Analytical Robust Tuning Approach for Two-Degree-of-Freedom PI/PID Controllers

R. Vilanova, V.M. Alfaro, O. Arrieta

***Abstract*—This paper's aim is to present a tuning approach for full Two-Degree-of-Freedom (2-DoF) PI and PID controllers for First-Order-Plus-Dead-Time (FOPDT) and Second-Order-Plus-Dead-Time (SOPDT) controlled processes. The tuning relations provide the value of the typical parameters for a PID controller plus the set-point weighting factor, being these relations driven by just one single design parameter to be selected by the user. This fact makes the approach easier to apply. The design procedure also considers the control-loop robustness by means of the maximum sensitivity requirements, allowing the designer to deal with the performance-robustness trade-off.**

***Index Terms*—PID control, Two-Degrees-of-Freedom, Robustness, Process Control**

I. Introduction

Most of the single-loop controllers used in practice are found under the form of a PI/PID controller. Effectively, since their introduction in 1940 [1], [2] commercial *Proportional - Integrative - Derivative* (PID) controllers have been with no doubt the most extensive option found on industrial control applications. Their success is mainly due to its simple structure and meaning of the corresponding three parameters. This fact makes PID control easier to understand by the control engineers than other most advanced control techniques. This fact has motivated a continuous research effort to find alternative tuning and design approaches to improve PI/PID based control system's performance.

With regard to the design and tuning of PID controllers, there are many methods that can be found in the literature over the last sixty years. Special attention is made of the *IFAC workshop PID'00 Past, Present and Future of PID Control* held in Terrassa, Spain, in April of 2000, where a glimpse of the state-of-the-art on PID control was provided. It can be seen that most of them are concerned with feedback controllers which are tuned either with a view to the rejection of disturbances [3], [4], [5] or for a well-damped fast response to a step change in the controller set-point [6], [7], [8]. O'Dwyer [9] presents a collection of tuning rules for PI and PID controllers, which show their abundance.

Recently, tuning methods based on optimization approaches with the aim of ensuring good stability robustness have received attention in the literature [10], [11]. Also, great advances on optimal methods based on stabilizing PID solutions have been achieved [12], [13]. However these methods, although effective, use to rely on somewhat complex numerical optimization procedures and do not provide tuning rules. Instead, the tuning of the controller is defined as the solution of the optimization problem.

Among the different approaches, the direct or analytical synthesis constitutes a quite straightforward approach to PID controller tuning. The controller synthesis presented by Martin [6] made use of zero-pole cancellation techniques. Similar relations were obtained by Rivera *et. al.* [7], [14], applying the IMC concepts of Garcia and Morari [15] to tuning PID controllers for low-order process models. A combination of analytical procedures and the IMC tuning can be found in [16], [17], [18]. With this respect, the usual approach is to specify the desired closed-loop transfer function and to solve analytically for the feedback controller. In cases where the process model is of simple structure, the resulting controller has the PI/PID structure. Most of the analytically developed tuning rules are related with the servo-control problem while the consideration of the load-disturbance specifications has received not so much attention. It is worth to mention the notable work of Chen and Seborg [19], where the importance of emphasizing disturbance rejection as the starting point for design is discussed. However it is well known that if we optimize the closed-loop transfer function for a step-response specification, the performance with respect to load-disturbance attenuation can be very poor [20]. This is indeed the situation, for example, for IMC controllers that are designed in order to attain a desired set-point to output transfer function presenting a sluggish response to the disturbance [18].

The need to deal with both kind of properties and the recognition that a control system is, inherently, a system with Two Degrees-of-Freedom (2-DoF) - two closed-loop transfer functions can be adjusted independently -, motivated the introduction of 2-DoF PI/PID controllers [21]. The 2-DoF formulation is aimed at trying to met both objectives, say good regulation and tracking properties. This second degree of freedom is aimed at providing additional flexibility to the control system design. See for example [22], [23], [24] and its characteristics revised and summarized in [25], [26] and [27], as well as different tuning methods that have been formulated over the last years [25], [28], [29], [30], [31], [32], [33], [34], [35]. There have also been some particular applications of the 2-DoF formulation based on advanced optimization algorithms (see for example [36], [37], [38],

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[39]). The point is that, with a few exceptions such as the AMIGO [34] and Kappa-Tau; $\kappa - \tau$; [40] methods, no analytical expressions are provided for all controller parameters (feedback and reference part) and, at the same time, ensure a certain robustness degree for the resulting closed-loop. To provide simple tuning expressions and, at the same time, guarantee some degree of robustness are the main contributions of the paper.This second degree of freedom is found on the presented literature as well as in commercial PID controllers under the form of the well known set-point weighting factor (usually called β) that ranges within $0 \leq \beta \leq 1.0$, being the main purpose of this parameter to avoid excessive proportional control action when a reference change takes place. Therefore the use of *just a fraction* of the reference.

However, performance with respect to load-disturbance attenuation is just one of the drawbacks of the analytical approaches to PI/PID controller design. In fact, the known analytical approaches do not include any consideration on the control system robustness. The usual approach is to measure the robustness of the resulting design (usually in terms of the peak value of the sensitivity function M_s) instead of specifying a desired robustness level from the very beginning. It is with this respect that this paper provides its main contribution: a load-disturbance based analytical design being the *only* design parameter the desired robustness level of the resulting control system. At this point, the performance-robustness tradeoff arises and has to be introduced into the design procedure. As for set-point performance the desired closed-loop time constant is to be chosen as fast as possible (robustness permitting) the presented procedure characterizes, for each possible peak value of the sensitivity function (within its usual [1.2 - 2.0] range), the lowest allowable time constant. This first analysis conducts to a design approach that is divided in two steps: first of all, an equation is provided that generates the desired closed-loop time constant from the specified robustness; on a second step this time-constant is introduced on the parameterized controller parameters relations. It is worth to stress that at this point the approach is presented here just for PI controller design, being the full PID case more involved and its full derivation is to be presented separately.

As the design is based on a load-disturbance specification, in order to improve the resulting step-response performance, the available second degree of freedom under the form of a set-point weighting factor will be fully included into the design. While in [19] just some ad-hoc values are used that show that better step response can be obtained, in this work a selection rule is provided on the basis of a desired set-point to output transfer function. Therefore providing the a full tuning for a 2-DoF PI/PID controller.

Although the 2-DoF controller design approach presented hereafter may seem simple and straightforward it has not been fully detailed. Also, it is the authors opinion that this idea has in its simplicity one of its main attractiveness (as well as the λ-tuning method of Dahlin [41], the IMC approach developed by Morari and coworkers [7], [14] or the work by Gorez [42]) and this motivates the extension presented in this work by providing full tuning rules that also include the set-point processing components, then the second Degree-of-Freedom, for PI and PID controllers. In addition, the formulation was raised in order to obtain a control system with a dynamic performance that would be simultaneously considered optimum and robust.

Therefore, the work presented in this paper constitutes a direct extension of the ideas initiated in [19], providing a single-parameter driven Robust Tuning for 2-DOF PI and PID controllers. Therefore called Analytical Robust Tuning (ART_2).

The organization of the paper is as follows. Next section introduces the framework and notation related to the control system as well as how the design problem is formulated. Section III summarizes the early developed ART_2 method for the PI_2 case, whereas in section III there is the PID_2 tuning rules. Section V presents application examples and, finally, in section VI conclusions are conducted.

II. Framework and Problem Formulation

This section will present the controller structure we will work with as well as how the design problem is posed. The basic design relations that will be used on following sections will be obtained. Considerer the *Two-Degree-of-Freedom* (2-DoF) feedback control system of Fig. 1 where $P(s)$ is the controlled process transfer function, $C_r(s)$ the *set-point controller* transfer function, $C_y(s)$ the *feedback controller* transfer function, and $r(s)$ the set-point, $d(s)$ the load-disturbance, and $y(s)$ the controlled variable. The output of the 2-DoF PI, PI_2, controller is given by

$$u(s) = C_r(s)r(s) - C_y(s)y(s) \tag{1}$$

For a PI_2 controller [43] it is

$$u(s) = K_c\left(\beta + \frac{1}{T_i s}\right)r(s) - K_c\left(1 + \frac{1}{T_i s} + T_d s\right)y(s) \tag{2}$$

where K_c is the *controller gain*, T_i the *integral time constant*, T_d the *derivative time constant*, and β the *set-point weighting factor* ($0 \leq \beta \leq 1$).

Then, the controller's transfer functions are

$$C_r(s) = K_c\left(\beta + \frac{1}{T_i s}\right) \tag{3}$$

and

$$C_y(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{4}$$

The closed-loop control system response to a change in any of its inputs, will be given by

$$y(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)}r(s) + \frac{P(s)}{1 + C_y(s)P(s)}d(s) \tag{5}$$

or in a compact form by

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s) \tag{6}$$

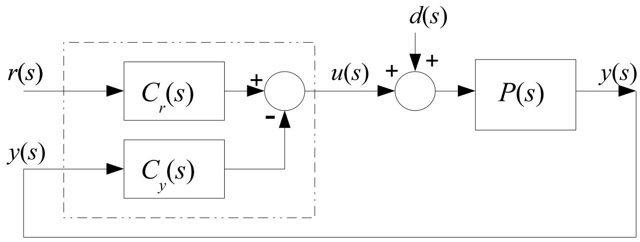

Fig. 1. 2-DoF Control System.

where $M_{yr}(s)$ is the transfer function from set-point to process variable: the *servo-control* closed-loop transfer function or complementary sensitivity function $T(s)$; and $M_{yd}(s)$ is the one from load-disturbance to process variable: the *regulatory control* closed-loop transfer function or disturbance sensitivity function $S_d(s)$.

If $\beta = 1$, all parameters of $C_r(s)$ are identical to the ones of $C_y(s)$. In such situation, it is impossible to specify the dynamic performance of the control system to set-point changes, independently of the performance to load-disturbances changes. Otherwise, if the contrary, $\beta < 1$, given a controlled process $P(s)$, the feedback controller $C_y(s)$ can be selected to achieve a target performance for the regulatory control $M_{yd}(s)$, and then use the set-point weighting factor in the set-point controller $C_r(s)$, to modify the servo-control performance $M_{yr}(s)$.

On the other hand, the characteristic polynomial of the closed-loop control system is

$$p(s) = 1 + C_y(s)P(s) \tag{7}$$

from where it can be seen that the closed-loop poles location; therefore the closed-loop stability; depends only on the $C_y(s)$ parameters, hence not affected by β.

The proposed *Analytic Robust Tuning of Two-Degree-of-Freedom PI/PID controllers* (ART_2) [44], [28], is aimed at producing a control system that responds fast and without oscillations to a step load-disturbance, with a maximum sensitivity lower than a specified value; in order to assure robustness; and which will also show a fast non oscillating response to a set-point step change, not requiring strong or excessive control effort variations (*smooth* control). Of course, the fact of imposing a non-oscillatory response introduces an additional constraint and may seem excessively conservative. It is known that other approaches based on minimizing some error based index (Integrated absolute error for example) generate slightly oscillatory responses that may be faster. However because one of the aims of the approach is to be able to explicitly introduce the robustness-performance tradeoff into the design relations, smooth signals are preferred. Therefore the use of non-oscillatory target responses.

A. Outline of Controller Design Procedure

The first step in the Two-Degree-of-Freedom controller synthesis consists of obtaining the feedback controller $C_y(s)$, required to achieve a target $M^t_{yd}(s)$ regulatory closed-loop transfer function. From (5) and (6) the regulator control closed-loop transfer function is given by

$$\frac{y(s)}{d(s)} = M_{yd} = \frac{P(s)}{1 + C_y(s)P(s)} \tag{8}$$

and the one for servo-control is

$$\frac{y(s)}{r(s)} = M_{yr}(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)} \tag{9}$$

which are related by

$$M_{yr}(s) = C_r(s)M_{yd}(s) \tag{10}$$

From (8) once the controlled process is given and the target regulatory transfer function, $M^t_{yd}(s)$, specified the required feedback controller can be synthesized. The resulting feedback controller design equation is

$$C_y(s) = \frac{P(s) - M^t_{yd}(s)}{P(s)M^t_{yd}(s)} = \frac{1}{M^t_{yd}(s)} - \frac{1}{P(s)} \tag{11}$$

Once, as a first step, the feedback controller $C_y(s)$, is obtained from (11), on a second step, the set-point controller $C_r(s)$ free parameter (β) can be used in order to modify the servo-control closed-loop transfer function $M_{yr}(s)$ (10).

The outlined design approach is in fact like the direct design as proposed within the IMC framework [7]. In IMC however, the designer has to choose the well known IMC design parameter in order to satisfy the performance/robustness tradeoff. What will be proposed in the formulation presented here is to avoid such step, by an automatic selection of the controller parameters in terms of the desired robustness. The selection of the control system bandwidth is done in such a way the closed-loop bandwith is as large as possible while meeting the robustness constraint. It could therefore be interpreted as an IMC controller with robustness considerations explicitly incorporated.

III. 2-DoF PI Robust Tuning for First-Order-Plus-Dead-Time Processes

Consider the First-Order-Plus-Dead-Time (FOPDT) controlled process given by

$$P(s) = \frac{K_p e^{-Ls}}{Ts + 1} \tag{12}$$

where K_p is the process gain, T the time-constant, and L its dead-time. From here and after, $\tau_o = L/T$ will be referred as the controlled process *normalized dead-time*. In this work process models with normalized dead-time $\tau_o \leq 2$ are considered. Processes with long dead-time will need some kind of dead-time compensation scheme (a Smith predictor, for example).

For the FOPDT process the specified regulatory and closed-loop control target transfer functions are chosen as

$$M^t_{yd}(s) = \frac{Kse^{-Ls}}{(\tau_c Ts + 1)^2} \tag{13}$$

and the closed-loop target function selected for the servo-control as

$$M_{yr}^t(s) = \frac{e^{-Ls}}{\tau_c T s + 1} \tag{14}$$

where τ_c will be the *dimensionless design parameter*. It is the ratio of the closed-loop control system time constant (T_c) to the controlled process time constant (T). The specified target closed-loop transfer functions (13) and (14) will provide non-oscillating responses to step changes in both, the set-point and the load-disturbance, with an adjustable speed.

A. Controller Parameters

In order to synthesize the 2-DoF PI controller for the FOPDT process it is necessary to use a rational function in s as an approximation of the controlled process dead-time. This approximation will affect the closed-loop response characteristics. Using the Maclaurin first order series for the dead-time

$$e^{-Ls} \approx 1 - Ls \tag{15}$$

and 12 and 13 in 11, the PI_2 controller tuning equations are obtained as

$$\kappa_c = K_c K_p = \frac{2\tau_c - \tau_c^2 + \tau_o}{(\tau_c + \tau_o)^2} \tag{16}$$

$$\tau_i = \frac{T_i}{T} = \frac{2\tau_c - \tau_c^2 + \tau_o}{1 + \tau_o} \tag{17}$$

where κ_c and τ_i are the controller *normalized parameters.*

In order to assure that the controller parameters (16) and (17) have positive values, the design parameter τ_c must be selected within the range

$$0 < \tau_c \leq 1 + \sqrt{1 + \tau_o} \tag{18}$$

The resulting regulatory control closed-loop transfer function is

$$M_{yd}(s) = \frac{T_i s e^{-Ls}}{K_c(\tau_c T s + 1)^2} \tag{19}$$

The variation of the resulting PI controller normalized parameters (16) and (17) is show in Fig. 2.

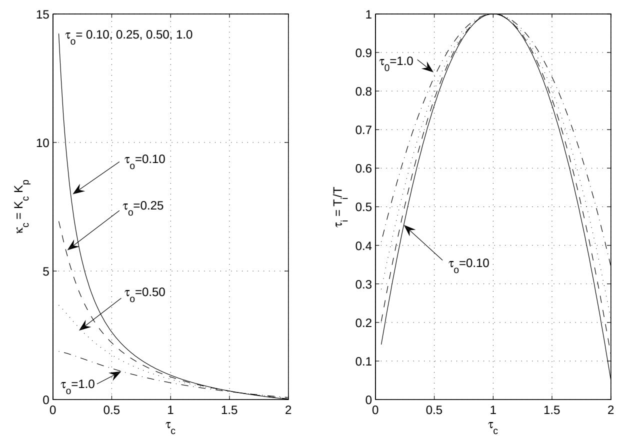

Fig. 2. PI Normalized Parameters.

B. Set-point Weighting Factor

As the closed-loop transfer functions are related by $M_{yr}(s) = C_{yr}(s)M_{yr}(s)$, by using controller $C_r(s)$, $M_{yr}(s)$ can be written as

$$M_{yr}(s) = \frac{K_c\,(\beta T_i s + 1)}{T_i s} M_{yd}(s) \tag{20}$$

Introducing in (20) the regulatory control closed-loop transfer function (19) and also the controller parameters (16) and (17), the servo control transfer function then becomes

$$M_{yr}(s) = \frac{(\beta T_i s + 1)\, e^{-Ls}}{(\tau_c T s + 1)^2} \tag{21}$$

As the servo-control target transfer function was specified in (14), from (14), (20) and (21) in order to obtain a non-oscillatory response, an adequate selection of the set-point weighting factor would be $\beta = \tau_c T/T_i$, and then

$$\beta = \frac{\tau_c T}{T_i}, \qquad 0 < \tau_c \leq 1 \tag{22}$$

outside this range

$$\beta = 1, \qquad 1 < \tau_c < 1 + \sqrt{1 + \tau_o} \tag{23}$$

Effectively, it can be verified that $\tau_i \leq 1$. Therefore, if $\tau_c > 1$, as $\beta = \tau_c(T/T_i)$ we will have $\beta = \tau_c/\tau_i > 1$. In addition if $\tau_c \leq 1$ τ_i is always larger than τ_c therefore assuring $\beta = \tau_c/\tau_i \leq 1$. The constraint $\beta \leq 1$ is introduced because in commercial controllers the set-point weighting factor (when available) is restricted to have a value lower than one. This selection for the $0 < \tau_c \leq 1$ range, will made the set-point controller zero to cancel one of the closed-loop poles. This weighting factor also has influence in the controller output when the set-point changes. Effectively, the instantaneous change on the control signal caused by a sudden change in the reference signal of magnitude Δr is given by $\Delta u_r = K_c \beta \Delta e = K_c \beta \Delta r$ therefore, when very fast regulatory control responses are desired, high controller gain values are required, and the controller instantaneous output change when the set-point changes may be high. Then the controller output will be limited to be not greater than the total change on the set-point and then the set-point weighting factor selection criteria becomes

$$\beta = \min\left\{\frac{1}{K_c}, \frac{\tau_c T}{T_i}, 1\right\} \tag{24}$$

C. Control System Robustness

The maximum sensitivity

$$M_s = \max_{\omega} |S(j\omega)| = \max_{\omega} \left| \frac{1}{1 + C_y(j\omega)P(j\omega)} \right| \tag{25}$$

will be used as an indication of the closed-loop control system robustness.

The use of the maximum sensitivity as a robustness measure, has the advantage that lower bounds to the gain and phase margins [40] can be assured according to

$$A_m > \frac{M_s}{M_s - 1} \tag{26}$$

$$\phi_m > 2\sin^{-1}\left(\frac{1}{2M_s}\right) \tag{27}$$

A robustness analysis has been performed and shown in Fig. 3. This analysis shows that the control system maximum sensitivity M_s depends of the model normalized dead time τ_o and the design parameter τ_c.

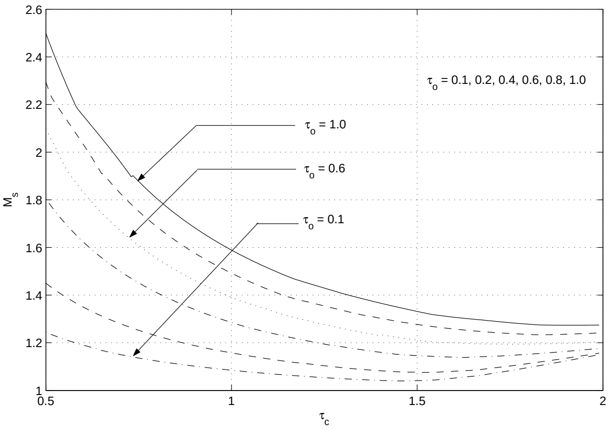

Fig. 3. Control System Robustness

In order to avoid the loss of robustness when a very low τ_c is used, it is necessary to establish a lower limit to this design parameter. This relative loss of stability is greater when the normalized model dead time τ_o is high.

Using the inverse function of Fig. 3; shown in Fig. 4, the lower limits to the design parameter for a specific robustness level can be obtained. These limits are shown in Fig. 5.

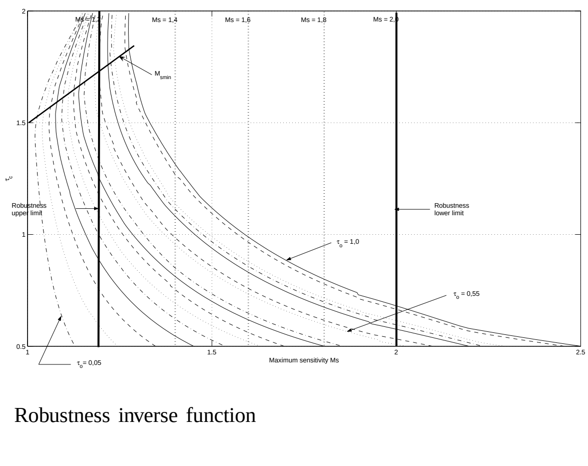

Fig. 4. Robustness inverse function

From Fig. 5 the design parameter lower limit for a given robustness level can be expressed in parameterized form as

$$\tau_{cmin} = k_1(M_s) + k_2(M_s)\tau_o \tag{28}$$

where the k_1 and k_2 are show in Table I.

TABLE I
EQUATION (28) CONSTANTS

M_s	1.2	1.4	1.6	1.8	2.0
k_1	0.4836	0.4152	0.3441	0.3254	0.3042
k_2	1.8982	0.9198	0.6659	0.4853	0.3822

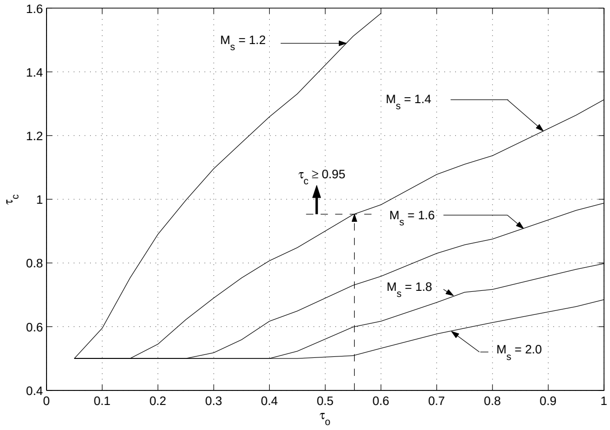

Fig. 5. Design Parameter Low Limits

The design parameter equations (28) can be expressed as a single equation as

$$\tau_{cmin} = k_{11}(M_s) + \left[\frac{k_{21}(M_s)}{k_{22}(M_s)}\right]\tau_o \tag{29}$$
$$k_{11}(M_s) = 1.384 - 1.063M_s + 0.262M_s^2$$
$$k_{21}(M_s) = -1.915 + 1.415M_s - 0.077M_s^2$$
$$k_{22}(M_s) = 4.382 - 7.396M_s + 3.0M_s^2$$

Also from Fig. 3 it can be seen that; as usual; as the system becomes slower its robustness increases but if very slow responses are specified the system robustness starts to decrease, therefore the upper limit of the design parameters τ_c also needs to be constrained as it is shown in Fig. 4. By combining the design parameter performance and robustness constraints it may be selected within the range

$$\max(0.50, \tau_{cmin}) \leq \tau_c \leq 1.50 + 0.3\tau_o \tag{30}$$

where τ_{cmin} is given by (29).

D. Control System Performance

The control system response will be given then by the equation

$$y(s) = \frac{(\beta T_i s + 1)e^{-Ls}}{(\tau_c T s + 1)^2} r(s) + \frac{Kse^{-Ls}}{(\tau_c T s + 1)^2} d(s) \tag{31}$$

with

$$K = K_p\left[\tau_c^2 T + \frac{(2\tau_c - \tau_c^2\tau_o)T\tau_o}{1+\tau_o}\right] \tag{32}$$

which reduces to

$$y(s) = \frac{e^{-Ls}}{\tau_c T s + 1} r(s) + \frac{Kse^{-Ls}}{(\tau_c T s + 1)^2} d(s) \tag{33}$$

if $\beta = \tau_c T / T_i$.

As it can be observed from (33) the obtained control system output corresponds to the regulatory and servo-control target closed-loop transfer functions specified in (13) and (14). In this case, the system responses to a step change in both, the set-point and the load-disturbance, will be non-oscillating. The performance (system speed) to robustness (M_s) trade-off may be resolved by the designer selecting the design parameter τ_c that guarantees a minimum desired robustness by (29).

IV. 2-DoF PID Robust Tuning for Second-Order-Plus-Dead-Time Processes

By using a similar procedure as the one presented in previous section for the PI controller, we will start right now with a Second-Order-Plus-Dead-Time (SOPDT) model of the form

$$P(s) = \frac{K_p e^{-L''s}}{(T''s+1)(aT''s+1)},\ \tau_o = \frac{L''}{T''} \tag{34}$$

$$0.1 \leq \tau_o \leq 1.0,\ 0.15 \leq a \leq 1.0$$

In this situation, a third mode will need to be introduced into the closed-loop system's target responses. In this case, the design parameter τ_c will denote the relation between the desired closed-loop time constant and T'' ($\tau_c = T_c/T''$).

The generated closed-loop relations will take the form:

$$\begin{aligned} y(s) =& \frac{(\beta T_i s+1)\, e^{-Ls}}{(\tau_c T''s+1)^2(T_{cx}s+1)} r(s) \\ &+ \frac{Kse^{-Ls}}{(\tau_c T''s+1)^2(T_{cx}s+1)} d(s) \end{aligned} \tag{35}$$

where T_{cx} is the time constant of the third pole of the closed-loop transfer function. This time constant was selected as $T_{cx} = 0.1\tau_c T''$ to reduce its influence on the control system dynamic behavior.

From (35), the regulatory control closed-loop transfer function is

$$M_{yd}(s) = \frac{Kse^{-Ls}}{(\tau_c T''s+1)^2(T_{cx}s+1)} \tag{36}$$

and the servo-control closed-loop transfer function is

$$M_{yr}(s) = \frac{(\beta T_i s+1)\, e^{-Ls}}{(\tau_c T''s+1)^2(T_{cx}s+1)} \tag{37}$$

that are related by

$$M_{yr}(s) = \frac{K_c\,(\beta T_i s+1)}{T_i s} M_{yd}(s) \tag{38}$$

As well as in the PI controller case, for the PID controller synthesis procedure was necessary to approximate the dead-time with the MacLaurin first order series (15).

It is worth to remark that it would be also possible to get a PID controller from a FOPDT model by approximating the dead-time by using a first order Padé approximation instead of the first order MacLaurin expansion. The derivation however is not included here but follows the same procedure.

A. Controller Parameters

The PID controller parameters are determined by the following equations for processes with parameters in the range $0.1 \leq \tau_o \leq 1.0$ and $0.15 \leq a \leq 1.0$.

$$\kappa_c = \frac{10\tau_i}{21\tau_c + 10\tau_o - 10\tau_i} \tag{39}$$

$$\tau_i = \frac{(21\tau_c + 10\tau_o)[(1+a)\tau_o + a] - \tau_c^2(\tau_c + 12\tau_o)}{10(1+a)\tau_o + 10a + 10\tau_o^2} \tag{40}$$

$$\tau_d = \frac{12\tau_c^2 + 10\tau_i\tau_o - (1+a)(21\tau_c + 10\tau_o - 10\tau_i)}{10\tau_i} \tag{41}$$

$$\beta = \min\left\{\frac{1}{K_c}, \frac{\tau_c T''}{T_i}, 1\right\} \tag{42}$$

The controller normalized parameters κ_c (K_cK_p), τ_i (T_i/T) and τ_d (T_d/T), and β depend on the model normalized dead-time τ_o and time constants ratio a, and on the design parameter τ_c.

To obtain positive controller parameters the design parameter upper value must be restricted to

$$\tau_c \leq 1.25 + 2.25a \tag{43}$$

Besides, due that the use of the dead-time first order MacLaurin series approximation made the system output to deviate from the target one when very fast responses are specified it is recommended to select the design parameter such that

$$0.065(2 - a + 10\tau_o + 10a\tau_o) \leq \tau_c \tag{44}$$

In addition to the performance of the resulting control system its robustness was also investigated.

A minimum system robustness level is incorporated into the design process estimating a recommended maximum speed (τ_{cmin}) of the resulting closed-loop control system parameterized in terms of the maximum sensitivity function (M_s) by using

$$\begin{aligned} \tau_{cmin} \quad = \quad & k_{11}(M_s) + k_{12}(M_s)a^{k_{13}(M_s)} \qquad (45) \\ & k_{11}(M_s) = 2.442 - 2.219M_s + 0.515{M_s}^2 \\ & k_{12}(M_s) = 10.518 - 8.990M_s + 2.203{M_s}^2 \\ & k_{13}(M_s) = 0.949 - 0.197M_s \end{aligned}$$

Combining the performance and robustness consideration above the design parameter may be selected in the range

$$\tau_{cmin} \leq \tau_c \leq 1.25 + 2.25a \tag{46}$$

The range limits for the design parameter selection (46) then combine the necessary restriction so that all controller parameters are positive and the accomplishment of a specified maximum sensitivity, with the necessity that the obtained response does not deviate too much away from the desired response, due of the dead-time approximation used in obtaining the tuning equations.

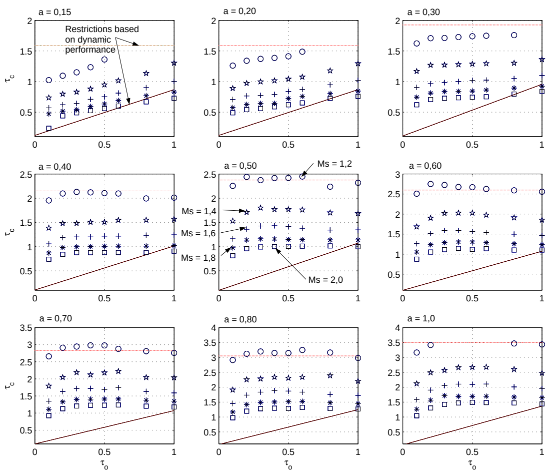

Fig. 6. Design Parameter Constraints

B. Control System Robustness

In order to analyze the resulting closed-loop robustness, the maximum sensitivity M_s was determined for τ_c within 0.25 and 3.5 allowing establishing (45), in order to estimate the lower τ_c value.

The relations between the design parameter τ_c, the closed-loop robustness M_s and the controlled process normalized dead-time τ_o and the time constant ratio a are shown in Fig. 6.

C. Control System Performance

Now the control system response is given by

$$\begin{aligned} y(s) =& \frac{(\beta T_i s + 1)\, e^{-Ls}}{(\tau_c T'' s + 1)^2 (0.1\tau_c T'' s + 1)} r(s) \\ &+ \frac{K s e^{-Ls}}{(\tau_c T'' s + 1)^2 (0.1\tau_c T'' s + 1)} d(s) \end{aligned} \tag{47}$$

with K given by

$$K = \frac{K_p T''[(21\tau_c + 10\tau_o)\tau_o^2 + \tau_c^2(\tau_c + 12\tau_o)]}{10[(1+a)\tau_o + a + \tau_o^2]} \tag{48}$$

and, as before, τ_c is the design parameter that expresses the relation between the closed-loop control system time constant and the controlled process dominant time constant.

If $\beta = \tau_c T / T_i$ (47) reduces to

$$\begin{aligned} y(s) =& \frac{e^{-Ls}}{(\tau_c T'' s + 1)(0.1\tau_c T'' s + 1)} r(s) \\ &+ \frac{K s e^{-Ls}}{(\tau_c T'' s + 1)^2 (0.1\tau_c T'' s + 1)} d(s) \end{aligned} \tag{49}$$

obtaining in such case the first-order and second-order target closed-loop transfer functions (13) and (14) (for PI case), plus a fast additional pole that will have a neglected influence over the system step responses.

V. Application of the ART_2 Tuning Method

This section provides an example of application of the presented tuning approach for a high order controlled process. The example starts showing the proposed method application in the case of PI_2 tuning from the process FOPDT model approximation followed with the PID_2 tuning from its SOPDT model, also a comparison of the proposed approach for tuning PID controller with other recognized tuning approaches is included.

In order to have simulation results more close to industrial practice, in all the examples it is assumed that all variables can vary in the 0 to 100% normalized range and that in the normal operation point, the controlled variable, the set-point and the control signal, have all values close to 70%.

The selected example will show, on one side, how the proposed ART_2 method performs by using the desired maximum sensitivity value as the system specification. On the other side, comparison with other well known direct synthesis methods such as the DS-d from [19] and SIMC from [18] will be outlined.

The maximum sensitivity value M_s will be used as a measure of the control system robustness. Recommended values for M_s are typically within the range 1.2 - 2.0. Although the DS-d method does not provide any relation between its design parameter T_c and the obtained control-loop robustness, for comparison purposes the design parameter for this method will be selected in such a way to obtain similar robustness levels. For the SIMC method its recommendation for robust tuning of using a design parameter equal to the model apparent dead-time will be followed.

Controlled Process: Considerer the fourth order system with the transfer function

$$P(s) = \frac{1}{(s+1)(0.4s+1)(0.16s+1)(0.64s+1)} \tag{50}$$

The FOPDT model approximation for this process is

$$P_1(s) = \frac{e^{-0.517s}}{1.149s + 1} \tag{51}$$

and the approximation with a SOPDT model

$$P_2(s) = \frac{e^{-0.147s}}{(0.856s+1)(0.603s+1)} \tag{52}$$

Both models were obtained using a three-point identification procedure [45].

Based on the previous approximations, a 2-DoF PI and a 2-DoF PID controller will be used respectively.

A. Proportional-Integral (PI) Controller

From model (51) we have $K_p = 1.0$, $T = 1.149$, $L = 0.517$ and $\tau_o = 0.450$. Using (29) and (30) the recommended range for the design parameter for this model is $\max(0.50, \tau_{cmin}) \leq \tau_c \leq 1.635$, where τ_{cmin} can be computed using (29) on the basis of a M_s specification.

TABLE II
EXAMPLE - ART_2 PI CONTROLLER PARAMETERS AND ROBUSTNESS

τ_c^d	K_c	T_i	β	M_{sm}^r	M_{sp}^r
0.50	1.330	0.951	0.604	1.854	1.704
0.60	1.170	1.022	0.674	1.667	1.542
0.80	0.902	1.117	0.823	1.439	1.394
1.00	0.690	1.149	1.0	1.315	1.286
1.20	0.518	1.117	1.0	1.231	1.219

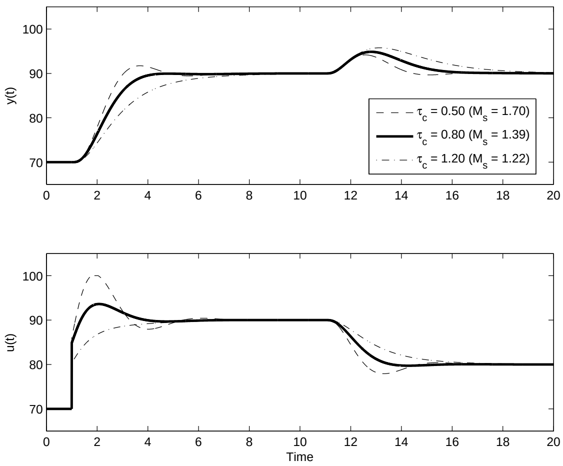

Fig. 7. Example - ART_2 PI System Responses

The PI_2 controller parameters and the control-loop robustness obtained with a selected set of parameters are shown in Table II. In this Table M_{sm}^r is the predicted robustness obtained using the model as the controlled plant and M_{sp}^r the one finally obtaining controlling the real high order process. As seen the obtained robustness are all slightly higher that ones predicted. Therefore confirming the safe way of choosing the time constant τ_c^d.

Fig. 7 shows the system responses to a 20% change in set-point followed by a 10% change in load-disturbance with three different design parameters.

The DS-d [19] PI_1 controller tuning equations are, in this case, the same as those of ART_2 for a PI_2 controller except with $\beta = 1.0$ in all cases. The design parameter of this method is the closed-loop time constant T_c, then using for design $T_c^d = \tau_c^d T$ same controller parameters are obtained. Control systems will have same robustness and response to a disturbance change but different response to a change in set-point. As shown in Fig. 8 in this particular example the controller parameters corresponding to $T_c = 0.575$ ($\tau_c = 0.50$) and $T_c = 0.689$ ($\tau_c = 0.60$) made the controller output to exceed its upper limit and may not be applied directly to a 1-DoF PI controller ($\beta = 1.0$). If a high speed and low robustness system is desired a weighting factor must be used ($\beta_{max} = 0.50$ and 0.60 for the two cases indicated above) or the control system operator must restrict the set-point changes to small increments to avoid controller output saturation.

Fig. 9 shows the time responses comparison for a given robustness level. In this case $M_s \approx 1.4$. This is the value we get if we apply the SIMC tuning. As it can be verified,

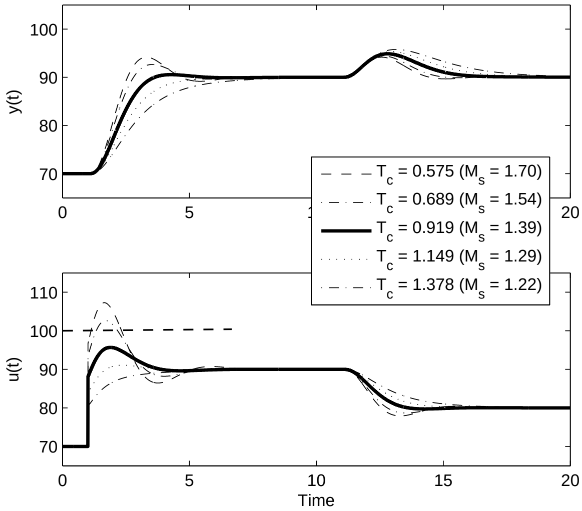

Fig. 8. Example - DS-d PI System Responses

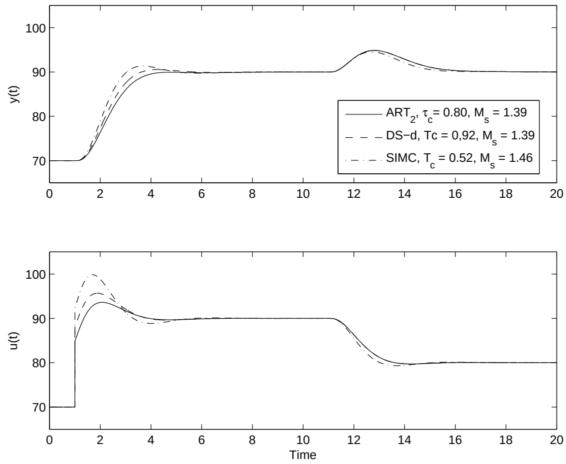

Fig. 9. Example - PI Controller System Responses

the outputs are reasonably similar but the proposed ART_2 method has lower control energy usage.

B. Proportional-Integral-Derivative (PID) Controller

From model (52) we have $K_p = 1.0$, $T_1 = 0.856$, $T_2 = 0.603$, $L'' = 0,147$, $a = 0.704$ and $\tau_o'' = 0.172$. Using (45) and (46) the recommended range for the design parameter for this model is $\tau_{cmin} \leq \tau_c \leq 2.834$ where τ_{cmin} can be computed using (45) on the basis of a M_s specification.

The PID_2 controller parameters and the control-loop robustness obtained with the ART_2 method and a selected set of design parameters are shown in Table III.

The PID_1 controller parameters and the control-loop robustness obtained with the DS-d method and a selected set of design parameters are shown in Table IV.

TABLE III
EXAMPLE - ART_2 PID CONTROLLER PARAMETERS AND ROBUSTNESS

τ_c^d	K_c	T_i	T_d	β	M_{sm}^r	M_{sp}^r
1.2	4.028	1.846	0.471	0.248	1.887	1.801
1.4	3.144	2.021	0.536	0.318	1.728	1.666
1.6	2.478	2.154	0.604	0.403	1.592	1.554
1.8	1.964	2.242	0.675	0.509	1.488	1.453
2.0	1.558	2.279	0.754	0.642	1.416	1.396
2.2	1.231	2.263	0.843	0.813	1.352	1.326
2.4	0.963	2.189	0.947	0.939	1.297	1.273
2.6	0.742	2.053	1.076	1.0	1.249	1.235
2.8	0.556	1.852	1.248	1.0	1.201	1.200

TABLE IV
EXAMPLE - DS-D PID CONTROLLER PARAMETERS AND ROBUSTNESS

T_c^d	K_c	T_i	T_d	β	M_{sm}^r	M_{sp}^r
0.35	6.335	1.034	0.272	1.0	1.934	2.045
0.40	5.191	1.129	0.291	1.0	1.737	1.820
0.45	4.293	1.214	0.307	1.0	1.593	1.635
0.50	3.574	1.287	0.322	1.0	1.484	1.507
0.55	2.992	1.347	0.333	1.0	1.398	1.432
0.60	2.514	1.393	0.342	1.0	1.326	1.356
0.65	2.116	1.424	0.348	1.0	1.269	1.295
0.70	1.782	1.439	0.350	1.0	1.221	1.243

As shown in Table III and IV the system robustness obtained with the ART_2 tuning are slightly higher than the ones predicted with the SOPDT model while the robustness obtained with the DS-d tuning are slightly lower than the ones expected. Considering the control system robustness the ART_2 tuning is safer than the DS-d tuning.

The recommended SIMC tuning for a PID_1 controller applied to this example provides a robustness level of $M_s \approx 1.8$ and will not be included in the comparison as higher robustness level are asked for.

For comparison purposes the ART_2 and DS-d tuning parameters, τ_c and T_c respectively, where adjusted in such a way to obtain same target robustness M_s^t in the range 1.2 to 2.0. The required controller parameters to do this are shown in Tables V and VI. With the DS-d tuning method there is no way to relate the tuning parameter T_c used with the resulting control system robustness (only the closed-loop speed is considered). On the other hand ART_2 recommended maximum speed for a target robustness τ_{cmin} (45) gives a safe estimation of the minimum value of the design parameter τ_c to use.

Fig. 10 and 11 show the time responses of both tuning

TABLE V
EXAMPLE - ART_2 PID CONTROLLER PARAMETERS

M_s^t	τ_c	K_c	T_i	T_d	β
2.0	1.00	5.243	1.633	0.407	0.191
1.6	1.51	2.756	2.100	0.573	0.363
1.2	2.80	0.556	1.852	1.248	1.0

TABLE VI
EXAMPLE - DS-D PID CONTROLLER PARAMETERS

M_s^t	T_c	K_c	T_i	T_d	β
2.0	0.360	6.082	1.054	0.276	1.0
1.6	0.465	4.060	1.237	0.312	1.0
1.2	0.750	1.498	1.438	0.347	1.0

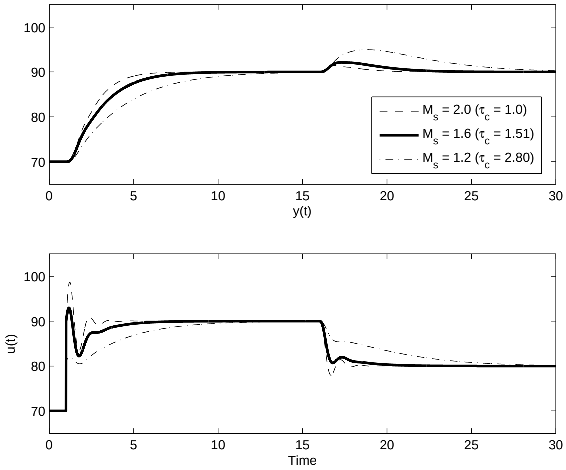

Fig. 10. Example - ART_2 PID Controller System Responses

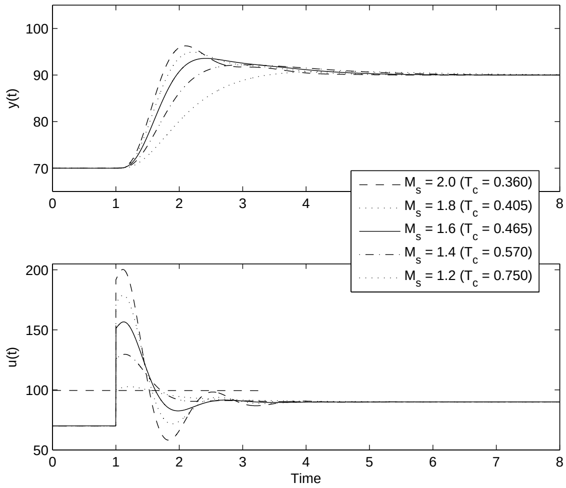

Fig. 11. Example - DS-d PID Controller System Responses

approaches. While the proposed method ensures the control variable do not exceed the 100%, as shown in Fig. 11 the PID_1 DS-d controller output to a 20% set-point change exceed its upper limit in all cases. For example for the $M_s = 2.0$ case the controller goes up to 202% (a change of 132%) and in the $M_s = 1.8$ case goes up to 180% (a 110% change) that are not physically possible in a real world application.

Fig. 12 shows a comparison of the system output for the $M_s = 2.0$ and 1.6 cases with ART_2 (PID_2) and DS-d (PID_1) settings.

VI. CONCLUSIONS

This paper has presented an analytically obtained method, ART_2, developed for Two-Degree-of-Freedom PID controllers. The method allows to obtain a control system that exhibits fast response to a load-disturbance step change yielding at the same time a desired minimum level of robustness. Selecting the design parameter τ_c the designer establishes the desired control system response speed (as the

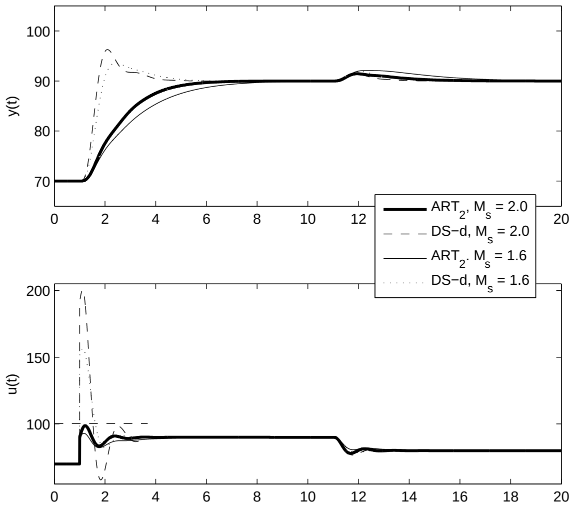

Fig. 12. Example - PID Controller System Responses

ratio between the closed-loop and model time constants). As the τ_c value becomes lower, the system response becomes faster, but its robustness decreases.

In order to establish the required control system robustness, given by the maximum sensitivity M_s, equations are provided for estimation of the minimum τ_c allowed.

The control system performance to a set-point step change can be modified by an adequate selection of the Two-Degree-of-Freedom controller set-point weighting factor β. The use of $\beta \leq 1$ values allows to decrease the servo-control response maximum overshot when very fast responses have been specified for the regulator control.

The examples presented show the advantages of the ART_2 tuning procedure. It is worth to mention the flexibility that allows the designer to take into consideration the regulatory control desired speed of response, control loop minimum required level of robustness and the resulting servo-control response characteristics.

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