Optimal Supplier Selection and Order Allocation for Multi-Period Multi-Product Manufacturing Incorporating Customer Flexibility

K.L.Mak, L.X. Cui, and W. Su

Abstract— **Effective and efficient supplier selection and order allocation are essential for a manufacturer to ensure stable material flows in a highly flexible and competitive supply chain. This paper attempts to solve the problem of optimal supplier selection and order allocation for multi-period multi-product manufacturing when customer flexibility exists. A new mathematical model in the form of a mixed integer programming (MIP) model is developed to describe the characteristics of the problem. The objective is to maximize the manufacturer's profit subject to the various operating constraints of the supply chain. In addition, a new hybrid algorithm based on the strengths of constraint programming (CP) and simulated annealing (SA) is developed to solve this complex combinatorial optimization problem which is NP-hard. The developed algorithm is applied to solve a set of randomly generated test problems to evaluate its performance. Comparison of the computational results obtained with those obtained by using the commercial software ILOG OPL clearly shows that the methodology developed in this paper is an effective and efficient approach to assist the manufacturer in formulating optimal supplier selection and order allocation decisions.**

Index Terms—**Constraint programming, customer flexibility, simulated annealing, supplier selection**

I. Introduction

With increasing product variety and escalating demand volatility, maintaining an efficient and flexible supply chain has become more critical for most manufacturers. In addition, it has been observed that customers are often indifferent to certain product specifications and are often willing to accept less desirable products given certain price discounts [11]. Mak and Cui [9] pointed out that this flexible customer behavior brings additional degree of freedom for manufacturers in handling customer orders and deploying available production resources. Indeed, the purpose of achieving high customer service level and low manufacturing cost in such a dynamic supply chain environment imposes a major challenge to manufacturers in selecting their suppliers and allocating orders to the selected suppliers.

Intuitively, customer flexibility provides a way for manufacturers to improve profit by making better utilization of manufacturing and supply resources as a result of the extra degree of flexibility in meeting customer specifications. Hence, it is important for manufacturers to exploit the advantages of customer flexibility to the full in selecting their suppliers and allocating orders to the selected suppliers. However, the challenging problem of developing methodologies for assisting manufacturers to make optimal decisions concerning supplier selection and order allocation incorporating customer flexibility has received very little attention in the research community. Kim et al. [8] have considered the supply network of a manufacturer which produces different types of products using a common set of inputs (e.g., raw materials and component parts). A mathematical model and an iterative algorithm have been developed to solve the configuration problem faced by the manufacturer. However, the study does not consider customer flexibility. Che and Wang [4] have developed an optimization model for integrated supplier selection and quantity allocation of common and non-common parts under a multiple products manufacturing environment. The model assumes that each product has a unique BOM structure. However, it ignores the impact of product families and customer flexibility.

In this connection, this paper explores the challenge from a new perspective by aiming to incorporate customer flexibility in tackling the problem of supplier selection and order allocation for a multi-product supply chain. The objective is to:

1) Determine the production quantity of each product variant
2) Select the most suitable suppliers based on the selection criteria and their capacity and split the orders among these suppliers
3) Maximize the manufacturer's profit.

A new mathematical model in the form of a mixed integer programming (MIP) model is firstly developed to represent the basic characteristics of the integrated supplier selection and order allocation problem. The problem is NP-hard, and thus could not be solved optimally in polynomial-bound time. In this research, a novel hybrid algorithm based on the strengths of both constraint programming technique and the simulated annealing algorithm is developed to solve the problem.

Constraint programming (CP) [2] is a powerful technique for solving large combinatorial problems. Its success has

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been demonstrated in solving large scale problems such as job shop scheduling problems, graph coloring problems. By efficient propagation and backtracking methods [13], the search space can be drastically reduced and feasible solutions can be obtained very quickly. However, the capability of CP in locating the global optimal solutions is inferior as compared to other meta-heuristics, such as simulated annealing, genetic algorithms [14], etc.

On the other hand, simulated annealing [9,10], a generic probabilistic meta-heuristic based on the manner in which liquids freeze or metals re-crystallize in the process of annealing, has been widely accepted and employed for global optimization problems due to its solution quality. The major hurdle of simulated annealing, however, is a large computation time due to lack of good initial solutions and its sequential nature of slow annealing process within the large solution space.

The strengths of both constraint programming and simulated annealing encourage the development of new efficient hybrid algorithms for solving large combinatorial optimization problems that are NP-hard and therefore intractable due to the size of a complete search tree. In the hybrid algorithm developed in this paper, a good feasible solution is firstly obtained quickly by constraint programming. Then simulated annealing is used to guide the search path to find the optimal solution. Unlike the traditional SA, in which the neighborhood solutions are obtained using local search methods, in the newly hybrid algorithm, the neighborhood solutions are obtained using the constraint programming approach. The performance of algorithm is further improved by memorizing the useful information which causes the infeasible solutions, thus can reduce the solution space drastically.

The remainder of the paper is organized as follows. Section 2 describes the problem scenario under investigation and presents the formulation of the mathematical model. The newly developed hybrid CP-SA algorithm is then detailed in Section 3. Extensive computation results on a set of randomly generated test problems are presented and the efficiency of the proposed algorithm is demonstrated in Section 4. Finally, Section 5 concludes this research.

II. Problem Scenario and Model Formulation

A. Problem Scenario

Figure 1 shows the supply chain network under consideration. A manufacturer aims to meet different needs of customers by producing multiple families of products, with multiple product variants in each family. These product families share common and non-common modules, such as raw materials and component parts. With limited capacity of suppliers, it is important to determine the supply quota to be allocated to different supplier groups to support the production of multiple products. The problem is further complicated by the multiple selection criteria for suppliers such as: price, quality, on-time delivery [5, 6] and trust [15].

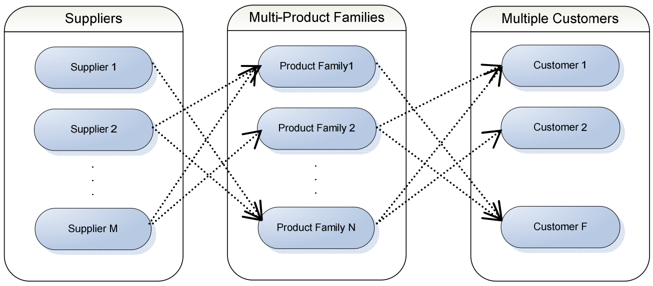

Fig.1 Supply chain network

B. Model Formulation

This part presents the development of a new mixed integer programming mathematical model describing the characteristics of the research problem. A manufacturer aims to provide n high variety products utilizing K OR modules and L AND modules provided by m capacity-constrained suppliers over a planning horizon with multiple time periods. Each product family has I^n product variants and these product variants cater for different customer requirements. To characterize the product structure, a genetic-bill-of-material (GBOM) method (see [7]) is adopted.

To facilitate the presentation, the notations are firstly listed as follows.

Indices:

k	OR module
S_k	number of options for OR module k
ks	option s of module k
l	AND module
n	product family
I^n	number of product variants in family n
ni	product variant i of family n
m	supplier
τ	time period
T	number of all time periods

Parameters:

z_{niks}	1 if ks is used for product ni, 0 otherwise
z'_{nl}	1 if l is used for product ni, 0 otherwise
V^{τ}_{mks}	capacity of supplier m for ks in period τ
G^{τ}_{ml}	capacity of supplier m for l in period τ
ω^{τ}_{mks}	supplier m's selling price for ks in period τ
b^{τ}_{ml}	Supplier m's selling price for l in period τ
$Q^{n\tau}$	market demand for family n in period τ
BOM_{nk}	units of k needed to produce one unit of final product variant in family n
BOM'_{nl}	units of l needed to produce one unit of final product variant in family n
F_{ni}	fixed cost for marking down the less desirable product ni
C_{ni}	production cost for one unit of product ni
S_{ni}	setup cost for one unit of product variant ni
B^{τ}_{m}	supplier m's minimum budget in period τ

T_m^τ	supplier m's trust level in period τ
O_m^τ	transaction cost of supplier m in period τ
p_{n1}^τ	retail price for the ideal variant of family n in period τ
p_{ni}^τ	retail price for the product ni in period τ
H_{ks}	inventory holding cost for ks
$H_l^{'}$	inventory holding cost for l
HH_{ni}	inventory holding cost for product ni
d_m^τ	late delivery days of m in period τ
TP_{ni}	unit tardiness penalty for product ni per day
QP	quality penalty for one unit of the modules per percent below 100%
QL_{mks}^τ	quality level of ks procured from m in period τ
$QL_{ml}^{'\tau}$	quality level of l procured from m in period τ
δ_{mks}^τ	1if m is capable of providing ks in period τ, 0 otherwise
$\delta_{ml}^{'\tau}$	1if m is capable of providing l in period τ, 0 otherwise

Continuous variables:

Q_{ni}^τ	quantity of product ni produced in period τ
A_{ni}^τ	quantity of product ni sold in period τ
x_{mks}^τ	order quantity of ks from m in period τ
$x_{ml}^{'\tau}$	order quantity of l from m in period τ
I_{ks}^τ	inventory level of ks in period τ
$I_l^{'\tau}$	inventory level of l in period τ
II_{ni}^τ	inventory level of product ni in period τ

Binary variables:

y_{mks}^τ	1 if ks is procured from supplier m in period τ, 0 otherwise
$y_{ml}^{'\tau}$	1 if l is procured from supplier m in period τ, 0 otherwise
Y_m^τ	1 if supplier m is selected in period τ, 0 otherwise
η_{ni}^τ	1 if product ni is produced in period τ, 0 otherwise
ζ_{ni}^τ	1 if product ni is sold in period τ, 0 otherwise

Mathematical model:

Objective:
Maximize: Total profit = Total revenue - Total cost

Total Revenue = $\sum_{\tau=1}^{T}\sum_{n=1}^{N}\sum_{i=1}^{I^n} A_{ni}^\tau \zeta_{ni}^\tau p_{ni}^\tau$

Total cost= $\sum_{c=1}^{9}$ Costc

Cost1:
Total purchasing cost of modules=

$$\sum_{\tau=1}^{\mathrm{T}}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{s=1}^{S_k} x_{mks}^\tau \omega_{mks}^\tau + \sum_{\tau=1}^{T}\sum_{m=1}^{M}\sum_{l=1}^{L} x_{ml}^\tau b_{ml}^\tau$$

Cost2:
Total transaction cost with both the module suppliers =

$$\sum_{\tau=1}^{\mathrm{T}}\sum_{m=1}^{M} O_m^\tau Y_m^\tau$$

Cost3:
Cost incurred by the efforts in promotion, advertising, to lure the customer to buy the products = $\sum_{\tau=1}^{\mathrm{T}}\sum_{n=1}^{N}\sum_{i=1}^{I^n} Q_{ni}^\tau \eta_{ni}^\tau F_{ni}$

Cost4:

Total quality penalty= $QP \times \sum_{\tau=1}^{\mathrm{T}}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{s=1}^{S_k}(1-QL_{mks}^\tau)x_{mks}^\tau + QP \times \sum_{\tau=1}^{T}\sum_{m=1}^{M}\sum_{l=1}^{L}(1-QL_{ml}^\tau)x_{ml}^\tau$

Cost5:
Total tardiness penalty= $\sum_{\tau=1}^{\mathrm{T}}\sum_{n=1}^{N}\sum_{i=1}^{I^n}\zeta_{ni}^\tau \times TP_{ni} \times PD$,

where

$$PD = \max\left\{\arg\max_{d_m^\tau}\left(z_{niks} \times y_{mks}^\tau \times d_m^\tau\right), \arg\max_{d_m^\tau}\left(z_{nl}^{'} \times y_{ml}^\tau \times d_m^\tau\right)\right\}$$

Cost6:
Total inventory holding cost for the modules =

$$\sum_{\tau=1}^{\mathrm{T}}\sum_{k=1}^{K}\sum_{s=1}^{S_k} I_{ks}^\tau H_{ks} + \sum_{\tau=1}^{T}\sum_{l=1}^{L} I_l^{'\tau} H_l^{'}$$

Cost7:
Total inventory holding cost for the final products=

$$\sum_{\tau=1}^{\mathrm{T}}\sum_{n=1}^{N}\sum_{i=1}^{I^n} II_{ni}^\tau HH_{ni}$$

Cost 8:

Total production cost = $\sum_{\tau=1}^{T}\sum_{n=1}^{N}\sum_{i=1}^{I^n} Q_{ni}^\tau \eta_{ni}^\tau C_{ni}$

Cost 9:

Total production setup cost= $\sum_{\tau=1}^{T}\sum_{n=1}^{N}\sum_{i=1}^{I^n} \eta_{ni}^\tau S_{ni}$

Subject to: (Constraints)

$$0 \le x_{mks}^\tau \le V_{mks}^\tau, \quad \forall m,k,s,\tau \tag{1}$$

$$0 \le x_{ml}^t \le G_{ml}^t, \quad \forall m,l,t \tag{2}$$

$$\sum_{k=1}^{K}\sum_{s=1}^{S_k} x_{mks}^\tau \omega_{mks}^\tau + \sum_{l=1}^{L} x_{ml}^\tau b_{ml}^\tau > B_m^\tau, \forall m,k,s,l,\tau \tag{3}$$

$$I_{ks}^{\tau-1} + \sum_{m=1}^{M} x_{mks}^\tau \ge \sum_{n=1}^{N}\sum_{i=1}^{I^n} z_{niks}\eta_{ni}^\tau Q_{ni}^\tau BOM_{nk}, \quad \forall m,k,s,n,i,\tau,\tau \ne \mathrm{T} \tag{4}$$

$$I_{ks}^{\mathrm{T}-1} + \sum_{m=1}^{M} x_{mks}^{\mathrm{T}} = \sum_{n=1}^{N}\sum_{i=1}^{I^n} z_{niks}\eta_{ni}^{\mathrm{T}} Q_{ni}^{\mathrm{T}} BOM_{nk}, \forall m,k,s,n,i \tag{5}$$

$$I_l^{'(\tau-1)} + \sum_{m=1}^{M} x_{ml}^{'\tau} \ge \sum_{n=1}^{N}\sum_{i=1}^{I^n} z_{nl}^{'}\eta_{ni}^\tau Q_{ni}^\tau BOM_{nl}^{'}, \quad \forall m,l,n,i,\tau,\tau \ne \mathrm{T} \tag{6}$$

$$I_l^{'(\mathrm{T}-1)} + \sum_{m=1}^{M} x_{ml}^{'\mathrm{T}} = \sum_{n=1}^{N}\sum_{i=1}^{I^n} z_{nl}^{'}\eta_{ni}^{\mathrm{T}} Q_{ni}^{\mathrm{T}} BOM_{nl}^{'}, \forall m,l,n,i \tag{7}$$

$$\sum_{i=1}^{I^n} \zeta_{ni}^\tau A_{ni}^\tau = Q^{n\tau}, \forall n,i,\tau \tag{8}$$

$$Q_{ni}^{\tau} + II_{ni}^{\tau-1} \geq A_{ni}^{\tau}, \forall n, i, \tau \quad (9)$$

$$\sum_{\tau=1}^{T} \sum_{i=1}^{I^n} \eta_{ni}^{\tau} Q_{ni}^{\tau} = \sum_{\tau=1}^{T} \sum_{i=1}^{I^n} \zeta_{ni}^{\tau} A_{ni}^{\tau} = \sum_{\tau=1}^{T} Q^{n\tau}, \forall n, i, \tau \quad (10)$$

$$II_{ni}^{\tau} = II_{ni}^{\tau-1} + Q_{ni}^{\tau} - A_{ni}^{\tau}, \forall n, i, \tau \quad (11)$$

$$II_{ni}^{0} = 0, II_{ni}^{T} = 0, \forall n, \forall i \quad (12)$$

$$p_{ni}^{\tau} = p_{n1}^{\tau} \sqrt[\alpha]{(u_{ni})^{\beta}} \quad (13)$$

$$y_{mks}^{\tau} = \min(1, x_{mks}^{\tau}), \forall m, k, s, \tau \quad (14)$$

$$y_{ml}^{'\tau} = \min(1, x_{ml}^{'\tau}), \forall m, l, \tau \quad (15)$$

$$Y_{m}^{\tau} = \min(1, y_{mks}^{\tau} + y_{ml}^{'\tau}), \forall m, k, s, l, \tau$$
(16)

$$\eta_{ni}^{\tau} = \min(1, Q_{ni}^{\tau}), \forall n, i, \tau \quad (17)$$

$$\zeta_{ni}^{\tau} = \min(1, A_{ni}^{\tau}), \forall n, i, \tau \quad (18)$$

$$0 \leq x_{mks}^{\tau} \leq \delta_{mks}^{\tau} \times M_{\infty}, \forall m, k, s, \tau \quad (19)$$

$$0 \leq x_{ml}^{'\tau} \leq \delta_{ml}^{'\tau} \times M_{\infty}, \forall m, l, \tau \quad (20)$$

The objective function is to maximize the manufacturer's total profit which equals to total revenue minus total cost. The total cost includes purchasing cost, transaction cost, discounting cost, quality penalty, tardiness penalty, inventory holding cost, production cost and production setup cost. Constraints (1) and (2) indicate that the suppliers have limited capacity for the OR and AND modules. Constraint (3) represents the lowest purchasing amount required by the suppliers. Constraints (4)-(7) imply the relationship between available resources and the production quantity of the final products over the planning horizon, i.e., GBOM constraints. The demand satisfaction requirement and the relationship between the production and sale quantity of the final products are governed by constraints (8)-(10). By constraints (11) and (12), the inventory balances of the final products are ensured. Price discounts for the less desirable product variants with customer flexibility considerations is given in equation (13), where p_{n1}^{t} is the retail price for the ideal product variant, α and β refer to price elasticity and utility elasticity, respectively. Constraints (14)-(18) govern that $y_{mks}^{t}, y_{ml}^{t}, Y_{m}^{t}, \eta_{ni}^{t}, \zeta_{ni}^{t}$ are 0,1 integer variables. Constraints (19) and (20) govern the procurement of modules from suppliers, where M_{∞} is a large positive number.

C. An Illustrative Example

A simple numerical example is presented to illustrate how the proposed integrated supplier selection and order allocation problem can be formulated and applied in a multi-product supply chain.

Consider a manufacturer who aims to produce two families of products to meet different customer needs. The customers have specifications regarding the shape, color and material used for the products.

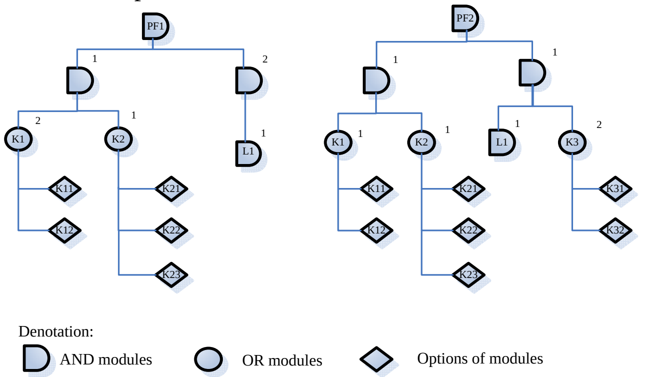

Fig.2 GBOM for Two Product Families

As shown in figure 2, a three-level GBOM is used to depict the product structure of the product families. The maximum number of the OR modules in the lowest level is set to 3, as indexed by $K1, K2, K3$. These modules embody the shape, color and material requirements of the specific modules, respectively. There is only one AND module ($L1$) in the lowest level. The details of the three OR modules are given as below.

Module $K1$	Module $K2$	Module $K3$
$K11$: rectangular $K12$: circular	$K21$: green $K22$: yello w $K23$: white	$K31$: plastics $K32$: steel

Hence, the total numbers of variants in each product family can be calculated as 2×3, $2\times3\times2$ respectively. The detailed mapping relationships between all product variants and the raw materials (in terms of the AND module and the OR modules) are presented below.

	P11	P12	P13	P14	P15	P16
K11	1	1	1	0	0	0
K12	0	0	0	1	1	1
K21	1	0	0	1	0	0
K22	0	1	0	0	1	0
K23	0	0	1	0	0	1
K31	0	0	0	0	0	0
K32	0	0	0	0	0	0
L1	1	1	1	1	1	1

	P21	P22	P23	P24	P25	P26
K11	1	1	1	1	1	1
K12	0	0	0	0	0	0
K21	1	1	0	0	0	0
K22	0	0	1	1	0	0
K23	0	0	0	0	1	1
K31	1	0	1	0	1	0
K32	0	1	0	1	0	1
L1	1	1	1	1	1	1

	P27	P28	P29	P210	P211	P212
K11	0	0	0	0	0	0
K12	1	1	1	1	1	1
K21	1	1	0	0	0	0
K22	0	0	1	1	0	0
K23	0	0	0	0	1	1
K31	1	0	1	0	1	0
K32	0	1	0	1	0	1
L1	1	1	1	1	1	1

Note that here "1" represents the raw materials are used for a certain product variant, "0" otherwise.

Using the proposed hybrid algorithm, the solutions to this example can be obtained as follows.

Production quantity of product variants:
In product family 1, only two product variants are produced, i.e.,
$Q_{11}^{1} = 85, Q_{14}^{1} = 48$,
The product variants produced in family 2 are:
$Q_{22}^{1} = 40, Q_{25}^{1} = 57$.
The production quantities of all the other product variants are zero.

Selection of suppliers:
All three suppliers are selected.

Order quantity of OR and AND modules:

	Supplier 1	Supplier 2	Supplier 3
K11	267	0	0
K12	0	96	0
K21	73	100	0
K22	0	0	0
K23	0	0	57
K31	0	54	50
K32	0	0	80
L1	200	0	63

III. Hybrid Algorithm

The proposed mathematical model describes a planning problem which is NP-hard and needs to be solved by an efficient method. To this end, this section focuses on the development of a new hybrid algorithm based on the strengths of both constraint programming and simulated annealing.

A. Notations

The following notations are listed to facilitate the presentation of the algorithm.

t	temperature iteration index ($t = 0,1,\ldots, max\_t$)
N_d	set of indices for all the product variants
d	index for the product variants, $d \in N_d$
N_e	N_e is the set of indices for all the feasible solutions in an iteration. N_e is also the Markov chain length of simulated annealing
e	index for a complete feasible solution, $e \in N_e$
E_Best	optimal solution among all the feasible sequences within an iteration (local optimum)
T_Best	optimal solution among all the iterations (global optimum)

B. Elements of the Hybrid CP-SA

Formulation of the production planning problem as a constraint network

It is not easy to generate the feasible solutions that satisfy all the relevant constraints, especially when the solution space is very large. Hence, this paper tackles this challenge by firstly formulating the problem as a constraint satisfaction problem [13].

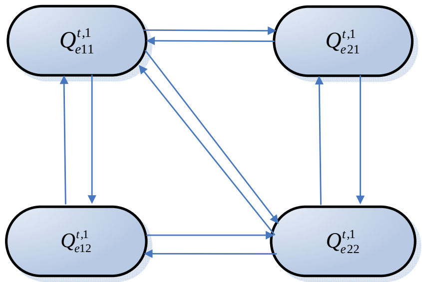

Fig. 3 Constraint network for a simple example

Figure 3 illustrates a detailed formulation of a constraint network for a simple numerical example over a planning horizon which comprises of only one time period. In this example, two product families are considered, and each family has two product variants, with their production quantities denoted by: Q_{e1}^{t}, Q_{e2}^{t}, Q_{e3}^{t} and Q_{e4}^{t}.

Components of the simulated annealing algorithm

The simulated annealing algorithm includes some basic components (see [3]):

(1) Configuration: A solution to the problem.

(2) Energy function: A measure of how good a solution is.

(3) Neighborhood move: A transition from one configuration to another that results in a neighboring solution.

(4) Acceptance-rejection of a solution: The neighborhood solution is either accepted or rejected with probability according to the following acceptance probability:

$$\text{Pac} = \begin{cases} 1 & \text{if Profit}_b \geq \text{Profit}_a \\ e^{(\text{Profit}_b - \text{Profit}_a)/Tem} & \text{if Profit}_b < \text{Profit}_a \end{cases}$$

where Tem is the control parameter known as temperature, "a" (current solution) and "b" (neighboring solution) are two solutions with profits Profit_a and Profit_b.

(5) Termination criterion: The algorithm terminates if the pre-determined parameters (e.g. maximum number of iterations) are reached.

(6) Cooling schedule (see [1]): The initial temperature, the rule for decreasing the value of temperatures, the number of transitions at each temperature, and the time at which annealing should be stopped are referred to as the cooling schedule. The efficiency and effectiveness of the algorithm depends on the cooling schedule. In this research, the cooling schedule proposed by [12] is adopted which has the following form: $Tem_t = \alpha Tem_{t-1}$.

C. Basic Steps of the Hybrid CP-SA

The basic procedures of the hybrid CP-SA algorithm are then outlined as follows.

Step 1: Set t =0, set the initial temperature of simulated annealing as $Tem_0 = \dfrac{\text{Cost}_{\min} - \text{Cost}_{\max}}{\ln Pac_0}$, here $\text{Cost}_{\min}$ and $\text{cost}_{\max}$ are the minimum and maximum bounds of problem complexity. The initial acceptance probability Pac_0 is set very close to 1. The resulting high initial temperature provides a high degree of randomness and most of the movements are accepted in the initial stage. Set $e = 0$.

Step 2: Select an input node d (product variant index) for constraint programming based on the retail price, i.e., $d^* = \arg\max_{d \in N_d} (\text{Price}_d)$, set $d^* = 1$. Generate the value of Q_{e1}^t within the feasible range bounded by demand, capacity of raw materials.

Step 3: Search for a complete feasible solution
a) if $d < |N_d|$, then let $d = d + 1$, use constraint programming algorithm to generate the value for Q_{ed}^t, reduce the search space.
b) else if $d = |N_d|$, then one complete feasible solution $q_e^t = \{Q_{e1}^t, Q_{e2}^t, ..., Q_{eN_d}^t\}$ has been found. Initialize this solution as the current optimal solution. $E\_Best = q_e^t$, $e = e + 1$.

Step 4: Generate a neighborhood solution starts from $d = d + 1$ using the constraint programming algorithm.

Step 5: Compare the two solutions using the proposed SA algorithm.
a) if the neighborhood solution replaces the current optimal solution, i.e., $\dfrac{\exp(-Fitness(q_e^t) - Fitness(E\_Best))}{Tem_t} > \rho$, where ρ is a real number randomly generated between 0 and 1, then $d = d + 1$,generate another neighborhood solution starting from d using constraint programming approach, $e = e + 1$;
b) else if the neighborhood solution doesn't replace the current optimal solution,
then $d = d - 1$, generate another neighborhood solution starting from d using constraint programming approach, $e = e + 1$.

Step 6: If $e < |N_e|$, then repeat Step 5 until $e = |N_e|$, then let $t = t + 1$, update T_Best, then go to step 7.

Step 7: Calculate the temperature of the new iteration, i.e., $Tem_t = \alpha Tem_{t-1}$, set $e = 0$. Here α is the cooling rate of the proposed simulated annealing algorithm, which belongs to (0,1). The cooling rate α is dependent on the variance ($Var_{Tem_{t-1}}$) of the objective function values provided by the feasible solutions at the temperature Tem_{t-1}. The mean value and variance ($Var_{Tem_{t-1}}$) of the objective function values is calculated as follows:

$$Mean_{Tem_{t-1}} = \frac{1}{|Ne|} \sum_{\forall e \in Ne} Fitness(q_e^t)$$

$$Var_{Tem_{t-1}} = \frac{1}{|Ne|} \sum_{\forall e \in Ne} (Fitness(q_e^t) - Mean_{Tem_{t-1}})^2$$

The definition of [8] for cooling rate α is then applied:

$$\alpha = \frac{1}{1 + [(Tem_{t-1} \times \ln(1+\delta))/3Var_{Tem_{t-1}}]}$$

where δ is a control rate and experimentally determined as 0.01.

Step 8: Repeat steps 2-5 until $e = |N_e|$.

D. Flowchart of the Hybrid CP-SA

Figure 4 presents the flowchart of the hybrid algorithm. It can be noted that the hybrid algorithm has two loops -- an inner loop which is a serial simulated annealing process and an outer loop where the temperature of the system decreases gradually.

IV. TEST PROBLEMS AND COMPUTATION RESULTS

A. Test Problems

The effectiveness of the proposed hybrid CP-SA algorithm is demonstrated by applying the algorithm to solve a set of randomly generated test problems. The computational results obtained are compared with those obtained by using ILOG OPL.

In these test problems, the number of product families considered ranges from 4 to 8. Each product family has a unique product structure depicted in its GBOM. The number of suppliers are randomly generated within the ranges [2,6]. The number of time periods is randomly generated within the range [1,6]. A set of small scale test problems and several medium to large scale test problems are used in the experiments.

In each experiments, CP-SA runs 50 iterations, and the value for N_e is experimentally determined as 60. The algorithm is programmed in C++ and implemented on a Pentium IV 3.2 GHz computer with 512M Ram.

B. *Convergence properties for the illustrative example*

As shown in figure 5, for the illustrative example used in the previous section, the search process evolves as the temperature of the system cools down, and eventually converges to the "global optima" as it terminates.

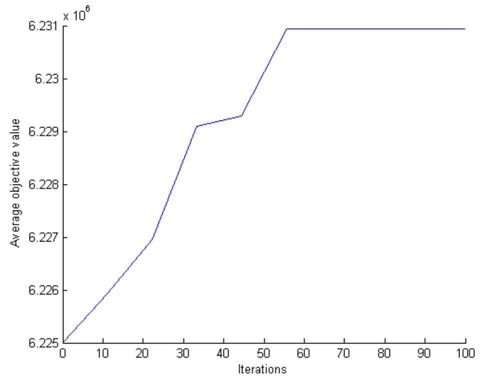

Fig.5 Convergence Behavior of the Hybrid Algorithm

C. *Computation Results and Discussions*

Table 1 summarizes the best and the average of the best solutions obtained for the small scale test problems by running the proposed hybrid algorithm 5 times. The average computation times (in seconds) needed to achieve the best solutions are also included.

Table 1 Computational Results for Small Scale problems

No	CP-SA			ILOG	
	avg	best	time	solution	time
1	6230942	6253590	0.16	6297116	0.38
2	865834	878028	0.27	878439	0.48
3	811032	816235	0.35	821093	0.52
4	1089033	1089037	0.45	1089733	0.82
5	544283	544611	0.66	545163	1.04
6	1198525	1204146	0.75	1216688	1.61
7	1403233	1409490	0.95	1433080	1.82
8	1422288	1426626	0.92	1481515	2.58
9	592299	592369	1.10	597152	2.88
10	2369706	2557677	1.25	2557999	3.09
11	1352883	1353818	1.33	1380407	3.72
12	17278360	17301138	5.35	17467460	13.9
13	1352883	1380142	6.35	1389411	17.3
14	1710104	1714143	8.00	1738481	24.3
15	1531070	1542378	20.3	1558840	44.5

The results in the above table clearly show that, the proposed hybrid CP-SA algorithm is able to locate near-optimal solutions with less computation efforts. For small scale problems, the differences are within 1% as compared to the optimal solutions obtained by ILOG OPL.

Table 2 Results for Medium to Large Scale Cases

No	CP-SA			ILOG	
	avg	best	time	solution	time
1	1259033	1300589	120.4	1147630*	163.3
2	2499031	2589007	152.1	2376304*	92.9
3	16352238	168020045	90.4	---	>
4	130015486	134578902	111.3	---	>
5	3522116	3895412	160.2	---	>
6	8054113	8105263	254.1	---	>
7	1519141	1588042	212.1	---	>
8	2503264	2587032	216.3	---	>
9	1654701	1689412	289.2	---	>
10	3941028	3989745	321.4	---	>

As shown in table 2, for medium to large scale cases, the hybrid algorithm can find better solutions (bold numbers) with less computation efforts. The --- indicates there is no solution found after running ILOG for more than 2000s (symbolized by ">") listed in the table. This indicates the hybrid algorithm is superior to ILOG OPL in determining near optimal solutions when applied to more complex real problems.

V. CONCLUSIONS

This paper has studied an integrated supplier selection and order allocation problem for a supply chain manufacturing multiple products over a planning horizon with multiple time periods when customer flexibility exists. A novel mixed integer mathematical programming model has been developed to maximize the manufacturer's profit by determining the production quantity of each product variant, and by selecting the most suitable suppliers based on the selection criteria and their capacity and splitting the orders among these suppliers. A new hybrid algorithm based on the strengths of both constraint programming and simulated annealing has also been developed for solving this complex NP-hard problem. In the hybrid algorithm, CP is used to generate the initial feasible solution and SA is used to guide the search path. Unlike traditional SA, CP is used to generate the neighborhood solutions for SA. Useful information obtained from CP helps to reduce the search space drastically. The efficiency of the proposed algorithm has been tested with a set of randomly generated test problems. Comparison of the results obtained from solving these test problems with those obtained by using ILOG OPL, a commercial software, clearly show that the methodology developed in this research is an efficient and effective optimization approach to solving the integrated supplier selection and order allocation problem for multi-period multi-product manufacturing when customer flexibility exists.

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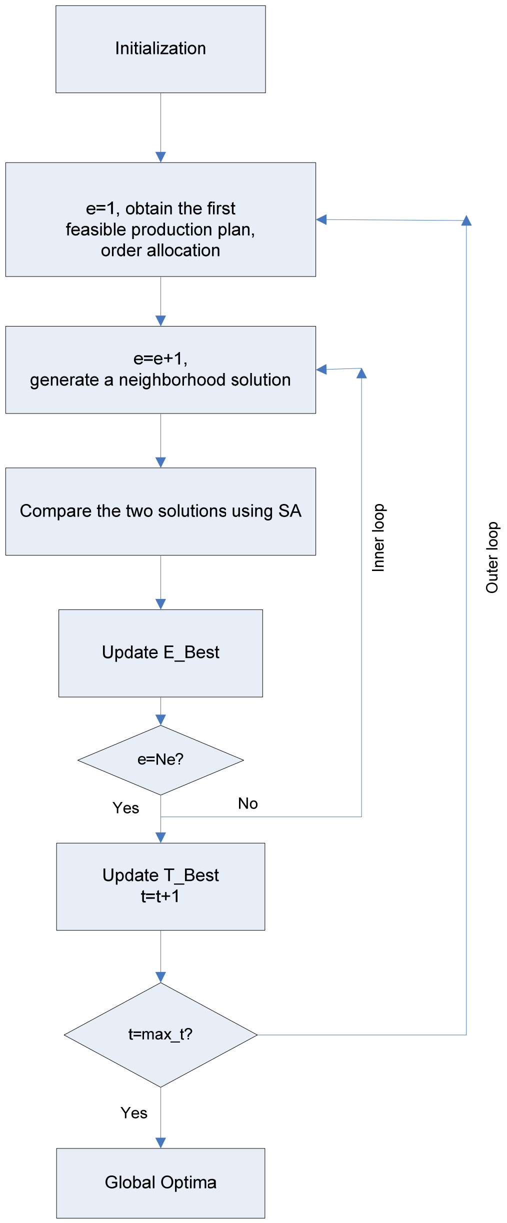

Fig.4 Flowchart of the Hybrid Algorithm