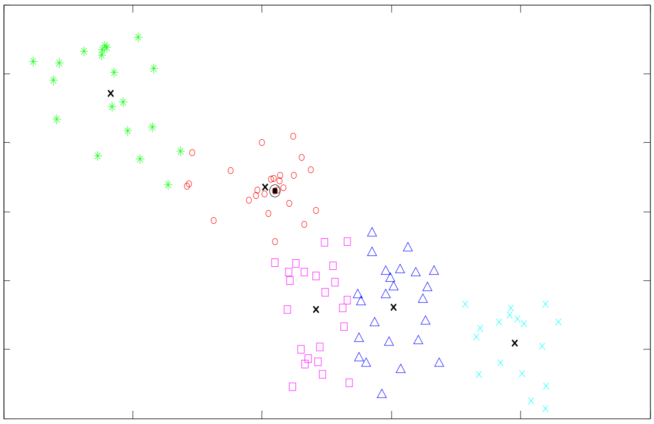
Our proposed heuristic approach is compared with other five approaches: the Old Strategy (strategy 1), 1-level strategy with TSP (strategy 2), Routing First and Cluster Second (RFCS) without the improvement technique, RFCS with SA, Cluster First and Routing Second (CFRS) without the improvement technique. Computational results by different approaches are shown in Fig. 13 respectively. We can see that our proposed approach outperforms the other five approaches in terms of the total distance (in kilometers).

V. Conclusion

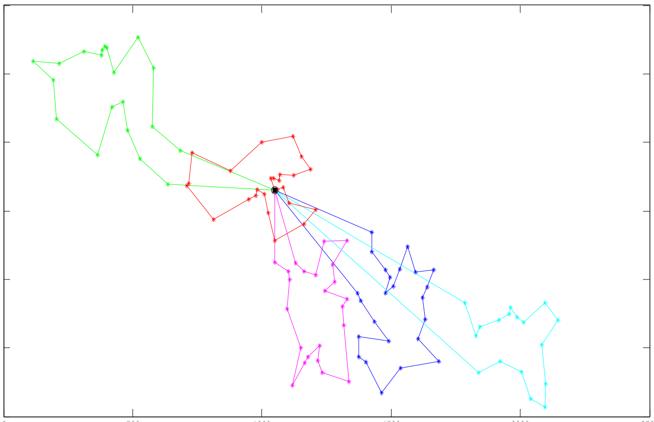
The decomposition technique divides the original problem into some sub-problems. In this way, the size of the problem is reduced and it is simpler to solve sub-problems with fewer nodes than to solve the original problem. Due to its advantages, a heuristic approach based on the decomposition technique is a promising way to find a good solution for large problem.

This study, firstly, presented the General Delivery Problem which is more general than the well-known delivery problem, VRP and TSP and a framework of a heuristic approach based on decomposition technique to solve GDP was proposed in order to assist the delivery management of the distribution centre. Then a heuristic approach with CCA and SA was designed to delivery problem instance in the north of France.

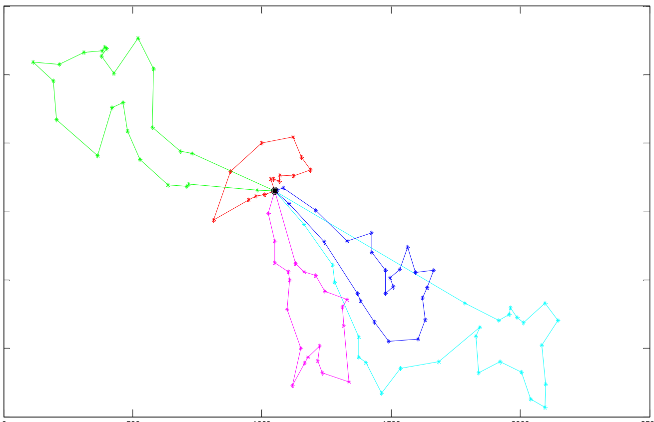
The proposed approach was evaluated by comparing with the other five approaches. Computational results showed that our approach outperformed these five approaches.



(a) Groups Produced by Capacitated Clustering



(b) Routing Sequence after Clustering the Destination Nodes



(c) Routing Sequence Improved by SA

Fig. 12 Performance of the Proposed Heuristic Approach

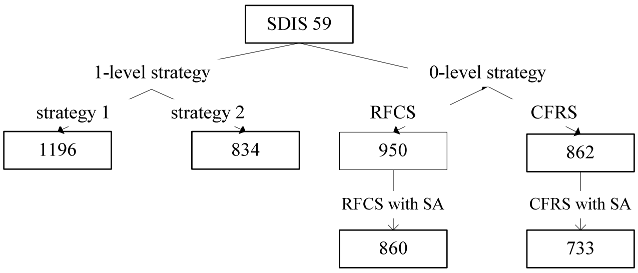


Fig. 13 Performance comparison between different approaches

Despite of these encouraging results, there are also many opportunities. For instance, further research includes in the following aspects. Firstly, summarize the existing approaches to solve GDP and improve the proposed framework in the paper. Secondly, test the proposed heuristic approach to solve GDP in more instances to demonstrate the performance of the proposed approaches. Meanwhile, apply the proposed framework to the other types of distribution problems. Moreover, in the aspect of decomposition technique, it is possible to introduce other better techniques and compare with the capacity clustering algorithm to improve the computational results. We attempt to improve the solution concerning delivery distance; nevertheless, the choice of the final solution doesn't only depend on the delivery distance in practical perspective. So in the future, removing the overlaps of the routing, reinforcing the workload balance and other practical aspects will be considered.

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