An Improved Spatial Smoothing Technique for DoA Estimation of Highly Correlated Signals

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***Abstract*—Spatial superresolution techniques have been investigated for several decades in many fields: Communication, radar, sonar, etc. One of the most investigated techniques is multiple signal classification (MUSIC) algorithm. Its simplicity has made it very attractive in such fields. However, the main drawback of this algorithm lies on its failure to resolve coherent/highly correlated signals. Many algorithms have been developed in order to overcome this problem. One of the famous algorithms is the spatial smoothing method. In this paper we have developed new algorithm that improves the spatial smoothing algorithm and raises the probability of signals resolution. The performances of our new algorithm were analyzed via extensive simulations under imperfect conditions of electrical calibration and mechanical deformations of phased array antenna.**

***Index Terms*—direction-of-arrival estimation, adaptive arrays, spatial smoothing, array signal processing, beamformer.**

I. Introduction

SUPERRESOLUTION methods refer to techniques that estimate the direction of arrival (DoA) of closely spaced signals which are received at phased array antenna. The motivation for superresolution of closely spaced/coherent signals can be found in many important fields. Multipath problem can be found for example at radar field. When a signal is received with multipath components (which are closely spaced signals) a high altitude estimation error occurs [1,2]. At air defence system there is a strong motivation for signals superresolution [3]. Earlier identification of attack cluster of targets will help to improve the overall performances of a defense system against difficult attack scenarios.

MUSIC algorithm [4,5,19] is the most known algorithm for superresolution technique. Its simplicity has made it very attractive in many fields. At this method the estimated DoA of signals are extracted from the peaks of MUSIC spectrum. This spectrum is composed of the eigenvectors of the noise subspace which is orthogonal to the signal subspace. These eigenvectors are extracted by singular value decomposition (SVD) applied on the array correlation matrix [20]. The main drawback of this algorithm lies on its failure to resolve closely spaced/coherent signals [6]. Alternative approach for closely spaced signals separation is the weighted subspace fitting (WSF) [7,8]. This approach works well at the environment of coherent signals. However since WSF method is based on multi-variable minimization, the computational load is very high. Many researches have been made in order to enhance MUSIC algorithm to resolve closely spaced signals [9,10,11]. The most known technique is the spatial smoothing [12]. At this technique the array is divided into multiple overlapping sub-arrays. The correlation matrix of each sub-array is being estimated from its sampled data. Then the final correlation matrix from which we estimate the signal and noise sub-space parameters is given as the average of all sub-array correlation matrices. This method does not work well as the signals are become highly correlated. When the signals are highly correlated the probability of signals resolution, by this method, is getting lower. The new method introduced here improves the probability of highly correlated signals resolution.

The paper is organized as follows: In section 2, we present the mathematical formulation of the DoA estimation problem. Section 3 presents the development of our new method. In section 4 we demonstrate the performances of new method compared to the ordinary spatial smoothing method. Performances of our new algorithm under electrical and mechanical errors are also introduced. Concluding remarks are reported in section 5.

II. Problem Formulation

The mathematical presentation of the signals received at uniform linear array (ULA) can be written as follows:

$$\underline{y}(m) = \sum_{n=1}^{N} \underline{a}(\theta_n) g^n(t_m) + \underline{v}(t_m) \tag{1}$$

where N is the number of received signals, θ_n and $g^n(t_m)$ are DoA and waveform (sampled at t_m) of signal n, respectively, $\underline{v}$ is considered to be zero-mean spatially and temporally white complex Gaussian vector with second moment:

$$R_n = E[\underline{v}(t)\underline{v}^H(s)] = \sigma^2 I \delta_{t,s} \tag{2}$$

where $\delta_{t,s}$ stands for Kronecker delta and is given by:

$$\delta_{t,s} = \begin{cases} 1, & t = s \\ 0, & \text{else} \end{cases} \tag{3}$$

I is the identity matrix, E stands for the expectation operation and $(\cdot)^H$ is the hermitian operation. The signal DoA vector is given by:

$$\underline{a}(\theta_n) = \begin{pmatrix} 1 \\ e^{-jksin(\theta_n)} \\ \vdots \\ e^{-jk(L-1)sin(\theta_n)} \end{pmatrix} \tag{4}$$

where k is the wave number given by $\frac{2\pi d}{\lambda}$, d is the element interspace, λ is the wavelength and L is the number of elements at ULA. The estimation problem is to extract the signals DoA $\theta_n, n = 1,...N$, from time samples of $\underline{y}(m), m = 1,...,M$ where M is the number of time samples. The correlation matrix of the array is given by [15]:

$$R = S\Lambda_s S^H + E\Lambda_e E^H \tag{5}$$

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where S denotes an L by N matrix with its N columns being the eigenvectors corresponding to the N largest eigenvalues of the array correlation matrix R. Λ_s is a diagonal matrix that contains the relevant eigenvalues at its diagonal. E denotes an L by $L-N$ matrix with its $L-N$ columns being the eigenvectors corresponding to the $L-N$ smallest eigenvalues of the array correlation matrix R. Λ_e is a diagonal matrix contains the corresponding eigenvalues at its diagonal. MUSIC algorithm is based on searching of signals DoA vectors that are orthogonal to the noise subspace. This is accomplish by searching for peaks in MUSIC spectrum Ψ given by:

$$\Psi = \frac{1}{\underline{a}^H(\theta)\widehat{E}_L\widehat{E}_L^H\underline{a}(\theta)} \tag{6}$$

P. Stoica et. al [16] have shown that MUSIC algorithm can be realized as a special case of Maximum Likelihood Method (MLM) if and only if the signals are uncorrelated. Large sample realization of the MLM estimator is given by the minimizer of the following function:

$$tr[A^H E E^H A]P \tag{7}$$

where $tr(\cdot)$ stands for trace operation and P is the signal correlation matrix given by:

$$P = E[\underline{g}(t_m)\underline{g}^H(t_m)] \tag{8}$$

where

$$\underline{g}(t_m) = [g^1(t_m)\dots g^N(t_m)]^T \tag{9}$$

and $(\cdot)^T$ stands for the transpose operation. A is DoA signals matrix and is given by:

$$A(\underline{\theta}) = [\underline{a}(\theta_1)\dots\underline{a}(\theta_N)] \tag{10}$$

A. Signals Correlation

In this section we will explain the failure of MUSIC algorithm to estimate the DoA of signals in the presence of correlative/coherent signals. The correlation ρ between two signals $x(t)$ and $y(t)$ is given by:

$$\rho(\tau) = \frac{R_{xy}(\tau)}{\sqrt{R_{xx}(\tau)R_{yy}(\tau)}} \tag{11}$$

where $R_{xy}(\tau) = E(x(t)y(t+\tau))$. The correlation matrix of the array can also be introduced as follows:

$$R = A^H P A^H + R_n \tag{12}$$

Explicit formulation for signal correlation matrix (for $N = 2$) is given by:

$$P = \begin{pmatrix} p_1 & \sqrt{p_1 p_2}\rho \\ \sqrt{p_1 p_2}\rho & p_2 \end{pmatrix} \tag{13}$$

where p_i is power of signal i. Note that if signals are uncorrelated, namely $\rho = 0$, P is a diagonal matrix guaranteeing R to be positive definite (assuming that vectors $\underline{a}(\theta_n)$ are linearly independent, $rank(A) = N$) and therefore invertible. If correlation matrix R is invertible then MUSIC algorithm can be applied (via SVD process). Signals correlation affects the rank of S and thus of R. If signals are correlated, correlation matrix R may not be a full rank matrix and thus not invertible, which can cause MUSIC algorithm to be failed.

III. Improved Spatial Smoothing Algorithm

In this section we will describe our new algorithm for superresolution of closely spaced signals. The algorithm has been developed in order to improve the resolution performances especially at scenarios of high correlated signals. The ordinary spatial smoothing method is based on extracting the array correlation matrix as the average of all correlation matrices from the sub-arrays. The estimated correlation matrix of this method can be written as follows:

$$\widehat{R} = \frac{1}{k}\sum_{k=1}^{K}\widehat{R}^l \tag{14}$$

where K is the number of sub-arrays and $\widehat{R}^l$ is the estimated correlation matrix extracted from the lth sub-array and is given by:

$$\widehat{R}^l = \frac{1}{M}\sum_{m=1}^{M}\underline{y}^l(m)\underline{y}^{lH}(m) \tag{15}$$

and

$$\underline{y}^l(m) = \begin{pmatrix} y_1^l(m) \\ \vdots \\ y_\xi^l(m) \end{pmatrix} \tag{16}$$

$y_\varepsilon^l(m)$ is the received data (sampled at time t_m) from the εth element at the lth sub-array, $\varepsilon = 1, ..., \xi$ and ξ is the length of sub-array. Note that all sub-arrays have the same length.

Our method is based on processing each of sub-array separately and then averaging the DoA estimations. From each sub-array we extract its correlation matrix and then applying eigenvalue decomposition in order to build MUSIC spectrum. From each sub-array we estimate independently signals DoA. The algorithm averages DoAs estimation only from those sub-arrays that have full targets resolution, namely, the number of picks at MUSIC spectrum equals to the estimated number of signals. This new algorithm can be realized as multiple estimators that work on different part of the sampled data which is received at the elements array. The new method can be summarized as follows:

- Divide the array into K overlapping sub-arrays.
- From each subarray estimate its correlation matrix $\widehat{R}^l$.
- Estimate the number of received signals [13].
- Apply MUSIC algorithm on each sub-array.
- Choose the estimators that have full resolution (namely, the number of MUSIC peaks equals the estimated number of received signals). This group will be denoted by Ω. Let us denote by $\widehat{\underline{\theta}}^\varsigma$ the estimated DoA of sub-array ς ($\varsigma \in \Omega$).
- The final estimation is given by:

$$\widehat{\underline{\theta}} = \frac{1}{size(\Omega)}\sum_{\varsigma=1}^{size(\Omega)}\widehat{\underline{\theta}}^\varsigma \tag{17}$$

IV. Simulations Results

We have analyzed the new proposed algorithm under different types of Mote-Carlo simulations. In this section we introduce the results of the analysis. We compared the new proposed algorithm to the ordinary spatial smoothing MUSIC algorithm. In the simulations we assumed that we knew a-priori the number of received signals N. At all simulations

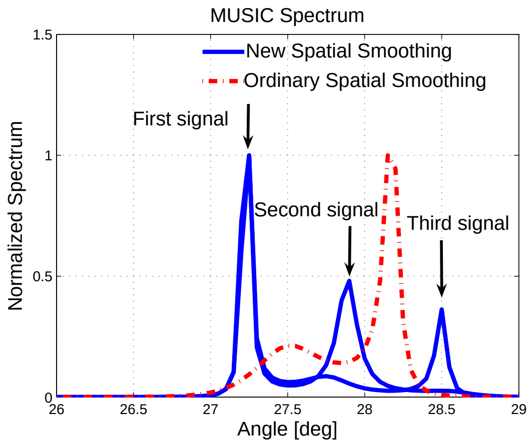

Fig. 1. MUSIC spectrum, M=64, K=4.

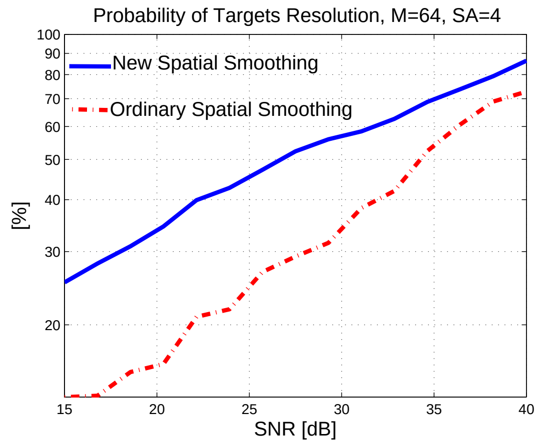

Fig. 2. Probability of resolution versus SNR.

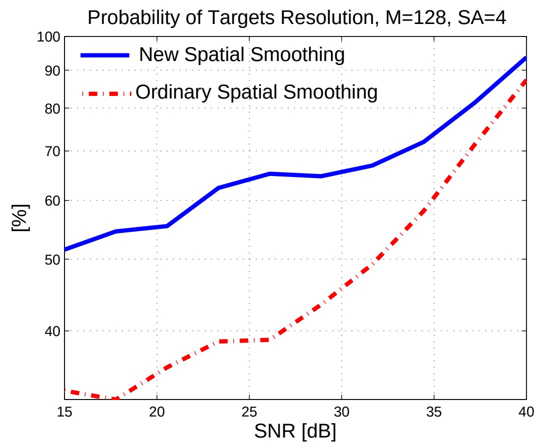

Fig. 3. Probability of resolution versus SNR.

a ULA with length of 2.8 [m] was taken. All signals at all simulations were fully correlated (with correlation coefficient equals one). The center frequency of the signals was 3.3 [Ghz]. The signals waveform $g(t)$ was taken to be a LFM pulse of 10 [μs] with duty cycle of 10 percent. At the receiver we used a matched filter to this waveform. The number of Monte-Carlo trials was 1500. The figure of merit in all Monte-Carlo simulations was targets probability of resolution. Fig. 1. demonstrates a scenario of three signals located at 27.1^o, 27.7^o and 28.3^o. The figure presents the spectrum of MUSIC algorithm applied at the ordinary spatial smoothing and at the new method from single trial. The SNR was taken to be 30 [dB] and the number of time samples was 64. Four sub-arrays were taken. As we can see, the new method has 2 out of 4 estimators that has three peaks $(|\Omega| = 2)$ at MUSIC spectrum (full resolution), while at the ordinary method there are only two peaks.

A. SNR Analysis

This part of analysis introduces the performances of new proposed method as a function of SNR. Two signals were simulated. The first one was located at 27.1^o and the second one was located at 27.9^o. We defined the SNR to be:

$$\begin{aligned} SNR &= \frac{(\sum_{n=1}^{N} \underline{a}(\theta_n) g^n(t_m))^H (\sum_{n=1}^{N} \underline{a}(\theta_n) g^n(t_m))}{E(\underline{v}^H(t_m)\underline{v}(t_m))} \\ &= \frac{b^H(A^H(\underline{\theta})A(\underline{\theta}))b}{L\sigma^2} \end{aligned} \quad (18)$$

σ^2 is the variance of the noise and $b = (1 \ldots 1)^H$ (Nx1 vector).

Fig.2. demonstrates the performances of new spatial smoothing technique compared to ordinary spatial smoothing at different levels of SNR. The number of time samples (transmitted pulses) was 64 and the number of sub-arrays was 4. As we can see, the new algorithm achieves better signals separation and below a SNR level of 30 [dB] it has about 25 percent of targets resolution probability more than the ordinary method. Fig. 3. introduces the performances of both methods with time samples equals 128. Note that as the number of time samples raises the performances of both algorithms are improved.

B. Targets Separation Analysis

In this section we analyze the performances of new algorithm as a function of signals DoA separation. The number of time samples that were taken to estimate the correlation matrices was 64 and the number of overlapping sub-arrays was 4. Three signals were simulated. The simulation was run with SNR equals 30 [dB]. As we can see from Fig. 4., at difficult scenarios with signals separation below 0.6^o (the 3-dB beam-width was 2^o) new method outperforms significantly the ordinary method and achieves about 30 percent of probability of targets separation more than the ordinary method.

C. Time Samples Analysis

In this section we analyze the performances of new algorithm as a function of the number of time samples. The number of overlapping sub-arrays was 4. Three signals were simulated with DoA separation of 0.6^o. The simulation was run with SNR equals 30 [dB]. As we can see from Fig. 5., when the number of time samples is below 100 the new method outperforms the ordinary method. At time samples above 100 the performances of both methods are not so different.

D. Electrical Calibration Errors Analysis

Phased array antenna must be calibrated before its operation. At calibration process we match phase and amplitude of

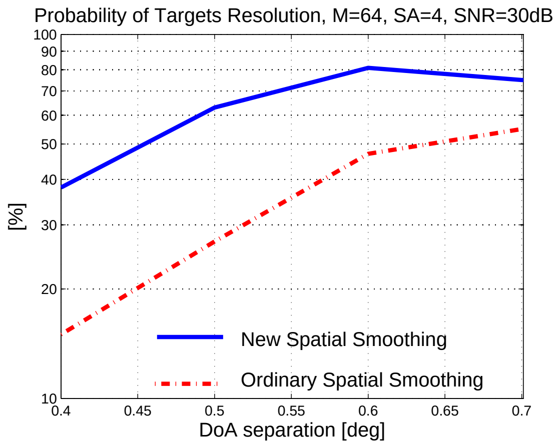

Fig. 4. Probability of resolution versus signals DoA separation.

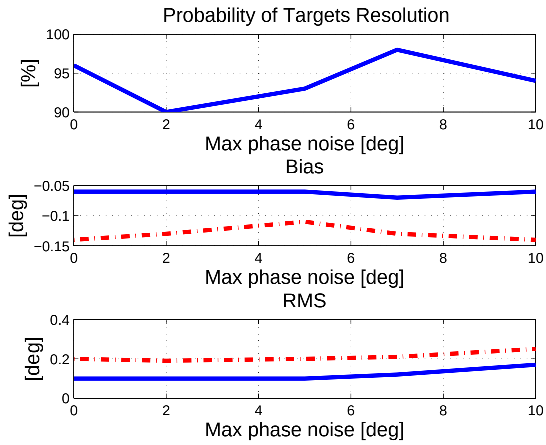

Fig. 6. Performances with calibration phase noise (two targets scenario).

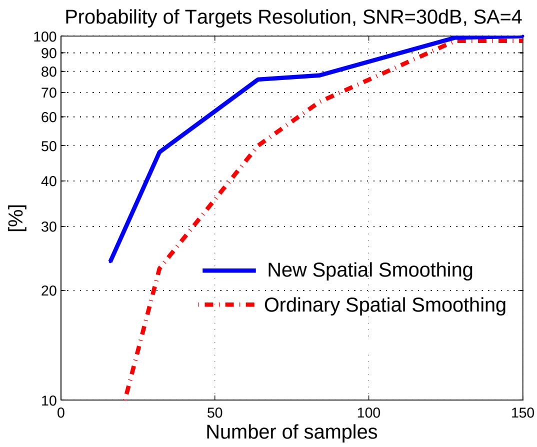

Fig. 5. Probability of resolution versus number of time samples (with DoA separation of 0.6 [deg]).

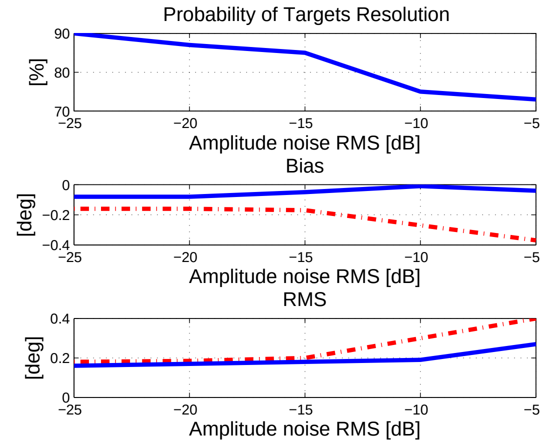

Fig. 7. Performances with calibration amplitude noise (two targets scenario).

all elements at the array [21]. Calibration errors degrade the performances of superresolution algorithms. Each superresolution algorithm may be affected differently due to calibration errors [18]. In this section we analyze the degradation of our proposed algorithm. The calibration error is represented by a complex number that describes the deviation of each Rx channel (of each element at the array) from its nominal value. The calibration errors are statistically independent from element to element. The phase error ψ is modeled as white noise uniformly distributed between $[-\phi_{max} : \phi_{max}]$. The amplitude error ξ is modeled as follows:

$$\xi = 1 + \delta \tag{19}$$

where $\delta \sim N(0, \sigma_\delta^2)$ (a white Gaussian noise). The total calibration error χ is given by $\xi e^{-j\psi}$.

1) Two Signals Analysis: At this analysis the number of pulses was taken to be 32. Signals DoA was 27.1 and 27.6 [deg]. Number of sub-arrays was three and the SNR was 20 [dB]. First signal and second signal are introduced with solid line and dash-dot line, respectively. As we can see from Fig. 6., phase noise has almost no effect on the performances of our algorithm. The probability of signals resolution is about 95 percent for almost all values of phase noise parameter. At Fig. 7., we can see the performances under amplitude error which is more dominant then phase noise. As we can see, the algorithm cannot perform well with RMS values more than -15 [dB].

2) Three Signals Analysis: At this analysis the number of pulses was 64. Signals DoA was 27.1, 27.8 and 28.6 [deg]. Number of sub-arrays was 4 and the SNR was 20 [dB]. First signal, second signal and third signal are introduced with solid line, dash-dot line and dashed line, respectively. As we can see from Fig. 8., phase noise has almost no effect on algorithm performances. These results are coincide with the results obtained with two signals. From Fig. 9., we can see that for three signals the algorithm can perform well until RMS error of -20 [dB].

From the above results we may conclude that as the number of signals is raised the calibration requirements from the phased array antenna are more stricter.

E. Mechanical Deformation Analysis

Phased array antenna may have mechanical deformations due to its heaviness or manufacturing limitations [22]. In this section we will analyze the performances of our algorithm with two main deformations: parabolic deformation and element position error. The element position error ψ describes the position error (along one dimension) with respect to the true element position. This error is modeled as a zero mean white Gaussian noise ($N \sim (0, \sigma^2)$) statistically independent from element to element. Parabolic deformation ξ is an

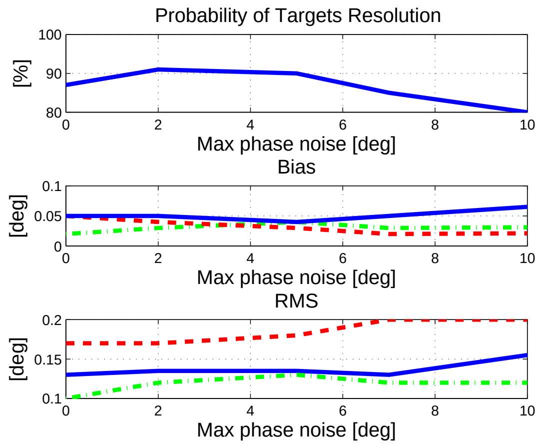

Fig. 8. Performances with calibration phase noise (three targets scenario).

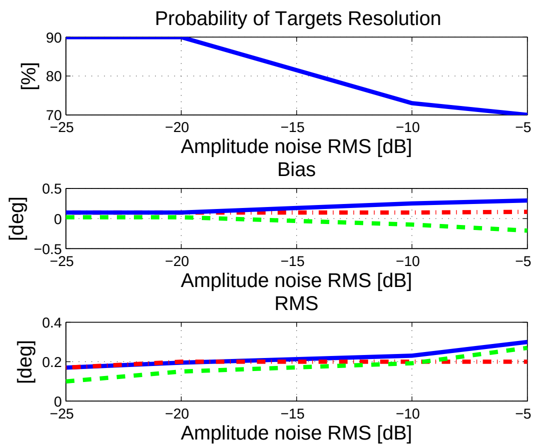

Fig. 9. Performances with calibration amplitude noise (three targets scenario).

unknown deterministic deformation along z-axis while X-Y plane is antenna plane (ξ represents the z-coordinate of element). The mathematical formulation of this deformation is given by:

$$\xi = \xi_{max}(1 - (\frac{2x}{L_a})^2) \qquad (20)$$

where L_a stands for the array length, x is x-coordinate of element and ξ_{max} is the maximum distortion of element position along z-axis.

We analyze the performances of our new algorithm under conditions of mechanical deformations via extensive simulations. At these simulations number of signals was three. The number of pulses was taken to be 64 and signals DoA was 27.1, 27.8 and 28.5 [deg]. Number of sub-arrays was 4 and the SNR was 20 [dB].

We can see from the results that the performances are not much affected by errors up to 1 [mm] under both element position error and parabolic deformation. For larger errors a compensation architecture should be added. The reason for degradation of our algorithm under mechanical deformation errors larger than 1 [mm] (for ULA) is that at these order of errors array symmetry breaks down. Spatial smoothing technique needs array symmetry namely, each element at the array located at position vector $\underline{\rho}$ should have identical element at position $-\underline{\rho}$ [17]. Since our algorithm works with multiple sub-arrays the symmetry property inside a sub-array under mechanical deformation with errors larger than 1 [mm] cannot hold.

TABLE I
RESULTS FOR SIGNAL NO.1

Mechanical distortion ($\xi_{max}[mm]$,σ [mm]) Probability of resolution	Bias [deg]	RMS [deg]
(0,0) 97.5	0.01	0.1
(1,0) 89.4	0.04	0.17
(2,0) 61	0.22	0.32
(0,1) 95.4	0.02	0.12
(0,2) 84	0.03	0.17
(1,1) 87	0.02	0.18

TABLE II
RESULTS FOR SIGNAL NO.2

Mechanical distortion ($\xi_{max}[mm]$,σ [mm]) Probability of resolution	Bias [deg]	RMS [deg]
(0,0) 97.5	0.01	0.1
(1,0) 89.4	0	0.19
(2,0) 61	0	0.3
(0,1) 95.4	0	0.15
(0,2) 84	0.01	0.18
(1,1) 87	0	0.2

TABLE III
RESULTS FOR SIGNAL NO. 3

Mechanical distortion ($\xi_{max}[mm]$,σ [mm]) Probability of resolution	Bias [deg]	RMS [deg]
(0,0) 97.5	0.01	0.06
(1,0) 89.4	0.01	0.12
(2,0) 61	0.06	0.2
(0,1) 95.4	0.01	0.08
(0,2) 84	0.01	0.12
(1,1) 87	0.03	0.15

V. CONCLUSION

High probability of signals separation is a very important task in many fields, as we mentioned before. Closely spaced signals problem can be found for example at communication and radar.

In this paper we introduced a new technique that improves signals separation. We compared our new technique to the ordinary spatial smoothing algorithm. The results that were introduced in this paper prove that the new method outperforms the ordinary spatial smoothing especially at difficult scenarios (very close sources, low SNR, etc). The new method raises the probability of closely spaced signals resolution at about 25 percent with comparison to the ordinary spatial smoothing. At scenarios of high SNR (more than 35 [dB]) or high signals spatial separation (more than 0.7 [deg]), the performances of both methods are close. The new method does not increase dramatically the computational load compared for example, to WSF algorithm and can be easily applied at real-time systems. Our new algorithm has been investigated under electrical and mechanical distortions. Analysis of these distortions reveals that our new algorithm can perform well under reasonable mechanical deformation and calibration errors.

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