Model Reference Adaptive Control of Biped Robot Actuators for Mimicking Human Gait

Pavan K. Vempaty, Ka C. Cheok, Robert N. K. Loh, and Safa Hasan

Abstract — **Many robotics problems do not take the dynamics of the actuators into account in the formulation of the control solutions. The fallacy is in assuming that desired forces/torques can be instantaneously and accurately generated. In practice, actuator dynamics may be unknown and can have significant transient effect on the overall results. This paper presents a Model Reference Adaptive Controller (MRAC) for the actuators of a biped robot that mimics a human walking motion. The MRAC self-adjusts so that the actuators produce the desired torques in accordance with an inverse reference model. Lyapunov stability criterion is used to provide the MRAC structure, and a rate of convergence analysis is provided. The control scheme for the biped robot is simulated on a sagittal plane to verify the MRAC scheme for the actuators.**

Keywords- **Model reference adaptive control, biped robot, inverse reference model, actuator dynamics, Lyapunov.**

I. Introduction

Biped walking dynamics is highly nonlinear, has many degrees of freedom and requires developing complicated model to describe its walking behavior. Many novel approaches have emerged in the field of biped walking to address this complicated control mechanism. Existing biped walking methods [4]-[7], give precise stability control for walking bipeds. However these methods require highly precise biped walking dynamics. In recent years, biped walking through imitation has been a promising approach, since it avoids developing complex kinematics of the human walking trajectory and gives the biped a human like walking behavior. These methods combine the conventional control schemes to develop the walking mechanism for the bipeds. Examples include, imitation based on intelligent control methods like genetic algorithm [8], [5], fuzzy logic [9], neural network approach [10], and other methods such as adaptation of biped locomotion [6], learning to walk through imitation [10] and reinforcement learning [12], [5], [9], [13]. But these methods cannot adapt their behavior to the changes in the dynamics of the process and the character of the disturbances [4]. Therefore, adaptive control approaches [14]-[17] are useful.

To address the problem of poorly known actuator dynamic characteristics and unpredictable variations of a biped system, we propose a Lyapunov based model reference adaptive control system (MRAC) method for the biped walking control. In MRAC, the presence of the reference model specifies the plants desired performance. The plant (biped actuator) adapts itself to the reference model (desired dynamics for the actuators). In this paper, reference model output represents the desired torques. The desired torques can be derived from a biped walking controller [1] that closes the loop of the biped motion to that of the desired motion or captured human gait. In the formulation of the MRAC, we will need to estimate the input of the reference model using an input estimator. The resulting *inverse reference model* in effect predicts the needed input to drive the actuator. Lyapunov's stability criterion is used to provide the architecture for parameter tuning. An analysis on determining the converge rate of the MRAC is also discussed. Through this scheme, a robot can learn its behavior through its reference models.

Manuscript received on February 10, 2010.

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II. Problem Description

A. Objective

Consider the objective of controlling a biped robot so that it imitates the movement of a person. Fig. 1 shows the basic idea where the human movement is represented by $\mathbf{y}_d$ and the biped movement by $\mathbf{y}$. The biped motion is determined by the actuators which are controlled by the inputs $\mathbf{u}_a$. The overall objective is to find the adaptive $\mathbf{u}_a$ such that $\mathbf{y} \to \mathbf{y}_d$.

In the present problem, we will consider the case where the actuator dynamics have uncertainties including nonlinearities, unknown parameter values and delays. Actuator dynamics have not been widely addressed. Fig. 2 shows the adaptive actuator objective where the actuator output moment $\mathbf{M}$ is made to follow a required $\mathbf{M}_d$, which will be computed from the desired requirement that

$\mathbf{y}$ tracks $\mathbf{y}_d$. The study presented in this paper deals with the formulation and simulation aspects of the MRAC actuator scheme.

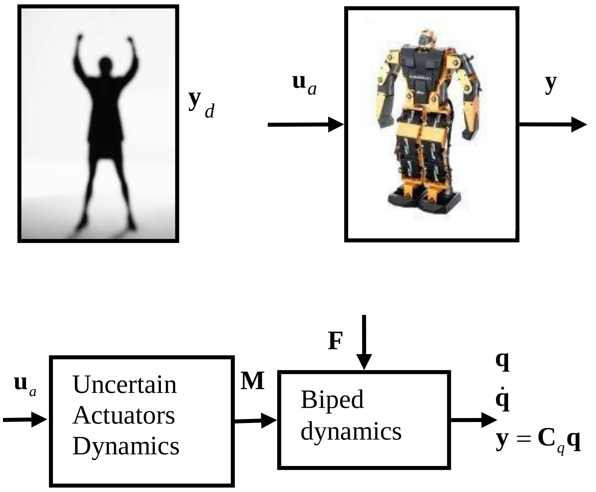

Fig. 1. Human movements, biped robot, its actuators

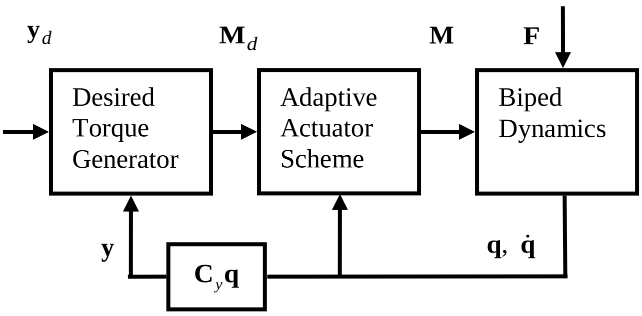

Fig. 2. MRAC scheme for the biped walker.

B. Biped Dynamics

Equations for describing the dynamics of a biped robot were introduced in [1], and can be summarized as follows.

$$\mathbf{A}_q(\mathbf{q})\ddot{\mathbf{q}} = B(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{M}, \mathbf{F}) \tag{1}$$

where $\mathbf{q}$ is the generalized coordinates of the robot, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are the first and second derivatives, $\mathbf{M}$ the moments/torques applied to the joints in the robot and $\mathbf{F}$ the reaction forces at the contact of the robot's feet and ground surface. The relationship (1) has been well studied [1].

C. Uncertain Actuator Dynamics

In the literature, it is assumed that $\mathbf{M}$ can be readily generated without considering the dynamics of the actuators. As an example, if a set of desired torques are calculated as $\mathbf{M}_d$, then it would assume that $\mathbf{M} = \mathbf{M}_d$ and applied directly as inputs to the robot. However, this is not a valid assumption since in practice the moments $\mathbf{M}$ will be generated by actuators which normally have unknown parameters, time delays and nonlinearities. The moments $\mathbf{M}$ can be modeled as the states of

$$\begin{aligned}\dot{\mathbf{x}}_a(t) &= \mathbf{A}_a\mathbf{x}_a(t) + \mathbf{B}_a\mathbf{u}_a(t) + \mathbf{d}_a(\mathbf{q}(t), \dot{\mathbf{q}}(t), \boldsymbol{\tau}_{ext}(t)) \\ \mathbf{M} &= \mathbf{C}_a\mathbf{x}_a\end{aligned} \tag{2}$$

where $\mathbf{u}_a$ are inputs of actuators, τ is the transport delay in the response of the actuator and $\mathbf{d}_a(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau}_{ext})$ represents disturbance torques to the actuators due to robot movements. $\boldsymbol{\tau}_{ext}$ is an external disturbance torque. We assume that the moments/torques $\mathbf{M}$ can be measured; for example, by measuring the currents in motors or pressure in hydraulics. In pre-tuned actuators, we can assume that $\mathbf{M} = \mathbf{x}_a$, i.e., $\mathbf{C}_a = \mathbf{I}$.

D. Desired Moments $\mathbf{M}_d$

The desired moments $\mathbf{M}_d$ can be derived as the output of a controller that operates on $\mathbf{y}_d$ and $\mathbf{y}$. For example,

$$\mathbf{M}_d(s) = \mathbf{G}_c(s)\big(\mathbf{y}_d(s) - \mathbf{y}(s)\big) \tag{3}$$

where s is the Laplace variable, $\mathbf{G}_c(s)$ is the controller transfer function. The controller $\mathbf{G}_c$ is designed to generate the desired moments $\mathbf{M}_d$, required for the adaptive actuator scheme by using the information from $\mathbf{y}$ and $\mathbf{y}_d$.

E. Adaptive Control Approach

The problem here is that we have to deal with unknowns and uncertainties in the dynamics and parameters of the actuators (2). More specifically, we would like the actuator output $\mathbf{M}$ to follow $\mathbf{M}_d$. Defining $\mathbf{e}(t) = \mathbf{M}_d(t) - \mathbf{M}(t)$ as the tracking error, we would like to see that $\mathbf{e}(t) \to 0$ as $t \to \infty$ with asymptotic stable dynamics. An MRAC scheme based on Lyapunov stability synthesis is proposed for dealing with the issue. The MRAC should generate an estimate for the unknown parameters with bounded tracking error and sufficient speed of adaptation in order to maintain the performance requirements of the mimicking robot. Fig. 2 shows the adaptation scheme.

III. Solution

A. Reference Model for Actuator

To apply the MRAC approach to the actuator (2), a reference model for the actuator is needed as follows. The computed desired moments $\mathbf{M}_d$ (3) will be represented as the states of the reference model

$$\begin{aligned}\dot{\mathbf{x}}_m(t) &= \mathbf{A}_m\mathbf{x}_m(t) + \mathbf{B}_m\mathbf{u}_m(t) \\ \mathbf{M}_d(t) &= \mathbf{x}_m(t)\end{aligned} \tag{4}$$

where $\mathbf{A}_m$ and $\mathbf{B}_m$ represents the desired dynamics for the actuator to follow. $\mathbf{u}_m(t)$ represents the command input to the reference model of the actuator and is required for the MRAC.

B. Inverse Model Reference

However, we do not know the input $\mathbf{u}_m$. So the approach here is to estimate the unknown $\mathbf{u}_m$ knowing $\mathbf{x}_m$. The unknown $\mathbf{u}_m$ can be represented as the output of an waveform shaping model, i.e.,

$$\begin{aligned} \dot{\mathbf{x}}_u &= \mathbf{A}_u \mathbf{x}_u + \mathbf{w}_u \\ \mathbf{u}_m &= \mathbf{C}_u \mathbf{x}_u \end{aligned} \tag{5}$$

where $\mathbf{A}_u$ and $\mathbf{C}_u$ represents approximate waveform characteristics and $\mathbf{w}_u$ is a sparse and small shaping input.

The reference model and shaping model can be augmented as

$$\begin{aligned} \dot{\mathbf{x}}_{mu} &= \mathbf{A}_{mu} \mathbf{x}_{mu} + \mathbf{B}_{mu} \mathbf{w}_u \\ \mathbf{x}_m &= \mathbf{C}_{mu} \mathbf{x}_{mu} \end{aligned} \tag{6}$$

where $\mathbf{x}_{mu} = \begin{bmatrix} \mathbf{x}_m \\ \mathbf{x}_u \end{bmatrix}$, $\mathbf{A}_{mu} = \begin{bmatrix} \mathbf{A}_m & \mathbf{B}_m \mathbf{C}_u \\ 0 & \mathbf{A}_u \end{bmatrix}$, $\mathbf{B}_{mu} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}$ and $\mathbf{C}_{mu} = [\mathbf{I} \quad \mathbf{0}]$; $\mathbf{0}$ and $\mathbf{I}$ are null and identity matrix, respectively, of appropriate dimensions. An estimate of the of $\mathbf{u}_m$ can be found using an observer of the form

$$\begin{aligned} \dot{\hat{\mathbf{x}}}_{mu} &= \mathbf{A}_{mu} \hat{\mathbf{x}}_{mu} + \mathbf{K}_{mu} \left[\mathbf{x}_m - \mathbf{C}_{mu} \hat{\mathbf{x}}_{mu} \right] \\ \hat{\mathbf{u}}_m &= [\mathbf{0} \quad \mathbf{C}_m] \hat{\mathbf{x}}_{mu} \end{aligned} \tag{7}$$

where $\mathbf{K}_{mu}$ is chosen such that $\mathbf{A}_{mu} - \mathbf{K}_{mu} \mathbf{C}_{mu}$ has exponentially stable eigenvalues. We refer to (7) as the *inverse reference model*, where the state $\mathbf{x}_m$ is given and the input $\mathbf{u}_m$ is estimated by the inverse model as the output $\hat{\mathbf{u}}_m$. $\hat{\mathbf{u}}_m$ can be interpreted as a prediction of $\mathbf{u}_m$ needed to produce $\mathbf{x}_m$. The dynamics of the prediction error is given by

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}_{mu} &= \left(\mathbf{A}_{mu} - \mathbf{K}_{mu} \mathbf{C}_{mu} \right) \tilde{\mathbf{x}}_{mu} + \mathbf{B}_{mu} \mathbf{w}_u \\ \tilde{\mathbf{u}}_m &= [\mathbf{0} \quad \mathbf{C}_u] \tilde{\mathbf{x}}_{mu} \end{aligned} \tag{8}$$

where $\tilde{\mathbf{u}}_m = \mathbf{u}_m - \hat{\mathbf{u}}_m$.

In the ensuing analysis we will assume that $\mathbf{w}_u$ is negligible so that the $\hat{\mathbf{u}}_m$ generated by the input observer converges to the unknown $\mathbf{u}_m$. $\hat{\mathbf{u}}_m$ will be used as the input to the MRAC to be shown in Section C.

C. MRAC Scheme

(i) Configuration of MRAC Actuator

The adaptive actuator scheme is shown in Fig 3 below, where the reference model is specified by

$$\dot{\mathbf{x}}_m = \mathbf{A}_m \mathbf{x}_m + \mathbf{B}_m \hat{\mathbf{u}}_m \tag{9}$$

and the control to the actuator by,

$$\mathbf{u}_a = \mathbf{L} \mathbf{x}_a + \mathbf{N} \hat{\mathbf{u}}_m \tag{10}$$

The adaptation algorithm in the MRAC will adjust the gains $\mathbf{L}$ and $\mathbf{N}$ based on Lyapunov stability criteria as follows.

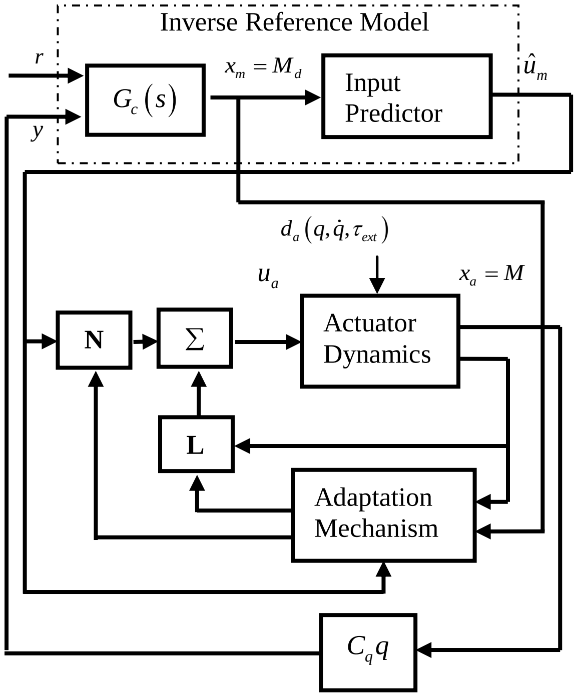

Fig.3. The MRAC with the input Predictor

(ii) Error Dynamics

Define the errors $\mathbf{e}$ between the actuator torque $\mathbf{x}_a = \mathbf{M}$ and the desired torque $\mathbf{x}_m = \mathbf{M}_d$ as

$$\mathbf{e} = \mathbf{x}_m - \mathbf{x}_a \tag{11}$$

It can be shown that

$$\begin{aligned} \dot{\mathbf{e}} = \mathbf{A}_m \mathbf{e} &+ \left[\mathbf{A}_m - \mathbf{A}_a - \mathbf{B}_a \mathbf{L} \right] \mathbf{x}_a \\ &+ \left[\mathbf{B}_m - \mathbf{B}_a \mathbf{N} \right] \hat{\mathbf{u}}_m - \mathbf{d}_a(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau}_{ext}) \end{aligned} \tag{12}$$

(iii) Lyapunov Stability Analysis

Define a candidate for a Lyapunov function as

$$\begin{aligned} v &= \mathbf{e}^T\mathbf{P}\mathbf{e} \\ &+ trace\left(\mathbf{A}_m - \mathbf{A}_a - \mathbf{B}_a\mathbf{L}\right)^T \mathbf{Q}\left(\mathbf{A}_m - \mathbf{A}_a - \mathbf{B}_a\mathbf{L}\right) \\ &+ trace\left(\mathbf{B}_m - \mathbf{B}_a\mathbf{N}\right)^T \mathbf{R}\left(\mathbf{B}_m - \mathbf{B}_a\mathbf{N}\right) \end{aligned} \tag{13}$$

where $\mathbf{P} = \mathbf{P}^T > 0$, $\mathbf{Q} = \mathbf{Q}^T > 0$ and $\mathbf{R} = \mathbf{R}^T > 0$ are positive definite matrices. Then

$$\begin{aligned} \dot{v} &= \dot{\mathbf{e}}^T\mathbf{P}\mathbf{e} + \mathbf{e}^T\mathbf{P}\dot{\mathbf{e}} \\ &+2\left(trace\left(\mathbf{A}_m - \mathbf{A}_a - \mathbf{B}_a\mathbf{L}\right)^T \mathbf{Q}\left(\mathbf{B}_a\dot{\mathbf{L}}\right)\right) \\ &+2\left(trace\left(\mathbf{B}_m - \mathbf{B}_a\mathbf{N}\right)^T \mathbf{R}\left(-\mathbf{B}_a\dot{\mathbf{N}}\right)\right) \\ &= \mathbf{e}^T\left[\mathbf{P}\mathbf{A}_m + \mathbf{A}_m^T\mathbf{P}\right]\mathbf{e} - 2\mathbf{e}^T\mathbf{A}_m^T\mathbf{d}_a(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\tau}_{ext}) \\ &+2\left(trace\left(\mathbf{A}_m - \mathbf{A}_a - \mathbf{B}_a\mathbf{L}\right)^T \left(\mathbf{P}\mathbf{e}\mathbf{x}_a^T + \mathbf{Q}\left(\mathbf{B}_a\dot{\mathbf{L}}\right)\right)\right) \\ &+2\left(trace\left(\mathbf{B}_m - \mathbf{B}_a\mathbf{N}\right)^T \left(\mathbf{P}\mathbf{e}\hat{\mathbf{u}}_m^T + \mathbf{R}\left(-\mathbf{B}_a\dot{\mathbf{N}}\right)\right)\right) \end{aligned} \tag{14}$$

From inspection we choose

$$\begin{aligned} \mathbf{B}_a\dot{\mathbf{L}} &= \mathbf{Q}^{-1}\mathbf{P}\mathbf{e}\mathbf{x}_a^T \\ \mathbf{B}_a\dot{\mathbf{N}} &= -\mathbf{R}^{-1}\mathbf{P}\mathbf{e}\hat{\mathbf{u}}_m^T \end{aligned} \tag{15}$$

so that

$$\dot{v} = \mathbf{e}^T\left[\mathbf{P}\mathbf{A}_m + \mathbf{A}_m^T\mathbf{P}\right]\mathbf{e} - 2\mathbf{e}^T\mathbf{A}_m^T\mathbf{d}_a(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\tau}_{ext}) \tag{16}$$

We next choose an $\mathbf{S} = \mathbf{S}^T > 0$ and solve $\mathbf{P}$ from

$$\mathbf{P}\mathbf{A}_m + \mathbf{A}_m^T\mathbf{P} = -\mathbf{S} \tag{17}$$

We arrive at

$$\dot{v} = -\mathbf{e}^T\mathbf{S}\mathbf{e} + 2\mathbf{e}^T\mathbf{A}_m^T\mathbf{d}_a(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\tau}_{ext}) \tag{18}$$

where $\dot{v}$ is negative under the assumption that

$$\mathbf{e}^T\mathbf{S}\mathbf{e} > 2\mathbf{e}^T\mathbf{A}_m^T\mathbf{d}_a(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\tau}_{ext}) \tag{19}$$

A sufficient condition to satisfy (19) is

$$\left(\boldsymbol{e}^T\boldsymbol{e}\right) > 2\frac{\lambda_{\max}\left[\textbf{-}\boldsymbol{A}_m\right]}{\lambda_{\min}\left[\boldsymbol{S}\right]}\left(\boldsymbol{e}^T\boldsymbol{d}_a(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\tau}_{ext})\right) \tag{20}$$

which implies that the magnitude of error should be larger than the disturbance. In practice this means that the system should be persistently excited. Hence we conclude that the overall dynamic system comprising of (12) and (15) has a candidate function that satisfies the Lyapunov stability criterion under condition (19).

IV. MRAC for Walking Biped Actuators

A. Dynamics of Walking Biped

The bipedal model in this project has five links connected by 4 pin joints as shown in the Fig. 4a One link represents the upper body and two links are for each lower limb. The biped has two hip joints, two knee joints and two ankles at the tips of the lower limbs. There is an actuator located at each join and all of the joints are considered rotating only in the sagittal plane. As the system can move freely in the x-y-plane and contains five links, it has seven degrees of freedom. The corresponding seven coordinates are selected according to Fig. 4a as

$$\mathbf{q} = \left[x_o, y_o, \alpha, \beta_L, \beta_R, \gamma_L, \gamma_R\right]^T \tag{21}$$

The coordinates $\left(x_o, y_o\right)$ fix the position of the center of mass of the torso, and the rest of the coordinates describe the joint angles. The link lengths are denoted as $\left(h_0, h_1, h_2\right)$ and masses as $\left(m_0, m_1, m_2\right)$. The centers of mass of the links are located at the distances $\left(r_0, r_1, r_2\right)$ from the corresponding joints.

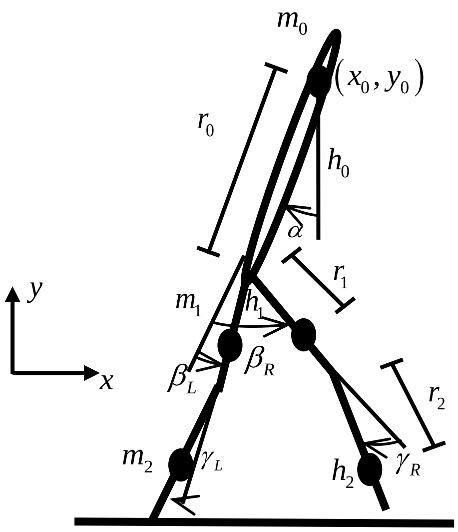

Fig. 4a. Biped robot with the corresponding 7 co-ordinates $\mathbf{q} = \left[x_o, y_o, \alpha, \beta_L, \beta_R, \gamma_L, \gamma_R\right]^T$

The model is actuated with four moments two of them acting between the torso and both thighs and two at the knee joints, which is shown in Fig. 4b.

$$\mathbf{M} = \left[M_{L1}, M_{R1}, M_{L2}, M_{R2}\right]^T \tag{22}$$

The walking surface is modeled using external forces that affect both leg tips shown in Fig. 4b.

$$\mathbf{F} = \left[F_{Lx}, F_{Ly}, F_{Rx}, F_{RY}\right] \tag{23}$$

When the leg should touch the ground, the corresponding forces are switched on to support the leg. As the leg rises, the forces are zeroed.

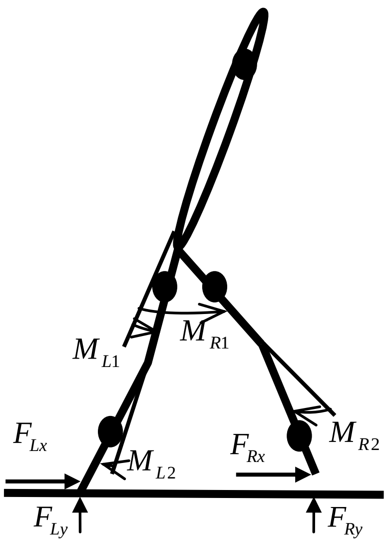

Fig. 4b. Biped robot with the moments $\mathbf{M} = \left[M_{L1}, M_{R1}, M_{L2}, M_{R2}\right]^T$ and the walking surface external forces $\mathbf{F} = \left[F_{Lx}, F_{Ly}, F_{Rx}, F_{RY}\right]$

Using Lagrangian mechanics, the dynamic equations for the biped system can be derived as shown in (1) where $\mathbf{A(q)} \in \Re^{7X7}$ is the inertia matrix and $\mathbf{b}\left(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{M}, \mathbf{F}\right) \in \Re^{7X1}$ is a vector containing the right hand sides of the seven partial differential equations. The closed form formulas for both **A** and **b** are presented in [1].

B. Computation of Desired Moments

The desired human movement angles will be measured and represented by

$$\mathbf{y}_d = [\alpha_m \quad \left(\beta_{Lm} - \beta_{Rm}\right) \quad \gamma_{Lm} \quad \gamma_{Rm}]^T \tag{24}$$

An output **y** is constructed from the biped feedback information **q** as

$$\mathbf{y} = [\alpha \quad \left(\beta_L - \beta_R\right) \quad \gamma_L \quad \gamma_R]^T = C_y \mathbf{q} \tag{25}$$

The desired torque is derived as the output of a controller that operates on the error between **r** and **y**. That is

$$\mathbf{M}_d(s) = \mathbf{G}_c(s)\left(\mathbf{r}(s) - \mathbf{y}(s)\right) \tag{26}$$

$$\mathbf{M}_d = \left[M_{d1} \;\; M_{d2} \;\; M_{d3} \;\; M_{d4}\right]^T \tag{27}$$

C. Dynamics of Actuators

We note that the actuator states $\mathbf{x}_a = \mathbf{M}$ are the torques that will drive the biped robot. The goal is to find $\mathbf{u}_a$ such that $\mathbf{M} \to \mathbf{M}_d$. We assume that dc motors are used as actuators. It follows that (2) can be decoupled into individual motors representing the first-order stator-rotor dynamics (a_{ai}, b_{ai}) that generates output torque (x_{ai}) while subject to disturbance (d_{ai}), that is

$$\begin{aligned}
\mathbf{x}_a &= \left[x_{a1} \;\; x_{a2} \;\; x_{a3} \;\; x_{a4}\right]^T \\
\mathbf{u}_a &= \left[u_{a1} \;\; u_{a2} \;\; u_{a3} \;\; u_{a4}\right]^T, \\
\mathbf{A}_a &= diag\left\{a_{a1},\, a_{a2},\, a_{a3},\, a_{a4}\right\} \\
\mathbf{B}_a &= diag\left\{b_{a1} \;\; b_{a2} \;\; b_{a3} \;\; b_{a4}\right\}^T \\
\mathbf{d}_a(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau}_{ext}) &= \left[d_{a1} \quad d_{a2} \quad d_{a3} \quad d_{a4}\right]^T
\end{aligned}$$

$\mathbf{A}_a$ $\mathbf{B}_a$ and $\mathbf{d}_a$ are the uncertain parameters vectors.

D. Inverse Reference Model

Estimation of $\mathbf{u}_m$ can be obtained from the input estimator given by (7)

$$\hat{\mathbf{u}}_m = \left[\hat{u}_{m1} \;\; \hat{u}_{m2} \;\; \hat{u}_{m3} \;\; \hat{u}_{m4}\right]^T \tag{28}$$

From (5),

$$\begin{aligned}
&\mathbf{A}_u = \begin{bmatrix} 0 & I_4 \\ 0 & 0 \end{bmatrix}, \; \mathbf{C}_u = \left[I_4 \quad 0\right], \\
&\mathbf{A}_m = diag([-20 \;\; -20 \;\; -20 \;\; -20]), \\
&\mathbf{B}_m = diag([-20 \;\; -20 \;\; -20 \;\; -20]).
\end{aligned}$$

The adaptation gain matrix K_{mu} is a Hurwitz matrix with stable eigenvalues $\mathrm{Re}[s] < 0$ to ensure stability for the tracking error, the choice of the eigenvalues depends on the application.

E. Configuration of MRAC Actuator

The control is specified by

$$\mathbf{u}_a = \mathbf{L}\mathbf{x}_a + \mathbf{N}\hat{\mathbf{u}}_m \tag{29}$$

where,

$$\begin{aligned}
\mathbf{L} &= diag\left\{l_1, \quad l_2, \quad l_3, \quad l_4\right\} \\
\mathbf{N} &= diag\left\{n_1, \quad n_2, \quad n_3, \quad n_4\right\}
\end{aligned}$$

It follows from the Lyapunov design that the gains $\mathbf{L} = diag\{l_1,\ l_2,\ l_3,\ l_4\}$ and $\mathbf{N} = diag\{n_1,\ n_2,\ n_3,\ n_4\}$ are adjusted according to

$$\begin{aligned} l_i &= \frac{1}{b_{ai}q_i} p_i (M_{di} - x_{ai}) x_{ai} \\ n_i &= -\frac{1}{b_{ai}r_i} p_i (M_{di} - x_{ai}) \hat{u}_{mi} \end{aligned} \qquad (30)$$

F. Convergence Analysis of MRAC

The convergence of the MRAC depends on the following dynamics

$$\begin{aligned} \dot{e}_i &= a_{mi} e_i + b_{ai} x_{ai} \tilde{l}_i - b_{ai} \hat{u}_{mi} \tilde{n}_i - d_{ai}(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau}_{ext}) \\ \dot{\tilde{l}}_i &= -\frac{p_i x_{ai}}{b_{ai} q_i} e_i \\ \tilde{n}_i &= \frac{p_i \hat{u}_{mi}}{b_{ai} r_i} e_i \end{aligned} \qquad (31)$$

where $e_i = x_m - x_{ai}$, $\tilde{l}_i = l_i^* - \hat{l}_i$ & $\tilde{n}_i = n_i^* - \hat{n}_i$, and l_i^* and n_i^* are the true value of parameters l_i and n_i. The convergence dynamics is characterized by the polynomial

$$\lambda^3 + c_1\lambda^2 + c_2\lambda + c_3 = 0$$

where $c_1 = -a_{mi}$, $c_2 = \dfrac{p_i x_{ai}^2}{q_i} + \dfrac{p_i \hat{u}_{mi}^2}{r_i}$ and $c_3 = 0$.

We would assign c_1 and c_2 to define the convergence characteristics. c_1 in turn defines a_{mi}. We next choose a large value for s_i to ensure that the Lyapunov rate is satisfied,

$$\dot{v}_i = -s_i e_i^2 - 2a_{mi} e_i d_{ai}(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau}_{ext}) < 0 \qquad (32)$$

We can now compute p_i, q_i, and r_i for the algebraic Lyapunov function as

$$\begin{aligned} p_i &= -\frac{s_i}{2a_{mi}} \\ q_i &= \frac{2p_i x_a^2}{c_2} \\ r_i &= \frac{2p_i u_m^2}{c_2} \end{aligned}$$

The analysis here was used to assist tuning of the parameters.

V. Simulation Results

The proposed control scheme for the 5 link biped model is simulated with the MRAC scheme. The biped is excited with a walking cycle that is manually developed and provided as the desired human gait. This walking cycle is repeated for every 1.5 seconds. A display of the biped walking cycle is shown in Fig. 5.1.

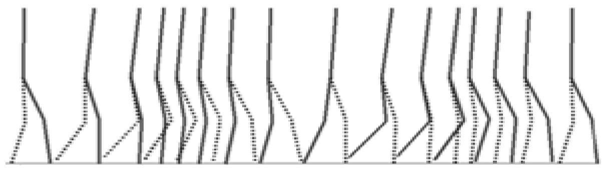

Fig. 5.1. A 2-step animation of biped walking on the sagittal plane

In the simulation, the desired movement angles $\mathbf{y}_d = [\alpha_m \ \ (\beta_{Lm} - \beta_{Rm}) \ \ \gamma_{Lm} \ \ \gamma_{Rm}]^T$ are given (24). The biped movements $\mathbf{y} = [\alpha \ \ (\beta_L - \beta_R) \ \ \gamma_L \ \ \gamma_R]^T$ (25) are measured and used to generate the desired torques $\mathbf{M_d} = [M_{d1}\ M_{d2}\ M_{d3}\ M_{d4}]^T$ (27). Equations (28), (29) and (30) were implemented as the MRAC scheme. Two sets of simulation runs are shown below. The first simulation does not include the disturbance torque $\mathbf{d}_a(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau}_{ext})$. We introduced the disturbance in the second simulation.

A. First Simulation (without disturbance)

(i) Results of the feedback biped movement angles $\mathbf{y}$ *and desired biped movement angles* $\mathbf{y}_d$

Fig. 5.2 to Fig. 5.5 shows the output states $\mathbf{y}$ plotted against desired output states $\mathbf{y}_d$. It can be observed that the both the desired and observed output states of the biped converged smoothly.

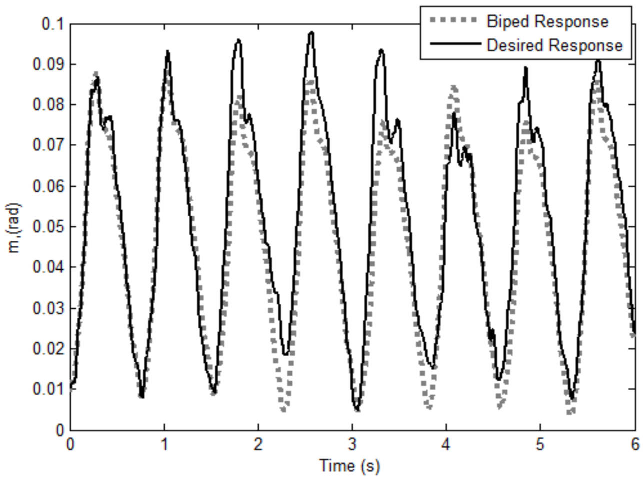

Fig. 5.2. Biped torso angle α plotted against the desired torso angle α_m

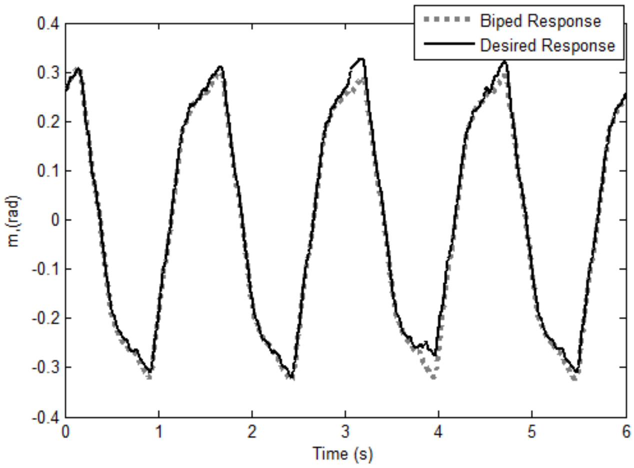

Fig. 5.3. Biped thigh angles $(\beta_L - \beta_R)$ plotted against the desired thigh angles $(\beta_{Lm} - \beta_{Rm})$

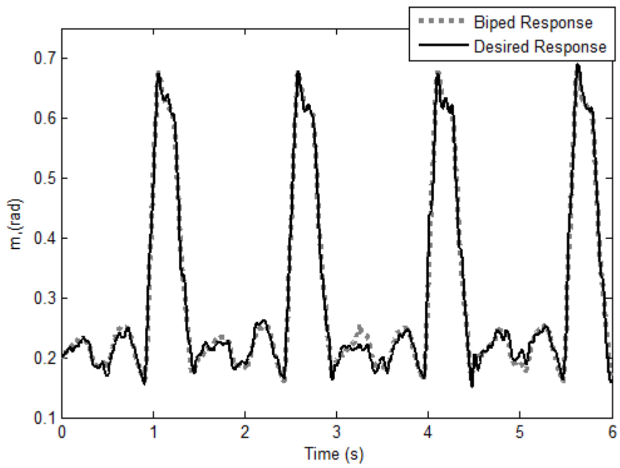

Fig. 5.4. Biped left leg knee angle γ_L plotted against the desired left leg knee angle γ_{Lm}

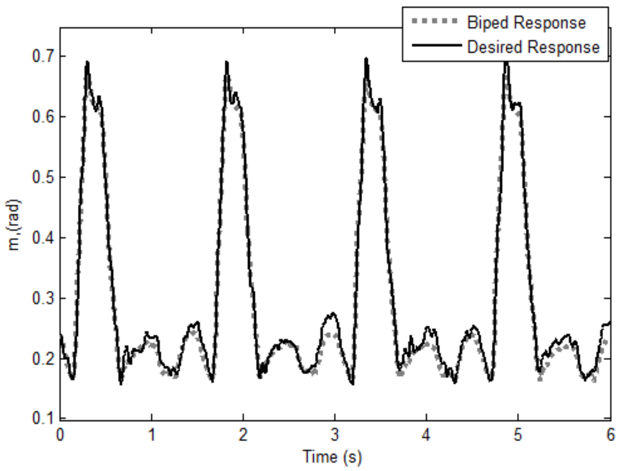

Fig. 5.5. Biped right leg knee angle γ_R plotted against the desired right leg knee angle γ_{Rm}

(ii) Results of the biped moments $\mathbf{M}$ *and desired biped moments* $\mathbf{M}_d$

Fig. 5.6 to Fig 5.9 show the torques $\mathbf{M} = [M_{L1} \quad M_{R1} \quad M_{L2} \quad M_{R2}]^T$ converging to the desired torques $\mathbf{M}_d = [M_{d1} \; M_{d2} \; M_{d3} \; M_{d4}]^T$ for one step cycle. It can be noted that the biped moments $\mathbf{M}$ converges to the reference model's behavior immediately.

Therefore this gives the biped walker a human like gait as described by $\mathbf{M}_d$.

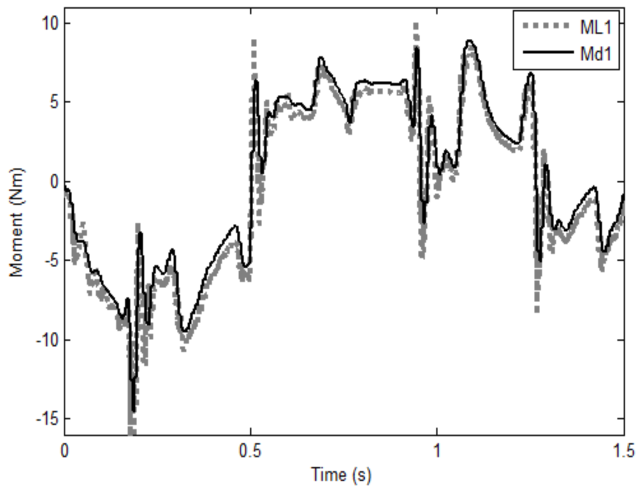

Fig. 5.6. Response of M_{L1} tracking M_{d1}

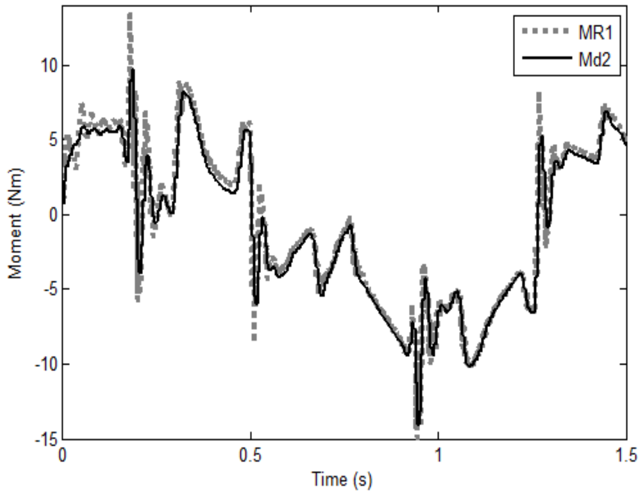

Fig. 5.7. Response of M_{R1} tracking M_{d2}

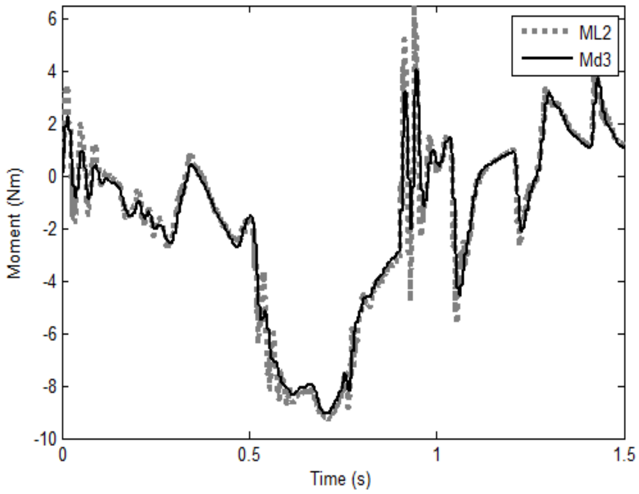

Fig. 5.8. Response of M_{L2} tracking M_{d3}

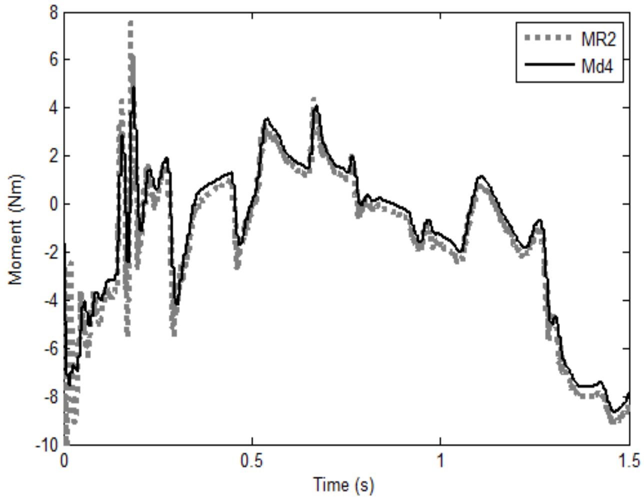

Fig. 5.9. Response of M_{R2} tracking M_{d4}

(iii) Results of the biped walking simulation

Fig. 5.10 plots the stable torso height y_o (see Fig 5) of the biped walking as the result of the adapted torques M.

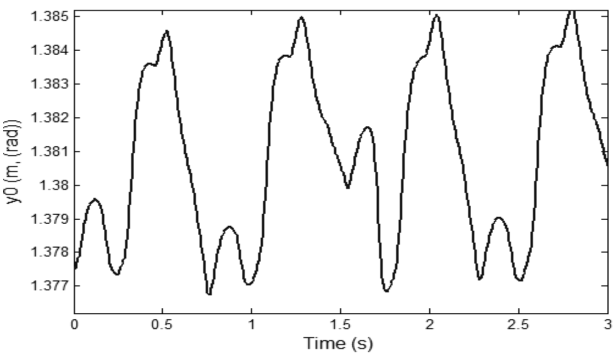

Fig. 5.10. Height of the biped torso y_o

(iv) Results of the MRAC gain adaptation

The value of $a_{ai} = -10$ and $b_{bi} = 10$ were used to specify the actuator model, while $a_{mi} = -100$ and $b_{mi} = 100$ were used for the reference model (Sec. IV.C). These values express that we would like to adapt the (slower) actuator to response like the (faster) reference model. Since we want $\frac{b_{ai} n_i}{s + a_{ai} + b_{ai} l_i}$ converge to $\frac{b_{mi}}{s + a_{mi}}$, it can be shown that the expected $n_i = 10$, and $l_i = 9$.

Fig. 5.11 and 5.12 show the case for i = 1.

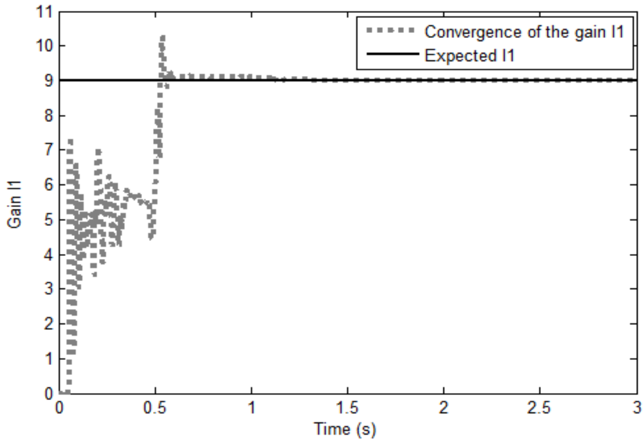

Fig. 5.11. Convergence of the gain l_1 to approximately the expected value 9

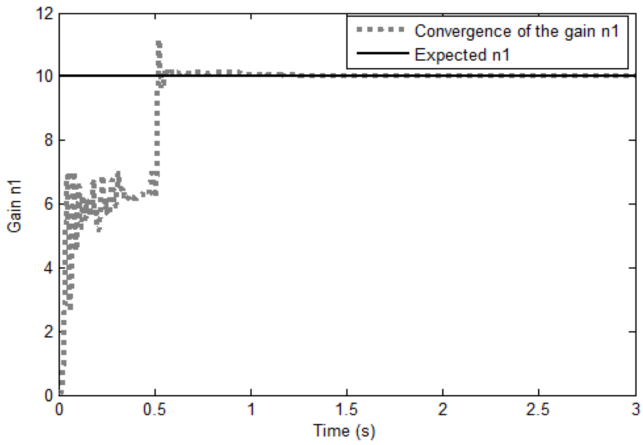

Fig. 5.12. Convergence of the gain n_1 to approximately the expected value 10

B. Second Simulation (disturbance)

(i) Results of the biped walking with torque disturbances

Equation (2) includes disturbance torques $\mathbf{d}_a(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau}_{ext})$ which represents torque feedback and other external moments. This causes the dynamics of the actuator to vary.

Fig. 5.13a and Fig. 5.13b show the biped moment M_{L1}, and biped torso height y_o, recovering from the impact due to the external disturbance to the biped walker, introduced by four impulses at 0.05 (s), 0.3 (s), 1.34 (s), and 2.2 (s) with a magnitude of 5 Nm and 6 Nm as shown in Fig. 5.13c.

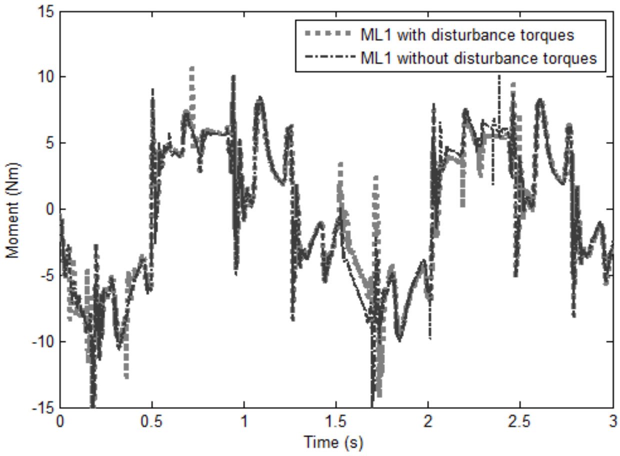

Fig. 5.13a. M_{L1} with and without disturbance torques $\mathbf{d}_a(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau}_{ext})$, simulated over 3 (s)

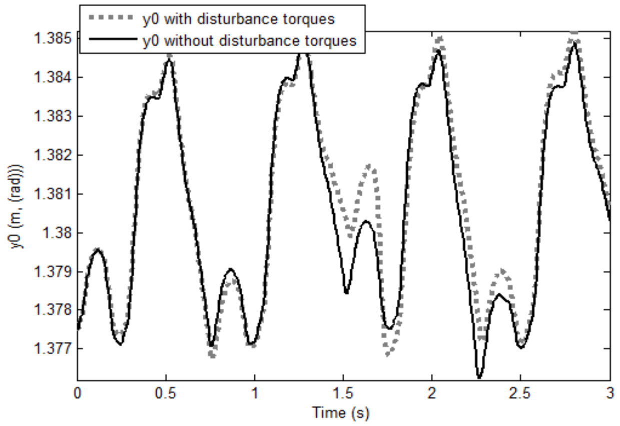

Fig. 5.13b. Height of the biped torso with and without disturbance torques $\mathbf{d}_a(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau}_{ext})$, simulated over 3 (s)

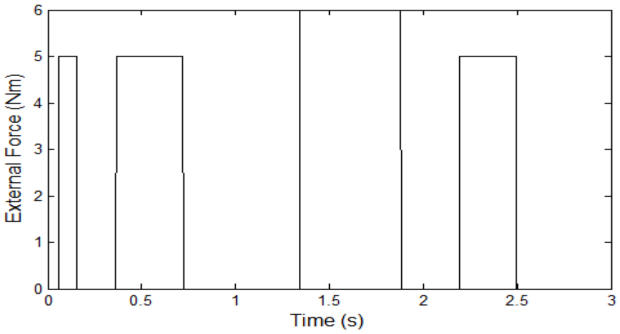

Fig. 5.13c. Simulated external disturbance to (2), at t=0.05 (s), t=0.3 (s), t=1.34 (s), and t=2.2 (s)

VI. Conclusions

In this paper, we presented an MRAC technique to ensure that the actuators faithfully produce desired torques necessary for a walking robot. An observer was used to predict an anticipated state of the desired torque, thus causing the adaptive actuators to anticipate motion torques. We provided a proof to show that the MRAC scheme results in a stable system in the sense of Lyapunov when errors between the desired and actuator torques are significant. Also, the convergence analysis for tuning p, q, and r is provided. Simulation results verify that the system is robust when tested with various parameters and unknown coefficients. We plan to implement the MRAC scheme with the transport and time delays in our future work.

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