An Optimal Placement of a Liaison between Two Levels in an Organization Structure of a Complete K-ary Tree

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***Abstract*—This paper proposes a model of placing a liaison which forms relations to two members of different levels in a pyramid organization structure such that the communication of information between every member in the organization becomes the most efficient. For the model of adding a node of liaison which gets adjacent to a node with a depth M and its descendant with a depth N in a complete K-ary tree of height H which can describe a pyramid organization structure with K subordinates, we obtained an optimal pair of depth $(M, N)^*$ which maximizes the sum of shortening lengths of the shortest paths between every pair of all nodes in the complete K-ary tree.**

***Index Terms*—organization structure, liaison, complete K-ary tree, shortest path.**

I. Introduction

A pyramid organization [1] is a formal organization structure which is a hierarchical structure based on the principle of unity of command [2] that every member except the top in the organization should have a single immediate superior. There exist relations only between each superior and his direct subordinates in the pyramid organization. The pyramid organization structure can be expressed as a rooted tree, if we let nodes and edges in the rooted tree correspond to members and relations between members in the organization respectively.

The pyramid organization structure is characterized by the number of subordinates of each member, that is, the number of children of each node and the number of levels in the organization, that is, the height of the rooted tree [3], [4]. Moreover, the path between a pair of nodes in the rooted tree is equivalent to the route of communication of information between a pair of members in the organization, and adding edges to the rooted tree is equivalent to forming additional relations other than that between each superior and his direct subordinates.

We have proposed some models [5], [6], [7] of forming additional relations between members in a pyramid organization structure such that the communication of information between every member in the organization becomes the most efficient. For each model we have obtained a set of additional edges to a complete K-ary tree minimizing the sum of lengths of the shortest paths between every pair of all nodes.

Liaisons [8], [9] which have roles of coordinating different sections are also placed as a means to become effective in communication of information in an organization. However, it has not been theoretically discussed which members of an organization should form relations to the liaisons.

We have obtained an optimal set for each of the following two models of placing a liaison which forms relations to members of the same level in a pyramid organization structure which is a complete K-ary tree of height H: (i) a model of adding a node of liaison which gets adjacent to two nodes with the same depth [10] and (ii) a model of adding a node of liaison which gets adjacent to all nodes with the same depth [11]. A complete K-ary tree is a rooted tree in which all leaves have the same depth and all internal nodes have $K(K = 2, 3, \ldots)$ children [12]. The depth of a node is the number of edges from the root to the node.

The above models (i) and (ii) correspond to the formation of additional relations between a liaison and members in the same level. These models give us optimal levels when we add relations to the liaison in one level of the organization structure which is a complete K-ary tree of height H, but these models cannot be applied to placing a liaison between different levels.

This paper proposes a model of placing a liaison which forms relations to two members of different levels in a pyramid organization structure which is a complete K-ary tree of height $H(H = 3, 4, \ldots)$ [13]. We obtain the two levels of which the liaison forms relations to two members such that the communication of information between every member in the organization becomes the most efficient. This means that we obtain the optimal pair of depth $(M, N)^*$ minimizing the sum of lengths of the shortest paths between every pair of all nodes when an added node of liaison gets adjacent to a node with a depth $M(M = 0, 1, \ldots, H-3)$ and its descendant with a depth $N(N = M + 3, M + 4, \ldots, H)$ in a complete K-ary tree of height H.

If $l_{i,j}(= l_{j,i})$ denotes the path length, which is the number of edges in the shortest path from a node v_i to a node v_j $(i, j = 1, 2, \ldots, (K^{H+1} - 1)/(K - 1))$ in the complete K-ary tree of height H, then $\sum_{i<j} l_{i,j}$ is the total path length. Furthermore, if $l'_{i,j}$ denotes the path length from v_i to v_j after getting adjacent in this model, $l_{i,j} - l'_{i,j}$ is called the shortening path length between v_i and v_j, and $\sum_{i<j}(l_{i,j} - l'_{i,j})$ is called the *total shortening path length*. Minimizing the total path length is equivalent to maximizing the total shortening path length.

In Section 2 we formulate the total shortening path length in this model of adding a node of liaison which gets adjacent to a node with a depth M and its descendant with a depth N in a complete K-ary tree of height H. In Section 3 we obtain an optimal depth N^* which maximizes the total shortening

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path length for a fixed value of M, and in Section 4 we obtain an optimal pair of depth $(M,N)^*$ which maximizes the total shortening path length.

II. Formulation of Total Shortening Path Length

This section formulates the total shortening path length when a node of liaison is added and gets adjacent to a node with a depth $M(M=0,1,\ldots,H-3)$ and its descendant with a depth $N(N=M+3,M+4,\ldots,H)$ in a pyramid organization structure which is a complete K-ary tree of height $H(H=3,4,\ldots)$. Since we don't consider efficiency of communication of information between the liaison and the other members, the total shortening path length doesn't include the shortening path length between the node of liaison and nodes in a complete K-ary tree.

Let v_M and v_N denote the node with a depth M and the node with a depth N which get adjacent to the node of liaison, respectively. The set of descendants of v_N is denoted by V_1. (Note that every node is a descendant of itself [12].) Let V_2 denote the set obtained by removing V_1 from the set of descendants of the node which is a child of v_M and is an ancestor of v_N. Let V_3 denote the set obtained by removing V_1 and V_2 from all nodes of the complete K-ary tree.

Since that the node of liaison gets adjacent to v_M and v_N doesn't shorten path lengths between pairs of nodes in V_1 and between pairs of nodes in V_3, the total shortening path length can be formulated by adding up the following four sums of shortening path lengths: (i) the sum of shortening path lengths between every pair of nodes in V_1 and nodes in V_3, (ii) the sum of shortening path lengths between every pair of nodes in V_1 and nodes in V_2, (iii) the sum of shortening path lengths between every pair of nodes in V_2 and nodes in V_3 and (iv) the sum of shortening path lengths between every pair of nodes in V_2.

The sum of shortening path lengths between every pair of nodes in V_1 and nodes in V_3 is given by

$$\begin{aligned} &A_H(M,N) \\ &= W(H-N)\{W(H)-W(H-M-1)\} \\ &\quad\times (N-M-2), \end{aligned} \tag{1}$$

where $W(h)$ denotes the number of nodes of a complete K-ary tree of height $h\,(h=0,1,2,\ldots)$. The sum of shortening path lengths between every pair of nodes in V_1 and nodes in V_2 is given by

$$\begin{aligned} &B_H(M,N) \\ &= W(H-N) \\ &\quad\times \sum_{i=1}^{\left\lfloor\frac{N-M-1}{2}\right\rfloor-1}\{(K-1)W(H-M-i-1)+1\} \\ &\quad\times (N-M-2i-2), \end{aligned} \tag{2}$$

and the sum of shortening path lengths between every pair of nodes in V_2 and nodes in V_3 is given by

$$\begin{aligned} &C_H(M,N) \\ &= \{W(H)-W(H-M-1)\} \\ &\quad\times \sum_{i=1}^{\left\lfloor\frac{N-M-1}{2}\right\rfloor-1}\{(K-1)W(H-N+i-1)+1\} \\ &\quad\times (N-M-2i-2), \end{aligned} \tag{3}$$

where $\lfloor x\rfloor$ denotes the maximum integer which is equal to or less than x and we define $\sum_{i=1}^{0}\cdot=0$. Furthermore, the sum of shortening path lengths between every pair of nodes in V_2 is given by

$$\begin{aligned} &D_H(M,N) \\ &= \sum_{i=1}^{\left\lfloor\frac{N-M-1}{2}\right\rfloor-2}\{(K-1)W(H-M-i-1)+1\} \\ &\quad\times \sum_{j=1}^{\left\lfloor\frac{N-M-1}{2}\right\rfloor-i-1}\{(K-1)W(H-N+j-1)+1\} \\ &\quad\times (N-M-2i-2j-2), \end{aligned} \tag{4}$$

where we define $\sum_{i=1}^{-1}\cdot=0$.

From these equations, the total shortening path length $S_H(M,N)$ is given by

$$\begin{aligned} &S_H(M,N) \\ &= A_H(M,N)+B_H(M,N)+C_H(M,N) \\ &\quad+D_H(M,N) \\ &= W(H-N)\{W(H)-W(H-M-1)\} \\ &\quad\times (N-M-2)+W(H-N) \\ &\quad\times \sum_{i=1}^{\left\lfloor\frac{N-M-1}{2}\right\rfloor-1}\{(K-1)W(H-M-i-1)+1\} \\ &\quad\times (N-M-2i-2) \\ &\quad+\{W(H)-W(H-M-1)\} \\ &\quad\times \sum_{i=1}^{\left\lfloor\frac{N-M-1}{2}\right\rfloor-1}\{(K-1)W(H-N+i-1)+1\} \\ &\quad\times (N-M-2i-2) \\ &\quad+ \sum_{i=1}^{\left\lfloor\frac{N-M-1}{2}\right\rfloor-2}\{(K-1)W(H-M-i-1)+1\} \\ &\quad\times \sum_{j=1}^{\left\lfloor\frac{N-M-1}{2}\right\rfloor-i-1}\{(K-1)W(H-N+j-1)+1\} \\ &\quad\times (N-M-2i-2j-2). \end{aligned} \tag{5}$$

Since the number of nodes of a complete K-ary tree of height h is

$$W(h)=\frac{K^{h+1}-1}{K-1}, \tag{6}$$

$S_H(M,N)$ of Equation (5) becomes

$$\begin{aligned} &S_H(M,N) \\ &= \frac{1}{(K-1)^2}\left(K^{H+1}-K^{H-M}\right)\left(K^{H-N+1}-1\right) \\ &\quad\times (N-M-2)+\frac{K^{H-N+1}-1}{K-1} \\ &\quad\times \sum_{i=1}^{\left\lfloor\frac{N-M-1}{2}\right\rfloor-1} K^{H-M-i}(N-M-2i-2) \\ &\quad+\frac{K^{H+1}-K^{H-M}}{K-1} \end{aligned}$$

$$\times \sum_{i=1}^{\lfloor \frac{N-M-1}{2} \rfloor -1} K^{H-N+i}(N-M-2i-2)$$
$$+ \sum_{i=1}^{\lfloor \frac{N-M-1}{2} \rfloor -2} K^{H-M-i} \sum_{j=1}^{\lfloor \frac{N-M-1}{2} \rfloor -i-1} K^{H-N+j}$$
$$\times (N-M-2i-2j-2). \quad (7)$$

III. An Optimal Depth N^* for a Fixed Value of M

In this section, we seek $N = N^*$ which maximizes $S_H(M,N)$ in Equation (7) for a fixed value of M.

Lemma 1:

$$S_H(M, M+2L+1) \;>\; S_H(M, M+2L+2) \quad (8)$$

for $L = 1, 2, \ldots, \lfloor \frac{H-M-2}{2} \rfloor$.

Proof: Since

$$\begin{aligned}
&S_H(M, M+2L+1) - S_H(M, M+2L+2)\\
&= \frac{1}{(K-1)^2}\big(K^{H+1} - K^{H-M}\big)\\
&\quad \times \Big[K^{H-M-2L-1}\{K(2L-1)-2L\}+1\Big]\\
&\quad + \frac{1}{K-1}\sum_{i=1}^{L-1} K^{H-M-i}\Big[K^{H-M-2L-1}\\
&\quad \times \big\{K(2L-2i-1)-(2L-2i)\big\}+1\Big]\\
&\quad + \frac{1}{K-1}\big(K^{H+1}-K^{H-M}\big)\sum_{i=1}^{L-1} K^{H-M-2L+i-2}\\
&\quad \times \big\{K(2L-2i-1)-(2L-2i)\big\}\\
&\quad + \sum_{i=1}^{L-2} K^{H-M-i} \sum_{j=1}^{L-i-1} K^{H-M-2L+j-2}\\
&\quad \times \big\{K(2L-2i-2j-1)-(2L-2i-2j)\big\}\\
&> 0, \qquad (9)
\end{aligned}$$

we have $S_H(M, M+2L+1) > S_H(M, M+2L+2)$. The proof is complete.

Lemma 1 indicates that $N = M + 2L^* + 1$ maximizes $S_H(M,N)$ when $L = L^*$ maximizes $S_H(M, M+2L+1)$ for a fixed value of M. Let $R_{H,M}(L) \equiv S_H(M, M+2L+1)$, so that we have

$$\begin{aligned}
&R_{H,M}(L)\\
&= \frac{1}{(K-1)^3}\big\{K^{2H-2M-3L+1} - 2K^{2H-M-2L+1}\\
&\quad - K^{2H-2M-L} + (K+1)K^{2H-M-L}\\
&\quad - (K+1)K^{H-M-L+1} + 2K^{H-M+1}\\
&\quad - (2L-1)(K-1)K^{H+1}\big\} \qquad (10)
\end{aligned}$$

for $L = 1, 2, \ldots, \lfloor \frac{H-M-1}{2} \rfloor$. Let $\Delta R_{H,M}(L) \equiv R_{H,M}(L+1) - R_{H,M}(L)$, so that we have

$$\begin{aligned}
&\Delta R_{H,M}(L)\\
&= \frac{1}{(K-1)^2}\Big[\big\{-(K^2+K+1)K^{-2M-3L-2}\\
&\quad + 2(K+1)K^{-M-2L-1} + K^{-2M-L-1}\\
&\quad - (K+1)K^{-M-L-1}\big\}K^{2H}\\
&\quad + \big\{(K+1)K^{-M-L} - 2K\big\}K^{H}\Big] \qquad (11)
\end{aligned}$$

for $L = 1, 2, \ldots, \lfloor \frac{H-M-1}{2} \rfloor - 1$.

Let us define a continuous variable x which depends on H as

$$x = K^H \;, \quad (12)$$

then $\Delta R_{H,M}(L)$ in Equation (11) becomes

$$\begin{aligned}
&T_{L,M}(x)\\
&= \frac{1}{(K-1)^2}\Big[\big\{-(K^2+K+1)K^{-2M-3L-2}\\
&\quad + 2(K+1)K^{-M-2L-1} + K^{-2M-L-1}\\
&\quad - (K+1)K^{-M-L-1}\big\}x^2\\
&\quad + \big\{(K+1)K^{-M-L} - 2K\big\}x\Big] \qquad (13)
\end{aligned}$$

which is a quadratic function of x.

From the sign of the coefficient of x^2 in Equation (13), the following two cases can be discussed:

(I) When $K = 2$ and $L = 1$, then $-(K^2+K+1)K^{-2M-3L-2} + 2(K+1)K^{-M-2L-1} + K^{-2M-L-1} - (K+1)K^{-M-L-1} > 0$ which indicates that $T_{L,M}(x)$ is convex downward.

(II) When $K = 2$ and $L = 2, 3, \ldots, \lfloor \frac{H-M-1}{2} \rfloor - 1$ or $K = 3, 4, \ldots$, then $-(K^2+K+1)K^{-2M-3L-2} + 2(K+1)K^{-M-2L-1} + K^{-2M-L-1} - (K+1)K^{-M-L-1} < 0$ which means that $T_{L,M}(x)$ is convex upward.

In the case of (I), $T_{L,M}(x)$ becomes

$$T_{L,M}(x) = 2^{-2M-5}x^2 + \big(3 \cdot 2^{-M-1} - 4\big)x \;. \quad (14)$$

Since $T_{L,M}(x) < 0$ for $0 < x \le 2^{2M+6}$ and $T_{L,M}(x) > 0$ for $x \ge 2^{2M+7}$ in Equation (14), we have $\Delta R_{H,M}(1) < 0$ for $H \le 2M+6$ and $\Delta R_{H,M}(1) > 0$ for $H \ge 2M+7$.

In the case of (II), since

$$T_{L,M}(0) = 0 \quad (15)$$

and

$$\begin{aligned}
\frac{d}{dx}T_{L,M}(0) &= \frac{1}{(K-1)^2}\big\{(K+1)K^{-M-L} - 2K\big\}\\
&< 0, \qquad (16)
\end{aligned}$$

we have $T_{L,M}(x) < 0$ for $x > 0$. Therefore, we have $\Delta R_{H,M}(L) < 0$ for $H = 3, 4, \ldots$.

From the above results, the optimal depth N^* for a fixed value M can be obtained and is given in Theorem 2.

Theorem 2:
(i) If $K = 2$, then we have the following:
  (a) If $H \le 2M+6$, then $N^* = M+3$.
  (b) If $H \ge 2M+7$, then $N^* = M+5$.
(ii) If $K = 3, 4, \ldots$, then $N^* = M+3$.

Proof:
(i) Assume $K = 2$.

(a) If $H = M + 3$ or $H = M + 4$, then $L^* = 1$; that is, $N^* = M + 3$ trivially. If $M + 5 \leq H \leq 2M + 6$, then $L^* = 1$; that is, $N^* = M + 3$ since $\Delta R_{H,M}(L) < 0$ for $L = 1, 2, \ldots, \lfloor \frac{H-M-1}{2} \rfloor - 1$.

(b) If $H \geq 2M + 7$, then $L^* = 2$; that is, $N^* = M + 5$ since $\Delta R_{H,M}(1) > 0$ and $\Delta R_{H,M}(L) < 0$ for $L = 2, 3, \ldots, \lfloor \frac{H-M-1}{2} \rfloor - 1$.

(ii) Assume $K = 3, 4, \ldots$.
If $H = M + 3$ or $H = M + 4$, then $L^* = 1$; that is, $N^* = M + 3$ trivially. If $H \geq M + 5$, then $L^* = 1$; that is, $N^* = M + 3$ since $\Delta R_{H,M}(L) < 0$ for $L = 1, 2, \ldots, \lfloor \frac{H-M-1}{2} \rfloor - 1$.
The proof is complete.

IV. An Optimal Pair of Depth $(M, N)^*$

In this section, we seek $(M, N) = (M, N)^*$ which maximizes $S_H(M, N)$ in Equation (7).

Let $Q_{1,H}(M)$ denote the total shortening path length when $N = M + 3$, so that we have

$$\begin{aligned} Q_{1,H}(M) & \\ \equiv \; & S_H(M, M+3) \\ = \; & R_{H,M}(1) \\ = \; & \frac{1}{(K-1)^2}\bigl(-K^{2H-2M-2} + K^{2H-M-1} \\ & + K^{H-M} - K^{H+1}\bigr) \end{aligned} \tag{17}$$

for $M = 0, 1, \ldots, H-3$. Let $\Delta Q_{1,H}(M) \equiv Q_{1,H}(M+1) - Q_{1,H}(M)$, so that we have

$$\begin{aligned} \Delta Q_{1,H}(M) & \\ = \; & \frac{1}{K-1}\bigl\{(K+1)K^{2H-2M-4} - K^{2H-M-2} \\ & - K^{H-M-1}\bigr\} \\ < \; & 0 \end{aligned} \tag{18}$$

for $M = 0, 1, \ldots, H-4$.

Let $Q_{2,H}(M)$ denote the total shortening path length when $N = M + 5$, so that we have

$$\begin{aligned} Q_{2,H}(M) & \\ \equiv \; & S_H(M, M+5) \\ = \; & R_{H,M}(2) \\ = \; & \frac{1}{(K-1)^2}\bigl\{-(K^2+K+1)K^{2H-2M-5} \\ & + (K+2)K^{2H-M-3} + (2K+1)K^{H-M-1} \\ & - 3K^{H+1}\bigr\} \end{aligned} \tag{19}$$

for $M = 0, 1, \ldots, H-5$. Let $\Delta Q_{2,H}(M) \equiv Q_{2,H}(M+1) - Q_{2,H}(M)$, so that we have

$$\begin{aligned} \Delta Q_{2,H}(M) & \\ = \; & \frac{1}{K-1}\bigl\{(K+1)(K^2+K+1)K^{2H-2M-7} \\ & - (K+2)K^{2H-M-4} - (2K+1)K^{H-M-2}\bigr\} \\ < \; & 0 \end{aligned} \tag{20}$$

for $M = 0, 1, \ldots, H-6$.

From the above results, we have the following lemma.

Lemma 3:
(i) If $N = M + 3$, then $M^* = 0$.
(ii) If $N = M + 5$, then $M^* = 0$.

Proof:
(i) If $H = 3$, then $M^* = 0$ trivially. If $H \geq 4$, then $M^* = 0$ since $\Delta Q_{1,H}(M) < 0$.
(ii) If $H = 5$, then $M^* = 0$ trivially. If $H \geq 6$, then $M^* = 0$ since $\Delta Q_{2,H}(M) < 0$.
The proof is complete.

From Theorem 2 and Lemma 3, the optimal pair of depth $(M, N)^*$ can be obtained and is given in Theorem 4.

Theorem 4:
(i) If $K = 2$, then we have the following:
(a) If $3 \leq H \leq 6$, then $(M, N)^* = (0, 3)$.
(b) If $H \geq 7$, then $(M, N)^* = (0, 5)$.
(ii) If $K = 3, 4, \ldots$, then $(M, N)^* = (0, 3)$.

Proof .
(i) Assume $K = 2$.
(a) Since $N^* = M + 3$ for $H \leq 2M + 6$ from (i)-(a) of Theorem 2 and $M^* = 0$ for $N = M + 3$ from (i) of Lemma 3, $(M, N)^* = (0, 3)$ for $3 \leq H \leq 6$.
(b) Since $N^* = M + 5$ for $H \geq 2M + 7$ from (i)-(b) of Theorem 2 and $M^* = 0$ for $N = M + 5$ from (ii) of Lemma 3, $(M, N)^* = (0, 5)$ for $H \geq 7$.
(ii) Assume $K = 3, 4, \ldots$.
Since $N^* = M + 3$ from (ii) of Theorem 2 and $M^* = 0$ for $N = M + 3$ from (i) of Lemma 3, $(M, N)^* = (0, 3)$.
The proof is complete.

Table I shows the optimal pair of depth $(M, N)^*$ and the total shortening path lengths $S_H(M, N)^*$ in the case of $K = 2, 3, 4$ and $H = 3, 4, \ldots, 10$.

TABLE I
Optimal Pair of depth $(M, N)^*$

K	H	$(M,N)^*$	$S_H(M,N)^*$
2	3	(0, 3)	8
	4	(0, 3)	48
	5	(0, 3)	224
	6	(0, 3)	960
	7	(0, 5)	4160
	8	(0, 5)	17536
	9	(0, 5)	71936
	10	(0, 5)	291328
3	3	(0, 3)	27
	4	(0, 3)	324
	5	(0, 3)	3159
	6	(0, 3)	29160
	7	(0, 3)	264627
	8	(0, 3)	2388204
	9	(0, 3)	21513519
	10	(0, 3)	193680720
4	3	(0, 3)	64
	4	(0, 3)	1280
	5	(0, 3)	21504
	6	(0, 3)	348160
	7	(0, 3)	5586944
	8	(0, 3)	89456640
	9	(0, 3)	1431568384
	10	(0, 3)	22906142720

V. Conclusions

This study considered the placement of a liaison which forms relations to two members of different levels in a pyramid organization structure such that the communication of information between every member in the organization becomes the most efficient. For the model of adding a node of liaison which gets adjacent to a node with a depth M and its descendant with a depth N in a complete K-ary tree of height H which can describe a pyramid organization structure with K subordinates, we obtained an optimal pair of depth $(M, N)^*$ which maximizes the total shortening path length.

The final result in Theorem 4 reveals that the most efficient pair of members of different levels which form relations to the liaison is a pair of the top and a node of the third level below the top or a pair of the top and a node of the fifth level below the top depending on the number of subordinates and the number of levels in the organization structure.

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