The Identification Problem Applied to Periodic Systems Using Hartley Series

Isidro I. Lázaro, Alberto Álvarez, Juan Anzurez

Abstract—**The identification problem of periodic systems is considered in this paper. It is demonstrated that the systems that have parameters with periodic behavior can be estimated using orthogonal functions; in this case the model can be approximated by Hartley series in the continuous range. The main advantage of using orthogonal functions is that the differential equations that describe the system to identify can be transformed into a set of algebraic equations, facilitating the identification process. The proposed methodology was used to estimate parameters in the main rotor model of a helicopter, with satisfactory results.**

Index Terms— **System identification, periodically varying parameters, Hartley series, basis function, operational matrix.**

I. Introduction

In a wide variety of practical applications, the characteristics of the plant dynamics are changing through time due to aging of components or the variation of the reference. When the dynamics of the system is changing at a slow pace, the least squares algorithm (with and without forgetting factor) is quite effective when it is used to follow this type of behavior. Nevertheless, the efficiency of the algorithm diminishes when the dynamics is changing rapidly. In the past years, systems with fast changing dynamics are being used more often in modern engineering applications; in particular, linear periodic time-varying systems (LPTVS) have appeared in different areas such as Control, Power Electronics, Aeronautics, Communications and Signal Processing [1]. For example, in aeronautics, the equations that describe the flight dynamics in aircrafts have coefficients that are functions of time. Recently, the high speeds and accelerations that modern aircrafts can handle, have contributed to show that the parameters of the system that depend on speed will change in the same fashion. For instance, the dynamic of a main rotor in a helicopter, which can be modeled by a linear periodic system, is important from the control perspective due to the fact that a precise control of certain variables can help to attenuate vibrations in the rotor components, that eventually may cause high levels of stress and fatigue [2]. Thus, there are important reasons to develop parametric identification techniques for this type of systems, some of these methodologies can be found in [3]-[6]. Also, the use of basis functions and operational matrices allows the approximation of system parameters of a previously known model; therefore, the LPTVS can be estimated through an approximation using orthogonal series [7]-[12].

The use of orthogonal series expansions is well known as an alternative for approximation and representation of functions. This procedure has been used during the past two decades to yield algebraic equations for the solution of problems described by differential equations, such as analysis of linear-time invariant and time-varying systems, model reduction, optimal control and system identification for linear and some nonlinear cases.

The main purpose of this paper is to provide an alternative method for parametric identification. It is based on the transformation of the differential equations that are used to describe the system into an algebraic set of equations. Some other advantages of this methodology are: easy implementation of the algorithm in computational packages (Matlab, Mathematica, C code, etc.) and the easy control of the identification procedure and its algorithm.

The structure of this paper is as follows: Section 2 is dedicated to the general background on approximation using Hartley series expansions and operational matrices. In section 3, the algorithm for the parametric identification for linear periodically time-varying systems is presented. The application of the methodology and the associated results can be found in Section 4. The last section is devoted to the conclusions of this work.

II. Orthogonal Series Expansions and Operational Propierties via Hartley Series

In this section a brief review of the Hartley series and the approximation using orthogonal series expansions is given. It can be stated that the kernel function of the familiar Fourier series is complex exponential ($e^{-j\omega t}$). The Hartley series in the other hand, uses a similar frequency based kernel, the function $\cos(\omega t)+\sin(\omega t)$, also known as the cosine and sine function or $cas(\bullet)$ [13].

A. Hartley Series

An integrable function *f(t)* defined over the interval $(0,T)$ can be approximated in terms of Hartley series as

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$$f(t) = F\Phi(t) \tag{1}$$

where

$$F = \begin{bmatrix} F_{-n} & \cdots & F_{-1} & F_0 & F_1 & \cdots & F_n \end{bmatrix}_{(1,\,2n+1)} \tag{2}$$

$$\Phi(t) = \begin{bmatrix} \phi_{-n} & \cdots & \phi_{-1} & \phi_0 & \phi_1 & \cdots & \phi_n \end{bmatrix}' \tag{3}$$

The basis functions for the Hartley series can be defined as $\phi_n = cas(n\omega t) = \cos(n\omega t) + \sin(n\omega t)$. Each coefficient of the vector F is calculated as follows

$$F_n = k\int_o^t f(t)\Phi(t)\,dt \tag{4}$$

Therefore, every function that complies with the Dirichlet conditions in the interval $(0,T)$ can be approximated by means of a matrix product, as indicated by (1). The number of terms in the series is related to the error level; in this case, more terms mean a better approximation and a reduction in the error. The operational properties of the orthogonal series can be written in terms of an integration matrix, where the main idea is the fact that the integral of an orthogonal series can be expressed also as an orthogonal series; in general, this property can be defined as

$$\underbrace{\int_0^t \cdots \int_0^t}_{n} \Phi(\tau)\,d\tau^n = P^n\Phi(t) \tag{5}$$

In the case of the Hartley series, the structure of the integration matrix P is presented in [13] and it is shown in the following equation

$$P = \frac{1}{\pi}\begin{bmatrix} & & & & -\frac{1}{n} & & & & \frac{1}{n} \\ & & & & \vdots & & & \iddots & \\ & & & & -\frac{1}{2} & & \frac{1}{2} & & \\ & & & & -1 & 1 & & & \\ \frac{1}{n} & \cdots & \frac{1}{2} & 1 & \pi & -1 & -\frac{1}{2} & \cdots & \frac{1}{n} \\ & & & -1 & 1 & & & & \\ & & -\frac{1}{2} & & \frac{1}{2} & & & & \\ & \iddots & & & \vdots & & & & \\ -\frac{1}{n} & & & & \frac{1}{n} & & & & \end{bmatrix}_{(2n+1)\times(2n+1)} \tag{6}$$

For the analysis of LPTV Systems, two additional matrices must be defined, the product matrix and the coefficient matrix. The product of the vector of basis functions and its transpose yields the product matrix, this is

$$\Pi(t) = \Phi(t)\Phi'(t) \tag{7}$$

The coefficient matrix is defined in terms of the product matrix and the vector of basis functions. This matrix satisfies the following relation

$$[C]\Phi(t) = \Pi(t)c \tag{8}$$

The matrix $[C]$, of order $2n+1$, is the coefficient matrix when a vector c is given. The next representation of the $[C]$ matrix is displayed only to appreciate its form when only three terms in the series are considered.

$$[C] = \frac{1}{2}\begin{bmatrix} 2c_0 & c_{-1}+c_1 & c_{-2}+c_2 & 2c_{-3} & c_{-2}-c_2 & c_{-1}+c_1 & 0 \\ c_{-1}+c_1 & 2c_0 & \begin{matrix} c_{-1}+c_1 \\ +c_{-3}-c_3 + \end{matrix} & \begin{matrix} 2c_{-2} \\ + \end{matrix} & \begin{matrix} c_{-1}-c_1 \\ c_{-3}\ \ c_3 \end{matrix} & 0 & c_1-c_{-1} \\ c_{-2}+c_2 & \begin{matrix} c_{-1}+c_1 \\ +c_{-3}-c_3 \end{matrix} & 2c_0 & 2c_{-1} & c_{-2}+c_2 & \begin{matrix} c_{-1}-c_1 \\ +c_3+c_{-3} \end{matrix} & c_2\ \ c_{-2} \\ 2c_{-3} & 2c_{-2} & 2c_{-1} & 2c_0 & 2c_1 & 2c_2 & 2c_3 \\ c_{-2}-c_2 & \begin{matrix} c_{-1}-c_1 \\ +c_{-3}+c_3 \end{matrix} & c_{-2}+c_2 & 2c_1 & \begin{matrix} 2c_0 \\ +c_2-c_{-2} \end{matrix} & \begin{matrix} c_{-1}+c_1 \\ +c_2-c_{-2} \end{matrix} & c_2+c_{-2} \\ c_{-1}+c_1 & 0 & \begin{matrix} c_1-c_{-1} \\ +c_3+c_{-3} \end{matrix} & 2c_2 & \begin{matrix} c_1+c_{-1} \\ +c_3-c_{-3} \end{matrix} & 2c_0 & c_{-1}+c_1 \\ 0 & c_1-c_{-1} & c_{-2}+c_2 & 2c_3 & c_{-2}+c_2 & c_{-1}+c_1 & 2c_0 \end{bmatrix} \tag{9}$$

Also, the vector c has the following general structure

$$c = \begin{bmatrix} c_{-n} & c_{-(n-1)} & \cdots & c_0 & \cdots & c_{n-1} & c_n \end{bmatrix}' \tag{10}$$

where n is the number of terms in the series used for approximation.

III. Parameter Estimation for Linear Periodically Time-Varying Systems

Any given differential equation of degree n can be expressed as a set of state-space equations. This same set can be transformed into an algebraic set of equations using operational matrices. In this section, a methodology to solve the identification problem and estimate the unknown parameters using the input-output data is presented [14] [15].

A. Identification Algorithm

Linear periodically time-varying systems can be described in general sense by the following set of state-space equations

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{11}$$

In this case, $x(t) \in \mathbb{R}^{n\times 1}$ and $u(t) \in \mathbb{R}^{q\times 1}$ are the state and input vectors respectively, and $A(t) \in \mathbb{R}^{n\times n}$ and $B(t) \in \mathbb{R}^{n\times q}$ are the corresponding periodic coefficients with the following properties considering that $k \in \mathbb{Z}$

$$\begin{aligned} A(t) &= A(t) \text{ for } kT \le t \le (k+1)T \\ B(t) &= B(t) \text{ for } kT \le t \le (k+1)T \end{aligned} \tag{12}$$

Using function approximations via orthogonal series, the expressions for the elements $a_{ij}(t)$ and $b_{ij}(t)$ of matrices $A(t)$ and $B(t)$ that satisfy the Dirichlet conditions in the interval $(0,T)$, can be expressed as

$$a_{ij}(t) \cong A_{ij}\Phi(t) \qquad b_{ij}(t) \cong B_{ij}\Phi(t) \tag{13}$$

and the matrices A_{ij} and B_{ij} of equation (13), have the following structure

$$\begin{aligned} A_{ij} &= \begin{bmatrix} A_{-n} & \cdots & A_0 & \cdots & A_n \end{bmatrix} \\ B_{ij} &= \begin{bmatrix} B_{-n} & \cdots & B_0 & \cdots & B_n \end{bmatrix} \end{aligned} \tag{14}$$

Similarly, the elements of vector $x(t)$ and $u(t)$ can be approximated as

$$x_i(t) \cong X_i\Phi(t) \quad u_i(t) \cong U_i\Phi(t) \tag{15}$$

With this in mind, the orthogonal series of the product $A(t)x(t)$ is

$$A(t)x(t) = \begin{bmatrix} A_{11}\Phi(t)X_1\Phi(t) + \cdots + A_{1n}\Phi(t)X_n\Phi(t) \\ A_{21}\Phi(t)X_1\Phi(t) + \cdots + A_{2n}\Phi(t)X_n\Phi(t) \\ \vdots \\ A_{n1}\Phi(t)X_1\Phi(t) + \cdots + A_{nn}\Phi(t)X_n\Phi(t) \end{bmatrix} \tag{16}$$

where every factor $A_{ij}\Phi(t)X_j\Phi(t)$ may be rearranged using the product and coefficient matrix as follows

$$A_{ij}\Phi(t)X_j\Phi(t) = A_{ij}\Phi(t)\Phi'(t)X_j'(t) = A_{ij}\left[X_j'\right]\Phi(t) \tag{17}$$

Matrix $[X_j'] \in \mathbb{R}^{m\times m}$ is the coefficient matrix of the vector $x(t)$ where $m = 2n+1$. It may be shown that with the above simplification, the product $A(t)x(t)$ may be written as

$$A(t)x(t) = \underbrace{\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}}_{\overline{A}} \underbrace{\begin{bmatrix} [X_1'] \\ [X_2'] \\ \vdots \\ [X_n'] \end{bmatrix}}_{[\overline{X}]} \Phi(t) \tag{18}$$

in other words

$$A(t)x(t) = \overline{A}\left[\overline{X}\right]\Phi(t) \tag{19}$$

The matrix $[X_j']$ is the coefficient matrix corresponding to each coefficient, $[\overline{X_j}]$ is the matrix formed by all the coefficient matrices $[X_j']$. A is formed by every vector $[A_{ij}]$ and $\Phi(t)$ is the basis vector. A similar procedure is used for $B(t)x(t)$, yielding

$$B(t)u(t) = \overline{B}\left[\overline{U}\right]\Phi(t) \tag{20}$$

where $B(t) \in \mathbb{R}^{n\times q(m)}$, $U(t) \in \mathbb{R}^{q(m\times m)}$ and $\Phi(t) \in \mathbb{R}^{(m\times 1)}$. Integrating in both sides (11), the following is obtained

$$x(t) - x_0(t) = \int_0^t A(t)x(t)\,dt + \int_0^t B(t)u(t)\,dt \tag{21}$$

Using operational matrices and substituting (19) and (20) into (21) yields

$$X\Phi(t) = X_0\Phi(t) + \overline{A}\left[\overline{X}\right]\overline{P}\Phi(t) + \overline{B}\left[\overline{U}\right]\overline{P}\Phi(t) \tag{22}$$

The matrix X_0 is the same size of X and contains the initial conditions of every state.

$$X_0 = \begin{bmatrix} 0 & \cdots & x_1(0) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & x_n(0) & \cdots & 0 \end{bmatrix} \tag{23}$$

Equation (22) can be rewritten as

$$X = \theta Z \tag{24}$$

therefore, the matrices in (24) are defined in the following way

$$\theta = \begin{bmatrix} A & B & X_0 \end{bmatrix} \quad Z = \begin{bmatrix} \left[\overline{X}\right]\overline{P} & \left[\overline{U}\right]\overline{P} & e \end{bmatrix}' \tag{25}$$

$$e = \begin{bmatrix} 0 & \cdots & 1 & \cdots & 0 \end{bmatrix} \tag{26}$$

Using the above equations and solving for the unknown parameters we obtain [14]:

$$\theta = XZ'\left(ZZ'\right)^{-1} \tag{27}$$

From the equation (27) the parameter coefficients can be obtained provided that $(ZZ')^{-1}$ exists.

B. Analytical Redundancy

In dynamic systems, it is common to find physical redundancy, for example in the implementation of several devices of the same characteristics within the same measurement and control system. Another example consists in the use two or more sensors to anticipate every eventuality before a failure that causes system instability occurs.

An alternative to this, that is often more economical, is the application of analytical redundancy. This kind of redundancy involves the ability to run multiple experiments on the system to identify; the use of more than one input enables the excitation of the different operation modes in the system for every experiment. Some other options include the application of the same signal at different frequencies and changing the phase and/or magnitude. Subsequently, all this information is used for parameter estimation, as shown in Fig. 1.

One restriction about this procedure is that the identification has to be done off-line. There are systems that are better suited for off-line identification than others, and this depends entirely on the purpose and behavior of the application.

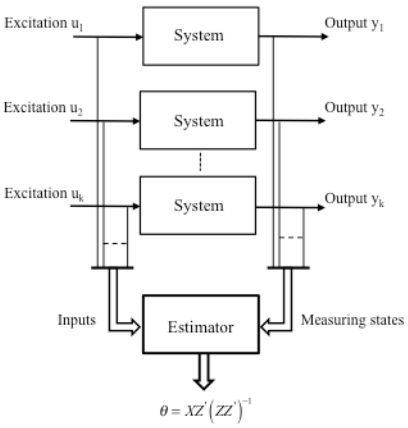

Fig. 1. Analytical redundancy scheme.

IV. Applications

A simple theoretical second order periodic system is used as an introductory example in order to illustrate the methodology described in section III. Then, the application of the procedure is shown using the equations of the dynamics of a helicopter main rotor in forward flight.

A. Second Order Periodic System

Consider a second order linear system with a periodic coefficient as described in [16].

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -f_1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{28}$$

where f_1 is a system with periodically varying parameters given by

$$f_1(t) = 1 + \sin(5\omega t) \tag{29}$$

The system described by (28) is excited by a periodic input $u(t) = sign(\sin(\omega t))$ with $\omega = 2\pi$. In order to perform the parametric identification of the system, (27) was used considering only the first 15 terms of the Hartley series and an analytical redundancy of 3. The system was excited off-line, with the purpose of having simulation data corresponding to the input-output behavior, with the following signals

$$u_1(t) = sign\left(\sin\left(\omega t\right)\right)$$

$$u_2(t) = 1.2\cos\left(3\omega t\right) + 0.5\cos\left(5\omega t\right) \tag{30}$$

$$u_3(t) = 1 + 0.2\cos\left(3\omega t\right) + 0.05\cos\left(5\omega t\right)$$

The simulation results obtained with the help of Matlab are shown in Figures 2 and 3. In these figures, the parametric identification of matrix $A(t)$ is displayed when using only 15 terms of the Hartley series, the differences between the estimated and real values can seen. Fig. 4 shows the results for the constant coefficients of matrix $B(t)$

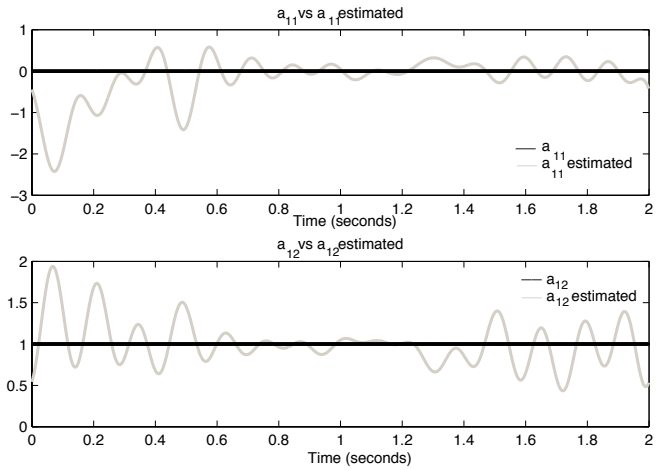

Fig. 2. Parameter estimates of $a_{11}(t)$ and $a_{12}(t)$ with n = 15.

In Fig. 2, the identification of the constant parameters in the system is shown, and a considerable error between the estimated and real values is observed. In this figure, 15 terms of the series were taken into account and it can be seen that about the first second of evolution in the system the error decreases.

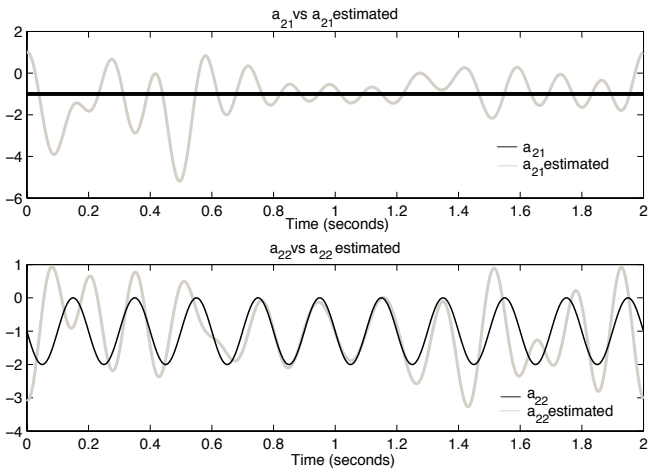

Fig. 3. Parameter estimates of $a_{21}(t)$ and $a_{22}(t)$ with n = 15.

In the other hand, Fig. 3 shows the identification of the only time-varying and periodic parameter of the system. As in Fig. 2, an error at the first part of the evolution can be observed for both parameters, then the error decreases significantly after approximately 0.8 seconds and stays in that level until 1.2 seconds.

Fig. 4 consists in the identification of the two constants in matrix B and also an error is present through the evolution of the system. Showing the same pattern as in figures 2 and 3, the error decreases substantially after the transitory effect of the identification test and maintains this level before increasing again after 1.2 seconds.

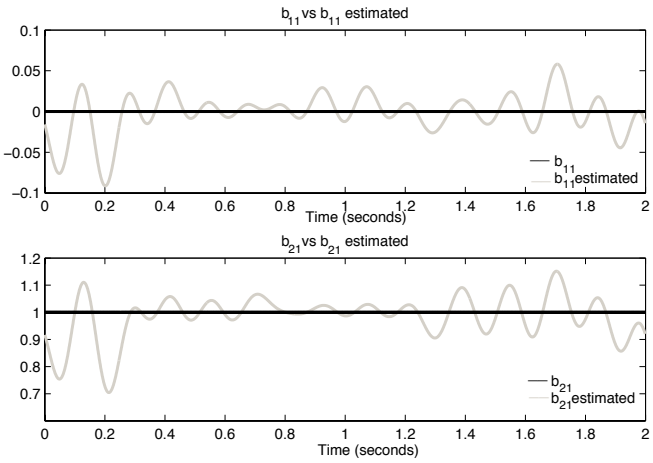

Fig. 4. Parameter estimates of $b_{11}(t)$ and $b_{21}(t)$ with $n = 15$.

With the purpose of improving the approximation between the estimated and the real parameter values, another test was performed considering 35 terms of the Hartley series.

Figures 5, 6 and 7 show the improvements of the identification when adding terms in the approximation process. In this case, the varying nature of the $a_{22}(t)$ parameter is sinusoidal; therefore, the use of Hartley series for parametric identification is adequate since a low number of terms are used in the estimation.

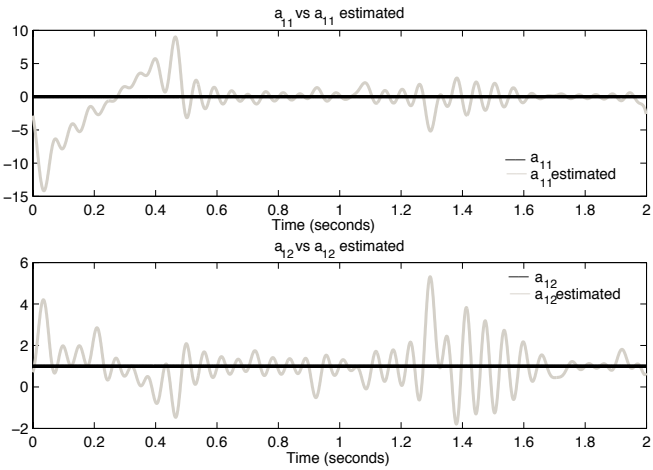

Fig. 5. Parameter estimates of $a_{11}(t)$ and $a_{12}(t)$ with $n = 35$.

In Fig. 6, the reduction in the error level can be seen in comparison with the approximation made with only 15 terms in the Hartley series, especially in parameter $a_{22}(t)$, where the identification is closer to the real value for a wider window of time compared to Fig. 3. (with $n = 15$).

From these results, we can conclude that the fact of adding terms to the Hartley series decreases the error level in the approximation. Also, a small increase in series terms leads to a considerable reduction in the error of the identification of time-varying parameters.

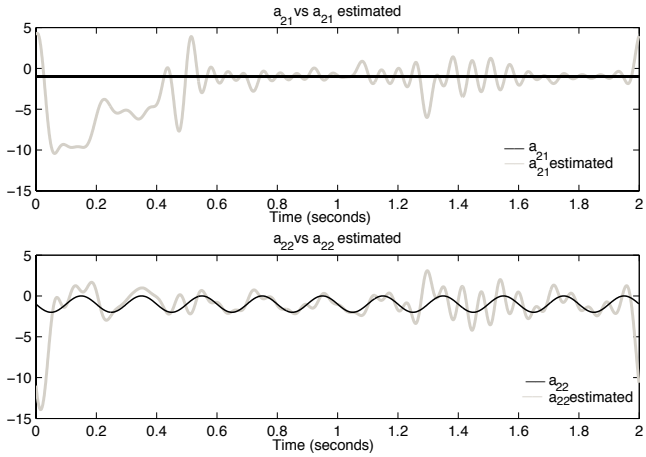

Fig. 6. Parameter estimates of $a_{21}(t)$ and $a_{22}(t)$ with $n = 35$.

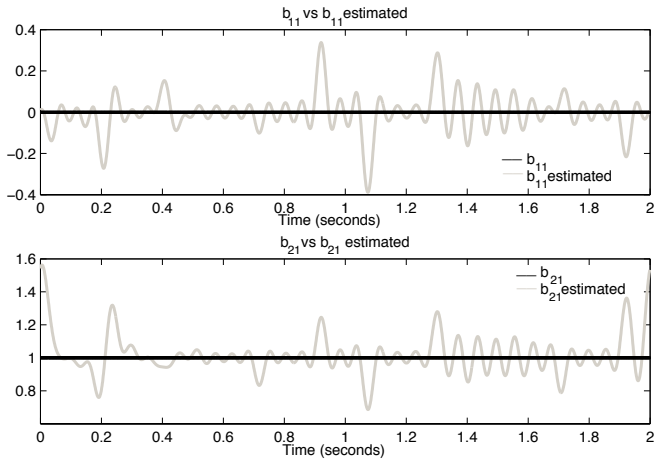

Fig. 7. Parameter estimates of $b_{11}(t)$ and $b_{22}(t)$ with $n = 35$.

B. *Helicopter Main Rotor Model*

The equations that describe the dynamics in frontal flight of a helicopter main rotor are presented in [17]-[18]. The model for one of the blades can be written in the form of a linear periodic system, and has the following form

$$\dot{x}(\psi) = A(\psi)x(\psi) + B(\psi)u(\psi) \tag{31}$$

Each matrix in (31) is described by

$$A(\psi) = \begin{bmatrix} 0 & 1 \\ -\left\{1 + \frac{\gamma}{8}\left(\frac{4}{3}\mu\cos\psi + \mu^2\sin 2\psi\right)\right\} & -\frac{\gamma}{8}\left(1 + \frac{4}{3}\mu\sin\psi\right) \end{bmatrix} \tag{32}$$

$$A(\psi) = \begin{bmatrix} 0 \\ \frac{\gamma}{8}\left(1 + \mu^2 + \frac{8}{3}\mu\sin\psi + \mu^2\cos 2\psi\right) \end{bmatrix} \tag{33}$$

$$x(\psi) = \begin{bmatrix} \beta \\ \dot{\beta} \end{bmatrix} \tag{34}$$

The value $\mu = 0.3$ represents the advance ratio of the helicopter and $\gamma = 8$ is the Lock number. In this case,

$\psi = \omega t$, and ω is the angular velocity. The first state variable β represents the angle of the blade with respect to the rotation plane and it is shown in Fig. 8.

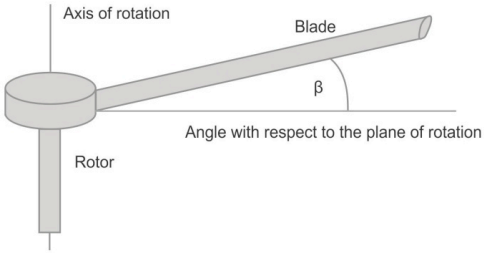

Fig. 8. Main rotor simplified diagram.

In order to illustrate the identification procedure of LPTV systems, the aforementioned rotor model (31) is used. The analytical redundancy value used in the simulation was 4, and the off-line applied signals that were used to obtain input-output measurements were

$$
\begin{aligned}
u_1(t) &= 1 + 0.1\cos(5t) \\
u_2(t) &= 0.8\cos(2t) + 0.1\cos(5t) \\
u_3(t) &= 3t + 0.1\cos(3t) \\
u_4(t) &= 1.1\cos(t) + 0.2\sin(3t) + 0.15\cos(3t) + 0.1\sin(5t)
\end{aligned}
\tag{35}
$$

As in the previous theoretical system described by (28), 15 and 35 terms of the Hartley Series are used to perform the identification procedure. Four different signals generate the set of measurements (state $x(t)$ and input $u(t)$) when the analytical redundancy value is 4.

The algorithms to identify the periodic parameters in the helicopter main rotor model were developed in Matlab. The results obtained from comparing the estimated vs. real parameters of matrices $A(t)$ and $B(t)$ can be seen in figures 9, 10, and 11, when the number of terms in the series is $n = 15$.

Fig. 9 shows the identification made on the constant parameters of matrix $A(t)$. In this case, the error when approximating constants in the helicopter model is substantially less than the one found in the previous example of a theoretical second order system (Fig. 2).

Also, in Fig. 10, both time-varying parameters of matrix $A(t)$ are approximated, and an error between the estimated and real values is present. This error is reduced after the second operation cycle and stays in that level for both parameters.

Finally, in Fig. 11, the comparison is made for the parameters of matrix $B(t)$ between the estimated and real values. From this figure it can be seen that the error got reduced significantly for almost the entire time span, considering that only 15 terms were used in this case.

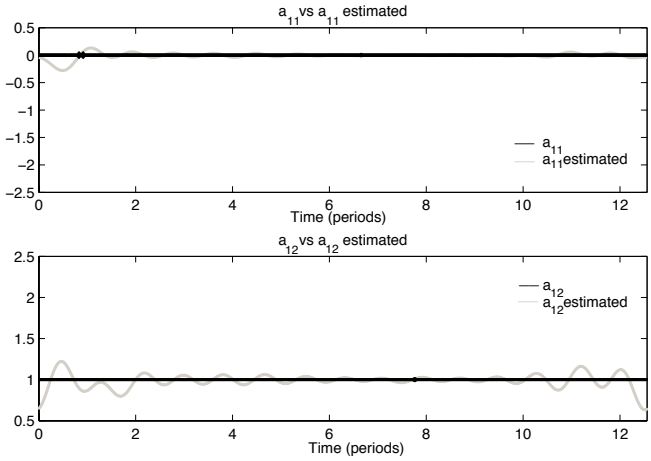

Fig. 9. Parameter estimates of $a_{11}(t)$ and $a_{12}(t)$ with $n = 15$.

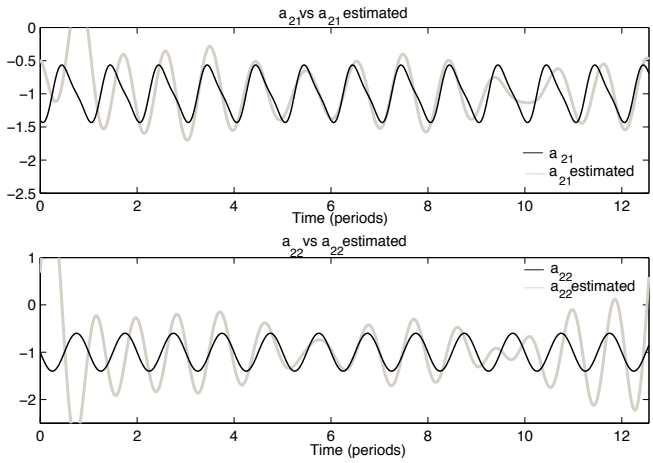

Fig. 10. Parameter estimates of $a_{21}(t)$ and $a_{22}(t)$ with $n = 15$.

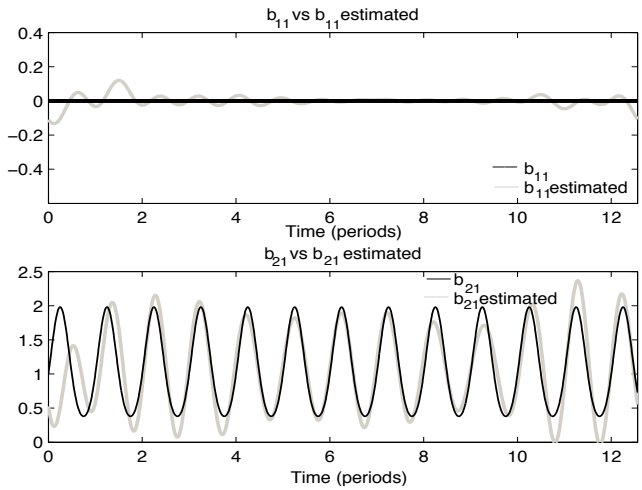

Fig. 11. Parameter estimates of $b_{11}(t)$ and $b_{21}(t)$ with $n = 15$.

Figures 12, 13 and 14 show the estimated parameters for matrices $A(t)$ and $B(t)$ when $n = 35$. Again, the correlation between the real values and the estimated ones, for both invariant and periodically variant parameters, can be seen.

In Fig. 12, the approximation over the constants of matrix $A(t)$ is presented. After the transitory effect, the estimated parameter tends to follow the real value with a small error and this behavior is maintained for the rest of the evolution of the system.

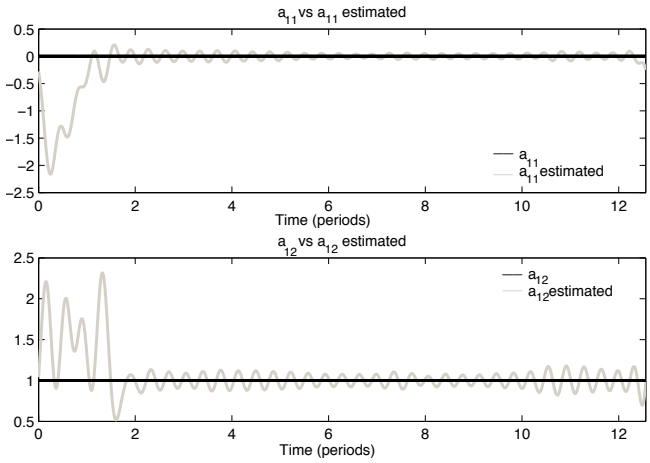

Fig. 12. Parameter estimates of $a_{11}(t)$ and $a_{12}(t)$ with $n = 35$.

The real advantage of this procedure can be seen in Fig. 13 where the time-varying part of matrix $A(t)$ is shown. Once the transitory effect is over, an almost exact reproduction of the periodic parameters is achieved and held over the entire simulation.

Thus, the conclusion of having an effective methodology for the identification of periodic parameters can be stated, having in mind that only 35 terms of the Hartley series were used.

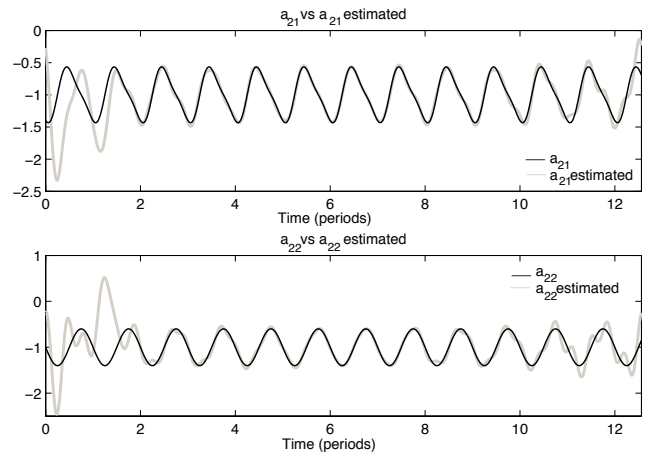

Fig. 13. Parameter estimates of $a_{21}(t)$ and $a_{22}(t)$ with $n = 35$.

The real value of the periodic parameter of matrix $B(t)$ is also matched exactly by the estimation made with this identification methodology. This can be observed in Fig. 14. The constant part of this matrix still has a small error around the real value.

In Fig. 15, the absolute error of the periodic parameters in matrix $A(t)$ is displayed. From this graphic, it can be seen that the error between estimated and real values decreases drastically after the transitory effect of the identification test and when the number of terms used in approximation increases. This contributes to show the effectiveness of this procedure that is based heavily on the use of orthogonal series, operational matrices and their properties. The Hartley series selection over rest of the available orthogonal series is based on its kernel, which has a natural periodic behavior.

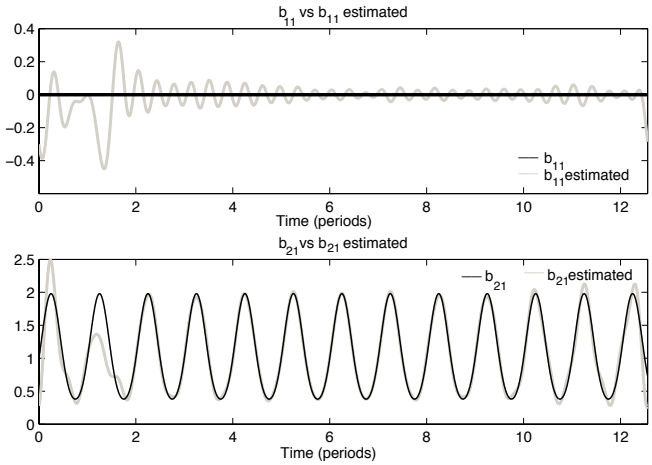

Fig. 14. Parameter estimates of $b_{11}(t)$ and $b_{21}(t)$ with $n = 35$.

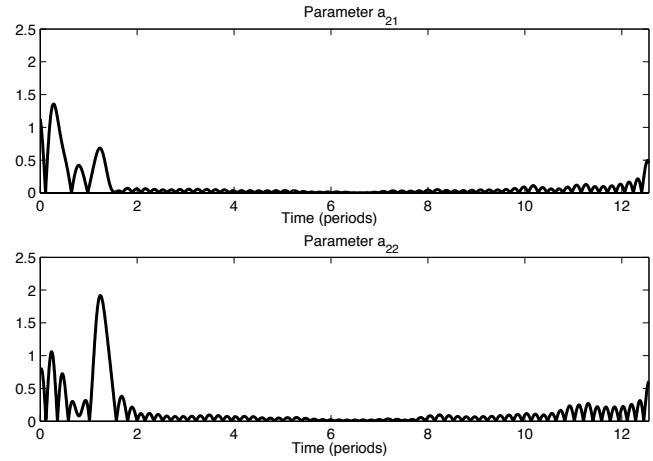

Fig. 15. Absolute error of $a_{21}(t)$ and a $a_{22}(t)$ with $n = 35$.

It is worth mentioning that not necessarily all the terms in the orthogonal series that are used in the identification process have great significance. For instance, the estimated parameter $a_{22}(t)$ has only 15 terms that are significant; the values of the rest of the terms are less than 10% of the Hartley series coefficient of higher magnitude. Therefore, to estimate the parameter, only an expansion in orthogonal series of 15 out of 35 terms is needed.

V. Conclusions

A frequency domain technique for off-line parameter estimation of LPTV systems has been presented. The operational matrices tools and the integration matrix in particular, are used to yield algebraic equations that allow the parameter identification procedure to take place. These algebraic equations can be solved through the use of a pseudoinverse.

The presented procedure is applied to two different cases using Hartley series as the basis function. These properties are used to estimate the parameters of a second order theoretical system with only one time-varying parameter, and also of the model that describes the dynamics of the main rotor of a helicopter in forward flight.

From the comparison made between the estimated and real parameters, for both cases, it can be seen that the average error decreases when the number of terms used in the series is increased. It is also important to notice that the number of terms to be considered in the estimation process can be reduced if only the parameters with higher magnitudes are taken into account.

Another important fact that is worth mentioning is that the selection of the Hartley series is especially convenient for the identification of periodic systems due to the time-varying behavior of its kernel.

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