Design of Optimal Robust PI Controller for Electro-Hydraulic Servo System

Piyapong Olranthichachat and Somyot Kaitwanidvilai

Abstract— **This paper proposes a new technique to design an optimal robust PI-controller for electro-hydraulic servo system to achieve both the robustness and performance. Comparative study between the conventional robust control and the proposed technique is included in the paper. Particle Swarm Optimization (PSO) is adopted to simplify the solving of structured robust control problem to realize the practical robust controller. Simulation results show that the proposed controller has simpler structure than that of the conventional robust loop shaping controller, and the stability margin obtained indicates the robustness of the proposed controller. Simulation and experimental results verify the effectiveness of the proposed algorithm.**

Index Terms—**Robust PI Controller, Particle Swarm Optimization, Electro-hydraulic servo system**

I. Introduction

The Electro-hydraulic servo system is an attractive choice for being used in both industrial and non-industrial applications because of its advantages of fast dynamic response, high power to inertia ratio, etc. Many approaches have been proposed to control this system to achieve good performance and robustness. One among them is robust control which the controlled system can perform well even under the conditions of disturbance and uncertainties. In the control design problem, several linear mathematic equations need to be solved to find the optimal robust controller. Unfortunately, the resulting controller from the conventional techniques is usually complicated with high order. In practical work, the model reduction methods such as Hankel Norm model reduction technique, Balance Trunc Realization, etc. have been adopted for reducing the controller order. However, in many cases, the stability margin obtained from the reduced order controller is not satisfied. Moreover, the structure of controller is not selectable; in practical control engineering, it is preferable if the structure of the controller can be selected. To overcome this problem, this paper applies the technique called structure specified robust controller to design a robust PI controller which gains both high stability margin and performance. The simple structure controller, PI, is today's most commonly used controller in servo systems. To reduce the gap between the theoretical and practical approaches mentioned above, the proposed technique adopts the particle swarm optimization technique for solving the robust stabilization control problem with specified controller structure.

Recently, many artificial intelligent techniques have been adopted to design a robust controller. In [1], a robust H_∞ optimal control problem with a structure specified controller was solved by genetic algorithm (GA). Mixed-sensitivity approach was adopted for indicating the performance of the designed controller. As results indicated [1], GA is a feasible method to design a structure specified H_∞ optimal controller. B.S. Chen. *et. al.* [2] proposed a PID design algorithm for mixed H_2/H_∞ control. In their paper, H_2 is mixed with the H_∞ to achieve both performance and robustness when designing the PID controller. In addition, the controller parameters were tuned in the stability domain which was analyzed based on the concepts of Routh Hurwitz and sampling technique. Similar method was proposed in [3] by applying the intelligent GA to solve the similar problem, mixed H_2/H_∞ optimal control. Clearly, the results in their papers [1-3] showed the robustness of the designed systems.

Although the methods in [1-3] are efficient to design a structure specified robust controller; however, the selection of uncertainty weight in their methods is not easy and straightforward. Especially, in the MIMO system, the difficulty of uncertainty weight selection becomes a dominant issue. To overcome the disadvantage of H infinity optimal control, McFarlane and Glover[4] proposed an alternative technique called H_∞ loop shaping control to design a robust controller. This technique is based on the concept of loop shaping which only 2 compensator weights need to be selected. Fortunately, the weight selection method in this technique is very clear by the classical control theorem. However, the problem of high order of the resulting controller is also occurred in this technique, and the reduction methods for reducing the controller order are sometimes inefficient. Many techniques proposed in the previous research works were adopted to solve this problem. Umut Genc [5] adopted the concept of state space approach and BMI optimization to design a robust PID controller. His technique is based on the concept of robust loop shaping technique. As shown in this paper, specifying of the initial solution has a huge effect on the optimal solution because of the local minima problem. S. Patra *et.al.* [6] designed an output feedback robust controller that has the same structure as the pre-compensator weight which is normally selected by PI. However, this technique is based on the concept of LMI approach which cannot avoid the problem of local minima. Several artificial intelligent techniques have been adopted to design a fixed-structure robust loop shaping in

This work was supported by the King Mongkut's Institute of Technology Ladkrabang Research Fund,

Piyapong and Somyot are with the Department of Electrical Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand. Email: drsomyotk@gmail.com. Somyot is also with the Center of Excellence for Innovative Energy Systems, King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand.

many kinds of systems [7-9, 13-14]. Somyot and Manukid [7] proposed the using of Genetic Algorithms to design a robust PID/PI controller for a SISO pneumatic servo system. Implementation in real system was shown in their paper to verify the robustness of their proposed technique. Somyot [8] stated that the problem of local minima in [5] and [6] can be reduced by using the PSO technique.

In this paper, the technique in [8, 9] was adopted to design a robust PI controller for a MIMO Electro-hydraulic Servo system. The PSO is employed to find the parameters of the optimal controller and the stability margin is adopted as the fitness function. Simulation results show that the controller designed by the proposed approach has a good performance and robustness as well as simple structure. The remainder of this paper is organized as follows. Section II illustrates the modeling of Electro-hydraulic Servo system studied. Section III describes the conventional robust loop shaping and the proposed technique. The design examples and results are demonstrated in section IV. Finally, Section V summarizes the paper.

II. Electro hydraulic servo modeling

Typical electro-hydraulic servo system is shown in Fig. 1 which consists of a position control system and a force control system [10]. The position control system is adopted to control the actuator movement and the force control system is applied to supply a required force to the system load. The main objective of the servo system is to satisfy the specified requirements; for example, zero steady state errors in motion of the actuator and force output.

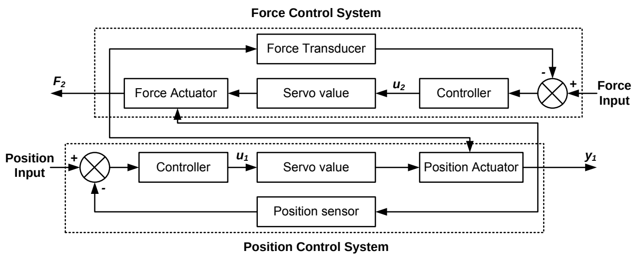


Fig.1. Electro-hydraulic servo system

The state-space model of the electro-hydraulic servo system can be shown in the following equations [10]:

$$\dot{x} = Ax + Bu \qquad y = Cx \tag{1}$$

where

$$A = \begin{bmatrix} -C_{pp}+K_{qp} & 0 & 0 & \frac{-4\beta_e A_1}{V_p} \\ 0 & -C_{pf}+K_{qf} & 0 & \frac{-4\beta_e A_1}{V_f} \\ 0 & 0 & 0 & 1 \\ \frac{A_1}{M} & \frac{A_2}{M} & 0 & \frac{-B_{vp}+f_{vp}}{M} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{4C_{qp}\beta_e K_{ap}}{V_p} & 0 \\ 0 & \frac{4C_{qf}\beta_e K_{af}}{V_p} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & A_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

As seen in the above mentioned dynamic, the system is MIMO system which has 2 outputs, F_2 – force of the system and y_1 – position of the actuator, and 2 inputs (u), u_1 – input servo valve of the position control system, and u_2 – input servo valve of the force control system.

III. Conventional robust Loop Shaping Control and the Proposed Technique

A. Conventional Robust Loop Shaping Control

Conventional robust loop shaping control [4] is an efficient approach to design a full order robust controller. This approach requires only two weighting functions, pre-compensator (W_1) and post-compensator (W_2) to shape the singular values of the original plant. Based on the concept in [4], the robust stabilization problem with the uncertainty model as normalized co-prime factors is solved by adopting the Ricatti equation. As shown in Fig.2, the co-prime factor uncertainty model divides the shaped plant (G_s) into nominator factor (N_s) and denominator factor (M_s. Consequently, the shaped plant can be written as:

$$G_s = W_1 G W_2 \tag{2}$$

$$G_s = (N_s + \Delta_{Ns})(M_s + \Delta_{Ms})^{-1} \tag{3}$$

where Δ_{Ns} and Δ_{Ms} are the uncertainty transfer functions in the nominator and denominator factors, respectively. $\|\Delta_{Ns}, \Delta_{Ms}\|_\infty \leq \varepsilon$, where ε is the stability margin. The determination of the normalized co-prime and the solving of the H_∞ loop shaping control can be seen from [11].

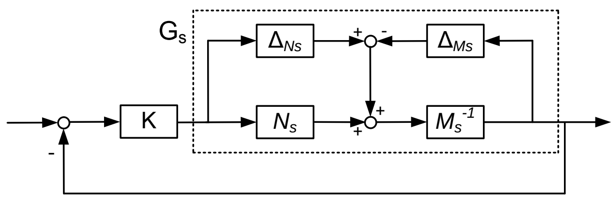


Fig.2. Co-prime factor robust stabilization problem.

As proposed by [4], the pre-compensator (W_1) and post-compensator (W_2) weights for achieving the desired loop shape are specified based on the concept of classical loop shaping technique. Then, the optimal stability margin (ε_{opt}) is obtained by solving the following equation.

$$\gamma_{opt} = \varepsilon_{opt}^{-1} = \inf_{stab\,K} \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I + G_s K)^{-1} M_s^{-1} \right\|_\infty \tag{4}$$

Too low optimal stability margin means that the selected weights are incompatible with the robust control design problem. If the (ε_{opt}) is too low, then select new weighting functions. Select the stability margin ($\varepsilon < \varepsilon_{opt}$) and then synthesize the controller, K_∞, by solving the following inequality.

$$\|T_{zw}\|_\infty = \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I+G_sK_\infty)^{-1} M_s^{-1} \right\|_\infty = \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I+G_sK_\infty)^{-1} \begin{bmatrix} I & G_s \end{bmatrix} \right\|_\infty \le \varepsilon^{-1} \quad (5)$$

The feedback controller (K) is

$$K = W_1 K_\infty W_2 \quad (6)$$

B. The Proposed Technique

The proposed technique fixes the structure of the controller ($K(p)$) and then the PSO is adopted to find the optimal parameter, p, to achieve the maximum stability margin. In the proposed technique, the stability margin (ε) shown in (7) is a single index to indicate the robust performance of the designed controller.

$$\|T_{zw}\|_\infty^{-1} = \varepsilon = \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I-G_sK_\infty)^{-1} \begin{bmatrix} I & G_s \end{bmatrix} \right\|_\infty^{-1} \quad (7)$$

K_∞ can be found by $K_\infty = W_1^{-1}K(p)W_2^{-1}$. Suppose that W_1 and W_2 are invertible. Generally, W_2 is chosen as identity matrix I. Therefore, the objective function can be written in the following form:

$$\textit{Objective function} = \varepsilon = \|T_{zw}\|_\infty^{-1} = \left\| \begin{bmatrix} I \\ W_1^{-1}K(p) \end{bmatrix} (I-G_sW_1^{-1}K(p))^{-1} \begin{bmatrix} I & G_s \end{bmatrix} \right\|_\infty^{-1} \quad (8)$$

For the design, the controller $K(p)$ will be designed to minimize the infinity norm from disturbance to state ($\|T_{zw}\|_\infty$) or maximize the stability margin (ε) by the PSO method. The PSO is based on the concept of swarm's movement as shown in Fig. 3 [12]. As seen in this figure, a bird represents a particle and the position of each particle represents the candidate solution. When applying the PSO, PSO parameters, i.e. the population of swam(n), lower and higher boundary (p_{min} , p_{max}) of the problem, minimum and maximum velocity of particles (v_{min} , v_{max}), minimum and maximum iteration(i_{max}), minimum and maximum inertia weight, need to be specified. In an iteration of the PSO, the value of fitness or objective function (fs) of each particle is evaluated. The particle which gives the highest fitness value is kept as the answer of current iteration. The inertia weight (Q), value of velocity (v) and position (p) of each particle in the current iteration (i) are updated by using equations (9), (10) and (11), respectively.

$$Q = Q_{\max} - \left(\frac{Q_{\max} - Q_{\min}}{i_{\max}} \right) i \quad (9)$$

$$v_{i+1} = Qv_i + \alpha_1[\gamma_{1i}(P_b - p_i)] + \alpha_2[\gamma_{2i}(U_b - p_i)] \quad (10)$$

$$p_{i+1} = p_i + v_{i+1} \quad (11)$$

Where α_1 , α_2 are acceleration coefficients,
γ_{1i} , γ_{2i} are any random numbers in (0→1) range.

Fig. 3. The movement of a swarm.

Based on the PSO technique, in this problem, a set of controller parameters p is formulated as a particle and the fitness can be written as:

$$\textit{Fitness function} = \left\| \begin{bmatrix} I \\ W_1^{-1}K(p) \end{bmatrix} (I - G_sW_1^{-1}K(p)^{-1} \begin{bmatrix} I & G_s \end{bmatrix}) \right\|_\infty^{-1} \quad (12)$$

Fitness value is specified as a very small value if the controlled system is unstable. The flow chart diagram of the proposed technique are shown in Fig.4.

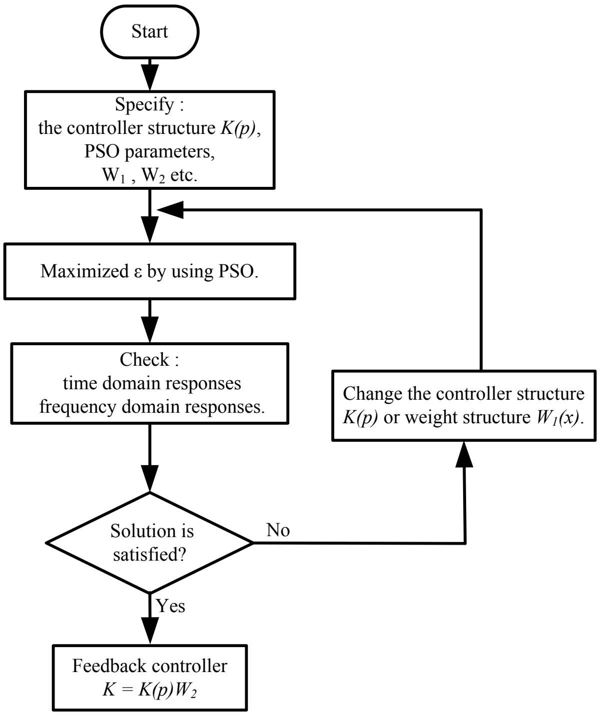


Fig. 4. Flow chart of the proposed design.

IV. Simulation Results

The state-space model of a nominal plant of the electro-hydraulic servo system can be seen in [10]. The state vector of this plant consists of four variables which are the supply pressure in force control system, the supply pressure in position control system, the position of actuator and the velocity of the actuator. Based on [11], the pre- and post-compensator weights are chosen as:

$$W_1 = \begin{bmatrix} \dfrac{0.53s+62}{s+0.001} & 0 \\ 0 & \dfrac{0.53s+62}{s+0.001} \end{bmatrix}, \quad W_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (13)$$

In this paper, the structure of controller is selected as:

$$K(p) = \begin{bmatrix} \dfrac{p_1 s + p_2}{s+0.001} & \dfrac{p_3 s + p_4}{s+0.001} \\ \dfrac{p_5 s + p_6}{s+0.001} & \dfrac{p_7 s + p_8}{s+0.001} \end{bmatrix} \qquad (14)$$

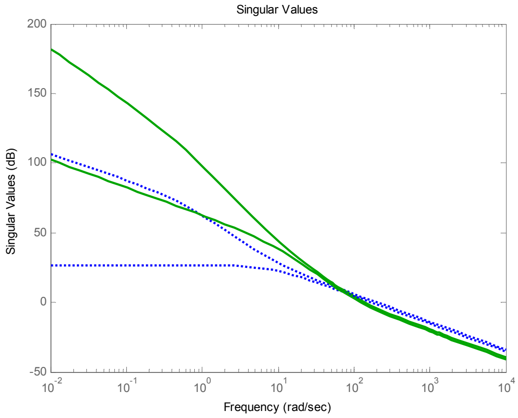


Fig.5. Singular values (--plant), (— Shaped plant) of MIMO electro-hydraulic servo system.

Singular values of the electro-hydraulic servo system and the desired loop shape are plotted in Fig.5. As seen in this figure, the bandwidth and performance are significantly improved by the compensator weights. The shaped plant has large gains at low frequencies for achieving good performance and small gains at high frequencies for noise attenuation. By using (13), the optimal stability margin of the shaped plant is found to be 0.5522. This value indicates that the selected weights are compatible with robust stability requirement. To design the conventional H_∞ loop shaping controller, stability margin 0.5217 is selected. The detail of the full order controller is given in appendix A. As shown in the result, the final controller (full order H_∞ loop shaping controller) is 8^{th} order and complicated.

Table 1 PSO parameters and controller parameter ranges.

Parameter	value
minimum velocities	0
maximum velocities	0.2
acceleration coefficients	2.1
minimum inertia weights	0.6
maximum inertia weights	0.9
maximum iteration	100
population size	500
$p_{1\text{-}8}$	[-60, 60]

Next, the proposed technique is adopted to design the robust PI controller. In the optimization problem, the upper and lower bounds of control parameters and the PSO parameters are given in Table 1.

After running the PSO for 56 iterations, the optimal solution is obtained as:

$$K(p) = \begin{bmatrix} \dfrac{-0.5284s - 36.683}{s+0.001} & \dfrac{-0.0717s - 8.0878}{s+0.001} \\ \dfrac{-0.0322s - 3.2293}{s+0.001} & \dfrac{-0.8966s - 38.441}{s+0.001} \end{bmatrix} \qquad (15)$$

Fig.6 shows the fitness or stability margin (ε) of the best controller obtained in each iteration. As seen in this figure, the best controller evolved by the PSO has a stability margin of 0.3905.

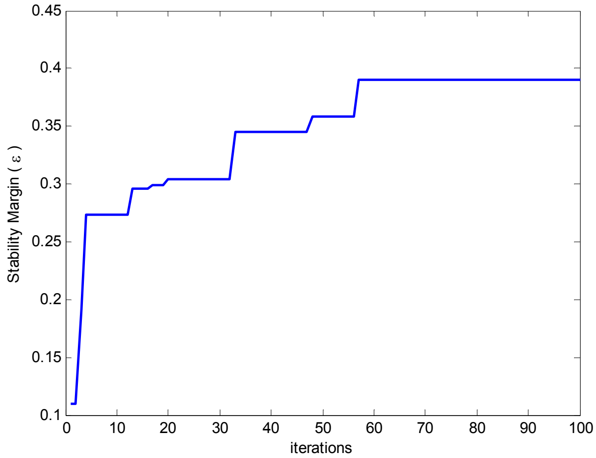
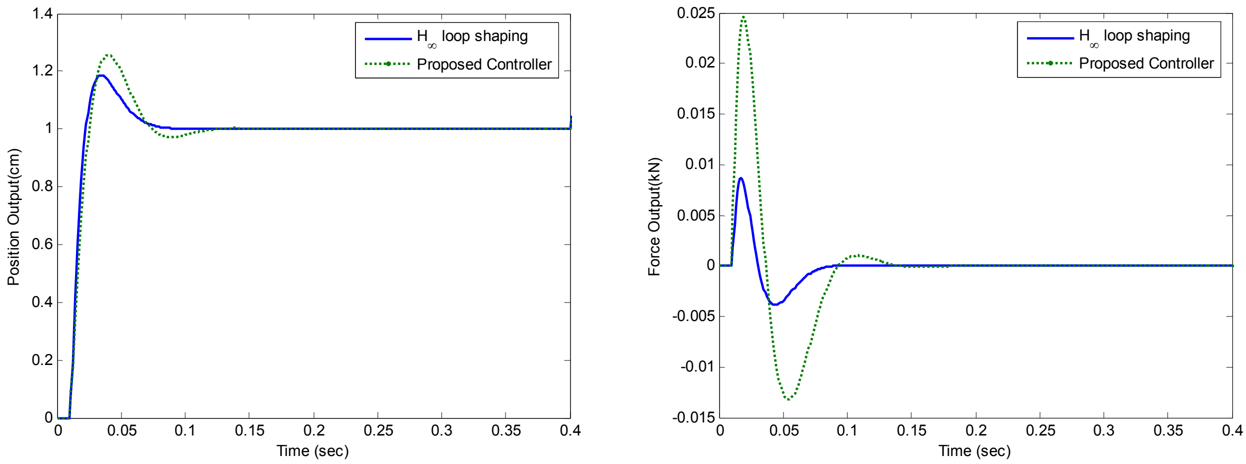


Fig.6. Stability margin (ε) versus iteration.

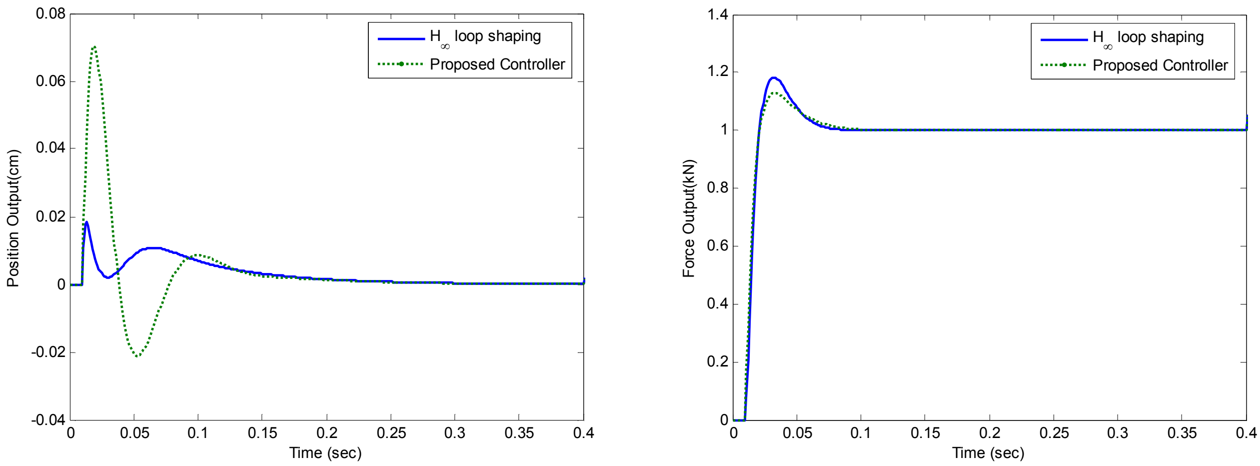
Fig.7 shows the responses of the outputs of the system in 2 channels (position and force outputs) when the unit step command input is fed into the position command. As seen in this figure, the proposed controller performs well as similar as the conventional H_∞ loop shaping controller. Fig.8 shows the responses of the system when unit step is fed into the force command input. As seen in Figs. 7 and 8, the overshoots appeared in the force channel by the proposed controller seem larger than that of the conventional technique; however, the maximum values are still small. This indicates that the proposed technique is applicable to the real system.

In Figs.9 and 10, the responses under the perturbed conditions (the MIMO electro-hydraulic servo system's parameters such as: mass increasing, torque motor gain variation. disturbances, etc.) verify that the proposed controller performs a good performance in terms of fast settling time, low oscillation and robustness. Clearly, the responses of the proposed technique are close to the responses from the robust loop shaping controller.



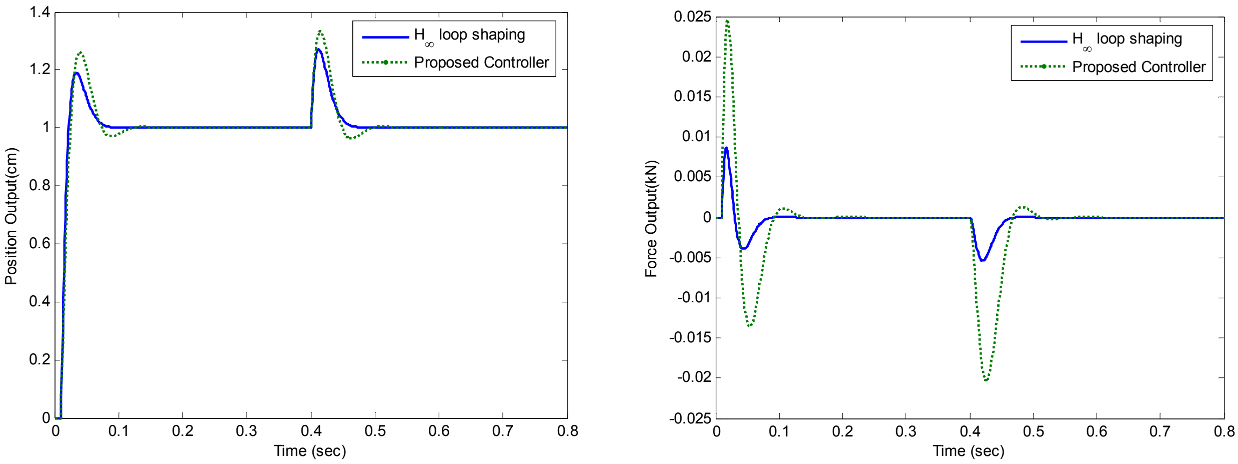
(a) (b)

Fig.7. Output responses of the system when the unit step is entered to position command.



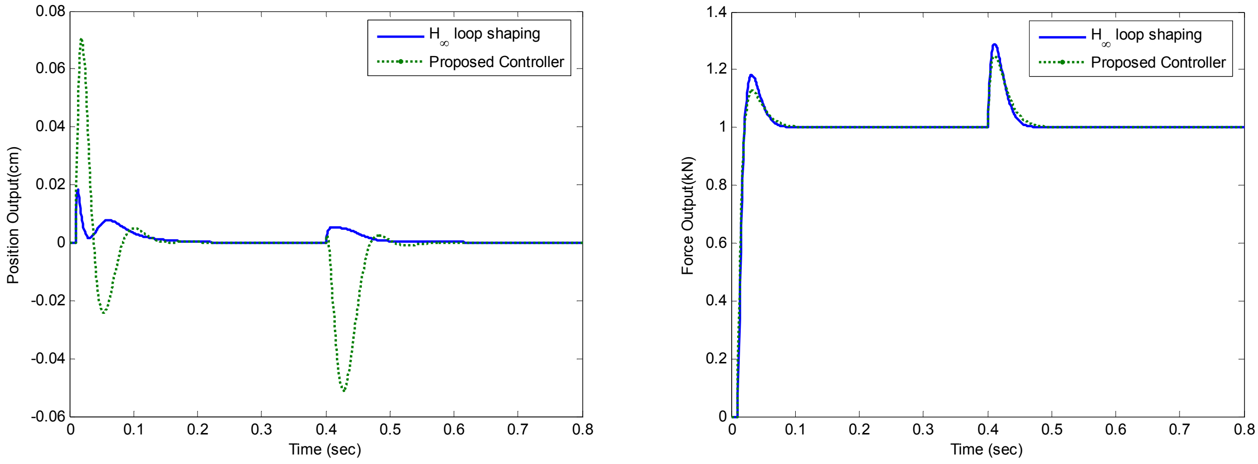
(a) (b)

Fig.8. Output responses of the system when the unit step is entered to the force command input.



(a) (b)

Fig.9. Output responses of the perturbed plant when the unit step and disturbance (*0.3u(t)*) are entered to the position command.



(a) (b)

Fig.10 Output responses of the perturbed plant when the unit step and disturbance (*0.3u(t)*) are entered to the force command.

V. Conclusion

This paper presents a new technique for designing a fixed structure robust loop shaping controller. As shown in the results, the proposed technique can be applied to control a MIMO electro-hydraulic servo system. The stability margin (ε) obtained by the proposed technique indicates the robustness and the performance of the proposed controller. Simulation results verify that the proposed technique is a feasible approach and reduces the gap between theoretical and practical schemes.

Acknowledgment

This research work is financially supported by the King Mongkut's Institute of Technology Ladkrabang Research Fund, King Mongkut's Institute of Technology Ladkrabang.

Appendix A

The state space representation of the robust loop shaping controller is given by $A_{H_\infty \text{ loop shaping controller}}$, $B_{H_\infty \text{ loop shaping controller}}$, $C_{H_\infty \text{ loop shaping controller}}$ and $D_{H_\infty \text{ loop shaping controller}}$ matrices as shown in the followings.

$$A_{H_\infty \text{ loop shaping controller}} = \begin{bmatrix} -1\times10^{-5} & -6.3998\times10^{-13} & 7.1725\times10^{-5} & -7.9386\times10^{-5} & -2.6913\times10^{-6} & 6.6929\times10^{-6} & 9.5343\times10^{-7} & 6.7209\times10^{-8} \\ 3.4197\times10^{-13} & -1\times10^{-5} & 9.8404\times10^{-5} & 7.7479\times10^{-5} & -7.2385\times10^{-6} & 1.5541\times10^{-7} & -4.2988\times10^{-6} & 6.6381\times10^{-8} \\ 5.8084\times10^{-5} & 0.00010724 & -1860.4 & -428.12 & 210.9 & -118.41 & 86.791 & -2.8545 \\ -8.926\times10^{-5} & 6.5538\times10^{-5} & -16.553 & -1622.7 & 71.963 & 135.01 & 103.1 & 0.0902 \\ 1.7123\times10^{-6} & -7.5377\times10^{-6} & 154.51 & 161.86 & -74.138 & 40.517 & -54.428 & 1.632 \\ 5.8492\times10^{-6} & 3.2979\times10^{-6} & -169.13 & 76.577 & 17.45 & -123.37 & -3.732 & -2.6062 \\ 3.1181\times10^{-6} & -3.0726\times10^{-6} & 33.193 & 123.55 & -54.957 & -43.631 & -81.739 & 1.6655 \\ -8.4247\times10^{-8} & 4.1953\times10^{-8} & 0.1339 & -2.7237 & 1.1233 & 2.0641 & 3.2602 & -0.6734 \end{bmatrix}$$

$$B_{H_\infty \text{ loop shaping controller}} = \begin{bmatrix} -3.6803 & 5.0342 \\ -4.9712 & -3.6342 \\ 37.343 & 4.8654 \\ -0.1353 & 34.377 \\ -1.5585 & -1.8007 \\ 1.8961 & -0.8730 \\ -0.1899 & -1.3432 \\ -0.00507 & 0.02882 \end{bmatrix}$$

$$C_{H_\infty \text{ loop shaping controller}} = \begin{bmatrix} -3.8459 & -4.8475 & 37.643 & 3.5133 & -2.2719 & 1.3247 & -0.8585 & 0.02901 \\ 4.9089 & -3.7977 & 1.0807 & 34.197 & -0.71389 & -1.6132 & -1.0503 & -0.0038 \end{bmatrix}$$

$$D_{H_\infty \text{ loop shaping controller}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

REFERENCES

[1] B. S. Chen and Y. M. Cheng. (1998), A structure-specified optimal control design for practical applications: a genetic approach," *IEEE Trans. on Control System Technology*, Vol. 6, No. 6, pp. 707-718.

[2] B. S. Chen, Y.-M. Cheng, and C. H. Lee. (1995), A genetic approach to mixed H_2/ H_∞ optimal PID control, *IEEE Trans. on Control Systems*, pp.51-60.

[3] S. J. Ho, S. Y. Ho, M. H. Hung, L. S. Shu, and H. L. Huang, Designing structure-specified mixed H_2/ H_∞ optimal controllers using an intelligent genetic algorithm IGA (2005), *IEEE Trans. on Control Systems*, 13(6), pp.1119-24.

[4] McFarlane, D., Glover, K. (1992). A loop shaping design procedure using H_∞ synthesis. *IEEE Transactions on Automatic Control*, 37(6), pp. 759-769.

[5] A. U. Genc, "A state-space algorithm for designing H_∞ loop shaping PID controllers," technical report., Cambridge University, Cambridge, UK, Oct. 2000.

[6] S. Patra, S. Sen and G. Ray. (2008) Design of static H_∞ loop shaping controller in four-block framework using LMI approach. *Automatica*, 44, pp. 2214-2220.

[7] S. Kaitwanidvilai, M. Parnichkun, Genetic Algorithm based Fixed-Structure Robust H infinity Loop Shaping Control of a Pneumatic Servo System, *International Journal of Robotics and Mechatronics,* 16, (4), *2004.*

[8] S. Kaitwanidvilai, "Particle Swarm Optimization based fixed-structure H∞ loop shaping control of MIMO system," *in Proc. of the 27th IASTED International Modeling, Identification and Control*, Innsbruck, Austria, 2008.

[9] P. Olarnthichachart, S. Kaitwanidvilai, Structure Specified Robust H_∞ Loop Shaping Control of a MIMO Electro-hydraulic Servo System using Particle Swarm Optimization, Lecture Notes in Engineering and Computer Science: Proceedings of The International MultiConference of Engineers and Computer Scientists 2011, IMECS 2011, 16-18 March, 2011, Hong Kong, pp 879-883.

[10] Hao Zhang, Neural adaptive control of nonlinear MIMO eletrohydraulic servo system, Spring Book Series, 1997.

[11] S. Skogestad & I. Postlethwaite, *Multivariable Feedback Control Analysis and Design*. 2nd ed. New York: John Wiley & Son, 1996.

[12] J. Kennedy and R. Eberhart, Particle swarm optimization, *IEEE International Conference on Neural Networks,* 4, 1995, 1942-1948.

[13] S. Kaitwanidvilai and Manukid Parnichkun, "Design of Structured Controller Satisfying H_∞ Loop Shaping using Evolutionary Optimization: Application to a Pneumatic Robot Arm", Engineering Letters, , Vol. 16 No.2, 2008.

[14] S. Kaitwanidvilai and Piyapong Olarnthichachart**,** **"**Fixed Structure Robust Loop Shaping Controller for a Buck-Boost Converter using Genetic Algorithm**",** Engineering Letters, an international journal, Vol. 16 No.3, 2008.