Design of Optimal Robust PI Controller for Electro-Hydraulic Servo System

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Abstract—This paper proposes a new technique to design an optimal robust PI-controller for electro-hydraulic servo system to achieve both the robustness and performance. Comparative study between the conventional robust control and the proposed technique is included in the paper. Particle Swarm Optimization (PSO) is adopted to simplify the solving of structured robust control problem to realize the practical robust controller. Simulation results show that the proposed controller has simpler structure than that of the conventional robust loop shaping controller, and the stability margin obtained indicates the robustness of the proposed controller. Simulation and experimental results verify the effectiveness of the proposed algorithm.

Index Terms—Robust PI Controller, Particle Swarm Optimization, Electro-hydraulic servo system

I. INTRODUCTION

The Electro-hydraulic servo system is an attractive choice for being used in both industrial and non-industrial applications because of its advantages of fast dynamic response, high power to inertia ratio, etc. Many approaches have been proposed to control this system to achieve good performance and robustness. One among them is robust control which the controlled system can perform well even under the conditions of disturbance and uncertainties. In the control design problem, several linear mathematical equations need to be solved to find the optimal robust controller. Unfortunately, the resulting controller from the conventional techniques is usually complicated with high order. In practical work, the model reduction methods such as Hankel Norm model reduction technique, Balance Trunc Realization, etc. have been adopted for reducing the controller order. However, in many cases, the stability margin obtained from the reduced order controller is not satisfied. Moreover, the structure of controller is not selectable; in practical control engineering, it is preferable if the structure of the controller can be selected. To overcome this problem, this paper applies the technique called structure specified robust controller to design a robust PI controller which gains both high stability margin and performance. The simple structure controller, PI, is today’s most commonly used controller in servo systems. To reduce the gap between the theoretical and practical approaches mentioned above, the proposed technique adopts the particle swarm optimization technique for solving the robust stabilization control problem with specified controller structure.

Recently, many artificial intelligent techniques have been adopted to design a robust controller. In [1], a robust $H_\infty$ optimal control problem with a structure specified controller was solved by genetic algorithm (GA). Mixed-sensitivity approach was adopted for indicating the performance of the designed controller. As results indicated [1], GA is a feasible method to design a structure specified $H_\infty$ optimal controller. B.S. Chen. et al. [2] proposed a PID design algorithm for mixed $H_2/H_\infty$ control. In their paper, $H_2$ is mixed with the $H_\infty$ to achieve both performance and robustness when designing the PID controller. In addition, the controller parameters were tuned in the stability domain which was analyzed based on the concepts of Routh Hurwitz and sampling technique. Similar method was proposed in [3] by applying the intelligent GA to solve the similar problem, mixed $H_2/H_\infty$ optimal control. Clearly, the results in their papers [1-3] showed the robustness of the designed systems.

Although the methods in [1-3] are efficient to design a structure specified robust controller; however, the selection of uncertainty weight in their methods is not easy and straightforward. Especially, in the MIMO system, the difficulty of uncertainty weight selection becomes a dominant issue. To overcome the disadvantage of $H$ infinity optimal control, McFarlane and Glover[4] proposed an alternative technique called $H_\infty$ loop shaping control to design a robust controller. This technique is based on the concept of loop shaping which only 2 compensator weights need to be selected. Fortunately, the weight selection method in this technique is very clear by the classical control theorem. However, the problem of high order of the resulting controller is also occurred in this technique, and the reduction methods for reducing the controller order are sometimes inefficient. Many techniques proposed in the previous research works were adopted to solve this problem. Umut Genc [5] adopted the concept of state space approach and BMI optimization to design a robust PID controller. His technique is based on the concept of robust loop shaping technique. As shown in this paper, specifying of the initial solution has a huge effect on the optimal solution because of the local minima problem. S. Patra et.al. [6] designed an output feedback robust controller that has the same structure as the pre-compensator weight which is normally selected by PI. However, this technique is based on the concept of LMI approach which cannot avoid the problem of local minima. Several artificial intelligent techniques have been adopted to design a fixed-structure robust loop shaping in

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many kinds of systems [7-9, 13-14]. Somyot and Manukid [7] proposed the using of Genetic Algorithms to design a robust PID/PI controller for a SISO pneumatic servo system. Implementation in real system was shown in their paper to verify the robustness of their proposed technique. Somyot [8] stated that the problem of local minima in [5] and [6] can be reduced by using the PSO technique.

In this paper, the technique in [8, 9] was adopted to design a robust PI controller for a MIMO Electro-hydraulic Servo system. The PSO is employed to find the parameters of the optimal controller and the stability margin is adopted as the fitness function. Simulation results show that the controller designed by the proposed approach has a good performance and robustness as well as simple structure. The remainder of this paper is organized as follows. Section II illustrates the modeling of Electro-hydraulic Servo system studied. Section III describes the conventional robust loop shaping and the proposed technique. The design examples and results are demonstrated in section IV. Finally, Section V summarizes the paper.

II. ELECTRO HYDRAULIC SERVO MODELING

Typical electro-hydraulic servo system is shown in Fig. 1 which consists of a position control system and a force control system [10]. The position control system is adopted to control the actuator movement and the force control system is applied to supply a required force to the system load. The main objective of the servo system is to satisfy the specified requirements; for example, zero steady state errors in motion of the actuator and force output.

\[
\begin{align*}
    x &= Ax + Bu \\
    y &= Cx
\end{align*}
\]

where

\[
A = \begin{bmatrix}
-\frac{C_{pp} + K_{pp}}{V_p} & 0 & 0 & -\frac{4\beta A_p}{V_p} \\
0 & -\frac{C_{pf} + K_{pf}}{V_p} & 0 & -\frac{4\beta A_f}{V_p} \\
0 & 0 & 0 & 1 \\
\frac{A_z}{M} & \frac{A_\gamma}{M} & 0 & -\frac{B_{op} + f_{op}}{M}
\end{bmatrix}
\]

As seen in the above mentioned dynamic, the system is MIMO system which has 2 outputs, \( F_2 \) – force of the system and \( y_1 \) – position of the actuator, and 2 inputs \( u_1 \), \( u_2 \) – input servo valve of the position control system, and \( u_2 \) – input servo valve of the force control system.

III. CONVENTIONAL ROBUST LOOP SHAPING CONTROL AND THE PROPOSED TECHNIQUE

A. Conventional Robust Loop Shaping Control

Conventional robust loop shaping control [4] is an efficient approach to design a full order robust controller. This approach requires only two weighting functions, pre-compensator \( (W_1) \) and post-compensator \( (W_2) \) to shape the singular values of the original plant. Based on the concept in [4], the robust stabilization problem with the uncertainty model as normalized co-prime factors is solved by adopting the Ricatti equation. As shown in Fig.2, the co-prime factor uncertainty model divides the shaped plant \( (G_s) \) into nominator factor \( (N_s) \) and denominator factor \( (M_s) \). Consequently, the shaped plant can be written as:

\[
G_s = W_1 G W_2
\]

\[
G_s = (N_s + \Delta_{nm})(M_s + \Delta_{nm})^{-1}
\]

where \( \Delta_{nm} \) and \( \Delta_{nm} \) are the uncertainty transfer functions in the nominator and denominator factors, respectively. \( \| \Delta_{nm}, \Delta_{nm} \|_\infty \leq \epsilon \) where \( \epsilon \) is the stability margin. The determination of the normalized co-prime and the solving of the \( H_\infty \) loop shaping control can be seen from [11].

As proposed by [4], the pre-compensator \( (W_1) \) and post-compensator \( (W_2) \) weights for achieving the desired loop shape are specified based on the concept of classical loop shaping technique. Then, the optimal stability margin \( (\epsilon_{opt}) \) is obtained by solving the following equation.

\[
\gamma_{opt} = \epsilon_{opt}^{-1} = \inf_{\text{sub} K} \left\| \frac{1}{K} (I + G_s K) M_s^{-1} \right\|_\infty
\]
Too low optimal stability margin means that the selected weights are incompatible with the robust control design problem. If the ($\varepsilon_{opt}$) is too low, then select new weighting functions. Select the stability margin ($\varepsilon<\varepsilon_{opt}$) and then synthesize the controller, $K_{sc}$ by solving the following inequality.

$$\left\| T_{sc} \right\|_\varepsilon = \left\| \begin{bmatrix} I \\ K_{sc} \end{bmatrix} \left( I + G_s K_{sc} \right)^{-1} M_s \right\|_\varepsilon \leq \varepsilon^{-1}$$  \hspace{1cm} (5)

The feedback controller ($K$) is

$$K = W_1 K_{sc} W_2$$  \hspace{1cm} (6)

**B. The Proposed Technique**

The proposed technique fixes the structure of the controller ($K(p)$) and then the PSO is adopted to find the optimal parameter, $p$, to achieve the maximum stability margin. In the proposed technique, the stability margin ($\varepsilon$) shown in (7) is a single index to indicate the robust performance of the designed controller.

$$\left\| T_{sc} \right\|_\varepsilon = \varepsilon = \left\| \begin{bmatrix} I \\ K_{sc} \end{bmatrix} \left( I - G_s K_{sc} \right)^{-1} \left[ I \quad G_s \right] \right\|_\varepsilon$$  \hspace{1cm} (7)

$K_{sc}$ can be found by $K_{sc} = W_1^{-1} K(p) W_2^{-1}$. Suppose that $W_1$ and $W_2$ are invertible. Generally, $W_2$ is chosen as identity matrix $I$. Therefore, the objective function can be written in the following form:

$$Objective\ function = \varepsilon = \left\| T_{sc} \right\|_\varepsilon$$  \hspace{1cm} (8)

For the design, the controller $K(p)$ will be designed to minimize the infinity norm from disturbance to state ($\left\| T_{sc} \right\|_\varepsilon$) or maximize the stability margin ($\varepsilon$) by the PSO method. The PSO is based on the concept of swarm’s movement as shown in Fig. 3 [12]. As seen in this figure, a bird represents a particle and the position of each particle represents the candidate solution. When applying the PSO, PSO parameters, i.e. the population of swarm($n$), lower and higher boundary ($p_{min}$, $p_{max}$) of the problem, minimum and maximum velocity of particles ($v_{min}$, $v_{max}$), minimum and maximum iteration($i_{max}$), minimum and maximum inertia weight, need to be specified. In an iteration of the PSO, the value of fitness or objective function ($f$) of each particle is evaluated. The particle which gives the highest fitness value is kept as the answer of current iteration. The inertia weight ($Q$), value of velocity ($v$) and position ($p$) of each particle in the current iteration ($i$) are updated by using equations (9), (10) and (11), respectively.

$$Q = Q_{max} - \left( \frac{Q_{max} - Q_{min}}{i_{max}} \right) i$$  \hspace{1cm} (9)

$$v_{i+1} = v_i + \alpha_1 [\gamma_1 (P_i - p_i)] + \alpha_2 [\gamma_2 (U_i - p_i)]$$  \hspace{1cm} (10)

$$P_{i+1} = P_i + v_{i+1}$$  \hspace{1cm} (11)

Where $\alpha_1$, $\alpha_2$ are acceleration coefficients, $\gamma_1$, $\gamma_2$ are any random numbers in (0→1) range.

Based on the PSO technique, in this problem, a set of controller parameters $p$ is formulated as a particle and the fitness can be written as:

$$Fitness\ function = \left\| \begin{bmatrix} I \\ W_1^{-1} K(p) \end{bmatrix} \left( I - G_s W_1^{-1} K(p) \right)^{-1} \left[ I \quad G_s \right] \right\|_\varepsilon$$  \hspace{1cm} (12)

Fitness value is specified as a very small value if the controlled system is unstable. The flow chart diagram of the proposed technique are shown in Fig.4.

**IV. SIMULATION RESULTS**

The state-space model of a nominal plant of the electro-hydraulic servo system can be seen in [10]. The state vector of this plant consists of four variables which are the supply pressure in force control system, the supply pressure in position control system, the position of actuator and the velocity of the actuator. Based on [11], the pre- and post-compensator weights are chosen as:

![Flow chart of the proposed design](image-url)
In this paper, the structure of controller is selected as:

\[
K(p) = \begin{bmatrix}
    p_1 s + p_2 \\
    p_3 s + p_4 \\
    s + 0.001
\end{bmatrix}
\begin{bmatrix}
    s + 0.001 \\
    s + 0.001 \\
    s + 0.001
\end{bmatrix}
\begin{bmatrix}
    s + 0.001 \\
    s + 0.001 \\
    s + 0.001
\end{bmatrix}
\]

(14)

and lower bounds of control parameters and the PSO parameters are given in Table 1.

After running the PSO for 56 iterations, the optimal solution is obtained as:

\[
K(p) = \begin{bmatrix}
    -0.5284s - 36.683 \\
    -0.0322s - 3.2293 \\
    -0.8966s - 38.441
\end{bmatrix}
\begin{bmatrix}
    s + 0.001 \\
    s + 0.001 \\
    s + 0.001
\end{bmatrix}
\begin{bmatrix}
    s + 0.001 \\
    s + 0.001 \\
    s + 0.001
\end{bmatrix}
\]

(15)

Fig. 6 shows the fitness or stability margin (ε) of the best controller obtained in each iteration. As seen in this figure, the best controller evolved by the PSO has a stability margin of 0.3905.

Singular values of the electro-hydraulic servo system and the desired loop shape are plotted in Fig. 5. As seen in this figure, the bandwidth and performance are significantly improved by the compensator weights. The shaped plant has large gains at low frequencies for achieving good performance and small gains at high frequencies for noise attenuation. By using (13), the optimal stability margin of the shaped plant is found to be 0.5522. This value indicates that the selected weights are compatible with robust stability requirement. To design the conventional \(H_\infty\) loop shaping controller, stability margin 0.5217 is selected. The detail of the full order controller is given in appendix A. As shown in the result, the final controller (full order \(H_\infty\) loop shaping controller) is 8th order and complicated.

Table 1 PSO parameters and controller parameter ranges.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum velocities</td>
<td>0</td>
</tr>
<tr>
<td>maximum velocities</td>
<td>0.2</td>
</tr>
<tr>
<td>acceleration coefficients</td>
<td>2.1</td>
</tr>
<tr>
<td>minimum inertia weights</td>
<td>0.6</td>
</tr>
<tr>
<td>maximum inertia weights</td>
<td>0.9</td>
</tr>
<tr>
<td>maximum iteration</td>
<td>100</td>
</tr>
<tr>
<td>population size</td>
<td>500</td>
</tr>
<tr>
<td>(p_1-s)</td>
<td>[-60, 60]</td>
</tr>
</tbody>
</table>

Next, the proposed technique is adopted to design the robust PI controller. In the optimization problem, the upper
Fig. 7. Output responses of the system when the unit step is entered to position command.

Fig. 8. Output responses of the system when the unit step is entered to the force command input.

Fig. 9. Output responses of the perturbed plant when the unit step and disturbance (0.3u(t)) are entered to the position command.
V. CONCLUSION

This paper presents a new technique for designing a fixed structure robust loop shaping controller. As shown in the results, the proposed technique can be applied to control a MIMO electro-hydraulic servo system. The stability margin ($\varepsilon$) obtained by the proposed technique indicates the robustness and the performance of the proposed controller. Simulation results verify that the proposed technique is a feasible approach and reduces the gap between theoretical and practical schemes.

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APPENDIX A

The state space representation of the robust loop shaping controller is given by $A_{H_{\infty}}$, $B_{H_{\infty}}$, $C_{H_{\infty}}$ and $D_{H_{\infty}}$ matrices as shown in the followings.

\[
A_{H_{\infty}} = \begin{bmatrix}
-1 \times 10^8 & -6.3998 \times 10^{-13} & 7.1725 \times 10^{-7} & -7.9386 \times 10^{-4} & -2.6913 \times 10^{-6} & 6.6929 \times 10^{-8} & 9.5343 \times 10^{-7} & 6.7209 \times 10^{-8} \\
3.4197 \times 10^{-13} & -1 \times 10^{-6} & 9.8404 \times 10^{-4} & 7.7479 \times 10^{-7} & -7.2385 \times 10^{-6} & 1.5541 \times 10^{-7} & -4.2988 \times 10^{-8} & 6.6381 \times 10^{-8} \\
5.8084 \times 10^{-6} & 0.00010724 & -1860.4 & -428.12 & 210.9 & -118.41 & 86.791 & -2.8545 \\
-8.926 \times 10^{-6} & 6.5538 \times 10^{-8} & -16.553 & -1622.7 & 71.963 & 135.01 & 103.1 & 0.0902 \\
1.7123 \times 10^{-6} & -7.5377 \times 10^{-6} & 154.51 & 161.86 & -74.138 & 40.517 & -54.428 & 1.632 \\
5.8492 \times 10^{-6} & 3.2979 \times 10^{-6} & -169.13 & 76.577 & 17.45 & -123.37 & -3.732 & -2.6062 \\
3.1181 \times 10^{-6} & -3.0726 \times 10^{-6} & 33.193 & 123.55 & -54.957 & 40.517 & -54.428 & 1.6655 \\
-8.4247 \times 10^{-4} & 4.1953 \times 10^{-6} & 0.1339 & -2.7237 & 1.1233 & 2.0641 & 3.2602 & -0.6734
\end{bmatrix}
\]

\[
B_{H_{\infty}} = \begin{bmatrix}
-3.6803 & 5.0342 \\
-4.9712 & -3.6342 \\
37.343 & 4.8654 \\
-0.1353 & 34.377 \\
-1.5585 & -1.8007 \\
1.8961 & -0.8730 \\
-0.1899 & -1.3432 \\
-0.00507 & 0.02882
\end{bmatrix}
\]

\[
C_{H_{\infty}} = \begin{bmatrix}
-3.8459 & -4.8475 & 37.643 & 3.5133 & -2.2719 & 1.3247 & -0.8585 & 0.02901 \\
4.9089 & -3.7977 & 1.0807 & 34.197 & -0.71389 & -1.6132 & -1.0503 & -0.0038
\end{bmatrix}
\]

\[
D_{H_{\infty}} = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}
\]
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