Program Evaluation and Review Techniques II (PERT II)

Siamak Haji Yakhchali

Abstract—**This paper proposes a novel approach to project scheduling in networks with stochastic activity durations which is called PERT II. Program evaluation and review techniques (PERT) used the conception of the random variable with the beta distribution to deal with uncertainty in a scheduling environment. The assumptions of the beta distribution, large enough number of activities and the way of approximating means and variances constitutes challenging problems which has received intensive attention. The exact calculation is generally intractable in PERT, except when the network is a series-parallel one. Although the Monte Carlo simulation has been suggested to cope with PERT difficulties and various probability distributions for activity durations, but it is often too expensive and is not able to determine the exact distribution of project completion time. To remedy the drawbacks connected with PERT and Monte Carlo simulation, an approach which determines the exact cumulative distribution functions of earliest and latest starting and finishing and floats of activities based on confidence interval is proposed. After computing the intervals of project quantities at each confidence level, the cumulative distribution functions of these quantities are reconstructed. The proposed approach was compared for accuracy and validation with Monte Carlo simulation.**

Index Terms— **Project management and scheduling; PERT/CPM; Probability.**

I. Introduction and Motivation

The critical path method-CPM [25] which is a network-based method is useful in practice and control of complex projects. The CPM introduced the concept of precedence which reflects the partial ordering that exists among activities of a project, due to technical or other reasons. The critical path analysis is based on the computation of latest starting times of activities from the knowledge of the earliest ending time of the project. The activity durations in the CPM are deterministic and known, although precise information about the durations of activities is seldom available. To deal quantitatively with imprecise durations, the Program Evaluation and Review Technique-PERT [32] based on the probability theory can be employed.

The originators of PERT modeled each activity duration as a stochastic variable with an appropriated the beta distribution and proposed an approximate method to calculate the expectation and the variance of the makespan of the project. They proposed to use three estimates for each activity duration (the optimistic, the most likely and the pessimistic estimate) to approximate the mean and the variance of the beta probability density function. The assumption of the beta distribution and the way of approximating the mean and the variance constitutes the important and challenging problem which has received intensive attention ([8], [13], [18], [21], [22], [28], [31], [33], [36], [3], [40]). Kamburowski [24] summarized the most of arguments about computing the mean and variance of the random activity duration. Elmaghraby [16] recommended the use of the uniform probability density function for the activity durations. Lootsma [29] proposed the use of the gamma probability density function.

PERT also assumed that the project has large enough number of activities to assume the normal distributions which is used to estimate the probability a critical path in desired time. Therefore, when the number of activities is not large, the analysis may be biased. A significant drawback of PERT is the assumption that the backward recursion is independent of the forward recursion, so the exact calculation of latest starting times and floats of activities is generally intractable in PERT, except when the network is a series-parallel one. The cumulative distribution function is calculated in series-parallel networks by using series-parallel reductions. Adlakha and Kulkarni [1] have listed researches on exact methods for computing the distribution of the project completion time. This is why approximation techniques have been proposed, e.g. transforming the original graph into a series–parallel one [12]. Most efforts in literature on PERT concentrate on the so-called stochastic resource-constrained project scheduling problem (for a detailed discussion, see Chapter 9 in [11]). For example, Golenko-Ginzburg and Gonik ([19], [20]) consider PERT where the activities require a constant amount of renewable resources during their execution and the objective is to minimize the expected project duration.

Fuzzy sets is an approach for measuring imprecision or vagueness in estimation, and may be preferred to probability theory in capturing activity duration uncertainty in situations where past data are either unavailable or not relevant, the definition of the activity itself is somewhat unclear, or the notion of the activity's completion is vague [27]. Shipley et al. [39] and Lootsma [30] have compared the fuzzy approach with PERT. (See [5] & [23] for surveys on fuzzy project scheduling).

Monte Carlo simulation is often preferred in practice when historical data about activity durations are available. Simulation involves the generation of artificial events or processes for the system and collects the observations to draw any inference about the real system. Monte Carlo

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simulation is able to cope with various probability distributions for activity durations. In fact in this situation, analytic approaches fail and researcher fall back on sampling techniques. Cook and Jennings [9] have provided heuristics for determining paths in the network which have little or no chance of becoming critical to reduce simulation time. Schonberger [38] has discussed the advantages of Monte Carlo simulation. Anklesaria and Drezner [2] have proposed an approach to estimate the distribution of completion times (see for instance [35] for a survey and [26]). Dawson [10] proposed a dynamic sampling technique for the simulation.

Despite merit of Monte Carlo simulation, numbers of execution affect the quality of results. By increasing the numbers of project activities, the number of simulation executions must increase and the exact distribution of project completion time can not be achieved. The exact distribution of project completion time, earliest and latest starting times and floats of activities in networks with stochastic activity durations are determined by the proposed approach. The concept of confidence interval is used to obtain networks with imprecise durations, represented by intervals Yakhchali [45] initially introduced this idea). After computing the intervals of project quantities at each confidence level, cumulative distribution functions of the quantities are reconstructed from their interval. Numerical example is used to compare the proposed approach with Monte Carlos simulation.

II. Terminology and Representation

The project scheduling problems to be dealt with throughout this paper can be stated as follows. A set $V=\{1,2, ..., n\}$ of *activities* has to be executed where the dummy activities 1 and n represent the beginning and the termination of the project, respectively. Dummy activities are only needed to satisfy the requirement that the network possesses only one initial and one terminal node. Activities can be represented by an *activity-on-node* (*AON*) network $G=<V, E>$ with node set V, arc set E. The arc set E define the zero-lag finish-start precedence relations among the activities. Assume without loss of generality that the activities topologically numbered such that an arc always leads from a smaller to a higher node number.

A. Activity Durations

In order to cope with uncertainties, the duration of an activity i, $i\in V$, is assumed to be a random variable d_i. Each random variable has its arbitrary probability distribution. $f_i(x)$ denotes the probability density function (pdf) of d_i. For discrete distributions, the pdf assigns a probability to each outcome, in this context the pdf is often called a probability mass function (pmf). For continuous distributions, the pdf assigns a probability density to each outcome. The probability of any single outcome is zero. The pdf must be integrated over a set of outcomes to compute the probability that an outcome falls within that set. $F_i(x)$ will denote the cumulative distribution function (cdf) of d_i (for discrete distributions $F_i(x)=\sum_{y\le x} f_i(y)$ and for continuous distributions $F_i(x)=\int_{-\infty}^{x} f_i(x)\,d_y$). An interval for the random variable d_i at a measure of probability $1-\alpha$ is defined as following:

$$P\{\underline{d}_i^{\alpha} \le d_i \le \overline{d}_i^{\alpha}\} = 1-\alpha$$

where $\underline{d}_i^{\alpha}$ and $\overline{d}_i^{\alpha}$ are the lower and upper the interval, respectively. The measure of probability $1-\alpha$ is called the *confidence level*. It should be noted that when it is said d_i lies in a $100(1-\alpha)$ percent interval, it means that on the average, d_i lies in the interval $100(1-\alpha)$ percent of the time. Thus, if the project is repeated many times and a $100(1-\alpha)$ percent interval is computed each time, then in $100(1-\alpha)$ percent of the project executions, the interval includes the true value of d_i.The interval of d_i at a confidence level of $1-\alpha$ is denoted by $[\underline{d}_i^{\alpha}, \overline{d}_i^{\alpha}]$. G^{α} will denote the network G at a confidence level of $1-\alpha$. It means that the network G with interval activity durations $[\underline{d}_i^{\alpha}, \overline{d}_i^{\alpha}]$, $i\in V$.

B. The notation of Configuration

The notation of *configuration* has been defined by Buckley [4] to relate the interval case to the deterministic case of classical CPM problems. A configuration at a confidence level of $1-\alpha$, denoted by Ω^{α}, is a tuple $(d_1, d_2, ..., d_n)$ of activity durations such that $\forall i\in V$, $d_i\in[\underline{d}_i^{\alpha}, \overline{d}_i^{\alpha}]$.

H^{α} denotes the set of all configurations at the confidence level of $1-\alpha$: $H^{\alpha} = \times_{i\in[1,n]}[\underline{d}_i^{\alpha}, \overline{d}_i^{\alpha}]$. For a configuration Ω^{α}, $d_i(\Omega^{\alpha})$ will denote the duration of the activity i.

A configuration defines an instance of deterministic project scheduling problem, to which the CPM can be applied. $ES_i(\Omega^{\alpha})$, $LS_i(\Omega^{\alpha})$ and $F_i(\Omega^{\alpha})$ will denote the earliest starting time, latest starting time and total float of the activity i in the configuration Ω^{α}, respectively. The earliest starting time of each activity is determined by a *forward recursion* procedure consists in applying formula (1).

$$ES_i(\Omega^{\alpha}) = \begin{cases} 0 & \text{for } i=1 \\ max_{j\in Pred(i)}\{ES_j(\Omega^{\alpha}) + d_j(\Omega^{\alpha})\} & \text{for } i\ne 1 \end{cases} \tag{1}$$

where $Pred(i)$ (resp, $Succ(i)$) refers to the set of activities that immediately precede (resp. follow) i, $PRED(i)$ while (resp. $SUCC(i)$) stands for the set of all activities that come before (resp. after) $i\in V$. A *backward recursion* procedure, under assumption $LS_n(\Omega^{\alpha}) = ES_n(\Omega^{\alpha})$, is used to determine the latest starting time of each activity.

$$LS_i(\Omega^{\alpha}) = \begin{cases} ES_n(\Omega^{\alpha}) & \text{for } j=n \\ min_{j\in Succ(i)}\{LS_j(\Omega^{\alpha})\} - d_i(\Omega^{\alpha}) & \text{for } j\ne n \end{cases} \tag{2}$$

The float $F_i(\Omega^{\alpha})$ is determined by means of the formula $F_i(\Omega^{\alpha}) = LS_i(\Omega^{\alpha}) - ES_i(\Omega^{\alpha})$.

III. The proposed approach

The crux of the proposed approach is the notion of the interval of random variables of activity durations at a confidence level of $1-\alpha$. We claim that the interval of random variables of activity durations can be used to solve problems of determining distributions of earliest starting times, latest starting times and floats of activities, by decomposition of activity distributions into confidence intervals. For each confidence level of $1-\alpha$, each interval of random variables of activity durations at the confidence level of $1-\alpha$ is obtain, thus the project scheduling problem in stochastic networks is converted to the project scheduling problem in networks with imprecise durations, represented by intervals. Then intervals of earliest starting times, latest starting times and floats of activities are computed by interval methods. Based on the definition of the cumulative distribution function, the cumulative distribution functions of these quantities are then reconstructed from their interval as show in Figure 1.

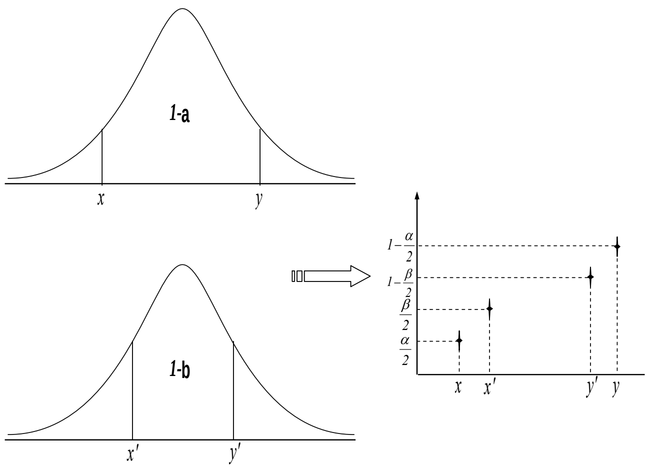

Figure 1: Reconstructing the cumulative distribution function of project quantities

In Figure 1, it is assumed that the activity duration obeys the normal distribution then two intervals at confidence levels of $1-\alpha$ and $1-\beta$ are made. Based on the intervals, the cumulative distribution function is reconstructed. In the following this approach elaborated for earliest starting times and latest starting times of activities.

A. *Determining the cdf of the Earliest Starting Times*

In the network *G* at a confidence level of $1-\alpha$, G^α, the bounds on the earliest starting times for a given activity $k \in V$, denoted by $ES_k^\alpha = [\underline{ES}_k^\alpha, \overline{ES}_k^\alpha]$, are equal $\underline{ES}_k^\alpha = min_{\Omega^\alpha \in H^\alpha} ES_k(\Omega^\alpha)$ and $\overline{ES}_k^\alpha = max_{\Omega^\alpha \in H^\alpha} ES_k(\Omega^\alpha)$. Proposition 1 provides the necessary and sufficient condition to compute $\underline{ES}_k^\alpha$ and $\overline{ES}_k^\alpha$. In order to compute these values, it is enough to use a forward recursion procedure in optimistic and pessimistic configurations at a confidence level of $1-\alpha$. The pessimistic configuration at the confidence level of $1-\alpha$, denoted by $\overline{\Omega}^\alpha$, is a configuration $\overline{\Omega}^\alpha \in H^\alpha$ that $d_i(\overline{\Omega}^\alpha) = \overline{d}_i^\alpha$ for all $i \in V$ and similarly $\underline{\Omega}^\alpha$, called the optimistic configuration at the confidence level of $1-\alpha$, is a configuration $\underline{\Omega}^\alpha \in H^\alpha$ that $d_i(\underline{\Omega}^\alpha) = \underline{d}_i^\alpha$ for all $i \in V$.

Proposition 1: The optimistic configuration at the confidence level of $1-\alpha$, $\underline{\Omega}^\alpha$, minimizes the earliest starting time of all the activities in G^α, $k \in V$, and the pessimistic configuration at the confidence level of $1-\alpha$, $\overline{\Omega}^\alpha$, maximizes their earliest starting times in G^α.

Proof: Assume on the contrary that $\underline{ES}_k^\alpha \neq ES_k(\underline{\Omega}^\alpha)$ so there exists a configuration Ω^*, $\Omega^* \in H^\alpha$, that $\underline{ES}_k^\alpha = ES_k(\Omega^*)$. It is concluded that $ES_k(\Omega^*) < ES_k(\underline{\Omega})$. Based on formula (2), there exist at least an activity $i \in PRED(k)$ which $d_i(\Omega^*) < d_i(\underline{\Omega}^\alpha)$. This contradicts that $d_i(\underline{\Omega}^\alpha) = \underline{d}_i^\alpha$, the definition of the optimistic configuration, thus $\underline{ES}_k^\alpha = ES_k(\underline{\Omega}^\alpha)$.

The proof of $\overline{ES}_k^\alpha = ES_k(\overline{\Omega}^\alpha)$ is the same as $\underline{ES}_k^\alpha = ES_k(\underline{\Omega}^\alpha)$.☐

Thus, it is enough to use forward recursion procedures to compute the bounds on the earliest starting times at a confidence level of $1-\alpha$. This proposition yields an approach which determines the cumulative distribution function of the project completion time and the earliest starting and finishing times of activities. To reconstruct the cumulative distribution functions of these quantities, the proposition can be applied only on a selection of suitably chosen confidence levels. This concept is illustrated in a numerical example in the following section.

B. *Determining the cdf of Latest Starting Times and Floats of Activities*

Analogously, the latest starting times and floats of activities are reconstructed from their intervals in the network *G* at confidence levels. The possible interval value of the latest starting times for a given activity $k \in V$ in the network *G* at a confidence level of $1-\alpha$, G^α, is defined as $LS_k^\alpha = [\underline{LS}_k^\alpha, \overline{LS}_k^\alpha]$ where $\underline{LS}_k^\alpha = min_{\Omega^\alpha \in H^\alpha} LS_k(\Omega^\alpha)$ and $\overline{LS}_k^\alpha = max_{\Omega^\alpha \in H^\alpha} LS_k(\Omega^\alpha)$. The float interval for a given activity $k \in V$ in the network *G* at a confidence level of $1-\alpha$, G^α, is defined as $F_k^\alpha = [\underline{F}_k^\alpha, \overline{F}_k^\alpha]$ where $\underline{F}_k^\alpha = min_{\Omega^\alpha \in H^\alpha} F_k(\Omega^\alpha)$ and $\overline{F}_k^\alpha = max_{\Omega^\alpha \in H^\alpha} F_k(\Omega^\alpha)$. These quantities can be calculated by configurations where activity durations are assigned to their minimal and maximal possible values. A configuration $\Omega^\alpha \in H^\alpha$ such that $\forall k \in V$, $d_i(\Omega^\alpha) = \overline{d}_i^\alpha$ or $\underline{d}_i^\alpha$ is called an *extreme configuration*. H_{ext}^α will denote the set of all extreme configurations. It is shown that the maximum and the minimum of the latest starting times and the floats of activities are attained on specific extreme configurations [14].

Unfortunately the backward recursion procedure fails to compute the possible values of the latest starting times. There are two common methods for determining the extreme configurations that contain the maximum and the minimum of the latest starting times and the floats of activities. The first method, called incremental method, the

duration of a given activity is increased by the free float of activities that the longest path from node 1 to that node traversing the given activity. This method computes the latest starting times of only one activity (see [46] and [43]). A path enumeration method, second method, is used to compute these quantities. Although this method is not polynomial, but experimental results have shown that this method can compute in a reasonable time the interval of the latest starting times and the floats of activities for project networks with more than one hundred activities [15].
The problem of computing the maximal float of a given activity in the network with interval duration has been solved [17]. Unfortunately the problem of determining the lower bound on floats of a given activity is strongly *NP*-Hard [6]. Chanas and Zielinski [7] have shown that this problem remains *NP*-Hard even in a network restricted to be planar. For this problem, Yakhchali and Ghodsypour [44] have proposed a hybrid genetic algorithm.
The above methods are used to compute the interval of the latest starting times and floats of activities at each confidence level and then their cumulative distributions are reconstructed as illustrated in the following numerical example.

IV. Numerical Examples

To illustrate the validity of the proposed, the following examples are investigated. Two examples have the same network (precedence relations) but different types of probability density functions. The activity durations in the first example are discrete probability densities and those in the second example are continuous probability distributions. To underscore the main advantages of the proposed approach, these two examples have to be investigated.

A. Example 1 (Discrete Distributions)

Assume that the network in Figure 2 is given where the dummy activities *1* and *n* represent the beginning and the termination of the project, respectively.

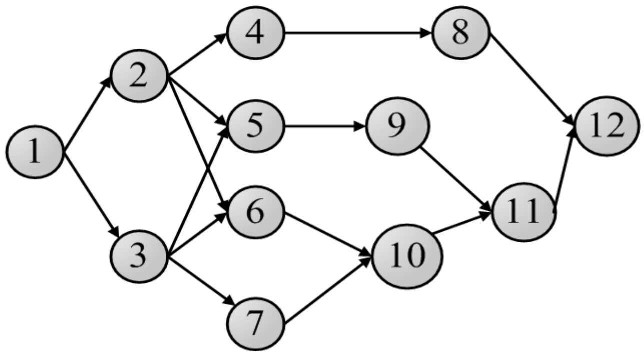

Figure 2: A project network for examples 1 and 2.

The activities durations are discrete probability densities as following (see for discrete probability densities [34]):

d_2 ~*Discrete Uniform* (*3,5*) d_3 ~*2+Binomial* (*5,0.5*)
d_4 ~*3+Poisson* (*7*) d_5 ~*4+Poisson* (*4*)
d_6 ~*2+Poisson* (*5*) d_7 ~*Discrete Uniform* (*2,6*)
d_8 ~*4+Binomial* (*8, 0.4*) d_9 ~*Discrete Uniform* (*2,7*)
d_{10} ~*Binomial* (*10, 0.7*) d_{11} ~*Poisson* (*9*)

It is worth noticing that the question of how the project manager chooses the appropriate probability distributions is important but is not treated in this paper. It is assumed that the probability distributions representing activity durations are already known. Several solutions have been proposed for this question.
Monte Carlo simulation is used to compare results of the proposed approach. Monte Carlo simulation is an attempt to create a series of randomly sample from the activity probability distributions. In the of the fact that the quality of simulation depends on numbers of executions, Monte Carlo simulation with 1000 and 10000 times of executions are reported (in as far as numbers suggested by Van Slyke, [41]). The cumulative probability distribution of the project completion time in example 1 is given in Figure 3. The results of the proposed approach and Monte Carlo simulation are shown in Figure 3.

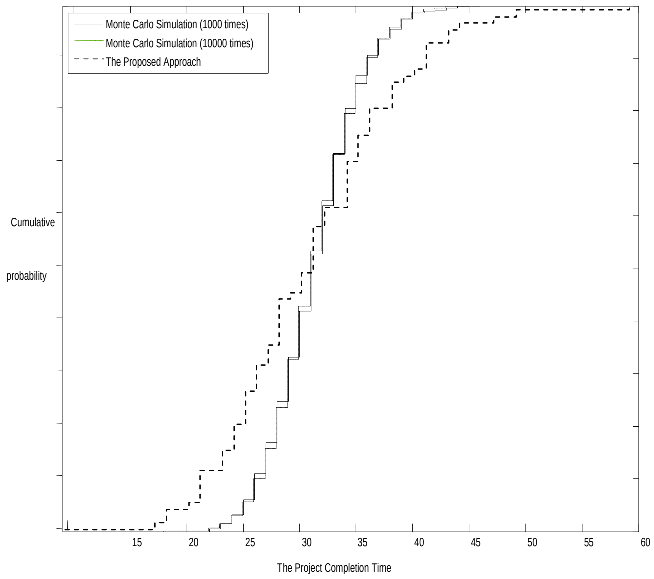

Figure 3: The cumulative distribution function of the project completion time in example 1.

The statistical indices, such as minimum, maximum, mean and etc, are listed in Table 1.

Table 1: The comparison between of project completion time results of the approach and simulations in Example 1.

	The Proposed Approach	Monte Carlo simulation (1000 times)	Monte Carlo simulation (10000 times)
Min	14	27	23
Max	64	50	51
Mean	35.7683	38.5	37
Median	36	38.5	37
Standard Deviation	8.022	6.9954	8.4397
Range	50	23	28

When the numbers of executions are increased in Monte Carlo simulation, better results are obtained (see Table 1).
The discrete uniform distribution is expected for the earliest starting time of activity (4) for its real predecessor is activity (2) with discrete uniform distribution. This conclusion is obtained by the proposed approach and Monte Carlo simulation as shown in Figure 4.

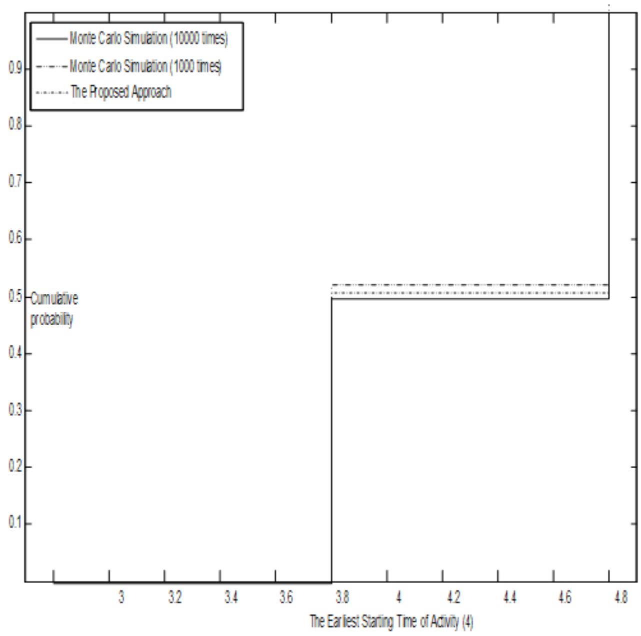

Figure 4: The cumulative distribution functions of the earliest starting time of activity (4) in example 1.

Figure 5 shows the cumulative distribution functions of the earliest finishing time of activity (4) computed by the proposed approach and Monte Carlo simulation in example 1.

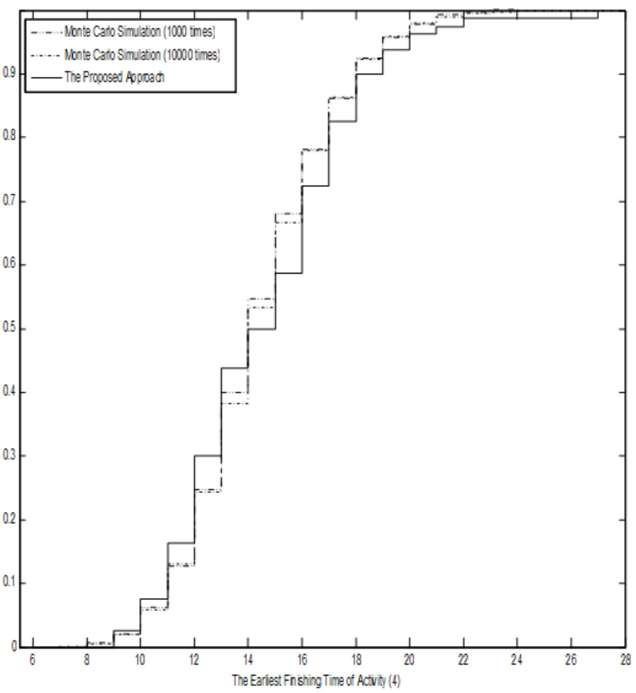

Figure 5: The cumulative distribution functions of the earliest finishing time of activity (4)

The cdf of the latest starting time of activity (4) is shown in Figure 6.

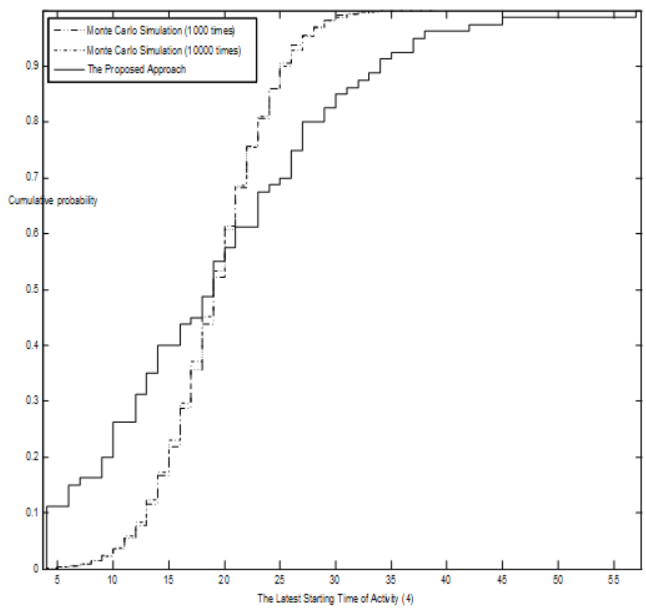

Figure 6: The cdf of the latest starting time of activity (4) example 1.

The earliest and latest starting and finishing times of other activities are omitted for sack of brevity.

B. *Example 2 (Continuous Distributions)*

The project network is similar to that in Example 1 Assume that the activity durations are continuous probability densities as following:

$d_2 \sim Uniform\ (3,5)$ $d_3 \sim 5+Exponential\ (4)$
$d_4 \sim 3+Gamma\ (2,4)$ $d_5 \sim 3+Normal\ (4,1)$
$d_6 \sim 2+Normal\ (6,1)$ $d_7 \sim 3+Normal\ (8,2)$
$d_8 \sim Uniform\ (5,8)$ $d_9 \sim Uniform\ (2,5)$
$d_{10} \sim Uniform\ (3,6)$ $d_{11} \sim Uniform\ (4,7)$

In example 2, the intervals of confidence level of 1 are not defined for some probability distributions, such as the normal distribution. The cumulative distribution function of the project completion time in example (2) is shown in Figure 7.

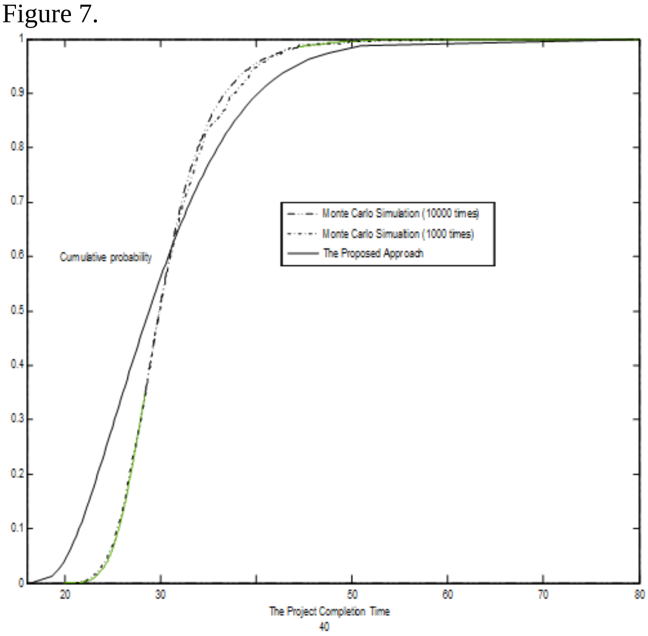

Figure 7: The cumulative distribution function of the project completion time in example (2).

Similarly, Monte Carlo simulation with 1000 and 10000 execution times is used to determine the cumulative distribution function of the project quantities in example (2) then the results are compared with the proposed approach as listed in Table 2.

Table 2: The comparison between of project completion time results of the proposed and simulations in Example 2

	The Proposed Approach	Monte Carlo simulation (1000 times)	Monte Carlo simulation (10000 times)
Min	16.281	21.1686	20.0182
Max	79.3593	59.9981	79.9189
Mean	30.3167	30.829	30.6765
Median	28.7726	29.9137	29.8477
Standard Deviation	9.0195	5.0202	4.7457

As an example of earliest starting times of activities, the cumulative distribution function of the earliest starting times of activity (7) is drawn in Figure 8.

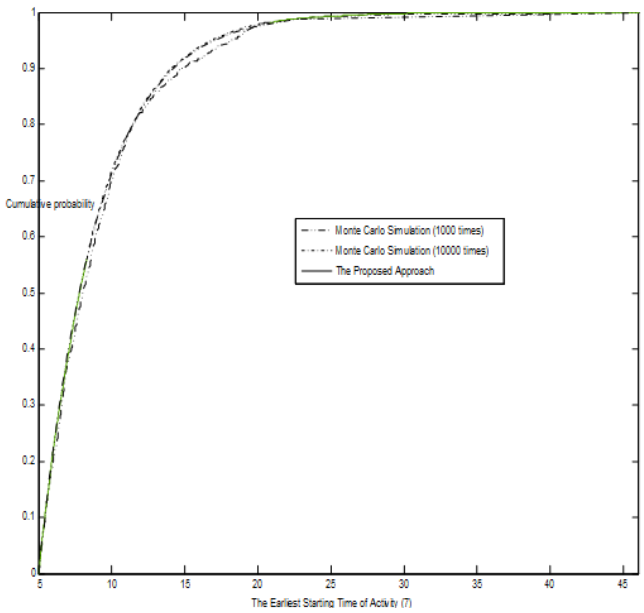

Figure 8: The cdf of the earliest starting times of activities in example (2).

C. *Comparisons*

Although both of the proposed approach and Monte Carlo simulation are related to probability theory, they are different on methodology and obtained results. The proposed approach is able to determine the exact cumulative distribution functions of project quantities with fix number of executions which is not depended on the number of activity and number of precedence relations among the activities. The proposed approach can be carryout only on a section of suitable chosen confidence levels. Monte Carlo simulation provides approximate probability theory for project quantities. To improve the quality of the Monte Carlo simulation results, the number of classical CPM executions must be increased. More observations in Monte Carlo simulation are needed if the number of activities increases. In above examples, the proposed approach invoked 200 executions of CPM calculations, although 1000 and 10000 executions of CPM calculations were invoked by Monte Carlo simulation.

The proposed approach copes with the exponential nature of the large project scheduling problem with stochastic activities. The proposed approach is easy to understand, due to simple principles. Another point to note is that since the proposed approach is based on interval confidence, it is applicable to the stochastic activity durations with reference distribution functions of different types.

V. Conclusions

A novel approach for project scheduling in networks with stochastic activity durations is introduced and analyzed in this paper. In the real world, project activities are subject to considerable uncertainty. To deal quantitatively with imprecise durations, PERT based on the probability theory can be employed. PERT assuming that the durations of activities are random variables of the beta distribution and the structure of the precedence network is deterministic, aim at providing the distribution function of the project completion time. PERT has been developed under assumptions that the probability distributions of activity durations are different to the beta distribution. Another drawback is the problem of finding the distribution of the project completion time that is intractable, due to the dependencies induced by network structures.

Therefore the Monte Carlo simulation has been suggested to cope with PERT difficulties. Monte Carlo simulation attempts to compute the distribution of the project completion time, but the exact distribution is not achievable. Even if the exact distribution of project quantities can be determined, it is admitted that an enormous execution of simulation is required. The important problem studied in this paper involves determining the exact distribution of project quantities.

The proposed approach determines the exact cumulative distribution functions of earliest and latest starting and finishing and floats of activities. The notion of confidence interval is used to obtain networks with imprecise activity durations, represented by intervals. The key to construct the exact distributions of project quantities is the confidence interval. The interval of earliest starting times of activities can be derived by applying the classical CPM on optimistic and pessimistic configurations at a specific confidence level. The path enumeration and incremental methods can be used to calculate the intervals of latest starting times and floats of activities. After computing the intervals of project quantities at each confidence level, the cumulative distribution functions of these quantities are reconstructed. The idea of reconstructions is based on the definition of the cumulative distribution function.

The proposed approach's output was compared for accuracy and validation with simulation output. This approach is more tractable that Monte Carlo simulation. Moreover, the principle of the proposed approach is simpler than the Monte Carlo simulation, and it copes with the exponential nature of the large project scheduling problem with stochastic activities. Thus, the proposed approach is easy to understand and apply.

As a hint for further research, the project scheduling

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problems without any resource constrains are considered in this paper, but the proposed approach can extent to the resource constrained project scheduling problems.

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