Active Modal Tuned Mass Damper for Smart Structures

G. Cazzulani, F. Resta, F. Ripamonti

Abstract— **This paper presents an active control logic for vibration suppression in flexible structures. This control logic, called Active Modal Tuned Mass Damper, is based on an active realization of the passive mechanical Tuned Mass Damper. Starting from the traditional TMD theory, a closed loop formulation is proposed to calculate the active control force. The possibility to describe every linear mechanical system or civil structure with a set of modal coordinates allows to act independently on each d.o.f. The technique is compared with different solutions already proposed in literature, such as the Independent Modal Space Control and the Positive Position Feedback. Numerical simulations, based on a FEM linear model, are carried out to investigate the pro and con of each logic. Finally an experimental campaign has been performed in order to validate the proposed control logic.**

Index Terms— **Active modal TMD, resonant control, vibration suppression**

I. Introduction

The necessity to reduce the vibrations in structures has always played a fundamental role not only in mechanics but also in many civil/architectonic applications. The stresses associated to the dynamic amplifications acting on these structures, in particular when they are forced in nearly resonance conditions, can affect their performances and integrity. These considerations assume even more importance when the same stresses lead to a component lifetime reduction and, as a consequence, to implications about the safety of persons and things in close contact with the structures under investigation.

For these reasons, the designers generally operated adopting passive devices able to reduce the vibrations level. The most intuitive solution is to apply viscous dampers for the energy dissipation. Anyway this approach implies some limits mainly due to the necessity of defining a fixed point to set to the ground the viscous forces. Moreover, the same fixed point becomes a critical element in the optimization procedure of damper parameters, since it actually modifies the dynamic response of the system. An alternative solution has been reached with the introduction of the mass damper theory. Thanks to its simplicity, although well known for many years, this passive device is still widely adopted in many civil and mechanical systems [1-4]. The working principle, based on the synchronization between the natural frequency of an auxiliary single d.o.f. coupled system and the one of the original structure (from which the name “Tuned Mass Damper”, TMD), involves that it operates applying an inertial force 90 degrees out of phase with respect to the displacement. The main limit of this device is the ability to protect the system within a given range of frequencies, while the others are practically uncontrolled. During the years, to improve their performances multiple resonances TMD have been created, as e.g. the Stockbridge [5], for the vibrations suppression in the cables of high voltage energy transmission lines, able to operate in a wider frequency range.

Anyway during the last decades, thanks to the improvements and the cost reduction of calculators and actuators systems, the active solutions have assumed more and more importance. In dynamic applications the solutions classified as active modal controls, able to independently act on each generic structure vibration mode, are particularly interesting. In this sense each vibration mode of the system under investigation can be analyzed as a single d.o.f. system. In general all the active control logics for vibrations suppression are based on several steps, summarizing as:

- Identification of the system vibratory state by means of modal filters or observers [6,7];
- Definition of the control law that, starting from the vibration level, returns the damping force[8];
- Actuation of the control forces through a suitable actuators system (piezoelectric, magneto-strictive, inertial electromechanical,…) [9];
- Evaluation of possible undesired effects associated to the implementation of the logic on a real system (for example spillover) [7,10].

The aim of present work is to investigate the second point, comparing different control logics and evaluating pro and con. Firstly two known-in-literature logics, the Independent Modal Space Control (IMSC) [11-14] and the Positive Position Feedback (PPF) [15-17], are briefly presented. Then an active TMD logic is proposed. Starting from the traditional TMD theory, it adopts a modal approach to calculate the control force, overcoming some of the TMD typical limits such as the imposed ratio between the system and the auxiliary masses, the static deflection due to the auxiliary mass,… In the following, the Active Modal TMD is tested on a numerical model of a clamped beam and compared with IMSC and PPF. Finally experimental tests are presented, showing the performances of the proposed

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control logic on a real system.

II. STATE OF THE ART SOLUTIONS

Consider a generic linear mechanical system

$$[\mathbf{M}]\underline{\ddot{\mathbf{x}}}+[\mathbf{R}]\underline{\dot{\mathbf{x}}}+[\mathbf{K}]\underline{\mathbf{x}}=\underline{\mathbf{f}}_c+\underline{\mathbf{f}}_d \tag{1}$$

where

- $\underline{\mathbf{x}}$ is the vector containing the n independent coordinates
- $[\mathbf{M}]$ and $[\mathbf{K}]$ respectively represent the inertial and elastic matrices;
- $[\mathbf{R}]$ is the damping matrix, assumed to be proportional to the elastic and inertial ones (Rayleigh assumption);
- $\underline{\mathbf{f}}_c$ is the vector containing the control contribution, while $\underline{\mathbf{f}}_d$ represents the generic disturbance forces applied to the system.

For a complex system (such as beam, plate, etc.), these matrices come from a discretization of the structure, for example using the Finite Element Method (FEM). For this reason they can be very large and un-useful for the synthesis of the control law. In this sense modal approach is very attractive, because it allows to describe the system through a limited set of modal coordinates. In fact, higher modes are typically very damped and difficult to excite and can be neglected in the control formulation. Defining $[\mathbf{\Phi}_{\text{tot}}]$ the $n\times n$ eigenvector matrix of $[\mathbf{M}]^{-1}[\mathbf{K}]$, the following coordinate change can be performed

$$\underline{\mathbf{x}}=[\mathbf{\Phi}_{\text{tot}}]\underline{\mathbf{q}}_{\text{tot}} \tag{2}$$

where $\underline{\mathbf{q}}_{\text{tot}}$ is an n vector containing all the system modal coordinates. Considering only the first m modes, the (2) becomes

$$\underline{\mathbf{x}}\simeq[\mathbf{\Phi}]\underline{\mathbf{q}} \tag{3}$$

where $[\mathbf{\Phi}]$ is an $n\times m$ matrix containing only the considered modal shapes. Substituting the (3) in the (1), a series of decoupled modal equations can be obtained as

$$m_i\ddot{q}_i+r_i\dot{q}_i+k_iq_i=u_{c,i}+f_{d,i} \tag{4}$$

where the subscript "i" indicates the i-th modal equation.

Therefore, through the (4), it is possible to define the control law $u_{c,i}$ independently for each mode. For this reason, in the following, a single-mode system is considered to describe the different control laws proposed. The effects of the control laws on the other modes will be discussed later.

In this section a brief overview of some important control theories developed in the last decades and based on the modal approach is presented. Two strategies, Independent Modal Space Control (IMSC) and Positive Position Feedback (PPF) are investigated.

A. Independent Modal Space Control

Considering the (4), the aim of the IMSC is to independently modify the dynamic behaviour (natural frequency and damping) of each controlled mode, without changing the parameters of the uncontrolled ones. The modal control force $u_{c,i}$ is defined as

$$u_{c,i}=-g_{v,i}\dot{q}_i-g_{p,i}q_i \tag{5}$$

and the closed loop equation of motion becomes

$$m_i\ddot{q}_i+\left(r_i+g_{v,i}\right)\dot{q}_i+\left(k_i+g_{p,i}\right)q_i=f_{d,i} \tag{6}$$

The two parameters $g_{p,i}$ and $g_{v,i}$ allow to set respectively the natural frequency and the damping ratio of the i-th controlled mode. In mechanical field, especially considering a vibration control problem, the position gain $g_{p,i}$ is often set to zero in order to avoid higher control forces and mechanical stress of the structure.

B. Positive Position Feedback

Another control strategy is the Positive Position Feedback (PPF), introduced by Goh and Caughey in 1985. In this method, the feedback control force is provided by a 2nd order compensator (fig. 1).

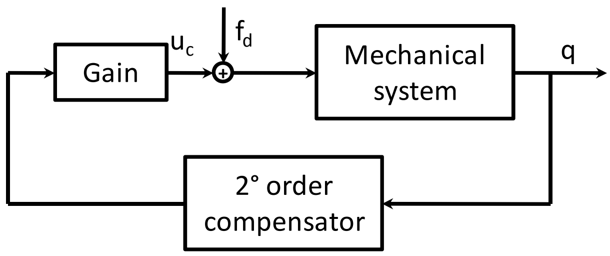

Fig. 1. Scheme of PPF controller for a single mode

Dividing the (4) by m_i

$$\ddot{q}_i+2\xi_i\omega_i\dot{q}_i+\omega_i^2q_i=\overline{u}_{c,i}+\overline{f}_{d,i} \tag{7}$$

the control law can be defined as

$$\overline{u}_{c,i}=g_i\omega_f^2\eta_i \tag{8}$$

where η_i is calculated through the 2nd order compensator defined by

$$\ddot{\eta}_i+2\xi_f\omega_f\dot{\eta}_i+\omega_f^2\eta_i=\omega_f^2q_i \tag{9}$$

In this formulation ξ_i and ω_i represent the damping ratio and natural frequency of the i-th mode, while ξ_f and ω_f

are those of the compensator. Combining the (7), (8) and (9), the equation of the closed loop system can be obtained as

$$\begin{Bmatrix}\dot{q}_i\\ \ddot{\eta}_i\end{Bmatrix}+\begin{bmatrix}2\xi_i\omega_i & 0\\ 0 & 2\xi_f\omega_f\end{bmatrix}\begin{Bmatrix}\dot{q}_i\\ \dot{\eta}_i\end{Bmatrix}+\begin{bmatrix}\omega_i^2 & -g_i\omega_f^2\\ -\omega_f^2 & \omega_f^2\end{bmatrix}\begin{Bmatrix}q_i\\ \eta_i\end{Bmatrix}=\begin{Bmatrix}\overline{f}_{d,i}\\ 0\end{Bmatrix} \tag{10}$$

The closed-loop system is stable if the stiffness matrix is positive-definite. This condition is verified if and only if

$$g_i < \frac{\omega_i^2}{\omega_f^2} \tag{11}$$

III. ACTIVE TUNED MASS DAMPER

In this paper, a control formulation combining the benefits in controlling independently the system modes with the know-how of the tuned mass damper theory is proposed. For this reason, for the sake of completeness, the traditional TMD for a mechanical system is presented. Subsequently this formulation is extended to a generic multi-modal case, considering an independent modal TMD control force.

A. Traditional TMD

Considering a generic single degree of freedom system, the classical tuned mass damper (TMD) consists of a mass-spring-damper system connected to the original one (fig. 2).

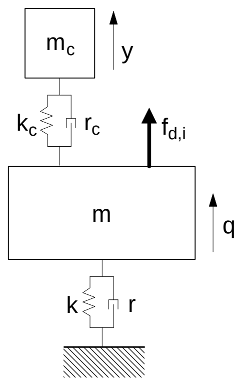

Fig. 2. A single degree of freedom system with TMD

Calling m_c, r_c and k_c the mass, damping and stiffness of the TMD, the equation of motion of the complete system (2 d.o.f.) becomes

$$\begin{bmatrix}m & 0\\ 0 & m_c\end{bmatrix}\begin{Bmatrix}\ddot{q}\\ \ddot{y}\end{Bmatrix}+\begin{bmatrix}r+r_c & -r_c\\ -r_c & r_c\end{bmatrix}\begin{Bmatrix}\dot{q}\\ \dot{y}\end{Bmatrix}+\begin{bmatrix}k+k_c & -k_c\\ -k_c & k_c\end{bmatrix}\begin{Bmatrix}q\\ y\end{Bmatrix}=\begin{Bmatrix}f_{d,i}\\ 0\end{Bmatrix} \tag{12}$$

Usually m_c is chosen between 5% and 10% of the system mass m to limit the "charge effect", while k_c is tuned so that $k_c/m_c = k/m$ and r_c is dimensioned in order to maximize the damping effect around the system resonance.

B. Active Modal TMD

As in the PPF technique (fig 1), it is possible to design a 2nd order compensator providing on the system a force calculated with the TMD equation. Considering the equation of the generic i-th mode (4), the active modal TMD compensator (AMTMD) can be designed as

$$u_{c,i} = k_{c,i}\left(y_i - q_i\right) + r_{c,i}\left(\dot{y}_i - \dot{q}_i\right) \tag{13}$$

where

$$m_{c,i}\ddot{y}_i + r_{c,i}\dot{y}_i + k_{c,i}y_i = k_{c,i}q_i + r_{c,i}\dot{q}_i \tag{14}$$

Under the assumption of knowing exactly the system modal coordinate q_i, the closed-loop is stable for any value of the parameters $m_{c,i}$, $r_{c,i}$ and $k_{c,i}$. Anyway, in order to achieve the best performances, their values should be chosen using the same approach, for each considered mode, of the single degree of freedom TMD, optimizing the phase between control force and displacement.

C. Extension to a multi-modal system

Until now, for every proposed method, the single modal coordinate has been considered under the assumption that it can be directly measured and controlled. The so-calculated forces $u_{c,i}$ represent the contributions of the actuator forces on the considered modes. In real cases, as said in the introduction paragraph, when a multi-mode system is considered, it becomes necessary to know the single modal contributions on system vibration and the actuator action on each mode.

Under the assumption of distributed actuators and sensors (for example piezoelectric patches) it is possible to measure directly each considered mode and to act directly on it, applying the previously calculated modal forces [8]. In all the other cases instead it becomes necessary to link the real forces and measurements with the considered modal contributions. In this case, knowing the generic n_m measurements vector $\underline{\boldsymbol{\mu}}$, the modal coordinates can be calculated as

$$\underline{\mathbf{q}} = \left(\left[\boldsymbol{\Lambda}_m\right]\left[\boldsymbol{\Phi}\right]\right)^{-1}\underline{\boldsymbol{\mu}} \tag{15}$$

where $\left(\left[\boldsymbol{\Lambda}_m\right]\left[\boldsymbol{\Phi}\right]\right)$ is an $m \times n_m$ matrix linking the modal coordinates with the measurements. This matrix must be invertible. It means that it must be square (the number of measurements must be equal to the number of considered modes) and nonsingular (the system must be observable). If $n_m < m$, modal observers [7] can be used to estimate the modal coordinates of the system.

On the other hand, the actuator forces can be calculated as

$$\underline{\mathbf{F}}_{\mathrm{act}} = \left(\left[\boldsymbol{\Phi}\right]^T\left[\boldsymbol{\Lambda}_{\mathrm{act}}\right]^T\right)^{-1}\underline{\mathbf{u}}_c \tag{16}$$

where $\underline{\mathbf{F}}_{\mathrm{act}}$ contains all the actuator forces, while $\underline{\mathbf{u}}_c$ all the modal action calculated by (13). The $n_{act} \times m$ matrix $\left([\boldsymbol{\Phi}]^T [\boldsymbol{\Lambda}_{\mathrm{act}}]^T\right)$ must be invertible too. It means that the condition $n_{act} = m$ must be satisfied and the system must be controllable (matrix is non-singular). If the number of considered modes is greater than the number of actuators, Moore-Penrose pseudo-inverse of $\left([\boldsymbol{\Phi}]^T [\boldsymbol{\Lambda}_{\mathrm{act}}]^T\right)$ can be used but, in this case, the control force will couple the system modes.

IV. NUMERICAL RESULTS

In this section, a numerical analysis is performed to compare the proposed control strategies. The FEM numerical model of a clamped beam is considered (Fig. 3).

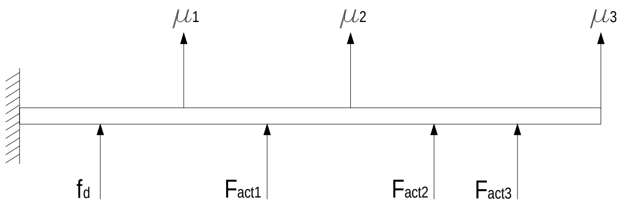

Fig. 3. The beam model considered for the numerical simulations

Table 1 resumes the main properties of the beam, while Table 2 shows the position of sensors and actuators.

TABLE I
CHARACTERISTICS OF THE CLAMPED BEAM

Length	1 m
Section	4E-4 m^2
J	3.3E-7
E	70 GPa

TABLE II
POSITION OF SENSORS AND ACTUATORS WITH RESPECT TO THE CLAMP

Actuator	Position [m]	Sensor	Position [m]
1	0.38	1	0.31
2	0.63	2	0.50
3	0.69	3	1.0

At first a comparison between the performances of IMSC and PPF on this system will be presented. In order to control the system modes independently, a 3-modes controller is implemented. In this way, the matrices of the (15) and (16) can be inverted. The control strategies performances are evaluated through the frequency response function (FRF) between the beam tip displacement and a vertical disturbance force, applied at 0.125 m from the clamp (fig. 3).

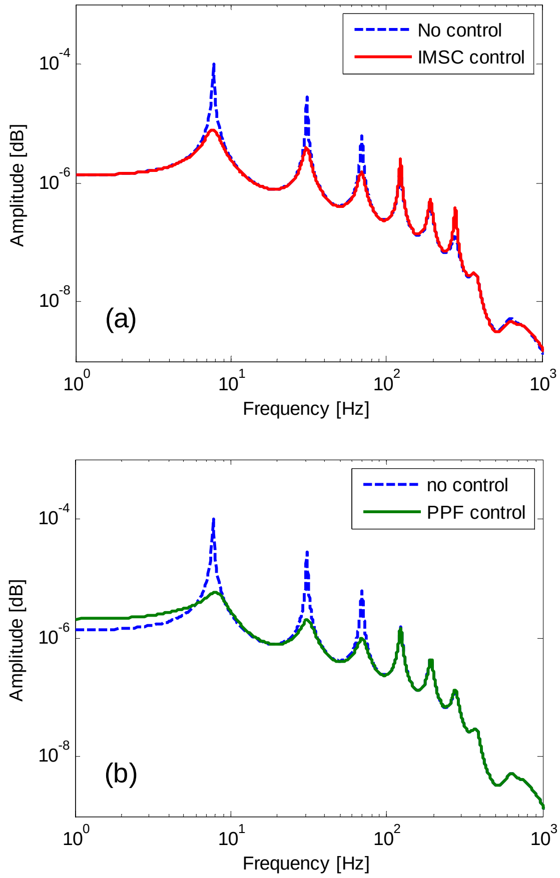

Fig. 4. Comparison between the transfer function of the uncontrolled system with IMSC (a) and PPF (b)

Figure 4 shows the comparison between IMSC and PPF. Considering only the modeled modes, the damping effect introduced by the IMSC should be greater than the damping effect of PPF because, from a theoretical point of view, under the assumption of ideal actuators, there is no limit to the damping increase introduced on the modeled modes. In this case instead it can be noticed that PPF, although it worsens the system response in the quasi-static range of frequencies (below the first system resonance), improves the performance in resonance conditions.

Moreover the great advantage of PPF is that the feedback loop is represented by a low-pass filter. For this reason, as shown in the pole diagram in fig. 5, the spillover risk on higher modes is lower. In particular, for this application, IMSC causes an important spillover effect on the sixth mode (about 300 rad/s), while using PPF control this effect is greatly reduced.

The AMTMD control is able to achieve the same performances of PPF in terms of effectiveness around the resonances and spillover rejection, but without causing the quasi-static amplification typical of the PPF. Figure 6 shows the FRF between tip displacement and disturbance force using AMTMD control (see, for comparison, fig. 4 and 5).

Moreover, AMTMD control outperforms classical TMD passive control since it avoids the deformations due to static preloads and it can be effective even if the considered system modes are very closed one to each other.

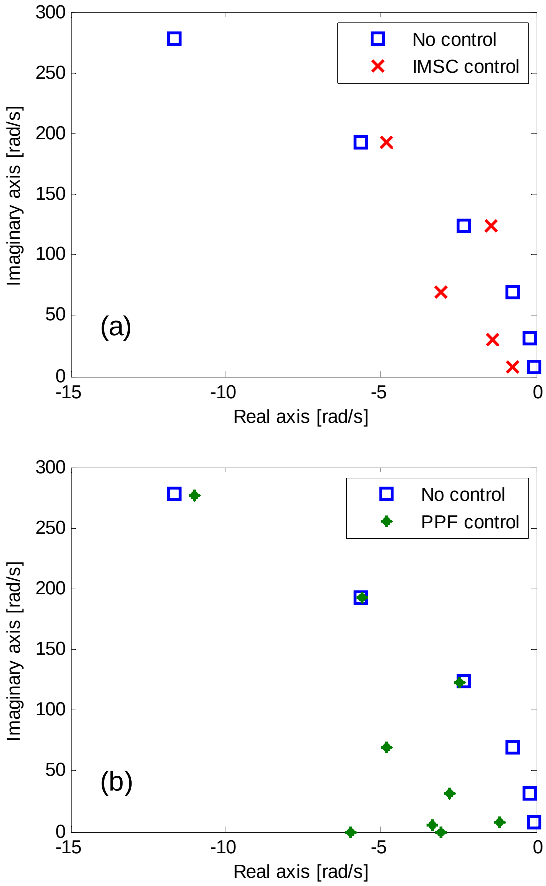

Fig. 5. Comparison between the poles of the uncontrolled system with IMSC (a) and PPF (b)

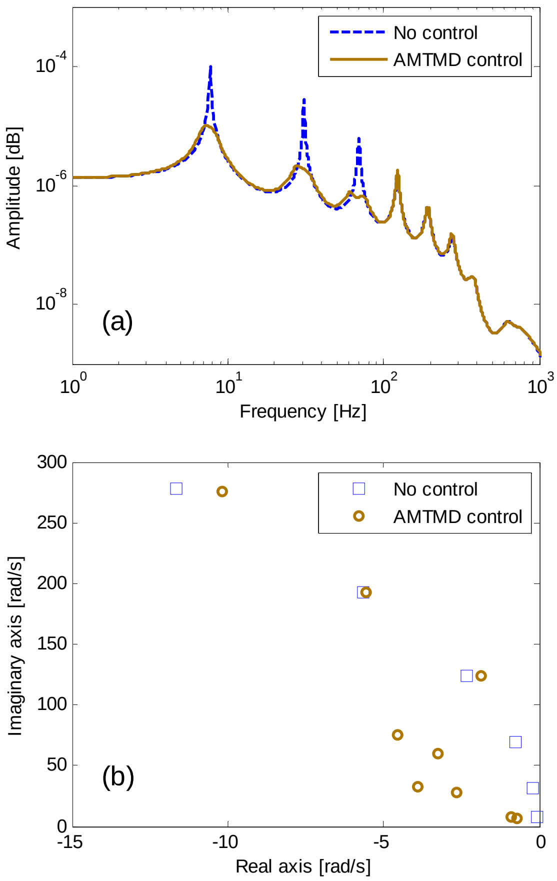

Fig. 6. Transfer function (a) and poles diagram (b) of the system controlled with AMTMD

V. EXPERIMENTAL SETUP

A. *The test rig*

The last part of the present work is dedicated to an experimental campaign carried out to verify the performances of the AMTMD on a real system. A clamped beam (figure 7), which main characteristics reproduce the ones of the beam considered for the numerical analysis (table 1), is considered. The measurement of the system vibration is performed using 3 piezoelectric accelerometers (Brüel&Kjær, mod. 5041), while 2 piezoelectric patches (MIDE, mod. QP20N) are considered for the control action application. A third piezoelectric patch applies a disturbance force on the beam. Actuators and sensors are not co-located, in order to consider the most generic case. Table 3 resumes the position of sensors and actuators with respect to the clamp origin.

Fig. 7. The experimental test rig

TABLE III
POSITION OF SENSORS AND ACTUATORS WITH RESPECT TO THE CLAMP

Actuator	Position [m]	Sensor	Position [m]
Control 1	0.10	1	0.40
Control 2	0.30	2	0.60
Disturbance	0.50	3	0.95

The first step of the experimental campaign is the validation of the numerical model considered in section 4. The importance of a good model is related to the tuning of the control parameters for the experimental tests. Since the control is defined using the modal approach, an identification of natural frequencies, damping ratios and modal shapes has been performed. Table 4 shows a comparison between the system numerical and estimated first natural frequencies and the corresponding estimated damping ratios.

TABLE IV
NUMERICAL AND EXPERIMENTAL NATURAL FREQUENCIES AND DAMPING RATIOS

Mode	Numerical frequency [Hz]	Experimental frequency [Hz]	Damping ratio [%]
1	5.00	5.03	0.42
2	30.84	30.93	0.16
3	87.26	87.27	0.19

Figure 8 shows the comparison between the first three modes in terms of mode shapes. The continuous lines represent the numerical mode shapes, while the markers represent the same mode shape measured at the accelerometers location.

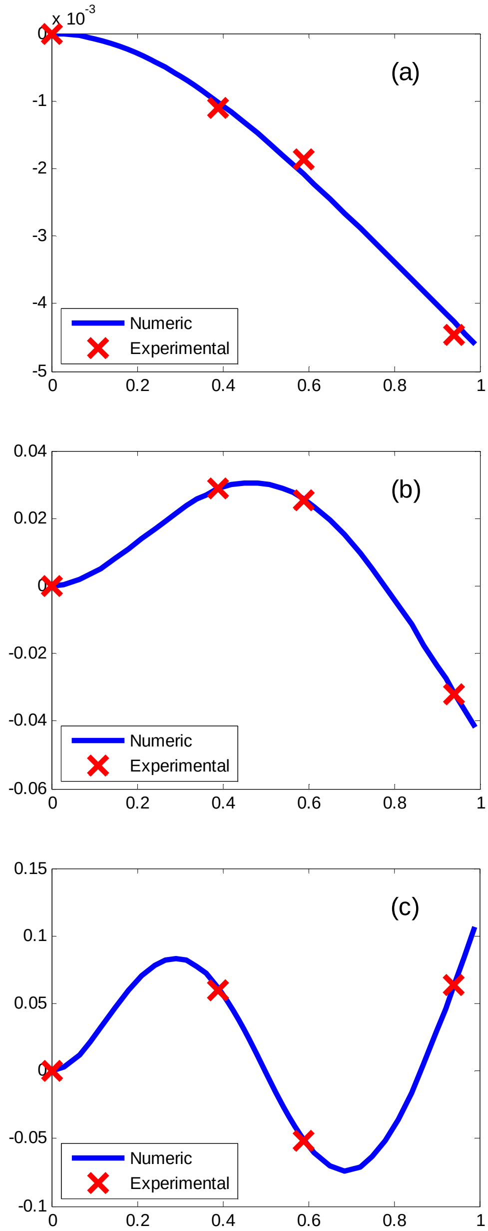

Fig. 8. Modal shapes of the system: mode 1 (a), mode 2 (b), mode 3 (c)

Figure 9 shows the frequency response functions (FRF) between the first actuator and the third sensor (Control 1 in table 3). It can be noticed that, while the phase is completely overlapped, the magnitude is shifted. In fact the numerical FRF represents the relationship between the actuator force (N) and the measurement, while the experimental one represents the relationship between the control board command (V) and the measurement.

This result allows to define the FRF between the control board command (V) and the control force (N) which, in the frequency range under investigation, is simply a gain.

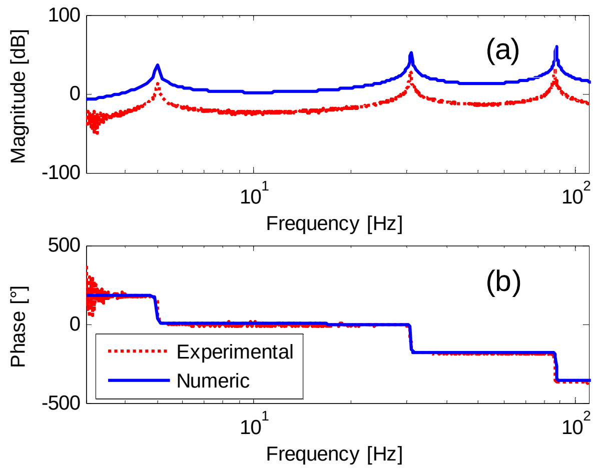

Fig. 9. Numerical/experimental FRF between actuator 1 and sensor 3

B. *Experimental tests on AMTMD*

Since two control actuators are available, two system modes can be independently controlled (16). In this application, the first and second modes are considered, while the higher modes remain uncontrolled. Figure 10 shows the result of a decay tests on the first (a) and the second (b) mode. In both cases the uncontrolled and controlled decays are compared, showing an increase in the damping ratio due to the AMTMD control. In particular the envelope analysis put in evidence an increase of the damping ratio of 3.5 times on the first mode and more than 4.5 times on the second one.

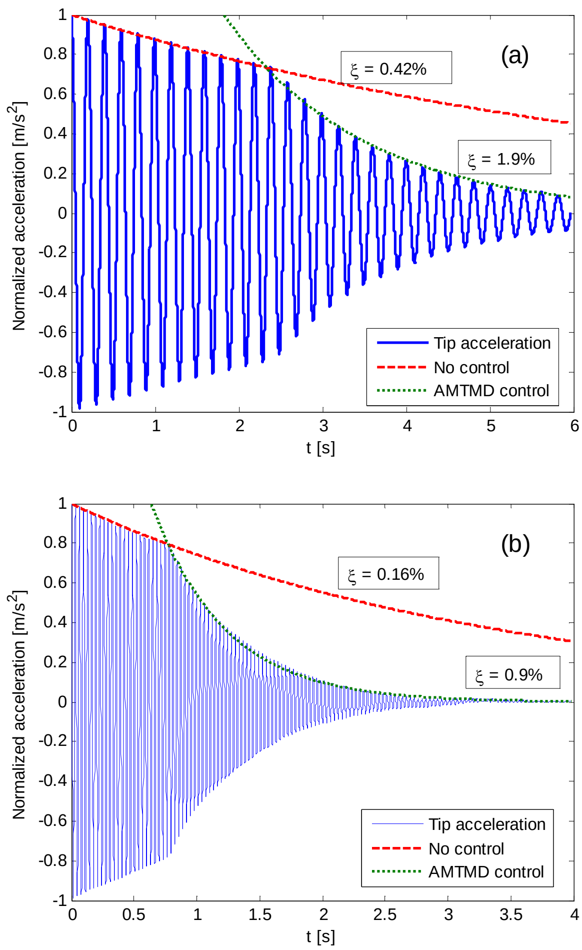

Fig. 10. Decay of the first (a) and second (b) mode with and without AMTMD control

VI. CONCLUSIONS

The paper proposes a control strategy merging the independent control of system modes (as IMSC) and the know-how of the tuned mass dampers. The result is an active control method, that has been called "Active Modal Tuned Mass Damper (AMTMD)", that achieves the same performances of classical resonant control methods around the system resonances, but outperforms IMSC in terms of robustness to control spillover and PPF in terms of low-frequency response.

Numerical tests have been carried out in order to compare the AMTMD with state-of-the-art resonant control techniques. Finally an experimental campaign has been carried out considering a clamped beam with piezoelectric actuators and accelerometers, showing the damping increase on the controlled modes due to the AMTMD.

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