A Hybrid Fuzzy Time Series Model for Forecasting

Saima Hassan, Jafreezal Jaafar, Brahim B. Samir, Tahseen A. Jilani

Abstract – **Researchers are finding their way to solve the chaotic and uncertain problems using the extensions of classical fuzzy model. At present Interval Type-2 Fuzzy logic Systems (IT2-FLS) are extensively used after the thriving exploitation of Type-2 FLS. Fuzzy time series models have been used for forecasting stock and FOREX indexes, enrollments, temperature, disease diagnosing and weather. In this paper an integrated fuzzy time series model based on ARIMA and IT2-FLS is presented. The propose model will use ARIMA to select appropriate coefficients from the observed dataset. IT2-FLS is utilized here for forecasting the result with more accuracy and certainty.**

Index Terms—**Type-1 Fuzzy sets (T1-FS), Type-2 Fuzzy sets (T2-FS), Fuzzy Logic System (FLS), Interval Type-II Fuzzy Logic Systems (IT2-FLS), Autoregressive Integrated Moving Average (ARIMA) models, Time Series Forecasting.**

I. INTRODUCTION

Traditionally, time series forecasting problems are being solved using a class of Autoregressive moving average models. Being linear statistical models, they cannot build relationship among the nonlinear variables/factors. Calculating the parameters for multi-variables is another issue faced by them. The strong relationship among these variables may result in large errors. Furthermore a model cannot be estimated correctly if the historical data is less. Handling the irregularity and uncertainty of real data are some more issues concerning to this group of models. Several statistical nonlinear models have been developed to overcome these limitations of linear models [1]. Because these models are developed for specific nonlinear patterns, they are not capable of modeling other types of nonlinearity in time series [2]. Consequently the researchers adopted hybrid approach i.e. mixing of two or more than two models to improve the accuracy of the forecasted result [3] [4] [5]. A hybrid approach of linear and nonlinear models was adopted in [2] for taking benefits of both the models.

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A hybrid model can increase the chance to capture different patterns in data and improve forecasting accuracy. In addition, the hybrid model is more robust with regard to the change in data. It has been revealed from empirical studies that the hybrid models outperform than the individual models [1] [6] [7].

Being nonlinear, the fuzzy logic systems (FLS) have also been mixed with time series models [4] [8] [9] to handle linguistic chaos and ambiguity present in data. Since the introduction of FLS, it has successfully been applied to a lot of time series problems including stock and FOREX indexes [10] [11] [12] [13], enrollments [14] [15] [4], temperature [16], weather forecasting [17] [18] [6] [5], and disease diagnosing [19].

Researchers are finding their way to solve the chaotic and uncertain problems using the extensions of classical fuzzy model. After the thriving exploitation of Type-2 FLS, currently, Interval Type-2 Fuzzy logic Systems (IT2-FLS) are being extensively used in hybrid models [6] [20] [21]. The non-interval secondary membership function of a general Type 2 Fuzzy sets (T2-FS) made it computationally more complex. While IT2 Fuzzy Sets are enjoying the center stage these days because of the non fuzziness of its secondary function [22] [23] [24]. The ability to deal with different sources of information (such as process data as well as expert knowledge) and nonlinearities, in a straightforward manner, makes the approach more suitable for modeling hybrid model for forecasting. Keeping this in view, the author in this paper, is presenting a hybrid fuzzy time series model. Based on ARIMA and Interval Type-2 Fuzzy Inference System (IT2-FIS) the proposed model will improve the forecasting result by handling the measurement and parametric uncertainties of ARIMA model using Fuzzy approach.

The rest of the paper is organized as follow. In section 2, a brief literature review on time series forecasting techniques is described. Section 3 and 4 consists of a short overview of ARIMA models and IT2-FIS respectively. A hybrid model is proposed in section 5 and future work is given in section 6.

II. LITERATURE REVIEW

Forecasting is a systematic way of anticipating future events and situations by estimating the past value of a variable(s). The management and scheduling done by forecasting helps to cope with uncertain situations of the future. Fuzzy time series is used to handle those forecasting problems where numerical

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as well as linguistic information related to them is available. Thus, fuzzy time series forecasting not only covers the statistical flavor but also the linguistic chaos analysis as well.

The idea of fuzzy time series based on the historical enrollments of the University of Alabama is practiced in [14]. A heuristic model of fuzzy time series model has been developed to improve forecasting [25]. The heuristic helped out the moving trend which supports forecasting without affecting the fuzzy relationships i.e. the improvement in forecasting result is achieved by introducing heuristic rules. The model is applied on only one variable where it can be applied on more heuristic variables. Weighted models are proposed in [11] to resolve the recurrence and weighting issues in fuzzy time series forecasting. These models demonstrate similarity to the weight functions in local regression models; though, both are dissimilar. The local regression models concentrated on fitting using a small portion of the data, whilst the weighted fuzzy time series models established fuzzy relationships using the promising data from the entire database. Two new multivariate fuzzy time series forecasting methods are presented in [3]. These methods assume m-factors with one significant main factor. Stochastic fuzzy dependence of order k is presumed to define general methods of multivariate fuzzy time series forecasting and control. A refined fuzzy time series model with improved defuzzification model is presented in [9] for stock exchange forecasting. Results with better accuracy were obtained by proposing a heuristic approach of fuzzy metric.

ARIMA, a popular statistical time series model is integrated with fuzzy regression model in [10] for forecasting the foreign exchange market. It is deduced from the results that it takes less observation to estimate a model than ARIMA. The forecasted results were made good by narrowing the fuzzy interval (Upper and lower bound). When fuzzy intervals were wide deleting its upper or lower bound the interval were made narrow. This gave better performance. However deleting the bounds may cause incomplete or missing interval/data. T2-FLS is applied to forecast Mackay-Glass time series [19]. Using T1-FLS, a T2-FLS is formed by incorporating noise information. The FLSs were designed on a single realization for practice. However, for different realizations of same data set, different FLSs parameters must be choose to obtain improved forecasting [19]. A novel hybridization model based on ARIMA and of soft computing methods such as Neural networks and evolutionary algorithm is explored in [26]. Fuzzy IF-THEN rules were evaluate by mamdani-Assilian technique and implemented by an evolutionary algorithm to determine the model structure. Unusually this attention-grabbing technique used the rules to select the best method instead of time series forecasting.

Reference [6] integrated ANFIS and an Evolution Optimizer to synchronize a First-order Interval Type 2 *Takagi-Sugeno*-Kang (IT2TSK) Fuzzy Logic System (FLS) to a hybrid model. Neuro-Evaluative IT2 TSK model has independent, normal distributed, uncorrelated and smaller residuals than other methods. It is observed as a good forecaster but in-depth search is required for replacing the worst data set with new ones. The developed Neuro-Genetic structure involves complex computation and to attain simplicity in such a model, an efficient algorithm(s) should be explored.

III. A BRIEF OVERVIEW OF AUTO REGRESSIVE INTEGRATED MOVING AVERAGE (ARIMA)

Time series is a unique sequence of data taken at a specific interval of time. This data may be stationary or non-stationary. The data must first be made stationary by numerous methods, as forecasting cannot be done with data comprising non-stationary behavior. ARIMA is a non-stationary model that can be reduced to a stationary time series by differencing. The model has three components i.e. Autoregressive, Integrated and Moving Average.

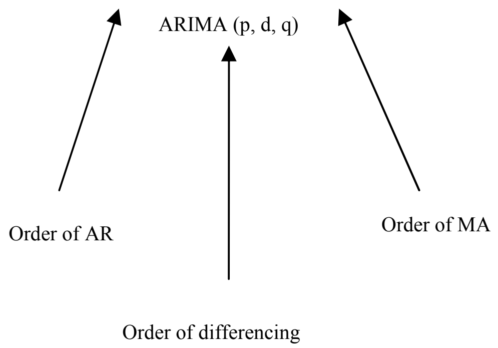

Fig. 1. A General View of ARIMA models.

A time-series Y_t is generated by an ARIMA (p, d, q) if

$$\varphi(L)(1-L)^d Y_t = \theta_q(L)\varepsilon_t \quad (1)$$

where $\varphi(L) = 1 - \varphi_1 L - \varphi_2 L^2 - \cdots - \varphi_p L^p$ and $\theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \cdots - \theta_q L^q$

are polynomials in L of degree p and q, L is the backward shift operator, p, d, q are integers, Y_t denotes the observed value of time-series data, $t = 1, 2, \ldots, k$, and time-series data are the observations.

Being one of the best forecasting model, it can be best illustrated by Box-Jenkins [27] four steps i.e.

1) Identification finds the appropriate value of differencing d. Practically, one or two levels of differencing are sufficient to make the time series stationary.
2) Estimation of the unknown model parameters.
3) Diagnostic statistics help to judge the adequacy of the model.
4) Forecasting from the selection mode.

IV. AN OVERVIEW OF INTERVAL TYPE 2 FUZZY LOGIC SETS AND SYSTEMS (IT2-FS AND IT2-FLS)

The T2-FS are described by membership functions that are characterized by more parameters than the T1-FS. Hence, the

T2-FS provide us with more design degree of freedom for handling uncertainties. IT2-FS are a limited version of the generalized T2-FS where the secondary membership grade is always 1. This limitation allows IT2-FS to be processed a great deal more quickly than generalized T2-FS. An IT2-FLS is quite similar to a T1-FLS [20] [28] instead with the defuzzifier block where it is replaced by the output processing block in an IT2-FLS. The output processing block of IT2-FLS consists of a type-reduction followed by defuzzification block. It can be thought of as a T1-FS with a blurred membership function. Uncertainty in the primary membership of an IT2-FS consists of a bounded area that is called the "footprint of uncertainty" (FOU). The FOU allows for the graphical depiction of an IT2-FS in two dimensions instead of three dimensions.

An IT2 FS can be defined as

*Definition***:** A T2-FS denoted by $\tilde{A}$ will be an IT2-FS, when all membership functions $\mu_{\tilde{A}}(x,u) = 1$, where $x \in X$ and $u \in J\_x \subseteq [0,1]$, i.e.

$$\tilde{A} = \{((x,u),1) \mid x \in X, u \in Jx, Jx \subseteq [0,1]\},$$

$\tilde{A}$ can also be expressed as

$$\tilde{A} = \int_{x\in X} \int_{u\in J_x} 1/(x,u), J_x \subseteq [0,1],$$

V. PROPOSED MODEL BASED ON IT2-FLS AND ARIMA MODELS

To develop the interval Type-2 Fuzzy ARIMA (IT2-FARIMA) model the methodology used in [2] will be adopted. In which a time series is supposed to be composed of a linear autocorrelation structure and a nonlinear component as follows:

$$y_t = L_t + NL_t$$

Where y_t is the actual time series, L_t and NL_t are the linear and nonlinear components respectively.

A. *The proposed model will consists of the following steps*

1) Identification and parameters estimation
2) Fuzzification of the estimated coefficients
3) Mapping of IT2-FS input into IT2-FS output
4) Defuzzify the forecasted result.

Identification and parameters estimation

The observed time series data must first be transformed into a reduced data set since dealing with large data requires high processing power and large amount of memory. The main tools in identification the ARIMA parameters are the autocorrelation function (ACF), the partial autocorrelation function (PACF), and the resulting correlogram, which is simply the plots of ACF and PACF against the lag length [27]. Now the stationary time series based on these functions may identify more than one model. An optimum model will be selected using statistical estimation such as the Akaike's Information Criterion (AIC) or the Bayesian Information Criterion (BIC) [27] [29].

Fuzzification of the estimated coefficients

The parameters of ARIMA $\varphi_1, \ldots, \varphi_p$ and $\theta_1, \ldots, \theta_q$ are crisp. Instead of using crisp, fuzzy parameters, $\tilde{\varphi}_1, \ldots, \tilde{\varphi}_p$ and $\tilde{\theta}_1, \ldots, \tilde{\theta}_q$ in the form of triangular fuzzy numbers [10] will be used. The fuzzy $ARIMA(p,d,q)$ model is described by a fuzzy function with fuzzy parameters; then Eq. (1) becomes:

$$\tilde{\varphi}(L)\, W_t = \tilde{\theta}_q\,(L)\varepsilon_t \quad (2)$$

$$W_t = (1-L)^d\, Y_t \quad (3)$$

$$W_t = \tilde{\varphi}_1 W_{t-1} + \tilde{\varphi}_2 W_{t-2} + \ldots + \tilde{\varphi}_p W_{t-p} + \varepsilon_t - \tilde{\theta}_1\varepsilon_{t-1} - \tilde{\theta}_2\varepsilon_{t-2} - \ldots - \tilde{\theta}_q\varepsilon_{t-q} \quad (4)$$

Where Y_t are observations, $\tilde{\varphi}_1, \ldots, \tilde{\varphi}_p$ and $\tilde{\theta}_1, \ldots, \tilde{\theta}_q$ are fuzzy numbers respectively. Eq. (4) can be modified as

$$W_t = \tilde{\beta}_1 W_{t-1} + \tilde{\beta}_2 W_{t-2} + \ldots + \tilde{\beta}_p W_{t-p} + \varepsilon_t - \tilde{\beta}_{p+1}\varepsilon_{t-1} - \tilde{\beta}_{p+2}\varepsilon_{t-2} - \cdots - \tilde{\beta}_{p+q}\varepsilon_{t-q} \quad (5)$$

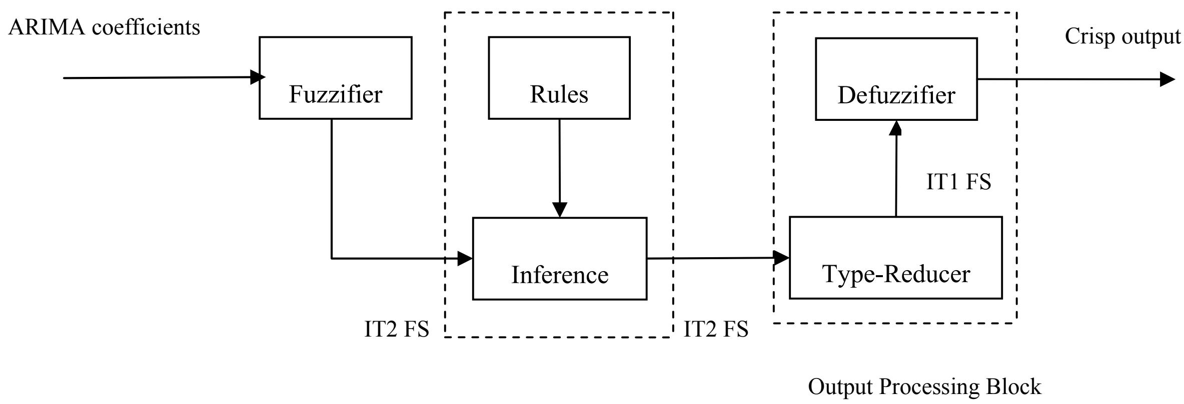

Fig. 2. Proposed IT2FS based Model

Fuzzy paratmeters in the form of triangular fuzzy numbers will be used:

$$\mu_{\tilde{L_i}}(\beta_i) = \begin{cases} 1 - \dfrac{\beta_i - \alpha_i}{c_i} & if\ \alpha_i - c_i \leq \beta_i \leq \alpha_i + c_i \\ 0 & other\ wise, \end{cases}$$

Where $\mu_{\tilde{L_i}}(\beta_i)$ is the membership function of the fuzzy sets that represents parameters β_i, α_i as the center of the fuzzy numbers, and c_i is the width or spread around the center of the fuzzy number.

Mapping of IT2-FS input into IT2-FS output

Let $\tilde{f}$ be an IT2 triangular fuzzy function then $\tilde{f}: U \rightarrow \widetilde{IT2FN}$ with $U \subseteq \widetilde{IT2FN}$ will be mapping between IT2 triangular fuzzy numbers (IT2FN) [21]. Takagi-Sugeno-Kang Interval Type-2 Fuzzy Inference System [30] will be used for the design of proposed model, which is better than Mamdani's model [31] in the sense that it reduces the number of required rules.

Defuzzify the forecasted result

Among various type-reduction methods the output will be reduced by using the Center of Sets type-reduction. As it has reasonable computational complexity that lies between the computationally expensive centroid type-reduction and the simple height and modified height type-reductions which have problems when only one rule fires [32]. Type-reduction technique extracts an interval for the output value from the uncertain region. At the end, the type-reduced IT1-FS output will be defuzzified by taking the average of two end-points to get a crisp value.

B. Problems that can be handled with the proposed Model

1) Less Historical Observations

One of the limitations of ARIMA model is that it cannot create accurate result if the observed data is too small. But sometimes in short term forecasting, the historical data may have less values [18]. In Fig.4, the graph has been built with 22 observations and it can be seen that even after first differencing, the result is not satisfactory, (see Fig.5). At first differe the graph become stationary at mean but still without constant variability (Non-homoscedastic). In such a situation the model cannot estimate the accurate coefficients, resulted in parameter uncertainity.

2) Missing Data

The second issue is of handling the missing data (see Fig. 6). In practical, missing data occurs frequently due to the non-functionality of the acquisition.

3) Sequence for Hybrid Model

Models are hybridized for the purpose of obtaining improved results than the individual. Since the real time series data contain linear and non linear patterns, the best approach would be the combination of linear and non linear model. The resulted single model would be able to deal with both types of relationships.

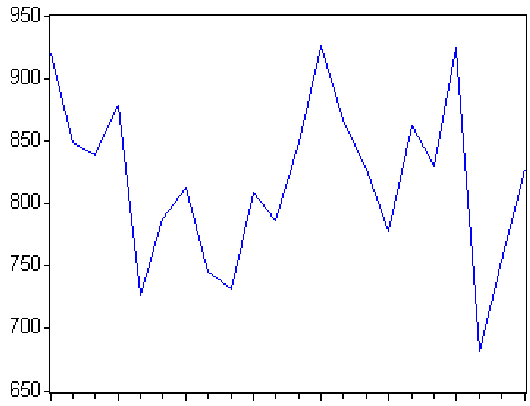

Fig. 3. Shows limitation of ARIMA when small dataset is used.

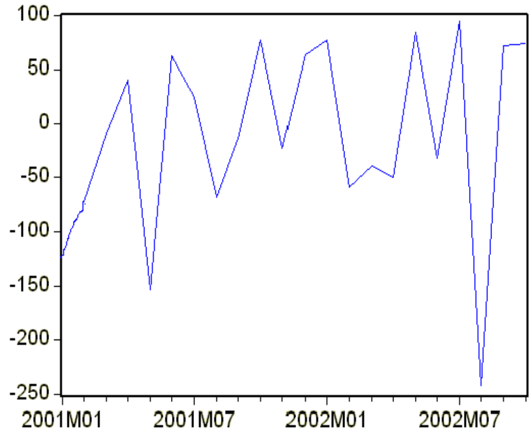

Fig. 4. Shows limitation of ARIMA when snall dataset is used.

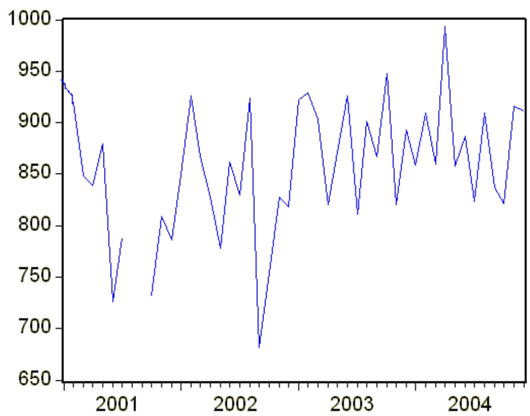

Fig. 5. Shows limitation of ARIMA when missing values are present in the dataset.

In [10] the Artificial Neural network is combined with Fuzzy Inference System to produce a hybrid ANFIS model for forecasting. Both the models are non-linear. The same data were used by ARIMA

TABLE 1
COMPARIS of ANFIS and ARIMA

	MAE		RMSE	
	ANFIS	ARIMA	ANFIS	ARIMA
Training data	1.2552	1.523	1.6312	2.075
Test data	1.3212	1.178	1.7176	1.0709

$(2, 1, 1)$ model to evaluate performance criteria. It is evidence from the result that ARIMA performed well at testing data where the performance of the ANFIS model was good in training data.

At feature extraction linear models perform well [12], where as in ANFIS model it is done by non linear model. That is the reason that testing data done by ARIMA gave good results as compared to ANFIS. Another weakness of ANFIS is its complex computations.

The performance comparisons of ANFIS and ARIMA models is done on Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Coefficient of Determination (R^2).are shown below

C. Research Approach for handling these problems

The above all three issues can be handled by the proposed hybrid model of Interval Type 2 Fuzzy model based on ARIMA (IT2FARIMA). Issue 1 and 2 are the limitations of ARIMA which would be effectively handled by IT2-FS. By using the fuzzy parameters for ARIMA, the requirement of historical data would be reduced. The issue of missing data could also be resolved by blurring the membership function and then select one by taking mean of the two points. The last issue of improving the accuracy would be solved by combining the two dissimilar models. The propose model will combine the ARIMA model with IT2-FLS, where the former is a linear model and later is a nonlinear one. The sequence for hybridizing the model will follow the conventional hybrid model, which starts with a linear model, followed by a nonlinear model to model the residual. The model will be evaluated by Mean Squared Error (MSE), RMSE and MAE.

Measurements formulas discuss in this paper are,

$$MSE = \frac{\sum_{i=1}^{N}(y_i - \widehat{y_i})^2}{N}, \qquad MAE = \frac{\sum_{i=1}^{N}|y_i - \widehat{y_i}|}{N}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(y_i - \widehat{y_i})^2}{N}}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{N}\left(y_i - \widehat{y_i}\right)^2}{\sum_{i=1}^{N}\left(y_i - \overline{y_i}\right)^2} = 1 - \frac{E}{\sum_{i=1}^{N}\left(y_i - \overline{y_i}\right)^2}$$

where y_i, $\widehat{y_i}$, $\overline{y_i}$ are actual, forecasted and mean values respectively, and N is the forecast number.

VI. FUTURE WORK

The proposed model is considering the observations of a single time series. The model can be extended to accept more than one factors affecting forecasting. Furthermore, different parameter optimization methods can be used to select the best model parameter. These optimization procedures include evolutionary techniques and particle swarm intelligence techniques that will make the model more efficient. Uses of fuzzy models are more transparent than neural networks, which make them useful in applications where transparency is required [33]. It is therefore of interest to investigate the use of fuzzy logic in hybrid modeling.

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