The Effect of Temperature Nonuniformities on Transient Behaviour of Three-Fluid Crossflow Heat Exchanger

Manish Mishra, and P.K.Sahoo

Abstract—A transient temperature response of three-fluid heat exchangers with finite core capacitance of the separating sheets is investigated numerically for the effect of temperature nonuniformity. Step perturbation is provided in the central (hot) fluid inlet temperature. The responses are found dependent on the nonuniformities present in the inlet temperature.

Index Terms — finite difference, temperature nonuniformity, three-fluid heat exchanger, transient behaviour.

I. INTRODUCTION

Three fluid and multi-fluid heat exchangers are widely used in cryogenics and different chemical processes, such as air separation, helium separation from natural gas, purification and liquefaction of hydrogen, and ammonia gas synthesis. Three fluid heat exchangers allow a more compact and economical design in various other applications also.

A wide literature is available on the steady state behavior of three fluid heat exchangers [1]. A pioneering effort in analyzing crossflow problem considering it to be a case of heat transfer with three thermal streams has been given by Rabinovich [2]. A general theory for two temperature effectiveness of three-fluid heat exchangers of parallel and counter flow type [3] and analysis with three thermal communications [4] was also presented. Further, a numerical method was used by Barron and Yeh [5] including the effect of longitudinal conduction of both the separating walls. Four possible arrangements for parallel and counterflow three-fluid heat exchanger with two thermal communications were presented by Sekulic and Kmecko [6]. Willis and Chapman [7] made an effort to present the performance of a three-fluid crossflow heat exchanger graphically in terms of the temperature effectiveness. An exact analytical solution of three-fluid crossflow heat exchanger was first tried by Baclie et al. [8] for unmixed flow arrangements using Laplace transforms. Sekulic and Shah [9] gave a very comprehensive review of methodologies for analyzing the steady state performance of three fluid heat exchangers for parallel, counter and crossflow arrangements. Further, the effect of longitudinal conduction in wall on thermal performance of three-fluid crossflow heat exchanger and a comparison of performance for different arrangements were numerically calculated by Yuan and Kou [10]-[12].

Although heat exchangers mostly operate under steady state conditions, steady state analysis is not adequate for situations like start-up, shutdown, failure and accidents. The transient response of heat exchangers needs to be known for designing control strategies and for taking care of thermal stresses in mechanical design. This has motivated to determine transient temperature fields and a few analytical, semi-analytical and numerical works [13]-[15] have been performed on dynamic behavior of three-fluid heat exchangers.

The case of nonuniform inlet temperature was first taken up by Kou and Yuan [16] for finding out the effects of longitudinal conduction for the two-fluid heat exchanger under steady state conditions. Ranganayakulu and Seetharamu [17]-[18] have also given the combined effect of longitudinal conduction, flow and temperature non-uniformity on steady state performance of crossflow plate-fin heat exchangers for two fluids. The effect of different flow maldistribution models on the thermal performance of three-fluid crossflow heat exchanger under steady state conditions has been studied by Yuan [19].

In the present work, the transient temperature response of the three-fluid crossflow heat exchanger having finite core capacity with all the fluids unmixed is investigated numerically for step change in the central fluid inlet temperature. Temperature nonuniformity has been considered only in the central fluid.

II. MATHEMATICAL MODELING

A direct-transfer, three-fluid, crossflow plate-fin heat exchanger with four possible arrangements are depicted in Fig. 1. For the mathematical analysis the two separating sheets having one fluid on either side are taken separately [Fig. 2(a)].

The following assumptions are made for the analysis.
1. The fluids are assumed single phase and unmixed without any source of heat generation.
2. The thermo-physical properties of the fluid streams and the walls are constant and uniform.
3. The central fluid is either the hottest or the coldest fluid.

Manuscript received March 18, 2010.

M.Mishra is with Indian Institute of Technology, Roorkee, INDIA-247667 (phone: +91-1332-285135; fax: +91-1332-285665/273560; e-mail: mishra_md@yahoo.com).

P.K.Sahoo, is with Indian Institute of Technology, Roorkee, INDIA-247667 (e-mail: sahoo_fme@iitr.ernet.in).

(Advance online publication: 19 August 2010)
4. The exchanger shell or shroud is adiabatic and the effects of the asymmetry in the top and bottom layers are neglected.

5. Flow is well mixed in any of the passages, so that variation of temperature and velocity in the fluid streams in a direction normal to the separating plate (z-direction) is negligible. Temperature non-uniformity present in the central fluid is assumed to be one-dimensional.

6. The primary and secondary areas of the separating plates have been lumped together, so that the variation of wall temperature is also two-dimensional.

7. Transverse conduction through fins between adjacent separating sheets is neglected. This implies there will be a temperature extremum in the fin temperature profile.

8. The thermal resistances on both sides, comprising film heat transfer coefficient of primary and secondary surface and fouling resistance, are constant and uniform.

9. Heat transfer area per unit base area and surface configurations are constant.

10. Transverse thermal resistance of the separating sheets in a direction normal to it (z-direction) is negligible.

Due to the introduction of a third fluid the process of energy exchange in a three-fluid heat exchanger is more complex compared to that in a conventional heat exchanger. The central fluid stream exchanges heat simultaneously with two adjacent streams. The exact distribution of this thermal energy plays an important role in steady state as well as in the dynamic behavior of the heat exchanger. This distribution depends upon the conditions of all the three fluids and the total area associated with them. As the thermo-physical properties of the top and the bottom fluid streams may be different in a general situation, it is likely that the two separating sheets will have different temperatures, and the fins in the central passage will have an asymmetric temperature profile. This indicates that the central stream may transfer heat to the top and the bottom separating sheets at different rates. To take care of this phenomenon it is assumed that part of the secondary surface is associated with the top separating sheet (w1), and the rest is associated with the bottom separating sheet (w2). This idealization is depicted in Fig. 2 (a) and 2 (b).

Assuming \((\eta hA)_{b-w1}\) and \((\eta hA)_{b-w2}\) are the convective conductances associated with the top and the bottom separating sheet respectively, the following relationship can be obtained.

\[
\frac{1}{(\eta hA)_{b-w1}} + \frac{1}{(\eta hA)_{b-w2}} = \frac{1}{(\eta hA)_b}
\]  

(1)

A non-dimensional parameter \(\phi\) as defined in (2) may be introduced.

\[
\frac{(\eta hA)_{b-w1}}{(\eta hA)_b} = \frac{1}{\phi}, \quad \text{and} \quad \frac{(\eta hA)_{b-w2}}{(\eta hA)_b} = \frac{1}{(1-\phi)}
\]

(2)

Proceeding with the same logic it may be assumed that the total thermal capacity of the separating sheets is also distributed amongst the upper and the lower sheet in the ratio \(\psi\) and \((1-\psi)\) respectively.

\[
(Mc)_{w1} + (Mc)_{w2} = (Mc)_w
\]

(3)

Then,

\[
\frac{(Mc)_{w1}}{(Mc)_w} = \psi, \quad \text{and} \quad \frac{(Mc)_{w2}}{(Mc)_w} = (1-\psi)
\]

(4)

The conservation of energy for the three fluid streams and the two separating sheets can be expressed in non-dimensional form for an infinitesimal small control volume as follows.

---

Fig. 1. FOUR POSSIBLE ARRANGEMENTS FOR THREE-FLUID SINGLE-PASS CROSSFLOW HEAT EXCHANGER [9].

Fig. 2. SCHEMATIC REPRESENTATION OF (a) FLOW AND SEPARATING SHEET WITH FINS, AND (b) DISTRIBUTION OF CONVECTIVE RESISTANCE OF FLUID b AND THE HEAT CAPACITY OF THE SEPARATING SHEET WITH FINS.

Assuming \((\eta hA)_{b-w1}\) and \((\eta hA)_{b-w2}\) are the convective conductances associated with the top and the bottom separating sheet respectively, the following relationship can be obtained.
For fluid streams a, b and c one gets (5), (6) and (7) respectively.

\[
\frac{V_c}{R_{ab}} \frac{\partial T_c}{\partial \theta} = T_{u_2} - T_a - \frac{E_{ab} \frac{\partial T_a}{\partial Y}}{Pe_a R_{ab} \frac{\partial \theta}{\partial Y^2}} + N_a E_{ab} \frac{\partial^2 T_a}{\partial Y^2}
\]

(5)

\[
V_b \frac{\partial T_b}{\partial \theta} = \frac{1}{\phi} (T_{s_1} - T_b) + \frac{1}{1-\phi} (T_{u_1} - T_b) - \frac{\partial T_b}{\partial X} + N_a \frac{\partial^2 T_b}{\partial X^2}
\]

(6)

When fluid c is moving in x-direction (C1, C2)

\[
\frac{V_c}{R_{ab}} \frac{\partial T_c}{\partial \theta} = T_{u_2} - T_c - \frac{E_{ab} \frac{\partial T_c}{\partial Y}}{Pe_c R_{ac} \frac{\partial \theta}{\partial Y^2}} + N_c E_{ab} \frac{\partial^2 T_c}{\partial Y^2}
\]

(7a)

When fluid c is moving in y-direction (C3, C4)

\[
\frac{V_c}{R_{ab}} \frac{\partial T_c}{\partial \theta} = T_{u_2} - T_c - \frac{E_{ab} \frac{\partial T_c}{\partial Y}}{Pe_c R_{ac} \frac{\partial \theta}{\partial Y^2}} + N_c E_{ab} \frac{\partial^2 T_c}{\partial Y^2}
\]

(7b)

Similarly for the two separating sheets w1 and w2, heat conduction equations are given in (8) and (9) respectively.

\[
\alpha \frac{\partial T_{wi}}{\partial \theta} = R_{wi}(T_{wi} - T) + \frac{1}{\phi} (T_{s_1} - T_{wi}) + \lambda_T \frac{\partial^2 T_{wi}}{\partial X^2} + \lambda_Y \frac{\partial^2 T_{wi}}{\partial Y^2}
\]

(8)

\[
(1-\alpha) \frac{\partial T_{wi}}{\partial \theta} = \frac{1}{1-\phi} (T_{u_1} - T_{wi}) + R_{wi}(T_{wi} - T) + \lambda_T \frac{\partial^2 T_{wi}}{\partial X^2} + \lambda_Y \frac{\partial^2 T_{wi}}{\partial Y^2}
\]

(9)

Here Pe is axial dispersive Peclet number given by (mc)A_D

where D is diffusion coefficient of the fluid representing the effect of axial dispersion.

It may be noted that five non-dimensional parameters for the steady state performance of a three-fluid heat exchanger were specified in earlier work [9]. On the other hand one needs nine non-dimensional parameters namely NTU, Ea-b, Ec-b, Ra-b, Rc-b, Va,b,c and Tc,in. The extra parameters are necessary in the present case to take care of the thermal capacity of the fluid streams and the separating sheets.

Further, one can introduce the number of transfer units (NTU) for the three-fluid heat exchanger and replace Na in terms of NTU. Conventionally NTU for three-fluid heat exchanger is defined considering the thermal interaction of the central fluid stream with any one of the streams [9]. According to this convention NTU can be defined as follows.

\[
NTU = \left( mc \right) \left\{ \frac{1}{(\eta A)_{a-w_1}} + \frac{1}{(\eta A)_{a-w_2}} \right\}^{-1}
\]

or

\[
\frac{1}{NTU} = C_a \left[ \frac{\phi}{(\eta A)_a} + \frac{1}{(\eta A)_a} \right]
\]

(10)

From the definition of R_{ab} and E_{ab}, it can be shown that

\[
Na = E_{a,b} \cdot NTU \left[ \phi + \frac{1}{R_{a,b}} \right]
\]

(11)

Equations (5)-(9) are subjected to following initial and boundary conditions.

\[
T_{a,Y,0} = T_{b,Y,0} = T_{c,Y,0} = T_{w,0} = 0
\]

(12)

(Advance online publication: 19 August 2010)
where $\mu$ is assumed to be unity.

III. TEMPERATURE NONUNIFORMITY

The hot and cold fluids enter their respective layers of the core by the header and flow distributors. In general the inlet temperatures of all the fluids are assumed to be uniform. Various researchers have considered the thermal performance of crossflow heat exchanger with uniform inlet temperatures. Many a times the fluid entering to the core have more than one stream and the complete mixing does not take place before entering the heat exchanger. The inlet temperature becomes nonuniform when two or more fluid streams at different temperatures enter into the heat exchanger core without complete mixing. The steady state thermal performance is affected due to nonuniformity of temperature [16]. At the same time its effect cannot be ignored in transient state also. To examine the effect of inlet temperature distribution on the transient performance of the heat exchanger three different cases have been considered. In all the three cases, the mean inlet temperature of the hot central fluid becomes nonuniform when two or more fluid streams at different temperatures enter into the heat exchanger core by the header and flow distributors. In general the inlet temperature distribution of the fluids a and c for all the three cases, the mean inlet temperature of the hot central fluid (fluid b) is the same. However, in first two cases the temperature distributions are nonuniform as depicted in Fig. 3. In the third case (case-III) the temperature distribution is uniform. The inlet temperature of the other two fluids are assumed to be entering uniformly at the respective sections.

\[ (22) \]

For both cases I and II a stepped temperature distribution specified by two temperature values $t_{b,1}$ and $t_{b,2}$ and a known dimension $y_0$ are taken. The dimensionless temperature is defined as

\[ T = \frac{t - t_{b,in}}{t_{b,2} - t_{b,1}} \]

Where, $t_{b,in} = t_{b,1}y_0 + t_{b,2}(L_b - y_0)$

Therefore in the case III, a uniform dimensionless inlet temperature is given by

\[ T_{b,in} = t_{b,in} = T_{b,1} + T_{b,2}(1-y_0) \]

The inlet temperature distribution of the fluids a and c for all the above cases will be

\[ T_a(X,0,Y) = T_{a,in} = 0, \]

\[ T_c(X,Y,0) = T_{c,in} \]

where $T_{c,in}$ is $T_c(0,Y,0)$, $T_c(N_a,Y,0)$, $T_c(X,N_c,0)$ and $T_c(X,0,0)$ for the arrangements C1, C2, C3 and C4 respectively.

To study the transient performance of the heat exchanger following input condition for the central fluid inlet temperature is considered

\[ T_{b,in} = T_{b,1}Y_0 + T_{b,2}(1-Y_0) = \bar{T} (\theta) \]

where $\bar{T} (\theta)$ is a specified function of temperature with respect to time. To get the temperature distribution at the inlet, one needs to supply the values of $T_{b,1}$ and $Y_0$.

IV. METHOD OF SOLUTION

The conservation equations are discretized using the finite difference technique. Forward difference scheme is used for time derivatives, while upwind scheme and central difference scheme are used for the first and second order space derivatives respectively [21]. The difference equations along with the boundary conditions are solved using Gauss-Seidel iterative technique. The convergence of the solution has been checked by varying the number of space grids and size of the time steps. Space and time grids equal to 70 and 20 respectively give the condition for grid independence. The solution gives the two-dimensional temperature distribution for all three fluids as well as for the separator plate.

V. RESULTS AND DISCUSSIONS

To check the validity of the numerical scheme, the solution for the steady state condition for the arrangement C4 at $E_{ab}=E_{b,ab}=1$, $R_{a,ab}=R_{b,ab}=1$, $V=\lambda=0$, $Pe=\alpha$, $T_{a,in}=0$, $T_{b,2}=1$ and $T_{c,in}=0.5$, was compared with the analytical solutions [8] without non-uniformity. Excellent agreement was observed as shown in Fig. 4.

The transient behavior of three-fluid crossflow heat exchanger has been studied for different excitations given to the central (hot) fluid inlet temperature. Performance of the heat exchanger was studied over a wide range of parameters.
as well as for sufficient time duration so that steady state conditions are obtained for each individual excitation. A preliminary result only for the step response of single pass co-current crossflow three fluid heat exchanger (arrangement C1) has been presented and discussed in this section. In the present analysis the effect of longitudinal conduction in separating wall and the effect of axial dispersion in fluids have not been considered.

For \( Y_0 = 0.2 \) and \( T_{b,1} = 0.1 \), the transient temperature response of the three fluids for step excitation in fluid-b are shown in Fig. 5 (a-c). It may be observed that the effect of nonuniformity is pronounced in case of mean exit temperature of fluid-a, while it has least effect on that of fluid-c. Further, case-I gives higher exit temperature response for fluid-a, while for other two fluids it gives lower responses compared to that in case III where nonuniformity is absent. Similarly, case-II gives higher exit temperature responses for fluids b and c compared to those in case III. This clearly shows that the fluid exit temperatures are decided not only by the mean inlet temperature of the heat exchanger, and the process and geometrical parameters, but also by the temperature distribution of the hot fluid inlet stream.

![Fig. 4. VALIDATION OF THE NUMERICAL RESULTS WITH THE ANALYTICAL STEADY STATE SOLUTION [8].](image)

![Fig. 5. EFFECT OF TEMPERATURE NON-UNIFORMITY ON MEAN EXIT TEMPERATURE OF (a) FLUID a, (b) FLUID b, AND (C) FLUID c FOR STEP INPUTS GIVEN TO FLUID b (ARRANGEMENT C1).](image)

VI. CONCLUSION

In the analysis, a typical case of co-current crossflow three-fluid heat exchanger (C1) with finite core capacity is analyzed based on a finite difference technique. Although the numerical solution considered above has been applied to a typical arrangement and nonuniformity model, other cases of transient heat transfer in three-fluid heat exchangers can also be analyzed. The analysis can be extended to the case of large core capacity i.e. gas-to-gas heat exchangers, in the presence of core longitudinal conduction and axial dispersion. Further, different arrangements for single pass crossflow heat exchangers can also be compared.

Moreover, from the given analysis one can determine the transient behavior of the core temperature at different time instants, which may be needed for mechanical design and calculation of thermal stresses.
NOMENCLATURE

- **A, \(A_{HT}\)**: Heat transfer area, m²
- **\(A_c\)**: Flow area, m²
- **C**: heat capacity rate (mc), W/K
- **c, \(C_p\)**: specific heat, J/kg-K
- **\(C1, C2, C3, C4\)**: crossflow arrangements
- **D**: diffusion coefficient, W/m-K
- **\(E_{a,c-b}\)**: capacity ratio
- **h**: heat transfer coefficient, W/m²-K
- **k**: thermal conductivity of the separating sheet, W/mK
- **L**: heat exchanger length, m
- **M**: mass of the separating sheet, kg
- **m**: mass flow rate of fluid, kg/s
- **Na**: number of transfer units, \(\frac{(UA)_{a,c}}{(mc)_b}\)
- **NTU**: number of transfer units
- **Pe**: axial dispersive Peclet number \(\frac{(mc)L}{A_cD}\)
- **Q, \(q\)**: rate of heat transfer, W
- **\(R_{a,c-b}\)**: conductance ratio \(\frac{(\rho A)_{a,c}}{(\rho A)_{b}}\)
- **Ru**: dimensionless temperature \(\frac{t-t_{in}}{t_{b,c} - t_{a,c}}\)
- **T**: temperature
- **\(\bar{T}\)**: dimensionless mean temperature
- **\(\bar{U}\)**: overall heat transfer coefficient, W/m²-K
- **u, v, W**: velocity of flow, m/s
- **V**: heat capacity ratio \(\frac{LA}{mc}\)
- **X**: \(\frac{(\rho A)_{a,c}}{(mc)_{a,c}}\)
- **Y**: \(\frac{(\rho A)_{b}}{(mc)_{b}}\)
- **Greek letters**
  - **\(\delta\)**: equivalent thickness of the separating sheet, m
  - **\(\eta\)**: overall surface efficiency \(\frac{(\rho A)_{a,c}}{(Mc)_{a,c}}\)
  - **\(\theta\)**: longitudinal heat conduction parameter
  - **\(\lambda\)**: \(\frac{kS_a}{L_c(mC)_b}\), \(\lambda_c = \frac{kS_c}{L_c(mC)_b}\)
  - **\(\mu\)**: dynamic viscosity, N-s/m²
  - **\(\rho\)**: density, kg/m³
  - **\(\tau\)**: time, s
  - **\(\phi()\)**: perturbation in hot fluid inlet temperature \(\frac{(Mc)_{a,c}}{(Mc)_{b}}\)
  - **\(\psi()\)**:

**Subscripts**
- **a, b, c**: fluid streams a, b and c
- **c, h**: cold and hot side
- **ex**: exit
- **i, in**: inlet

REFERENCES


(Advance online publication: 19 August 2010)