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Investigation of Using Dual Tree Complex Wavelet Transform (DT-CWT) to Improve the Performance of OFDM System

Mohamed H. M. Nerma, Nidal S. Kamel and Varun Jeoti Jagadish

***Abstract*— As demand for higher data rates is continuously rising, there is always a need to develop more efficient wireless communication systems. The work described in this paper is an effort in this direction. We have proposed a novel orthogonal frequency division multiplexing (OFDM) system base on dual – tree complex wavelet transform (DT-CWT). In the proposed scheme, DT-CWT is used in the place of fast Fourier transform (FFT). The proposed scheme offers the best peak – to – average power ratio (PAPR) performance than the conventional OFDM and wavelet packet modulation (WPM) systems at the expense of acceptable computational complexity without using any pruning techniques. The complementary cumulative distribution function (CCDF) of PAPR for the proposed scheme signal achieves about 3 dB improvement in PAPR over the traditional OFDM and WPM signals at 0.1% of CCDF. Also the proposed scheme achieves excellent improvements in bit error rate (BER) over conventional OFDM and WPM systems. The need for cyclic prefix (CP) is eliminated in the system design due to the good orthogonality and time frequency localization proprieties of the wavelet.**

***Index Terms*— OFDM; WPT; CWT; DT-CWT; FFT; Multicarrier Modulation; BER; PAPR.**

I. INTRODUCTION

OFDM and WPM have emerged as an efficient multicarrier modulation schemes for wireless, frequency selective, communication channels. Ease of implementation, high spectral efficiency, resilience to impulse noise and multipath are a few advantages of OFDM and WPM systems. However, a major drawback in the signals of these two systems are their large envelope function, which limits the efficiency of the non-linear power amplifiers specific to wireless communications by forcing them to operate at lower average power. This problem is quantified by the PAPR and results from the superposition of a large number of usually statistically independent sub-channels that can constructively sum up to high peaks. Also from the central limit theorem (CLT) [1], this causes the OFDM and WPM signals to have complex Gaussian process behavior and the instantaneous power is chi-square distributed. Various schemes have been developed to reduce high PAPR in OFDM [2] [3], and WPM signals [4].

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A. *Wavelet Modulation*

Wavelet Transform (WT) is a relatively new transform compared to the discrete Fourier transform (DFT). WT provides the time-frequency representation of signals, whereas DFT gives only the frequency representation. The properties of wavelet, such as localization in time and frequency, orthogonality across scale and translation presents to a new perspective in digital communication. It is used as a modulation technique in many communication fields including multicarrier modulation (MCM) and wireless communication [6]. The conventional multicarrier systems are DFT based systems. In MCM, the broadband channel splits into larger number of sub-channels. The high data rate bit-streams are divided into parallel sub streams with lower data rate [7].

However, DFT based MCM systems suffer from the high side lobes due to rectangular shaped DFT window and also these systems waste precious bandwidth due to the redundant CP. Moreover, the pulse shaping function used to modulate each subcarrier extends to infinity in the frequency domain this leads to high interference and lower performance levels. WT based MCM systems, namely wavelet based OFDM systems (WOFDM) can help to mitigate these problems. Therefore, conventional DFT based orthogonal systems are being replaced by wavelet based transceivers. The wavelet based transceiver uses quadrature mirror filters (QMF) in the synthesis and analysis filter banks (FBs). The wavelet filters posses the advantages of having greater side-lobe attenuation and requires no CP [8]. Wavelet packet transform (WPT) modulation in wireless communication has been proposed in [9]. The characteristics of multicarrier modulated signal are dependent on the basis functions being used in a modulation scheme. Therefore, using WPT as basis functions, the sensitivity to multipath channel distortion, inter symbol interference (ISI), inter carrier interference (ICI), and synchronization can be reduced as compared to traditional OFDM and the system performance of WOFDM with reference to ISI, ICI and signal-to-noise ratio (SNR) is shown to be far better than the conventional OFDM (DFT-OFDM) [9], [10].

B. *Peak – to – average power ratio (PAPR)*

The PAPR of the baseband transmitted signal $x(t)$ is defined as the ratio of the peak power ($P_{peak} = \max\{|x(t)|^2\}$); i.e., the maximum power of the transmitted signal over the average power ($P_{ave} = E\{|x(t)|^2\}$). In digital implementations of communications transceivers, rather than using the continuous time signal $x(t)$ in PAPR computation,

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we instead work with $x[n]$, the discrete time samples of *x(t)*, provided that an oversampling factor of at least 4 is used. PAPR is then expressed as [11]:

$$PAPR = \frac{\max\{|x[n]^2|\}}{\mathrm{E}\{|x[n]^2|\}} \tag{1}$$

where $E\{.\}$ denotes ensemble average calculated over the duration of the OFDM or WPM symbols.

In both OFDM and WPM systems, the signal going into the channel $x(t)$ is a sum of random symbols modulating orthogonal basis functions. Based on the CLT, it is claimed that $x(t)$ is complex Gaussian and its envelope follows a Rayleigh distribution. This implies a large PAPR. A high PAPR of the transmitted signals demands a very linear transmission path and limits the practical deployment of low-cost non-linear power amplifiers forcing them to operate with reduced power efficiency. Driving amplifiers operating close to saturation with signals of high PAPR results in the generation of unwanted spectral energy both in-band and out-of-band, which in turn reduces the systems performance and gives rise to adjacent channel interference (ACI).

In this work, the performance of the OFDM based on DT-CWT in PAPR reduction is demonstrated through the CCDF of PAPR, which is a performance metric independent of the transmitter amplifier. Given the reference level $PAPR_0 > 0$, the probability of a PAPR being higher than the reference value is the CCDF and is expressed as follows [12]:

$$CCDF(PAPR_0) = P_r\{PAPR > PAPR_0\} \tag{2}$$

To reduce the PAPR in OFDM and WPM systems, several techniques have been proposed, which basically can be divided in three categories. First, there are signal distortion techniques, which reduce the peak amplitudes simply by nonlinearly distorting the OFDM signal at or around the peaks. Examples of distortion techniques are clipping, peak windowing and peak cancellation. The second category is coding techniques that use a special forward-error correcting code set that excludes OFDM symbols with a large PAPR. The third technique is based on scrambling each OFDM symbol with different scrambling sequences and selecting that sequence that gives the smallest PAPR [13].

This paper is organized as follows: In section II we discuss the DT-CWT; in section III we discuss the half-sample delay condition; in section IV we make a comparisons between traditional OFDM and WPM systems; in section V we discuss the OFDM based on DT-CWT; in section VI we discuss the PAPR in OFDM based on DT-CWT; the BER results are presented in section VII; and we conclude this paper in section VIII by discussion the simulation results.

II. THE DUAL –TREE COMPLEX WAVELET TRANSFORM (DT-CWT)

Since the early 1990s the WT and WPT have received more and more attention in modern communications and have been widely used in wireless communication [14]. A number of modulation schemes based on wavelets have been proposed [15 - 23]. In fact, complex wavelet transform (CWT) is applied perfectly to digital image processing. Kingsbury [24 - 28] introduced the DT-CWT. The DT-CWT employs two real discrete WT (DWT); the upper part of the FB gives the real part of the transform while the lower one gives the imaginary part. This transform uses the pair of the filters ($h_0(n)$, $h_1(n)$ the low-pass/high-pass filter pair for the upper FB respectively) and ($g_0(n)$, $g_1(n)$ the low-pass/high-pass filter pair for the lower FB respectively) that are used to define the sequence of wavelet function $\psi(t)$ and scaling function $\phi(t)$ as follows

$$\psi_h(t) = \sqrt{2}\sum_n h_1(n)\phi_h(2t-n) \tag{3}$$

$$\phi_h(t) = \sqrt{2}\sum_n h_0(n)\phi_h(2t-n) \tag{4}$$

Where $h_1(n) = (-1)^n h_0(d-n)$, the wavelet function $\psi_g(t)$, the scaling function $\phi_g(t)$ and the high-pass filter for the imaginary part $g_1(n)$ are defined similarly. The two real wavelets associated with each of the two real transform are $\psi_h(t)$ and $\psi_g(t)$. To satisfying the perfect reconstruction (PR) conditions, the filters are designed so that the complex wavelet $\psi(t) \coloneqq \psi_h(t) + j\psi_g(t)$ is approximately analytic. Equivalently, they are designed so that $\psi_g(t)$ is approximately the Hilbert transform of $\psi_h(t)$.

$$\psi_g(t) = H\{\psi_h(t)\} \tag{5}$$

The analysis (decomposition or demodulation) and the synthesis (reconstruction or modulation) FBs used to implement the DT-CWT and their inverses are illustrated in fig. 1 and fig. 2 respectively. The inverse of DT-CWT is as simple as the forward transform. To invert the transform, the real part and the imaginary part are each inverted.

III. THE HALF-SAMPLE DELAY CONDITION

The two low pass filters should satisfy a very simple property: one of them should be approximately a half-sample shift of the other [29]

$$g_0(n) \approx h_0(n-0.5) \Longrightarrow \psi_g(t) \approx H\{\psi_h(t)\} \tag{6}$$

Since $g_0(n)$ and $h_0(n)$ are defined only on the integers, this statement is somewhat informal. However, we can make the statement rigorous using Fourier transform (FT). In [5] it is shown that, if $G_0(e^{j\omega}) = e^{-j0.5\omega}H_0(e^{j\omega})$, then $\psi_g(t) = H\{\psi_h(t)\}$.

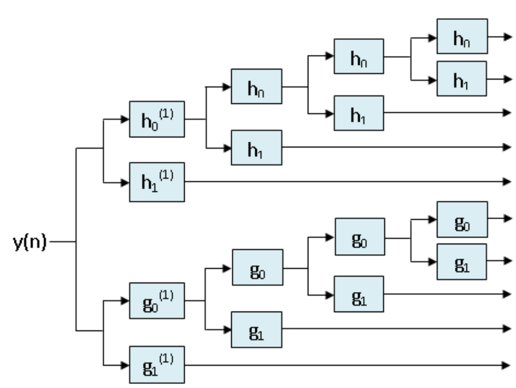

Figure 1. The dual tree discrete CWT (DT-DCWT) Analysis (demodulation) FB.

The converse has been proven in [30] [31], making the condition necessary and sufficient. The necessary and sufficient conditions for biorthogonal case were proven in [32]. We can rewrite the half-sample delay condition in terms of magnitude and phase function separately as shown in (7) and (8) as follows.

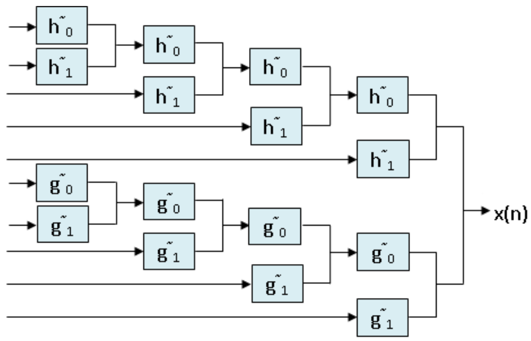

Figure 2. The Inverse dual tree discrete CWT (IDT-DCWT) Synthesis (modulation) FB.

$$\left|G_0\left(e^{j\omega}\right)\right| = \left|H_0\left(e^{j\omega}\right)\right| \tag{7}$$

$$\angle\left(e^{j\omega}\right) = \angle H_0\left(e^{j\omega}\right) - 0.5\omega \tag{8}$$

In practical implementation of the DT-CWT, the delay condition (7) and (8) will be satisfied only approximately; the wavelets $\psi_h(t)$ and $\psi_g(t)$ will form only an approximate Hilbert pair; and the complex wavelet $\psi_h(t) + j\psi_g(t)$ will be only approximately analytic. While the FT is based on complex valued oscillating cosine and sine components form a complete Hilbert transform pairs; i.e., they are 90^{o} out of phase with each other. Together they constitute an analytic signal $e^{j\theta t}$ that is supported on only one-half of the frequency axis $(\theta > 0)$ [24].

A. Filter design for DT-CWT

There are various approaches to the design of filters for the DT-CWT, such as linear-phase biorthogonal method, quarter shift method, and common factor method. These filters are satisfied the following desired properties:

1- Approximately half-sample delay property.
2- PR (orthogonal or biorthogonal).
3- Finite support (finite impulse response (FIR) filters).
4- Vanishing moments/good stop-band.
5- Linear phase filters.

It turns out that the implementation of the DT-CWT requires that the first stage of the dual-tree (DT) FB requires a different set of filters from the succeeding stages, and the succeeding stages can be used same sets of filters. See fig. (1). If the same PR filters are used for each stage, then the first several stages of the FB will not be approximately analytic [24].

IV. OFDM AND WPM SYSTEMS

Traditionally, OFDM is implemented using FFT. This transform however has the drawback that is uses a rectangular window, which creates rather high sidelobes. Moreover, the pulse shaping function used to modulate each subcarrier extends to infinity in the frequency domain this leads to high interference and lower performance levels. ISI and ICI can be avoided by adding a CP to the head of OFDM symbol. Adding CP can largely reduce the spectrum efficiency. The WPM system has a higher spectral efficiency and providing robustness with regard to inter-channel interference than the conventional OFDM system, because of the out-of-band energy (low sidelobes). Moreover WPM is able to decompose time – frequency plane flexibly by arranging FB constructions. In addition, using FFT in the traditional OFDM gives resolution only in the frequency domain while using WPT in WPM system gives resolution in both frequency domain and time domain [16].

WPM system do not required CP, thereby enhancing the spectrum efficiency. According to the IEEE broadband wireless standard 802.16.3, avoiding CP gives wavelet OFDM an advantage of roughly 20% in bandwidth efficiency. Moreover as pilot tones are not necessary for wavelet based OFDM system, they perform better in comparison to existing OFDM systems like 802.11a or HiperLAN, where 4 out of 52 sub-bands are used for pilots. This gives wavelet based OFDM system another 8% advantage over typical OFDM implementations [35]. We expected the OFDM based on DT-CWT will take all the advantages of WPM system.

However, a major problem of the common discrete WPT (DWPT) is its lack of shift invariance; this means that on shifts of the input signal, the wavelet coefficients vary substantially. The signal information may even not be stationary in the sub-bands so that the energy distribution across the sub-bands may change [15]. To overcome the problem of shift dependence, one possible approach is to simply omit the sub-sampling causing the shift dependence. Techniques that omit or partially omit sub-sampling are also known as cycle spinning, oversampled FBs or undecimated WT. However, these transforms are redundant [33], which is not desirable in multicarrier modulation. As an alternative, we used a non-redundant WT that achieves approximately shift invariance [34], this transform yields to complex wavelet coefficients that modulate the data stream in the same way that WPM do.

V. OFDM BASED ON DT-CWT

In this study, simulations are focusing on using DT-CWT in the OFDM system as a non-redundant WT that can achieves approximately shift invariance. Similar to the conventional OFDM and WPM systems, a functional block diagram of OFDM based on DT-CWT is shown in fig. (3). At the transmitter an inverse DT-CWT (IDT-CWT) block is used in place of inverse FFT (IFFT) block in conventional OFDM system or in place of inverse DWPT (IDWPT) block in WPM system. At the receiver side a DT-CWT is used in place of FFT block in conventional OFDM system or in place of DWPT block in WPM system. Data to be transmitted are typically in the form of a serial data stream. PSK or QAM modulations can be implemented in the proposed system the choice depends on various factors like the bit rate and sensitivity to errors. The transmitter accepts modulated data (in this paper we use 16 QAM). This stream is passed through a serial to parallel (S/P) converter, giving N lower bit rate data stream, and then this stream is modulated through an IDT-CWT matrix realized by an N-band synthesis FB. Before the receiver can demodulate the subcarriers, it has to perform the synchronization. For the proposed system, known data interleaved among unknown data are used for channel estimation. Then, the signal is down sampled by N and demodulated using elements of the DT-CWT matrix realized by an N-band analysis FB. The signal is equalized after DT-CWT stage.

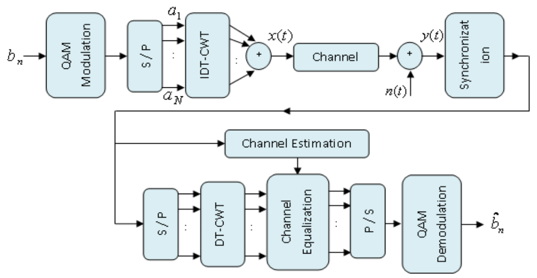

Figure 3. DT-CWT modulation (DT-CWTM) functional block diagram.

IDT-CWT works in a similar fashion to an IFFT or IDWPT. It takes as the input QAM symbols and outputs them in parallel time-frequency "subcarriers". In fig. (2) as the synthesis process, it can be shown that the transmitted signal, $x[n]$ is constructed as the sum of M waveform $\psi[k]$ individually modulated with the QAM or PSK symbols [34] as follows:

$$x[n] = \sum_{i} \sum_{j=0}^{M-1} a_{i,j}\, \psi_j[n - iM] \tag{9}$$

Where $a_{i,j}$ is a constellation encoded $i-th$ data symbol modulating the $j-th$ DT-CWT function. The IDT-CWT synthesis a discrete representation of the transmitted signal as sum of M waveforms shifted in time that embed information about data symbols. Those waveforms are built by j successive iterations of $\hat{h_0}, \hat{h_1}$ and $\hat{g_0}, \hat{g_1}$. The DT-CWT at the receiver recovers the transmitted symbols $a_{i,j}$ through the analysis formula exploiting orthogonality properties of DT-CWT and schematically represented in fig. (1).

In the baseband equivalent OFDM transmitter with mth frame of N QAM symbols, a_k^m , $k = 0,1,\dots,N-1$, the OFDM frame is given by:

$$x^m[n] = \sum_{k=0}^{N-1} a_k^m\, e^{j2\pi nk/N} \tag{10}$$

While for WPM system, the transmitted signal $x[n]$ is constructed as the sum of M wavelet packet function $\varphi_j[n]$ individually modulated with the QAM symbols as the case in this paper‡.

$$x[n] = \sum_{i} \sum_{j=0}^{M-1} a_{i,j}\, \varphi_j[n - iM] \tag{11}$$

The construction of discrete versions of transmitted waveforms for the conventional OFDM and WPM systems using (10) and (11) is quite similar. For any time index n, both waveforms are sum of random symbols a_k or $a_{i,j}$.

In order to achieve fair comparisons, same simulation parameters are used.

‡ We use $\varphi_j[n]$ for the wavelet packet function and $\psi_j[n]$ for the DT-CWT function to avoid any confusion.

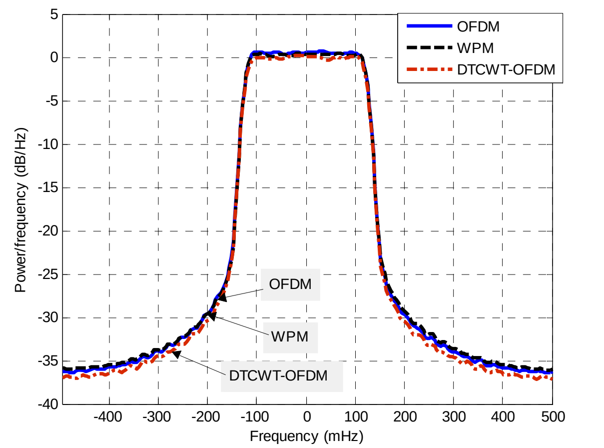

Figure 4. PSD of the Conventional OFDM, WPM and OFDM based on DT-CWT.

The simulations were carried out for conventional OFDM with a 64 subcarriers using a 16 QAM modulation. The Daubechies-1 (Daub-1) wavelet packet bases were used to construct the wavelet packet trees in WPM system. For the proposed scheme, near symmetric 13,19 tap filters and quarter sample shift orthogonal 14 tap filters were used to construct the real and the imaginary part of DT-CWT respectively. From the CLT, x[n] is complex Gaussian distributed, and the sequence x[n] has high PAPR. Furthermore, to demonstrate the similarities between power spectrum density (PSD) characteristic of conventional OFDM, WPM and the proposed scheme, the simulated PSD characteristics are presented in fig. 4. This fig. was shows that the proposed scheme performs better than the other two systems (more low out of band attenuation).

The simulation parameters are documented as follows: Modulation type is 16-QAM; the number of subcarriers are 64, 128, 256, 512, and 1024; a wavelet packet base is Daubechies-1 (DAUB-1); maximum tree depth (D = 7); PAPR threshold is 2dB; shaping filter is Raised Cosine (rolloff factor $\propto = 0.001$, upsampler = 4); DT-CWT using different filters (LeGall 5,3 tap filters (leg), Antonini 9,7 tap filters (anto), Near Symmetric 5,7 tap filters (n-sym-a), and Near Symmetric 13,19 tap filters (n-sym-b)) for the first stage of the FB and (Quarter Sample Shift Orthogonal 10,10 tap filters (q-sh-06) only 6,6 non-zero taps, Quarter Sample Shift Orthogonal 10,10 tap filters (q-sh-a) with 10,10 non-zero taps, unlike q-sh-06, Quarter Sample Shift Orthogonal 14,14 tap filters (q-sh-b), Quarter Sample Shift Orthogonal 16,16 tap filters (q-sh-c), and Quarter Sample Shift Orthogonal 18,18 tap filters (q-sh-d)) for the succeeding stages of the FB.

VI. PAPR ANALYSIS RESULTS

Consider the simple case where all the sub-symbols are independently and identically distributed (i.i.d). then, by the CLT, the real and imaginary parts of the N-point IFFT output have mutually independent Gaussian probability distribution function with zero mean and standard deviation to σ. The instantaneous power of baseband signal $x[n]$, is defined by

$$\lambda = Re\{x[n]\}^2 + Im\{x[n]\}^2 \tag{12}$$

We can characterize the instantaneous power as a chi-square distribution with two degrees of freedom [31]

$$f(\lambda) = \frac{1}{\sigma^2} e^{\left(-\frac{\lambda}{\sigma^2}\right)}, \lambda \geq 0 \tag{13}$$

If E $\{|x[n]^2|\}$ is normalized to unity, then the CCDF of the PAPR is given by:

$$\Pr[\lambda > \lambda_0] = 1 - \left(1 - e^{\left(-\frac{\lambda_0}{\sigma^2}\right)}\right)^N \tag{14}$$

Where N is the number of subcarriers. In order to analyze PAPR, we generate the transmitted waveforms using 16 QAM modulation with 64 subcarriers for all these systems. Fig. 5 shows, in the time domain, the envelope of the proposed system. For the comparison, we also plotted the envelope of the conventional OFDM and WPM waveforms corresponding to the same information symbol pattern. The transmitted envelopes for the conventional OFDM and WPM systems illustrate approximately similar behavior, where the peak is about 2.25, while the transmitted envelope for the proposed system demonstrates better behavior than the other two systems, where the peak is only about 1.25 and this is the reason for that the proposed system gives better result for PAPR than the other two systems.

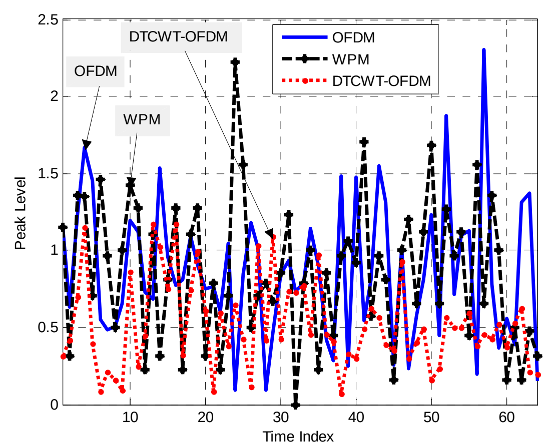

Figure 5. The Envelope of the Conventional OFDM, WPM and OFDM based on DT-CWT.

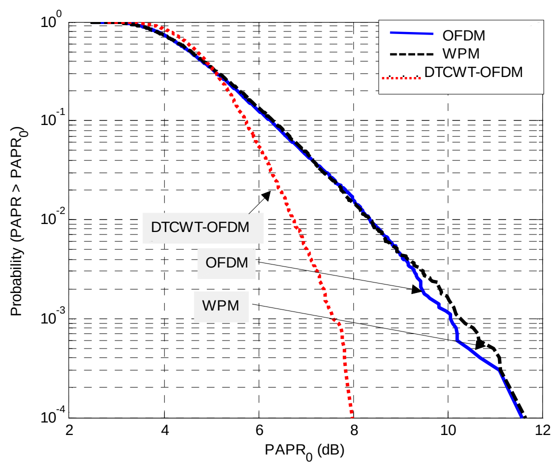

Figure 6. CCDF the Conventional OFDM, WPM and OFDM based on DT-CWT.

The results for PAPR are best quantified using CCDF. In fig. 6, for 64 subcarriers with 16 QAM modulation, the CCDF plots for the proposed system, conventional OFDM and WPM system are shown. This figure shows that the OFDM based on DT-CWT offers the best PAPR performance without using any reduction techniques. The proposed scheme signal achieves about 3 dB improvement in PAPR over the traditional OFDM and WPM signals at 0.1% of CCDF while the other two systems, are approximately given same results of PAPR.

The simulation results for PAPR were also repeated with 16 QAM modulation and 64 subcarriers (in fig. 7) using different set of filters. OFDM DT-CWT_1 illustrate the PAPR when using near-symmetric (n-sym) 13,19 tap filters in the first stage of the FB and quarter sample shift orthogonal (q-sh) 14 tap filters in the succeeding stages, OFDM DT-CWT_2 using (n-sym 13,19 with q-sha 10 (10 non zero taps) filters), OFDM DT-CWT_3 using (antonini (anto) 9,7 tap filters with q-sh0 10 (only 6 non zero taps) filters), OFDM DT-CWT_4 using (anto 9,7 with q-sh 14 filters), OFDM DT-CWT_5 using (n-sym 5,7 with q-sh 14 filters), OFDM DT-CWT_6 using (LeGall (leg) 5,3 tap filters with q-sh 14 filters), OFDM DT-CWT_7 using (n-sym 5,7 with q-sh 16 filters) and OFDM DT-CWT_8 using (leg 5,3 with q-sh 18 filters).

The results in fig. 7 show that there is no observed degradation as a result of using different set of mismatching filters in the design of the proposed scheme.

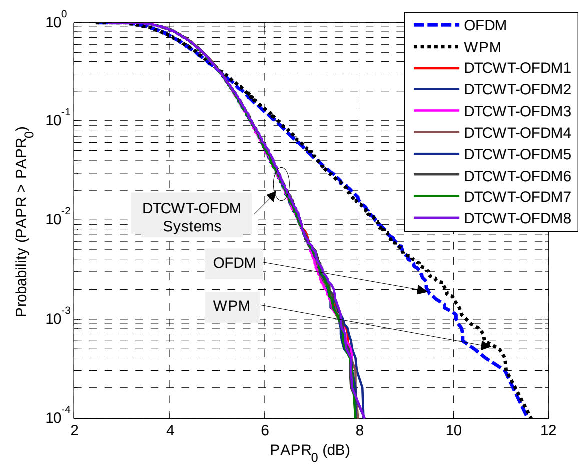

Figure 7. The effect of using different set of filters in design of the OFDM based on DT-CWT.

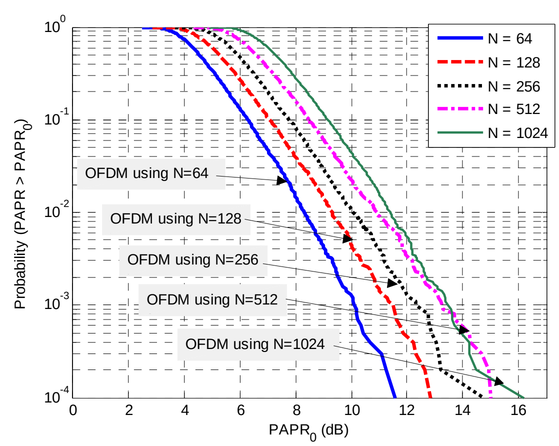

Figure 8. CCDF of PAPR for 16-QAM modulated conventional OFDM symbol with various values of subcarriers (N).

Again, the results for PAPR were repeated in both fig. 8 and fig. 9 for the conventional OFDM system and for the proposed system, respectively, using different numbers of subcarriers (64, 128, 256, 512, and 1024) with 16QAM modulation. We observe from these figures that the PAPR increases as the number of subcarriers numbers (N) increases. As shown in fig. 6, 7 and in fig. 8, 9 the PAPR performance of DT-CWT based OFDM systems is better than the conventional OFDM system or even WPM systems. As we saw in figure 5 showing the time domain signal envelop of all the systems, this improvement in PAPR performance is explained from lower peaks of DTCWT systems observed in the fig. 5.

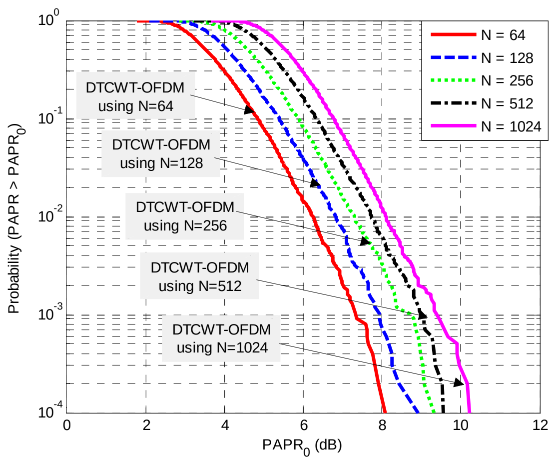

Figure 9. CCDF of PAPR for 16-QAM modulated OFDM based on DT-CWT symbol with various values of subcarriers (N).

VII. BER ANALYSIS RESULTS

The results given in this section compare the BER in the OFDM based on DT-CWT, with that for traditional OFDM, and WPM. Also, same simulation parameters are used to achieve a fair comparison. The results of BER in OFDM based on DT-CWT using different set of filters are also shown in this section.

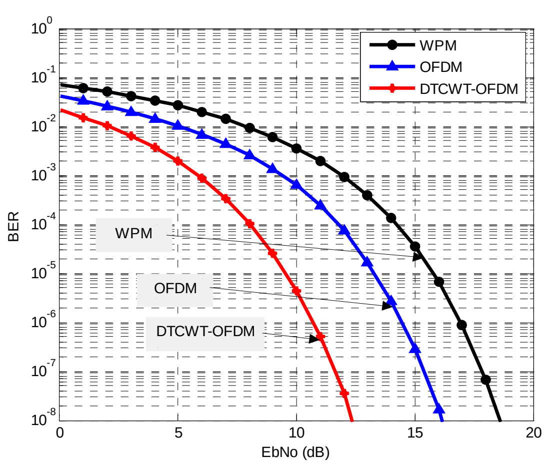

Figure 10. BER performance of conventional OFDM, WPM and OFDM based on DT-CWT.

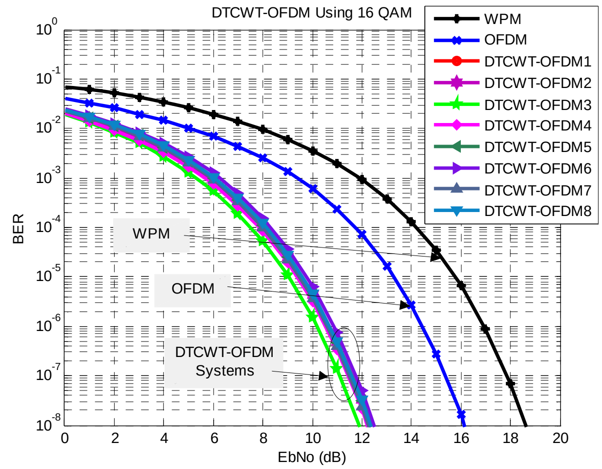

Figure 11. The effect of using different type of filters in BER for OFDM based on DT-CWT.

The results for BER are shown that the proposed scheme gives excellent improvements in BER over conventional OFDM and WPM systems. At the same time the conventional OFDM outperform the WPM system in term of BER. As shown in fig. 10.

The simulation results for BER were repeated using different set of filters (OFDM DT-CWT-1, OFDM DT-CWT-2, …, OFDM DT-CWT-8) and is shown in fig. 11. We see there is degradation of BER as a result of using different set of mismatching filters in the design of the proposed scheme.

VIII. CONCLUSION

In this paper a new OFDM scheme that is based on DT-CWT is proposed. Comparing the proposed scheme in terms of PAPR and BER with the traditional OFDM and WPM systems we see that the proposed scheme offers 3dB better PAPR performance over the conventional OFDM and WPM systems at 0.1% of CCDF. While the conventional OFDM and WPM systems shows similar behavior. Simulation results shows that there is no observed PAPR and BER degradation as a result of using different set of mismatching filters in DT-CWT based system. The proposed scheme outperforms the traditional OFDM and WPM systems in term of BER. Also we found that the conventional OFDM system gives better results of BER than WPM system.

APPENDIX

A. CCDF Approximations

We can characterize the instantaneous power as a chi-square distribution with two degrees of freedom [36]

$$f(\lambda) = \frac{1}{\sigma^2} e^{\left(-\frac{\lambda}{\sigma^2}\right)} \quad , \qquad \lambda \geq 0 \qquad (15)$$

As a result, the cumulative distribution function (CDF) is defined as:

$$Pr[\lambda < \lambda_0] = \int_0^{\lambda_0} f(\lambda)d\lambda = 1 - e^{(-\frac{\lambda_0}{\sigma^2})} \qquad (16)$$

If E $\{|x[n]^2|\}$ is normalized to unity, then the CCDF of the PAPR is given by:

$$Pr[\lambda > \lambda_0] = 1 - \left(1 - e^{(-\frac{\lambda_0}{\sigma^2})}\right)^N \qquad (17)$$

Where N is the number of subcarriers, however, this approximation is not close to the experimental results as the assumption made in deriving CCDF that the samples should be mutually uncorrelated is no longer valid when oversampling is employed [36]. There has been several attempts to determine the closed for approximations for the distribution of PAPR. Some of the approximations are shown in Table 1.

B. DT-CWT and IDT-CWT

If the two real DWTs are represented by the square matrices F_h for the upper part and F_g for the lower part, then the DT-CWT can be represented by the following form.

CCDF	Remarks
$Pr[\lambda > \lambda_0] \approx 1 - \left(1 - e^{(-\frac{\lambda_0}{\sigma^2})}\right)^{\propto N}$	$\propto$ = 2.8 and N ≥ 64 [36]
$Pr[\lambda > \lambda_0] \approx 1 - exp\left(-\sqrt{\frac{\pi\lambda_0}{3}} N\, e^{-\lambda_0}\right)$	[37]
$Pr[\lambda > \lambda_0] \approx 1 - exp\left(-\sqrt{\frac{\pi log N}{3}} N\, e^{-\lambda_0}\right)$	[38]
$Pr[\lambda > \lambda_0] \approx 1 - exp\left(-\frac{2N}{\sqrt{3}} e^{-\lambda_0/2}\right)$	N and $\boldsymbol{\lambda}$ are large [39]
$Pr[\lambda > \lambda_0] \approx 1 - \left(1 - erfc(\sqrt{\lambda_0/2})\right)^2 \left(1 - e^{-\lambda_0}\right)^{\frac{N-2}{2}}$	[40]

Table 1. Approximation to CCDF of PAPR.

$$F_c = \frac{1}{\sqrt{2}} \begin{bmatrix} I & jI \\ I & -jI \end{bmatrix} \cdot \begin{bmatrix} F_h \\ F_g \end{bmatrix} \qquad (18)$$

And the IDT-CWT is given by

$$F_c^{-1} = \frac{1}{\sqrt{2}} \left[F_h^{-1} \quad F_g^{-1}\right]^{-1} \cdot \begin{bmatrix} I & I \\ -jI & jI \end{bmatrix} \qquad (19)$$

Note that the complex sum/difference matrix in (18) is unitary (its conjugate transpose is its inverse).

$$\frac{1}{\sqrt{2}} \begin{bmatrix} I & jI \\ I & -jI \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} I & I \\ -jI & jI \end{bmatrix} = I \qquad (20)$$

Note that the identity matrix on the right-hand side of (20) is twice the size of those on the left-hand side. Therefore if the two real DWTs are orthonormal transforms, then the DT-CWT satisfies $F_c^*.F_c = I$, where * denotes the conjugate transpose.

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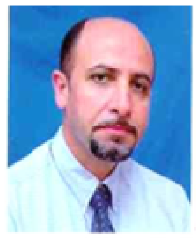

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