Free Vibration Analysis of Rotating Euler Beam by Finite Element Method

Kuo Mo Hsiao, Wen Yi Lin, and Fumio Fujii

***Abstract*—In this paper a corotational finite element formulation is employed to derive the equations of motion for a three dimensional rotating Euler beam with constant angular velocity. The steady state deformation and natural frequency of the infinitesimal free vibration measured from the position of the corresponding steady state deformation are investigated for rotating Euler beams with different setting angle. The governing equations for linear vibration are obtained by the first order Taylor series expansion of the equation of motion at the position of steady state deformation. Numerical examples are studied to demonstrate the accuracy of the proposed method and to investigate the effects of the steady twist deformation on the natural frequency of rotating beams with different setting angle.**

***Index Terms*—Rotating beam, Corotational formulation, Setting angle, Natural frequency.**

I. Introduction

Rotating beams are often used as a simple model for propellers, turbine blades, and satellite booms. Rotating beam differs from a non-rotating beam in having additional centrifugal force and Coriolis effects on its dynamics. It is well known that the spinning elastic bodies sustains a steady state deformations induced by constant rotation [1]. For doubly symmetric rotating beams with setting angle other than 0° and 90°, that steady state deformations include axial deformation and twist deformation. The bending vibration, torsional vibration, and axial vibration of rotating beams are coupled due to the Coriolis effects [2] and the steady state deformation [3]. However, to the authors' knowledge, the steady state deformation and its effects on the bending, torsional, and axial vibration of doubly symmetric rotating beams with setting angle other than 0° and 90° are not reported in the literature. The objective of this paper is to derive the equations of motion for a rotating doubly symmetric Euler beam with constant angular velocity using a corotational finite element formulation. The steady state deformation and natural frequency of the infinitesimal free vibration measured from the position of the corresponding steady state deformation are investigated using numerical examples.

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Kuo Mo Hsiao is with Department of Mechanical Engineering, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu, 300, Taiwan (phone: 886-3-5712121-55107; fax: 886-3-5720634; e-mail: kmhsiao@mail.nctu.edu.tw).

Wen Yi Lin is with Department of Mechanical Engineering, De Lin Institute of Technology, 1 Alley 380, Ching Yun Road, Tucheng, Taiwan (e-mail: wylin@dlit.edu.tw).

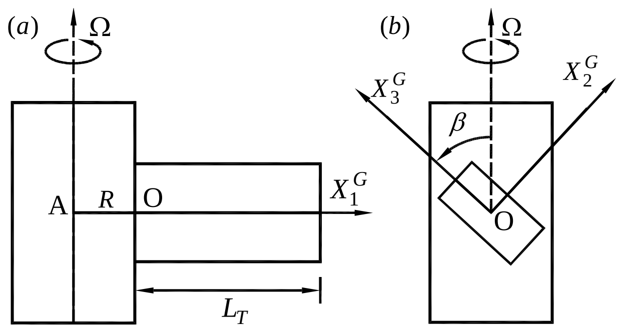
Fumio Fujii is with Department of Mathematical and Design Engineering, Gifu University, 1-1 Yanagido, Gifu City 501-1193, Japan (e-mail: ff@gifu-u.ac.jp).

II. Finite Element Formulation

A. Description of Problem

Consider a doubly symmetric uniform Euler beam of length L_T rigidly mounted with a setting angle β on the periphery of rigid hub with radius R rotating about its axis fixed in space at a constant angular velocity Ω as shown in Fig. 1. The axis of the rotating hub is perpendicular to the beam axis. The beam sustains a steady state axial and torsional deformation induced by constant rotation. In this study, large displacement and rotation with small strain are considered in the steady state deformation. The vibration of the beam is measured from the position of the steady state deformation, and only infinitesimal free vibration is considered. The kinematics of the beam element presented in [4] and the co-rotational finite element formulation proposed in [4, 5] are employed here. In the following only a brief description of the beam element is given.

Fig. 1. Rotating beam, (a) front view, (b) side view



B. Basic Assumptions

The following assumptions are made in derivation of the beam element behavior: (1) The beam is prismatic and slender, and the Euler-Bernoulli hypothesis is valid. (2) The cross section of the beam is doubly symmetric. (3) The unit extension of the centroid axis of the beam element is uniform. (4) The cross section of the beam element does not deform in its own plane and strains within this cross section can be neglected.

C. Coordinate Systems

In order to describe the system, we define three sets of right handed rectangular Cartesian coordinate systems:

1. A rotating global set of coordinates, X_i^G (i = 1, 2, 3)

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(see Figs. 1 and 2); the coordinates rotate about the hub axis at a constant angular speed Ω as shown in Fig. 1. The origin of this coordinate system is chosen to be the intersection of the hub and the centroid axis of the undeformed beam. The X_1^G axis are chosen to coincide with the centroid axis of the undeformed beam, and the X_2^G and X_3^G axes are chosen to be the principal directions of the cross section of the beam at the undeformed state. The nodal coordinates, nodal deformation displacements, nodal velocity, nodal acceleration, and equations of motion of the system are defined in this coordinates.

2. Element cross section coordinates, x_i^S (i = 1, 2, 3) (see Fig. 2); a set of element cross section coordinates is associated with each cross section of the beam element. The origin of this coordinate system is rigidly tied to the centroid of the cross section. The x_1^S axes are chosen to coincide with the normal of the unwrapped cross section and the x_2^S and x_3^S axes are chosen to be the principal directions of the cross section.

3. Element coordinates, x_i ($i = 1, 2, 3$) (see Fig. 2); a set of element coordinates is associated with each element, which is constructed at the current configuration of the beam element. The coordinates rotate about the hub axis at a constant angular speed Ω. The origin of this coordinate system is located at node 1, and the x_1 axis is chosen to pass through two end nodes of the element; the x_2 and x_3 axes are determined by the method proposed in [6]. The position vector, deformations, velocity, acceleration, internal nodal forces, stiffness matrices, and inertia matrices of the elements are defined in terms of these coordinates

D. Kinematics of Beam Element

In this study only the doubly symmetric cross section is considered. Let Q (Fig. 2) be an arbitrary point in the beam element, and P be the point corresponding to Q on the centroid axis. The position vector of point Q in the undeformed and deformed configurations may be expressed as [4]:

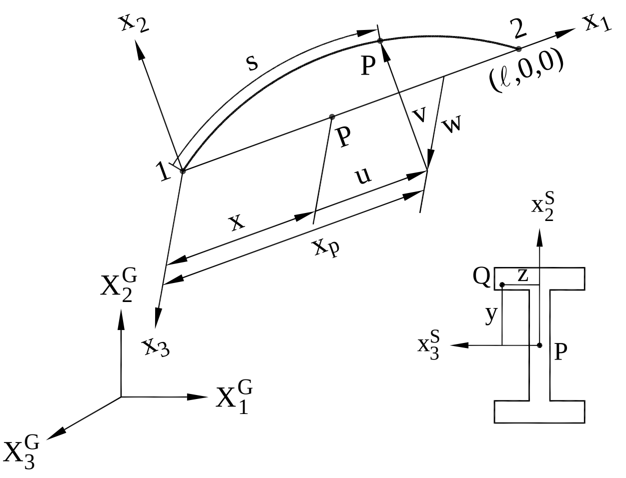

Fig. 2. Coordinate systems

$$\mathbf{r}_0 = x\mathbf{e}_1 + y\mathbf{e}_2 + z\mathbf{e}_3 \quad (1)$$

$$\mathbf{r} = x_p(x,t)\mathbf{e}_1 + v(x,t)\mathbf{e}_2 + w(x,t)\mathbf{e}_3 + \theta_{1,x}\omega\mathbf{e}_1^S + y\mathbf{e}_2^S + z\mathbf{e}_3^S \quad (2)$$

where $x_p(x,t)$, $v(x,t)$ and $w(x,t)$ are the x_1, x_2 and x_3 coordinates of point P, respectively, in the deformed configuration, $\theta_{1,x} = \theta_{1,x}(x,t)$ is the twist rate of the deformed centroid axis, $\omega(y,z)$ is the Saint Venant warping function for a prismatic beam of the same cross section, and $\mathbf{e}_i$ and $\mathbf{e}_i^S$ (i = 1, 2, 3) denote the unit vectors associated with the x_i and x_i^S axes, respectively. Note that $\mathbf{e}_i$ and $\mathbf{e}_i^S$ are coincident in the undeformed state. The relationship between $\mathbf{e}_i$ and $\mathbf{e}_i^S$ is given in [6] and not repeated here. Here, the lateral deflections of the centroid axis, $v(x,t)$ and $w(x,t)$, and the rotation about the centroid axis, $\theta_{1,x}$, are assumed to be the Hermitian polynomials of x.

The relationship among $x_p(x,t)$, $v(x,t)$, and $w(x,t)$, and x may be given as [5]

$$x_p(x,t) = u_1 + \int_0^x [(1+\varepsilon_c)^2 - v_{,x}^2 - w_{,x}^2]^{1/2} dx \quad (3)$$

where u_1 is the displacement of node 1 in the x_1 direction. Note that due to the definition of the element coordinate system, the value of u_1 is equal to zero. However, the variation and time derivatives of u_1 are not zero.

Making use of the assumption of uniform unit extension, ε_c and the axial displacements of the centroid axis may be calculated using (3) and the current chord length of the beam element.

The absolute velocity and acceleration vectors of point Q in the beam element may be expressed as

$$\mathbf{a} = \mathbf{a}_o + \dot{\mathbf{\Omega}} \times \mathbf{r} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}) + 2\mathbf{\Omega} \times \dot{\mathbf{r}} + \ddot{\mathbf{r}} \quad (4)$$

$$\mathbf{a}_o = \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{Ao}) \quad (5)$$

$$\mathbf{\Omega} = \mathbf{A}_{GE}^t \mathbf{\Omega}_G \quad (6)$$

$$\mathbf{r}_{Ao} = \mathbf{A}_{GE}^t \mathbf{r}_{AoG} \quad (7)$$

$$\mathbf{r}_{AoG} = \{R + X_o, 0, 0\} \quad (8)$$

where $\mathbf{r}$ is the position vector of point Q given in (2) referred to the current moving element coordinate system, the symbol $\dot{(\ )}$ denotes time derivative, $\mathbf{\Omega}$ is the vector of angular velocity referred to the current inertia element coordinates, $\mathbf{\Omega}_G = \Omega\{0 \quad \sin\beta \quad \cos\beta\}$ is the he angular velocity of the hub referred to the global coordinates, $\mathbf{A}_{GE}$ is the transformation matrix between the current global coordinates and the current element coordinates, $\mathbf{a}_o$ is the absolute acceleration of point o, the origin of the current element coordinates, X_o is coordinates of point o referred to the current global coordinates, R is the radius of the hub. $\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})$ and $2\mathbf{\Omega} \times \dot{\mathbf{r}}$ are centripetal acceleration and Coriolis acceleration, respectively. $\dot{\mathbf{r}}$ and $\ddot{\mathbf{r}}$ are the velocity and acceleration of point Q relative to the current moving element coordinates.

E. Element Nodal Force Vector, and Element Matrices

The element proposed here has two nodes with seven degrees of freedom per node. The nodal parameters are chosen to be u_{ij} ($u_{1j} = u_j$, $u_{2j} = v_j$, $u_{3j} = w_j$), the x_i (i = 1, 2, 3) components of the translation vectors $\mathbf{u}_j$ at node j (j = 1, 2), ϕ_{ij}, the x_i (i = 1, 2, 3) components of the rotation vectors $\boldsymbol{\phi}_j$ at node j (j = 1, 2), and β_j, the twist rate of the centroid axis at node j. Here, the values of $\boldsymbol{\phi}_j$ are reset to zero at current configuration. Thus, $\delta\phi_{ij}$, the variation of ϕ_{ij}, represents infinitesimal rotations about the x_i axes [6], and the generalized nodal forces corresponding to $\delta\phi_{ij}$ are m_{ij}, the conventional moments about the x_i axes. The generalized nodal forces corresponding to δu_{ij}, the variations of u_{ij}, are f_{ij}, the forces in the x_i directions. The generalized nodal forces corresponding to $\delta\beta_j$, the variations of β_j, are bimoment B_j.

The element nodal force vector is obtained from the virtual work principle and the d'Alembert principle in the current element coordinates. The virtual work principle requires that

$$\begin{aligned}&\delta W_{ext} = \delta\mathbf{q}^t\mathbf{f} = \delta W_{int}\\ &= \int_V (\sigma_{11}\delta\varepsilon_{11} + 2\sigma_{12}\delta\varepsilon_{12} + 2\sigma_{13}\delta\varepsilon_{13} + \rho\delta\mathbf{r}^t\mathbf{a})dV = \delta\mathbf{q}_\theta^t\mathbf{f}_\theta\end{aligned} \tag{9}$$

$$\mathbf{f} = \mathbf{f}^D + \mathbf{f}^I = \{\mathbf{f}_1, \mathbf{m}_1, \mathbf{f}_2, \mathbf{m}_2, \mathbf{B}\} \tag{10}$$

$$\mathbf{f}_\theta = \mathbf{f}_\theta^D + \mathbf{f}_\theta^I = \{\mathbf{f}_1^\theta, \mathbf{m}_1^\theta, \mathbf{f}_2^\theta, \mathbf{m}_2^\theta, \mathbf{B}\}$$

$$\begin{aligned}&\delta\mathbf{q}_\theta = \{\delta\mathbf{u}_1, \delta\boldsymbol{\theta}_1^*, \delta\mathbf{u}_2, \delta\boldsymbol{\theta}_2^*, \delta\boldsymbol{\beta}\}\\ &\delta\mathbf{q} = \{\delta\mathbf{u}_1, \delta\boldsymbol{\phi}_1, \delta\mathbf{u}_2, \delta\boldsymbol{\phi}_2, \delta\boldsymbol{\beta}\}\end{aligned} \tag{11}$$

$$\varepsilon_{11} = \frac{1}{2}(\mathbf{r}_{,x}^t\mathbf{r}_{,x} - 1),\ \varepsilon_{12} = \frac{1}{2}\mathbf{r}_{,x}^t\mathbf{r}_{,y}.\ \varepsilon_{13} = \frac{1}{2}\mathbf{r}_{,x}^t\mathbf{r}_{,z}. \tag{12}$$

where $\delta\mathbf{u}_j = \{\delta u_j, \delta v_j, \delta w_j\}$, $\delta\boldsymbol{\phi}_j = \{\delta\phi_{1j}, \delta\phi_{2j}, \delta\phi_{3j}\}$, $\delta\boldsymbol{\theta}_j^* = \{\delta\theta_{1j}, -\delta w_j', \delta v_j'\}$, $\mathbf{f}_j = \{f_{1j}, f_{2j}, f_{3j}\}$, $\mathbf{m}_j = \{m_{1j}, m_{2j}, m_{3j}\}$, $\mathbf{f}_j^\theta = \{f_{1j}^\theta, f_{2j}^\theta, f_{3j}^\theta\}$, $\mathbf{m}_j^\theta = \{m_{1j}^\theta, m_{2j}^\theta, m_{3j}^\theta\}$ (j = 1, 2), $\delta\boldsymbol{\beta} = \{\delta\beta_1, \delta\beta_2\}$ and $\mathbf{B} = \{B_1, B_2\}$. $\mathbf{f}^D$ and $\mathbf{f}^I$ are element deformation nodal force vector and inertia nodal force vector, respectively. V is the volume of the undeformed beam element, $\delta\varepsilon_{1i}$ (i = 1, 2, 3) are the variation of ε_{1i} in (12) corresponding to $\delta\mathbf{q}_\theta$. Note that because $\delta\varepsilon_{1i}$ are function of $\delta\mathbf{q}_\theta$, δW_{int} may be expressed by $\delta\mathbf{q}_\theta^t\mathbf{f}_\theta$. $\mathbf{f}_\theta^D$ and $\mathbf{f}_\theta^I$ are generalized deformation nodal force vector and inertia nodal force vector corresponding to $\delta\mathbf{q}_\theta$. σ_{1i} (i = 1, 2, 3) are the second Piola-Kirchhoff stress. For linear elastic material, $\sigma_{11} = E\varepsilon_{11}$, $\sigma_{12} = 2G\varepsilon_{12}$, and $\sigma_{13} = 2G\varepsilon_{13}$, where E is Young's modulus and G is the shear modulus. ρ is the density, $\delta\mathbf{r}$ is the variation of $\mathbf{r}$ in (2). $\mathbf{a}$ is the absolute acceleration given in (4). The higher order terms of nodal parameters in the element nodal forces are neglected by consistent second order linearization in this study.

The relation between $\delta\mathbf{q}$ and $\delta\mathbf{q}_\theta$, and the relation between $\mathbf{f}$ and $\mathbf{f}_\theta$ may be expressed as [4]

$$\delta\mathbf{q}_\theta = \mathbf{T}_{\theta\phi}\delta\mathbf{q},\quad \mathbf{f} = \mathbf{T}_{\theta\phi}^t\mathbf{f}_\theta \tag{13}$$

where $\mathbf{f}_\theta$ may be calculated using (2-12).

The element matrices considered are element tangent stiffness matrix $\mathbf{k}$, mass matrix $\mathbf{m}$, centripetal stiffness matrix $\mathbf{k}_\Omega$, and gyroscopic matrix $\mathbf{c}$. The element matrices may be obtained by differentiating the element nodal force vectors in (13) with respect to nodal parameters, and time derivatives of nodal parameters, The element matrices may be expressed as

$$\mathbf{k} = \frac{\partial\mathbf{f}^D}{\partial\mathbf{q}},\quad \mathbf{m} = \frac{\partial\mathbf{f}^I}{\partial\ddot{\mathbf{q}}},\quad \mathbf{k}_\Omega = \frac{\partial\mathbf{f}^I}{\Omega^2\partial\mathbf{q}},\quad \mathbf{c} = \frac{\partial\mathbf{f}^I}{\Omega\partial\dot{\mathbf{q}}} \tag{14}$$

F. Equations of Motion

The nonlinear equations of motion for a rotating beam with constant angular velocity may be expressed by

$$\boldsymbol{\varphi} = \mathbf{F}^D(\hat{\mathbf{Q}}) + \mathbf{F}^I(\Omega, \hat{\mathbf{Q}}, \dot{\hat{\mathbf{Q}}}, \ddot{\hat{\mathbf{Q}}}) = \mathbf{0} \tag{15}$$

$$\hat{\mathbf{Q}} = \mathbf{Q}_s + \mathbf{Q}(t) \tag{16}$$

where $\boldsymbol{\varphi}$, $\mathbf{F}^D$, and $\mathbf{F}^I$ are unbalanced force vector, deformation nodal force vector, and inertia nodal force vector of the structural system, respectively. $\mathbf{F}^I$ and $\mathbf{F}^D$ are assembled from the element nodal force vectors, which are calculated first in the current element coordinates and then transformed from element coordinate system to global coordinate system before assemblage using standard procedure. $\hat{\mathbf{Q}}$ is the nodal displacement vector of the rotating beam, $\dot{\hat{\mathbf{Q}}}$ and $\ddot{\hat{\mathbf{Q}}}$ are the nodal velocity vector and the nodal acceleration vector of the rotating beam, respectively, $\mathbf{Q}_s$ is the steady state nodal displacement vector induced by constant rotation speed Ω, $\mathbf{Q}(t)$ is the time dependent nodal displacements vector caused by the free vibration of the rotating beam. Here only infinitesimal vibration is considered.

G. Governing Equations for Steady State Deformation

For the steady state deformations, $\mathbf{Q}(t) = \mathbf{0}$. Thus (15) can be reduced to nonlinear steady state equilibrium equations and expressed by

$$\boldsymbol{\varphi} = \mathbf{F}_s^D(\mathbf{Q}_s) + \Omega^2\mathbf{F}_s^I(\mathbf{Q}_s) = \mathbf{0} \tag{17}$$

where $\mathbf{F}_s^D(\mathbf{Q}_s)$, and $\Omega^2\mathbf{F}_s^I(\mathbf{Q}_s)$ are the deformation nodal force vector, and the inertia nodal force (the centrifugal force) vector of the structural system corresponding to the steady state nodal displacement vector $\mathbf{Q}_s$, respectively. Note that $\Omega^2\mathbf{F}_s^I(\mathbf{Q}_s)$ is deformation dependent. Thus $\Omega^2\mathbf{F}_s^I(\mathbf{Q}_s)$ should be updated at each new configuration.

Here, an incremental-iterative method based on the Newton-Raphson method is employed for the solution of nonlinear steady state equilibrium equations at different rotation speed Ω.

H. Governing Equations for Free Vibration Measured From The Position of Steady State Deformation

Substituting (16) into (15), and setting the first-order Taylor series expansion of the unbalanced force vector $\boldsymbol{\varphi}$ around $\mathbf{Q}_s$ to zero, one may obtain the governing equations for linear free vibration of the rotating beam measured from the position of the steady state deformation as follows.

$$\mathbf{M}\ddot{\mathbf{Q}} + \Omega\mathbf{C}\dot{\mathbf{Q}} + (\mathbf{K} + \Omega^2\mathbf{K}_\Omega)\mathbf{Q} = \mathbf{0} \tag{18}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, and $\mathbf{K}_\Omega$ are mass matrix, gyroscopic matrix, tangent stiffness matrix, and centripetal stiffness matrix of the rotating beam, respectively. $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, and $\mathbf{K}_\Omega$ are assembled from the element mass matrix, gyroscopic matrix, tangent stiffness matrix, and centripetal stiffness matrix first in the current element coordinates and then transformed from element coordinate system to global coordinate system before assemblage using standard procedure.

We shall seek a solution of (18) in the form

$$\mathbf{Q} = (\mathbf{Q}_R + i\mathbf{Q}_I)e^{i\omega t} \tag{19}$$

where $i = \sqrt{-1}$, ω is natural frequency of rotating beam, and $\mathbf{Q}_R$ and $\mathbf{Q}_I$ are real part and imaginary part of the vibration mode.

Substituting (19) into (18), one may obtain a set of homogeneous equations expressed by

$$\mathbf{HZ} = \mathbf{0} \tag{20}$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{K} + \Omega^2\mathbf{K}_\Omega - \omega^2\mathbf{M} & \omega\Omega\mathbf{C}^t \\ \omega\Omega\mathbf{C} & \mathbf{K} + \Omega^2\mathbf{K}_\Omega - \omega^2\mathbf{M} \end{bmatrix} \tag{21}$$

Equation (20) is a quadratic eigenvalue problem. For a nontrivial $\mathbf{Z}$, the determinant of matrix $\mathbf{H}$ in (20) must be equal to zero. The value of K which make the determinant vanish are called eigenvalue of matrix $\mathbf{H}$. The bisection method is used here to find the eigenvalue.

III. Numerical Studies

To investigate the effect of angular speed on the natural frequency of rotating Euler beams with different setting angles, the beam of elliptical cross section as shown in Fig. 3 is considered. The twist inertia moment corresponding to the centrifugal force induced by the constant rotation may be expressed by $m = \rho\Omega^2(I_y - I_z)\sin(\beta + \phi_s)\cos(\beta + \phi_s)$, where ϕ_s is the steady state twist angle of the rotating beam, I_y and I_z are moment of inertia about the major axis and the minor axis of the cross section, respectively. The ratio of I_y and I_z is 25 for the elliptical cross section considered here. For convenience, the following dimensionless variables are used here:

$$r = \frac{R}{L_T},\ U_s = \frac{u_s}{L_T},\ k = \Omega L_T\sqrt{\frac{\rho}{E}},\ K = \omega L_T\sqrt{\frac{\rho}{E}},$$

where u_s is the steady state axial displacement of the rotating beam.

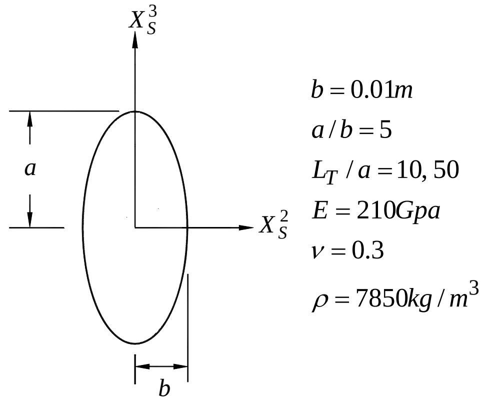

Fig. 3. Elliptical cross section of the rotating beam.

In practice, rotating structures are designed to operate in the elastic range of the materials. The maximum steady state axial strain of elastic rotating beam is $\varepsilon_{\max} = k^2(r + 0.5)$ [2], which occur at the root of the rotating beam with different setting angle β. The dimensionless radius of the rotating hub $r = 0$, and the maximum dimensionless angular speed $k = 0.1$ are considered here. In this section, K_i denotes that the dimensionless natural frequency of the rotating beam is the *i*th natural frequency at $k = 0$; B_i and C_i denote that the corresponding vibration modes are the *i*th lateral vibration modes in the X_2^G and X_3^G directions, respectively at $k = 0$; D_i denotes that the corresponding vibration mode is the *i*th twist vibration mode about the X_1^G axis at $k = 0$.

The results are obtained using 10 and 20 equal elements for cases $L_T/a = 10$ and 50, respectively. The dimensionless natural frequencies K_i (i = 1-4) for different setting angle at different angular speed are tabulated in tables I and II. The results given in [7] are also shown in Table I and II for comparison. To investigate the effect of the steady state twist deformation ϕ_s on the natural frequency of rotating Euler beams, cases with twist angle restrained are also studied for $\beta = 45^\circ$ at $k = 0.1$. The results are also shown in tables I and II. It can be seen that the difference between the lower dimensionless natural frequencies for the cases with and without considering the twist angle ϕ_s is insignificant for case $L_T/a = 10$, but is remarked for case $L_T/a = 50$. These may be partially attributed to that the twist angle ϕ_s of the rotating beam increases with increase of L_T/a. However, it seems that the effect of the twist angle ϕ_s on the higher natural frequencies of the rotating beam is not significant for all cases studied here.

TABLE I
DIMENSIONLESS FREQUENCIES FOR ROTATING BEAM
($a/b=5$, $L_T/a=10$))

k	β (deg)	K_1 (B1)	K_2 (C1)	K_3 (B2)	K_4 (D1)
0	0	0.03515	0.17479	0.22000	0.38751
0.01	0	0.03542	0.17512	0.22123	0.38754
	15	0.03552	0.17510	0.22125	0.38755
	30	0.03577	0.17505	0.22129	0.38760
	45	0.03612	0.17498	0.22135	0.38766
	60	0.03647	0.17490	0.22140	0.38771
	75	0.03672	0.17485	0.22144	0.38776
	90	0.03681	0.17483	0.22146	0.38777
0.05	0	0.04065	0.18291	0.24905	0.38818
	15	0.04253	0.18248	0.24939	0.38858
	30	0.04736	0.18126	0.25034	0.38967
	45	0.05344	0.17955	0.25161	0.39113
	60	0.05909	0.17775	0.25284	0.39257
	75	0.06305	0.17638	0.25373	0.39360
	90	0.06447	0.17586	0.25406	0.39397
0.1	0	0.04996	0.20515	0.32036	0.39016
	15	0.05489	0.20388	0.32154	0.39190
	30	0.06704	0.20020	0.32463	0.39645
	45	0.08200	0.19452	0.32860	0.40225
	45*	0.08652	0.19252	0.32817	-
	60	0.09647	0.18772	0.33232	0.40766
	75	0.10756	0.18155	0.33491	0.41138
	90	0.11192	0.17888	0.33583	0.41270

* Twist angle is restrained

TABLE II
DIMENSIONLESS FREQUENCIES FOR ROTATING BEAM
($a/b=5$, $L_T/a=50$)

k	β (deg)	K_1 (B1)	K_2 (C1)	K_3 (B2)	K_4 (B3)
0	0	0.00703	0.03515	0.04407	0.12338
0.01	0	0.00815	0.03681	0.04990	0.13001
	0 [7]	0.00815	0.03681	0.04990	0.13001
	15	0.00852	0.03672	0.04996	0.13003
	30	0.00948	0.03649	0.05015	0.13010
	45	0.01069	0.03615	0.05040	0.13020
	60	0.01182	0.03580	0.05064	0.13030
	75	0.01261	0.03552	0.05082	0.13037
	90	0.01290	0.03542	0.05089	0.13039
0.05	0	0.01495	0.06447	0.12360	0.23500
	15	0.01693	0.06398	0.12426	0.23536
	30	0.02166	0.06252	0.12606	0.23633
	45	0.02746	0.06020	0.12850	0.23765
	60	0.03328	0.05718	0.13093	0.23896
	75	0.05396	0.03828	0.13270	0.23992
	90	0.05220	0.04065	0.13335	0.24028
0.1	0	0.02033	0.11191	0.23214	0.40896
	15	0.02309	0.11136	0.23346	0.40987
	30	0.02952	0.10981	0.23708	0.41228
	45	0.03691	0.10752	0.24213	0.41540
	45*	0.07344	0.08652	0.24276	0.41515
	60	0.04356	0.10498	0.24737	0.41834
	75	0.10290	0.04824	0.25141	0.42040
	90	0.10208	0.04995	0.25295	0.42114

* Twist angle is restrained

The distributions of the dimensionless steady state axial displacement U_s and twist angle ϕ_s at different dimensionless angular speed for setting angle $\beta = 45^\circ$ are depicted in Fig. 4. The variation of dimensionless natural frequency with dimensionless angular speed is depicted in Fig. 5. It can be seen from Fig. 4 that the tip twist angle is about $3.8\times10^{-2}\,rad$ for case $L_T/a=50$ at $k=0.1$. It can be seen from Fig. 5 that the effect of the setting angle and angular speed on the natural frequencies of the torsional mode D_1 is not significant.

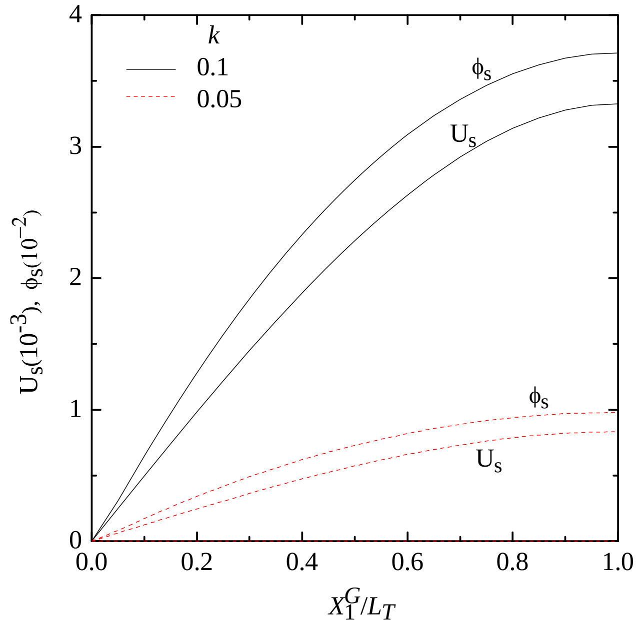

Fig. 4. The steady state deformation of rotating beam. ($a/b=5$, $L_T/a = 50$)

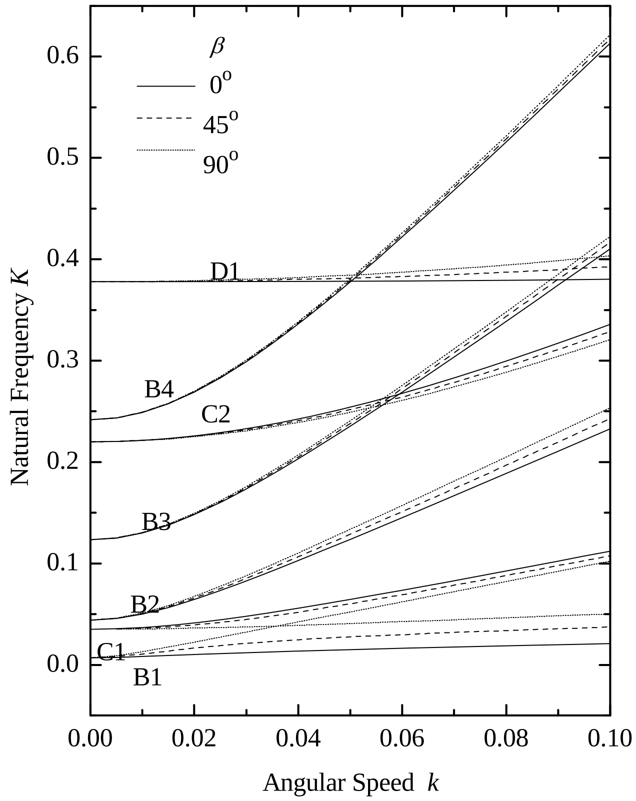

Fig. 5. Variation of natural frequency with angular speed ($a/b=5$, $L_T/a = 50$)

IV. Conclusions

In this paper, the steady state deformation and the natural frequency of infinitesimal free vibration measured from the position of the corresponding steady state deformation are investigated using corotational finite element formulation for the rotating Euler beams with different setting angles, slenderness ratios and angular speeds of the hub.

The element deformation and inertia nodal forces are systematically derived by the virtual work principle, the d'Alembert principle, and consistent linearization of the fully geometrically nonlinear beam theory in the current element coordinates. The equations of motion of the system are defined in terms of an inertia global coordinate system, which is coincident with a rotating global coordinate system rigidly tied to the rotating hub, while the total strains in the beam element are measured in an inertia element coordinate system, which is coincident with a rotating element coordinate system constructed at the current configuration of the beam element. The rotating element coordinates rotate about the hub axis at the angular speed of the hub. The results of numerical examples show that the geometrical nonlinearities that arise due to steady state twist angle and axial deformations should be considered for the natural frequencies of the rotating beams with different setting angle. It seems that the effect of the twist angle on the lower dimensionless natural frequencies of lateral vibration is remarked for slender beam with large ratio between moment of inertia about the major axis of the cross section and that of the minor axis.

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