A Subsidy Decision Procedure and its Impact on Universal Service and Access Fund Under Information Asymmetry

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Abstract-One of the main aims of telecommunication subsidies in developing countries is to extend the information and communication services to the information "have nots" through subsidized communication services. However, subsidies may have an impact on network resource utilization, quality of service and the amount of revenue generated. For example, subsidies may lead to low Quality of Service (QoS) and high resource utilization while in some instances unsubsidized services may lead to high quality of services and low utilization of resources. This see-saw effect may eventually lead to market failure and it may, now and then, destroy market efficiency. This phenomenon calls for a combined study, in which the relationship between subsidy, price, QoS and resource utilization is investigated. In this paper, the impact of subsidies on quality of service and resource utilization in multitier communities is investigated. We try to find a middle ground between implementation of subsidy policy and its effects on QoS and resource utilization in a network.

Index Terms-Pricing policy, Quality of service, Subsidy, ICTs

I. INTRODUCTION

I N one way or another, over-pricing or under-pricing of networks service provision in underserviced or unserved regions of developing countries has created resource problems associated with under-usage or over-exploitation of such resources [1] [5] [3]. Such under-usage or over-exploitation of resources in rural areas or underserviced regions of developing countries, arise from incompletely defined and enforced pricing policies and legal framework within such countries. This situation is further compounded by the problems associated with subsidy allocation by governments of developing countries when trying to promote social and economic agendas for its "needy" people [10].

Many arguments have risen on the usage of subsidies to promote social and economic agendas in developing [11]. On one hand literature points out that the usage of subsidies to enhance social and economical growth in a competitive market is not feasible and may distort market efficiency [4] [5] [6]. Other social-economical proponents of subsidies have argued, to the contrary, that subsidies are a necessity to promote, through lowering down of prices, social and economic growth in purely monopolistic market economies. One alternative study is, however, presented by Alesina and Rodrik [15] who showed that income disparities had an adverse effect on the country's economic growth, as such subsidies may promote economic growth. They showed that in more imbalanced communities, social and economic growth is lower because the demand for fiscal redistribution financed by distortionary taxation is higher [1] [8].

Amegashie [12] agrees that implementing a subsidy, "by reducing the price of the commodity, may increase the consumption of the commodity towards the equilibrium (perfectly) competitive quantity, given that output was initially too low" and if chosen properly a subsidy may "move the economy towards the perfectly competitive equilibrium quantity".



Fig. 1. Price discrimination may lead to: (a) underuse in Gautrain (tragedy of anti-commons) or (b) overuse in metro-rail (tragedy of the commons) of resources

Without doubt, income inequality in developing countries fuels social discontent and increases socio-political instability. The uncertainty in the politico-economic environment reduces investment which in turn reduces growth in underserviced areas or rural areas.

Subsidies, though seen as the means of promoting social and economic agendas in developing countries¹, can create the tragedies associated with public resources usage or somethingfor-nothing resources [3] [6]. Given a subsidy rate, consumers of developing countries usually anticipate a net social benefit derived from free resources due to subsidy or under pricing of such resources. Anticipation of net social benefits from such resources may generate a damaging rush from consumers to exploit the resource, which may result in the tragedy of the commons. By definition the tragedy of the commons is a

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¹"A novel idea to take voice and data services to the most rural areas could see Vodacom and MTN paid subsidies to do the job. The operators have to promise high-quality services even the poorest people can afford in return for having up to 80% of infrastructure subsidized. But the cost will not affect taxpayers as the cash will come from the Universal Service Fund, to which the operators themselves contribute" [16]

situation when "multiple owners are each endowed with the privilege to use a given resource and no one has the right to exclude another. When too many such people have the privileges to use, the resource is prone to over use [7].

On the contrary, when no subsidy is given, consumers face no differential between the perceived utility and the cost of the resource, as such very few consumers or users will create a damaging rush towards resource utilization creating a no social and pecuniary benefit to users. Generally, exogenous factors, such as exorbitant pricing or the absence of subsidy rates can decrease network resource usage and can make network usage in developing countries less effective. Heller in [2] defines the anti-commons as a situation where "multiple owners are each endowed with the right to exclude others from a scarce resource, and no one has an effective privilege of use".

Indeed, the implementation of subsidies may create a situation where the commodity is over utilized, because it is highly subsidized, or underutilized, because it under subsidized. This may create a problem that affects the market efficiency and social and economic growth of developing countries.

In light of the above problems, correct subsidy driven decision procedures are necessary in order to avoid the tragedy of the commons and the tragedy of the anti-commons. In this paper we will implement a subsidy decision procedure to prevent the tragedy of the commons or anti-commons in multi-tier communities. additionally this paper will look at the impact of subsidies on the USAF. This paper is organized as follows: Section II gives the necessity of subsidies in social and economic upliftment of developing countries, Section III and IV gives an introduction on subsidies in developing countries. We present the preliminaries in Section V and Section VI introduces the tyranny of subsidies on network resources. Section VII presents the model and graphicala and numerical analysis of our model. Section VIII presents the impact subsidies on universal service and access fund. We conclude our study in Section IX.

II. SUBSIDIES AS A RESPONSE TO MEETING SOCIAL OBJECTIVES

Subsidy usage in developing countries has both social and economic objectives, defined in this paper as reducing the effects of poverty and promoting social and economic development in rural or underserviced areas of developing countries. Having provided ICT infrastructure in underserviced regions of the country, the cumulative importance of service provision will, however, depend on the adoption and usage of communication services. In underserviced regions of developing countries very few people may adopt or use information resources because of the competing basic needs and uncompetitive market prices [18], [19].

Until such a time when the prices of ICTs are low and government intervention through subsidy, it is very unlikely that government efforts to bridging the digital divided may bear fruit. It is therefore necessary, if governments wants to promote social and economic objectives, for governments to "determine priority underserviced areas and accelerate the provision of Universal Service and Access through integrated government interventions" [17] such as sector specific subsidy injection.

When objectives for promoting social-economic growth in rural areas fall short of the intended objectives, they generally lack other complimentary government initiatives needed to undertake the economic aspects of such a development that ICT provision is suppose to compliment. Oyedemi [22] points out that "the failure of some of these policies is not solely inherent in the policies alone, but also in forces from other social factors that render many access program unrealistic". Therefore social-economic ICT provision directed at the poor rural people will probably fail without substantial complementary interventions by government, education institutions, regulatory institutions, and private institutions.

III. TOWARDS RURAL INFORMATION AND COMMUNICATION AFFORDABILITY

There are a number of ways to improve the affordability of information and communication in rural areas. When determining prices for communication services in rural areas, service providers and operators usually take into account both Capital Expenditure (CAPEX) and Operational Expenditure (OPEX) or costs. When these two costs are extremely high the price of telecommunication services tends to rise. It is for this reason that communication service providers and governments of developing countries must work hand in hand to cut costs through the use of subsidies and sector specific regulation of telecommunication services.

IV. PRICING AND SUBSIDIES IN INFORMATION AND COMMUNICATION PROVISION IN RURAL AREAS

Pricing of information and communication services in rural areas is of prime importance if social and economic development is to be realized [23]. Nevertheless, extremely poor households in developing countries, who live on less than ZAR 16 per day, find it difficult to afford telecommunications services which, in most of the rural areas, are overpriced. In pricing of such services, different pricing schemes and models may be used to cater for consumers or users with differing sensitivities towards price. Historically, especially in developed countries, such schemes have evolved from subsidy² driven pricing to competition as noted by Hayashi:

"Under state monopoly or regulated private monopoly, the mechanism was called cross subsidization that was built into the tariff structure. Usually long distance callers subsidized local callers, business users subsidized household users, and offices and houses in densely populated subsidized those in sparsely populated area" [21]. Hayashi [22] also notes that: "As countries moved into the privatization and competition phase, the cross subsidization scheme became increasingly unsustainable. Competition was introduced into the long distance market, as a result of which the source of subsidies dried up".

²Subsidies are normally applied in telecommunications to accomplish social objectives such as promoting the provision of universal service in telecom or providing special telecom access to specific users such as those living in remote areas.

Various pricing strategies must be applied in a heterogeneous or multi-tier society if social and economic objectives proposed by government are to be realized. However, in developing countries pricing for information services still remains a challenge to most communication and information services providers particularly in a community which has an unbalanced economic system between the information "haves" and "have nots" [20]. The problem governments should consider then is: how should governments of developing countries design a pricing model that will maximize or encourage social and economical development, and promote an inclusive information society, given an observable income disparity among users? While the more general problem is to determine the total subsidy budget as well as to maximize the net surplus (the benefit minus the cost), however, governments tend to focus only on the sub-problem of the subsidy rate path over a given specific budget. Such pricing mechanisms tend to be catastrophic, do not reflect costs involved and do not meet the social and economic objectives set forth by governments of developing countries.

V. DEFINING AND MODELING THE PARAMETERS

As defined in [3] [14], we consider a multi-tier community whose population profile comprises of heterogeneous type of consumers with an observable disparity in reservation prices; the information "haves" (N_2) and "have-nots" (N_1) . In many multi-tier communities, as in case of developing countries, customers have different service requirements. In such developing countries, service providers can offer different prices for each consumer and can choose a suitable price based on consumers' needs and acceptance of the quoted price and adjust their price accordingly so as to obtain an optimal price at which both types of consumers will be more willing to pay for the service. As Sumbwanyambe, Nel and Clarke [3] have stated, "these parameters can be learned through a market price adaptive research which estimates the number of consumers that accept a given price. In fact, the process of learning price parameters in such a market is a dynamic process with the aim of adjusting the quoted price in line with the consumers' behavior towards price dynamics". We will assume that the two types of consumers have differing reservation price.

Definition 1 Reservation price (sometimes called the threshold price), p_{thi} for $i \in (1, 2)$ is the price at which a customer is indifferent between subscribing to the current network or opting out of the current network and subscribing to the other network or dropping them all.

Furthermore we assume that p_1 and p_2 is the price that is payable to the service provider by the consumers N_1 and N_2 , respectively, where $p_1 = \beta p_2$, for $0 < \beta < 1$ and $p_2 \in (0, p_\infty)$ and $c_{1,2}$ is the cost of providing such services to N_1 and N_2 . β is the subsidy factor provided by the government. We use a simple cut-off price rule to determine whether the user would subscribe to the ISP or not. In the current context the cut-off rule is based on users' reservation price i.e. p_{th1} and p_{th2} , for N_1 and N_2 consumers. For any given price, those customers whose reservation price are greater than or equal to the given price will purchase the service provided by the ISP or the service provider. A price setting service provider in collaboration with government uses the information in the distribution of reservation prices to choose its optimal price.

VI. THE TYRANNY OF SUBSIDY AND PRICE SENSITIVITY ON NETWORK RESOURCES

Subsidies and price sensitivity has an adverse effect on resources utilization in multi-tier communities. From the users' point of view, subsidies affect their behavior in terms of network resources utilization, which is correlated to their price sensitivities. From the government and service provider's perspective, choosing the right subsidy is of great importance in maximizing revenue and enhancing the optimal usage of resources [24]. Therefore, correct subsidization of deserving customers or consumers in underserviced areas is of great importance if tragedies in such communities are to be avoided. In underserviced region of South Africa and Zambia willingness to pay is associated with a number of factors. Primarily, any heterogeneous user is assumed to maximize utility subject to a predetermined level of his or her income.

In a heterogeneous society (especially in developing countries), subsidy and price sensitivity can lead to the tragedy of the (anti) commons [13] [1]. If too large a subsidy ($\beta = 0$) is given and the price sensitivity is close to 1 (i.e. $\alpha = 1$), consumers will demand more than is available, creating the tragedy of the commons [5]. In order to prevent the tragedy of the commons, service providers and government policy makers have to cut the subsidy level and increase the price which will eventually reduce the number of consumers. The decrease in the number of consumers (especially rural consumers) may then lead to the tragedy of the anti-commons [1] [2], leaving a gap in government policy and objectives of promoting social and economic growth. In such an eventuality, the government will have to increase the subsidy and lower the prices yet again in order fulfill their objectives. This creates what is known as a see-saw effect. The following example demonstrates this effect:

Let's take that $0 < p_1 \le p_{th1}$ and $0 < p_2 \le p_{th2}$, there exists a NE at which all users i.e. N_1 and N_2 , will attempt to maximize their utilities, $u_{pth1,p1}$ and $u_{pth2,p2}$, given the reservation price p_{th1} and p_{th2} and the price p_1 and p_2 charged by the service provider. Such a development will lead to both users i.e. the information "haves" and "have nots" trying to maximize their utility. This utilization of resources without restraint will, more likely, result in the "tragedy of the commons". Once more, if $0 \le p_{th1} \le p_1$ and $0 \le p_{th2} \le p_2$ the tragedy of the anti-commons is a more likely outcome. If the utility of one user is positive and the other user is negative, then the free rider problem is likely to occur. The following definitions are applicable in equilibrium:

Definition 2 Given that $0 \le p_1 \le p_{th1}$ and $0 \le p_2 \le p_{th2}$ the population profile of N_1 and N_2 users are in equilibrium if it does not pay for any member of the group to opt out of any ISP.

Definition 3 Given that $0 \le p_{th1} \le p_1$ and $0 \le p_{th2} \le p_2$ the population profile of N_1 and N_2 users are in equilibrium

if it does not pay for any member of the group to subscribe to any ISP. $\hfill \Box$

Definition 4 Given any price p_1 and p_2 the population profile of N_1 and N_2 users are not in equilibrium if it pays for any member of the group to join another group.

Having put up the above definitions, the following Lemma will show that given the prices, p_1 and p_2 , and the reservation price, and if all users receive an equal negative or positive utility, users are unlikely to switch to another group or free ride or opt out.

Lemma 1 Given $0 \le p_1 \le p_{th1}$, $0 \le p_2 \le p_{th2}$ and $p_1 < p_2$, for any values of p_1 and p_2 the situation is in equilibrium if $u_{pth2,p2} - p_2 = u_{pth1,p1} - p_1$.

Proof: From definition 7.3 the situation is in equilibrium if the information "have-nots" and "haves"do not have a unilaterally decision to change from their present strategy to another. If the utility of the information "haves" is less than 0 i.e. $u_{pth2,p2} - p_2 < 0$, no N_2 users will subscribe to the ISP. Similarly if the utility of the information "havesnots" is less than zero i.e. $u_{pth1,p1} - p_1 < 0$, there will be no N_1 users who will subscribe to the ISP. Otherwise if $u_{pth2,p2} - p_2 < u_{pth1,p1} - p_1$ a free rider problem is likely to occur.

From the Lemma above we can draw the following conclusions:

- If there are no N_2 users subscribing to the ISP then the utility of such population group is zero i.e. $u_{pth2,p2}-p_2<0$
- If there are no N_1 users subscribing to the ISP then it means that the utility of the N_1 is zero i.e. $u_{pth1,p1}-p_1<0$

These two statements will lead us to the following corollary describing the structure of the equilibrium.

Corollary 1 Define $p_i = [p_1, p_2, p_{th1}, p_{th2}]$ an equilibrium has the following structure:

- It is the tragedy of the commons if $u_{pth2,p2} p_2 \ge 0$ and $u_{pth1,p1} p_1 \ge 0$ where $p_2 \approx 0$ and $p_1 \approx 0$.
- It is the tragedy of the anti-commons if u_{pth2,p2} − p₂ ≤ 0 and u_{pth1,p1} − p₁ ≤ 0 for all users in the network.
- It is likely to be a free rider problem when $u_{pth2,p2}-p_2 \le 0$ and $u_{pth1,p1}-p_1 \ge 0$ and $u_{pth2,p2}-p_2 \ge 0$ and $u_{pth1,p1}-p_1 \le 0$

VII. SUBSIDY, SOCIAL DILEMMAS, PRICE AND REVENUE MAXIMIZATION

In this section we provide a numerical and a graphical analysis of results on how the subsidy affects the number of users and ultimately how it affects the revenue of the service provider. We take into consideration the dynamics of a heterogeneous community with an observable difference between the reservation price of the information "haves" and information "have-nots".

A. A Local decision procedure for the tragedy of the commons and the tragedy of the anti-commons problem

This section gives a brief overview of a distributed algorithm to solve the problem of the tragedy of the commons and the tragedy of the anti-commons and also discusses the convergence of the algorithm to equilibrium. The algorithm can be used to determine the optimal values of α and β and the maximum revenue at which the utilization of network resources will be optimal. The algorithm is as follows:

Algorithm

Step 1: Define the initial values p_2 , p_{th1} , p_{th2} , α and β and bandwidth.

Step 2 Calculate the number of users by using equation below

$$N_{total} = \left(\frac{\beta p_2}{p_{th1}}\right)^{-\alpha_1} - 1 + \left(\frac{p_2}{p_{th2}}\right)^{-\alpha_2} - 1$$

Step 3:Calculate the amount of packets (T_{total}) sent by users by using equation below

$$T_{total} = (N_{total}\lambda)$$

Step 4: If (T_{total}) is greater than bandwidth add $\beta + 0.1$ and $p_2 + 1$

Step 5: If (T_{total}) is less or equal to 0 (bandwidth usage is minimal) start $\beta - 0.1$ and $p_2 - 1$

Step 6: Go to step 3, 4, 5

Step 7: Calculate the final value of p_2 and $\frac{p_1}{\beta}$

Step 8: If final $p_2 = \frac{p_1}{\beta} = p_{th2}$. STOP. Optimal β solution found

Step 9: Calculate maximum revenue Π using equation 1 and the value of p_2 by equating equation 2=0

Step 10 Government and ISP (service provider) maintains values of β and p_2 . Repeat step 8 and 9

Convergence to equilibrium: The system reaches equilibrium when all agents reach Step 9 of the algorithm. At this state, the government and the service provider feels that any increment or decrement in the value of β and p_2 will either reduce or increase its utility. Hence the number of packets in the system does not change. After step 4 the government and the service provider may either decrease the value of subsidy or increase the price p_2 . After step 3 the service provider calculates the number of packets or bandwidth used. (Note that in this case the number of packets used is proportional to the number of users subscribing to the ISP hence number of packets can be measured in terms of number of users). If the amount of packets sent is less than the total bandwidth, the government increments the subsidy factor and the ISP decrements the price p_2 . A rational government and ISP can reason after Step 3 that to prevent over-utilization of the resource (tragedy of the commons) it should either decrease the subsidy or increase the price by using Step 4. Similarly if the government wants to prevent the tragedy of the anticommons it should increase the subsidy or decrease the price p_2 by using step 5.

The government and ISP can use probabilistic searching scheme outlined in Steps 4 through 6 to reach its optimum

load. If the total number of packets sent is greater than the bandwidth, the government starts with an initial increment probability of subsidy factor by a factor of 0.1 and the price is increased by a determined increment factor. If the subsidy increment produces a lower utility for the users, the government and the service provider will follow step 5. Both the government and the ISP keep on probing in this manner until an optimal solution is reached where no users will deviate from his present status. At such a point government and ISP maintains the value of β and p_2 as shown in Figure 4. Our claim is that when equilibrium is reached, the combined load on the resource is exactly the critical load or capacity of the resource; i.e. the agents are using the resource optimally. They have reached this optimality through a distributed decision procedure using only local knowledge and with the directive of government.

B. Numerical and graphical analysis

In a developing country a complex relationship that exists between heterogeneous communities escalates the problem of developing a two tier workable pricing model. Customers have different income levels, sensitivities toward price and different reservation prices for a particular service. This problem is further compounded by the problem of subsidies and how to properly allocate subsidies to the "needy" or the information "haves nots", without creating the tragedy of commons and the tragedy of the anti-commons. When over-subsidization and over-pricing of network resources creates the undesirable outcomes of the tragedy of commons and tragedy of anticommons, it is necessary for the service provider and the government to subsidize and price network resources in an optimal way. In the next section we provide a numerical analysis and graphical analysis to make our statement more clearer.

Numerical analysis: We provide a numerical analysis by applying an algorithm and the pricing model as proposed by Sumbwanyambe and Nel in [2] as follows:

$$\Pi = \lambda \left(\left(\frac{\beta p_2}{p_{th1}} \right)^{-\alpha} - 1 \right) (p_2 - c) + \lambda \left(\left(\frac{p_2}{p_{th2}} \right)^{-\alpha} - 1 \right) (p_2 - c)$$
(1)

To find the optimal prices and an optimal subsidy rate at which no user will unilaterally change his decision, we consider the service provider's profit maximization problem (see equation 1). Often times finding such optimal prices entails checking all the likely values of p_2 and subsidy factor β and ask ourselves "is this a Nash Equilibrium?" At times it is possible to eliminate actions iteratively to narrow the cases that need to be checked. Since, profit functions are continuously differentiable, concave and the price p_2 is always positive, we can take the first order conditions of Π as follows:

$$\frac{\partial \Pi}{\partial p_2} = \frac{\lambda}{(\frac{\beta p_2}{p_{th1}})^{\alpha} - 1} + \frac{\lambda}{(\frac{p_2}{p_{th2}})^{\alpha} - 1}$$

$$+\frac{\lambda\alpha(c-p_2)}{p_{th2}(\frac{p_2}{p_{th2}})^{\alpha+1}} + \frac{\lambda\alpha\beta(c-p_2)}{p_{th1}(\frac{\beta p_2}{p_{t+1}})^{\alpha+1}}$$
(2)

To find the concavity of of the above profit function, we take the partial derivatives of equation 2. Clearly, the second order conditions imply that marginal profits should slope downwards with respect to the ISP's own action. Further differentiation of equation 2 provides us with an expression as follows:

$$\frac{\partial^2 \Pi}{\partial p_2^2} = -\left(\frac{2\lambda\alpha}{(p_{th2}\frac{p_2}{p_{th2}})^{\alpha} + 1}\right) - \left(\frac{2\lambda\alpha\beta}{(p_{th1}\frac{\beta p_2}{p_{th1}})^{\alpha} + 1}\right) - \left(\frac{\lambda\alpha(c - p_2)(\alpha + 1)}{(p_{th2}^2(\frac{p_2}{p_{th2}})^{\alpha+2}}\right) - \left(\frac{\lambda\alpha\beta^2(c - p_2)(\alpha + 1)}{p_{th1}^2(\frac{\beta p_2}{p_{th1}})^{\alpha+2}}\right)$$
(3)

To begin with, we assume that the government and the service provider lowers the price and the subsidy factor as shown in Table I. At NE selfish users will use the network resources without restraint. Given that the price is low, and government is heavily subsidizing the information "have nots", there will be an increase in the number of users trying to maximize their utility which may lead to the tragedy of commons. Consider again, that this time the service provider increases its price and the government heavily subsidizes the information "have nots". As observed from Table I, increasing the subsidy and increasing the price p_2 without consideration will result in the information "haves" failing to pay for the network services when the price set reaches the reservation price p_{th2} . For example when the price of network services is approximately equal or above the reservation price, the information "haves" won't be able to pay for the network services i.e. $p_{th2} = 100 = p_2$. This scenario may result in the free rider's problem and may disrupt the once stable telecommunication market or economic system of the country.

Table II paints a picture of how the value of subsidy factor β affects the number of information "have nots". At no subsidy or when subsidy from the government is approximately equal to zero, the numbers of information "have nots" users won't be able to pay for the service because $p_1 = p_{th1}$. Table III and Table IV shows the effect of users' sensitivity towards price at different values of β .

Graphical analysis: In Figure 1, 2 and 3, we evaluate the effect of subsidy on revenue maximization, given any price p_2 and the threshold price of consumers i.e. p_{th1} and p_{th2} at a constant sensitivity α . Figure 1, for example, shows that for a subsidy factor of $\beta = 1$ (in this case we assume that the government does not subsidize the information "have nots"), the revenue generated from the information "have nots" is, actually, dependent on the threshold value or the maximum reservation price p_{th1} that they are willing to pay. This phenomenon, especially in underserviced areas of developing countries, may sometimes result in the tragedy of anti-commons since very few information "have nots" will be more willing to subscribe to the ISP. For all intents and purposes, it is worthwhile to note that the information "haves nots" are contributing very little towards the ISP's overall revenue since no subsidy is being given by the government

TABLE I	
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Expected amount of revenue with varying p_2 : when $p_{th1} = 40$, $p_{th2} = 100$

Users	P_{th1}	P_{th2}	P_2	P_1	β	α	Revenue	Normalized number of users
N_1	40		20	5	0.1	0.3	291.30	1.4565
N_2		100	20	5	0.1	0.3	124.13	0.6207
N_1	40		50	20	0.1	0.3	692.85	0.8661
N_2		100	50	20	0.1	0.3	184.91	0.2311
N_1	40		90	35	0.1	0.3	903.02	0.5644
N_2		100	90	35	0.1	0.3	51.38	0.0321
N_1	40		100	40	0.1	0.3	928.28	0.5157
N_2		100	100	40	0.1	0.3	0	0

TABLE II Expected amount of revenue with varying $\beta :$ when $p_{th1}=40,\,p_{th2}=100$

Users	P_{th1}	P_{th2}	P_2	P_1	β	α	Revenue	Normalized number of users
N_1	40		50	5	0.1	0.3	692.00	0.8661
N_2		100	50	5	0.1	0.3	184.00	0.2311
N_1	40		50	20	0.4	0.3	184.91	0.2311
N_2		100	50	20	0.4	0.3	184.00	0.2311
N_1	40		50	35	0.7	0.3	32.69	0.0409
N_2		100	50	35	0.7	0.3	184.00	0.2311
N_1	40		50	40	0.8	0.3	0	0
N_2		100	50	40	0.8	0.3	184.00	0.2311

TABLE III Expected amount of revenue with varying $\alpha :$ when $p_{th1}=40,\,p_{th2}=100$

P_{th1}	P_{th2}	P_2	P_1	β	α	Revenue	Normalized number of users
40		50	5	0.3	0.1	57.41	0.1031
	100	50	5	0.3	0.1	57.41	0.0718
40		50	20	0.3	0.3	184.91	0.3421
	100	50	20	0.3	0.3	184.91	0.2311
40		50	35	0.3	0.7	499.60	0.9869
	100	50	35	0.3	0.7	499.60	0.6245
40		50	40	0.3	1	800.00	1.6667
	100	50	40	0.3	1	800.00	1
	P_{th1} 40 40 40 40 40	$\begin{array}{c c} P_{th1} & P_{th2} \\ \hline 40 & & \\ 100 \\ 40 & & \\ 100 \\ 40 & & \\ 40 & & \\ 100 \\ \end{array}$	$\begin{array}{c cccc} P_{th1} & P_{th2} & P_2 \\ \hline 40 & 50 \\ 100 & 50 \\ 40 & 50 \\ 100 & 50 \\ 40 & 50 \\ 100 & 50 \\ 40 & 50 \\ 100 & 50 \\ 100 & 50 \\ \end{array}$	$\begin{array}{c cccc} P_{th1} & P_{th2} & P_2 & P_1 \\ \hline 40 & 50 & 5 \\ 100 & 50 & 5 \\ 40 & 50 & 20 \\ 100 & 50 & 20 \\ 40 & 50 & 35 \\ 100 & 50 & 35 \\ 40 & 50 & 40 \\ 100 & 50 & 40 \\ \end{array}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

 $\begin{array}{c} \text{TABLE IV} \\ \text{Optimal subsidy rate, } \beta \text{, with varying } p_{th1} \text{ and } p_{th2} \text{: when } \alpha_1 = 0.4, \, \alpha_2 = 0.7 \end{array}$

Type of Users	P_{th1}	P_{th2}	P_2	P_1	β	α	ISP Revenue
N_1	10		50	5.5	0.11	0.4	216.119
N_2		90	50	5.5	0.11	0.7	407.2042
N_1	40		50	16.55	0.331	0.4	338.66
N_2		120	50	16.55	0.331	0.7	676.55
N_1	70		50	27	0.54	0.4	216.119
N_2		130	50	27	0.54	0.7	407.304

towards the ISP (see Figure 2). In order to obtain maximum revenue from the users (as shown in Figure 1), it will be meaningful for the ISP to charge the information "have nots" and "haves" an optimal price of 30 units so as to maximize revenue.

Since the main aim of the government is to promote social and economic growth in underserviced regions, Figure 2 would represent a much more desirable scenario in promoting social and economical development in underserviced areas (ICT access for all in developing countries). In actual fact, Figure 2 represents a desirable outcome that will objectively fulfill government policies, as it allows more information "have nots" to subscribe to the ISP at a subsidized price. However, such a situation would definitely promote the tragedy of the commons and may lead to market failure in once stable markets. A more close analysis of Figure 2, in reality, reveals that the information "haves" will fail to subscribe to the ISP, at a higher price e.g. at 100 units. Since this is the reservation price of information "haves". Thus, Figure 2 highlights the implications of heavy subsidy towards revenue maximization in a heterogeneous society, and shows that at an optimal price of 80 units (partially paid by the government) there will be a skewed information access between the information "haves" and the information "have nots" due to unbalanced subsidy distribution. Figure 4, on the other hand shows, that the revenue of an ISP is maximum at an optimal price of 40 units. In our view, Figure 3 represents an equilibrated game between the information "haves" and "have nots" as the price of both groups seem to follow the same trajectory. Actually, Figure 4 presents a case where both groups of users seem to follow



Fig. 2. Revenue versus price $alpha = 0.3 \ p_{th1} = 40 \ p_{th2} = 100 \ \beta = 0.1$



Fig. 3. Revenue versus price $alpha = 0.3 \ p_{th1} = 40 \ p_{th2} = 100 \ \beta = 1$



Fig. 4. Revenue versus price $alpha = 0.3 p_{th1} = 40 p_{th2} = 100 \beta = 0.4$

the same trajectory price and revenue path. Both users (as depicted in Figure 4) will not subscribe to the ISP at the same reservation price or threshold price of p_{th2} . Contrary to Figure 1 and 2, Figure 3 represents a desirable outcome between the information "haves" and "have nots" as it represents the same price versus revenue curve trajectory for both the information "haves" and "have nots". Generally, Figure 3, in point of fact,

may somehow prevent the tragedy of the commons and tragedy of the anti-commons in heterogeneous society as no member of a group is disadvantage due to skewed pricing.

VIII. ANALYSIS OF GOVERNMENT SUBSIDY COST

In this section we evaluate the practical implications of government subsidy cost as a function of users on the USAF. Consider equation 4 and 5 which gives the amount of total Expected Revenue (ER_{total}) and revenue from p_1 i.e. Revenue_{p1} generated from the information "have nots" given by:

$$ER_{total} = p_2 \left(\left(\frac{\beta p_2}{p_{th1}} \right)^{-\alpha_1} - 1 \right) \tag{4}$$

and consider also that the revenue paid by the information "have nots" is given by

$$Revenue_{p_1} = p_1 \left(\left(\frac{\beta p_2}{p_{th1}} \right)^{-\alpha_1} - 1 \right)$$
(5)

From the two equations, we can obtain the cost of subsidy by subtracting equation 5 from 4 to obtain equation 6

Subsidy
$$cost = (p_2 - p_1) \left(\left(\frac{\beta p_2}{p_{th1}} \right)^{-\alpha_1} - 1 \right)$$
 (6)

A. Discussion

From Figure 5, and considering equation 4, we can see that the expected revenue decreases as the value of β increases. The reason being that the number of information "have nots" decrease as β increases, resulting in few users subscribing to the ISP. Additionally, Figure 5 illustrates the fact that given the prices $p_2 = 70$, $p_2 = 50$ and $p_2 = 30$, the amount of revenue obtained as $\beta \rightarrow 1$ is highest when $p_2 = 30$. When $\beta \approx 0$, and to the contrary, the amount of revenue obtained is maximum when the price is high i.e. $p_2 = 70$. From the preceding statement, we can conclude that the service provider must always set up his price in accordance with the subsidy rate. At $\beta \approx 0$ it is advisable for the service provide to set his price high. However, at $\beta \approx 1$ the service provider must set his price low. Actually, this scenario is a bargaining process between the government and the service provider. Analysis of Figure 6 and considering equation 5, the amount of revenue obtained from the information "have nots", when the price is p_1 , tends to zero when $\beta \rightarrow 0$. This means that, at the value of $\beta \approx 0$, the value of $p_1 \approx 0$, meaning that the information "have nots" are paying nothing towards the cost of using the internet. Mathematically, we expect, from Figure 6, that as the value of $\beta \rightarrow 1$, the revenue from the information "have nots" must increase. However, that is not the case. The reason behind the decrease is that, as p_1 increases i.e. with no subsidy rate from the government, the number of users decline rapidly as $p_2 \rightarrow p_{th1}$.

Finally, Figure 7 shows the amount of subsidy cost with β . From Figure 7, we can see that the amount of subsidy that the government must supply, is the function of the price p_2 , the reservation price p_{th1} and it depends on the β value as

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Fig. 5. Expected ISP revenue against β , when $\alpha = 0.3$ and $p_{th1} = 70$



Fig. 6. Revenue against β , when $\alpha = 0.3$ and $p_{th1} = 70$



Fig. 7. Subsidy cost against β when $\alpha = 0.3$ and $p_{th1} = 70$

shown in Figure 7. Figure 7 clearly demonstrates that when the price is too low the level of subsidy is very low and vice versa. Literally, one can conclude, given any price, the amount of subsidy cost is much higher when $\beta \approx 0$, and low when $\beta \approx 1$. Note that as p_2 increases the value of subsidy rate, β , should decrease to prevent poor utilization. This, however, will make the subsidy cost to increase.

From a practical point of view, the government subsidy cost increases with an increasing number of users and has severe repercussions on the USAF. For example an increase in the number of users leads to an increased government cost subsidy towards users or the information "have nots". If not monitored closely, this phenomenon may have a negative impact on the USAF funds.

IX. CONCLUSION

In this paper we have analyzed the effect of subsidy driven procedure on the resource usage, revenue maximization and the impact of subsidies on universal service and access fund (USAF). We have shown that if implemented properly, subsidies can prevent the tragedy of the (anti) commons and could be the solution to promoting social and economical growth in developing countries. Our proposed subsidy driven pricing model addresses the importance of a balanced subsidy pricing scheme with a view of achieving social and economical growth while enhancing the usage of network resources efficiently. Using this framework, we have shown that a government subsidy in a developing country can result in the increase or decrease of information "have nots" leading to the tragedy of (anti) commons. We have also shown that a correct subsidy, given customer sensitivities, can promote desired revenue and that the revenue is a concave function of price. Additionally we have shown how such a subsidy can affect the USAF. Thus far, we have derived the optimal revenue given the price, subsidy factor β and α as shown in the Figures of 2, 3 and 4.

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