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An Optimal Power Distribution Algorithm for Two Time Slot Cooperative Diversity Protocols

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***Abstract*—We consider the general cooperative diversity protocols where transmission takes place in orthogonal two time slots. In first slot the transmitting node broadcasts, and in second slot some or all cooperating nodes, depending upon the strategy employed, transmit to destination. This letter proposes an algorithm for distributing the total broadcasting power in orthogonal time slots of a cooperative relaying protocol. This time slot based distribution of power constitutes the initial level of the two level power allocation strategy. The power allotted for the second slot can further be shared by the resources operating in this time slot using some available power allocation schemes. We consider that the total power to be broadcasted is constant, and compared the equal power distribution strategies of relay power allocation with and without our proposed algorithm.**

***Index Terms*—cooperative diversity, Nakagami-m fading, half duplex transmission.**

I. INTRODUCTION

THe vision of wireless communications supporting information exchange between people or devices is the communications frontier of the next few decades, and much of it already exists in some form. In digital wireless communication systems, information is transmitted through radio channels [1]. Radio-wave propagation through wireless channels is a complicated phenomenon characterized by various effects, such as multipath and shadowing. The transmitted signals are reflected and scattered, arriving at the receiver along multiple paths. They add constructively if these paths have similar delays, or destructively if path delays are different. This gives rise to fading. A large variety of fading channel models typical of communication links have been discussed in detail in [2]. In order to combat the adverse effects of fading, a technique known as *diversity* has been developed. Diversity relies on the use of multiple copies of the same signal, which the receiver can combine or select from [3], [4]. The idea behind is that, even if one copy of the signal is of poor quality, it is unlikely that all the copies will be so. This redundancy allows the communication quality to be maintained.

To provide transmit diversity in a system which cannot support multiple antennas due to size, power, complexity or any other constraint, cooperation among single antenna users to form a virtual array of antennas was suggested in [5], [6], as an efficient means. The fundamental idea was to use the resources of other nodes (relays or users) for transmission, in addition to direct source to destination transmission. These other nodes were later on described as the cooperative agent for source to destination transmission. Thus a new paradigm of cooperation at physical layer, in wireless communication started on, however the word *cooperation* in other layers can be found as in [7], but the approach and methodologies are often different from the concepts of cooperation referred here.

In cooperative diversity systems where orthogonal channels are used, the destination node receives two (or more) independently faded copies of from source and relay (or relays) either in one time slot with two frequencies, or in two time slot using single frequency band. In general orthogonal two time slot approach has been used for different protocols [8]- [11]. The energy and power efficiency of cooperative networks has been studied in [12]- [14]. In [15], author investigated the joint optimization of the power allocation and relay location under transmit power constraint in order to minimize outage and error probabilities. Most of the earlier works [15]- [19], explored the problem of power allocation assuming that only one relay is assisting the cooperative transmission. In [19], Phuyal et. al. discussed an optimal power allocation method, and explored the scenario of multiple amplify-and-forward (AF) relays. However, they assumed that source node transmits with a fixed power in first time slot, and the same amount of power is shared by all the relays in second time slot.

In this letter, we propose an algorithm which optimally distributes the power between the nodes on the basis of timing of their transmissions. We consider that the total power i.e. the power transmitted by source in first time slot, plus the power transmitted by relays in second time slot is kept constant. The flexibility with proposed scheme lies in the fact that the existing power allocation schemes may be separately employed for the distribution of power among the nodes operating in the second time slot. Further, the efficiency of this scheme is shown in terms of average symbol error rate (SER) for a multiple AF relays system over generalized Nakagami-m fading channels. We obtained an upper bound to error probability in case of M-ary phase shift keying (M-PSK) modulation, and compare the error performance of proposed scheme with equal power broadcasting [8]. Furthermore, we have obtained an optimal power distribution ratio which minimizes the average SER.

II. POWER DISTRIBUTION ALGORITHM

Problem statement: We distribute the total power to be broadcasted P, in a manner such that first time slot gets some fraction (say α_{tp}) of it i.e. $\alpha_{tp}P$ and remaining power goes to second slot transmission i.e. (1-α_{tp})P, where α_{tp} is transmitted power distribution ratio. Now we have to obtain an optimal α_{tp} so as to minimize average SER of a given system model.

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A. System Model

We have taken a simple model which consists of a source (s), destination (d) and multiple relays ($r_1, r_2, ..., r_n$), where ($n > 1$). The half duplex transmission is taking place in two phases, (i) source broadcasts, and all the relays and destination will listen to it, and (ii) each relay broadcasts the amplified version of the information, it received, to destination. The channel coefficients between source and destination f, source and i^{th} relay h_i, and that between ith relay and destination g_i, are assumed to be flat and distributed as Nakagami-m with parameters (m_f, Ω_f), (m_{h_i}, Ω_{h_i}), and (m_{g_i}, Ω_{g_i}) respectively. Here $\{i \in 1, ..., n\}$. At destination we employ maximum ratio combining (MRC), to detect the received information.

Now we know that In the first time slot source broadcasts with power $P_s = \alpha_{tp} P$ to all the relays and destination. In second slot from $P_R = (1 - \alpha_{tp})P$ power, the i^{th} relay transmits to destination the amplified versions of the signals it received in first phase, with power P_{r_i}. So,

$$P_R = \sum_{i=1}^{n} P_{r_i}, \tag{1}$$

The key idea is to distribute the total available power among the source node and cooperating relay node in a way to achieve the required system performance under given channel conditions. We further define transmitted power distribution ratio (α_{tp}) as

$$\alpha_{tp} = \frac{P_s}{P}. \tag{2}$$

this ratio (α_{tp}), signifies the fraction of total power transmitted by the source. In other words, the power transmitted by each of n relays, will be $\{(1 - \alpha_{tp})/n\}$ times the total transmitted power. The instantaneous received powers over various links are given as

$$\begin{aligned} P_f &= \mid f \mid^2 \alpha_{tp} P, \\ P_{h_i} &= \mid h_i \mid^2 \alpha_{tp} P, \\ P_{g_i} &= \mid g_i \mid^2 \frac{(1 - \alpha_{tp})}{n} P, \end{aligned} \tag{3}$$

where P_f is the instantaneous received power over source to destination link, P_{h_i} is the instantaneous received power over source to i^{th} relay link, and P_{g_i} is the instantaneous received power over i^{th} relay to destination link. Assuming maximum ratio combining at destination, the total received power can be given, as in [21], as

$$P_{eq} = P_f + \sum_{i=1}^{n} \frac{P_{h_i} P_{g_i}}{1 + P_{h_i} + P_{g_i}}. \tag{4}$$

to reduce the complexity, (4) can be approximated by its upper bound P_{ub} as

$$P_{eq} \leq P_{ub} = P_f + \sum_{i=1}^{n} P_i. \tag{5}$$

where

$$P_i = min(P_{h_i}, P_{g_i}).$$

This kind of approximation can be shown to be accurate enough at medium and high values of instantaneous power. Analytically the upper bound given in (5) is more tractable than the exact value as in (4).

III. ERROR ANALYSIS

In this section, the upper bound on the error performance is derived. We assume all the instantaneous powers (P_f, P_{h_i}, and P_{g_i}) to be independent to each other. The MGF of P_{ub} can be written as

$$M_{P_{ub}}(s) = M_{P_f}(s) \prod_{i=1}^{n} M_{P_i}(s). \tag{6}$$

where $M_{P_f}(s)$ and $M_{P_i}(s)$ are the MGF of P_f and P_i respectively. The MGF is defined as $M_P(s) = \mathbf{E}(e^{-sP})$, where $\mathbf{E}$ represents the expectation operator.

If we assume n relays to belong to two sets θ_1 and θ_2 such that for the relays belong to set θ_1, P_i as in (**??**) can be given as, $P_i = P_{h_i}$, and for the relays belong to set θ_2, $P_i = P_{g_j}$, where $i \neq j$ or the sets θ_1 and θ_2 are disjoint, $M_{P_{ub}}(s)$ given in (6) can be modified as,

$$M_{P_{ub}}(s) = M_{P_f}(s) \prod_{i \in \theta_1} M_{P_{h_i}}(s) \prod_{j \in \theta_2} M_{P_{g_j}}(s), \tag{7}$$

Now to further simplify (7), we assume $(m_{h_i}, \Omega_{h_i}) = (m_h, \Omega_h)$, and $(m_{g_i}, \Omega_{g_i}) = (m_g, \Omega_g)$, which imply that all the source to relay links have the same fading characteristics, and so do all the relay to destination links. This assumption is generally applicable in most of the practical scenarios. We further assume that the cardinality of set θ_1 is k and that of set θ_2 is $(n - k)$. Now (7) can be written as,

$$\begin{aligned} M_{P_{ub}}(s) = {} & \left(1 - \frac{\Omega_f}{m_f} \alpha_{tp} P s\right)^{-m_f} \left[\left(1 - \frac{\Omega_h}{m_h} \alpha_{tp} P s\right)^{-m_h}\right]^k \\ & \times \left[\left(1 - \frac{\Omega_g}{m_g} \frac{(1 - \alpha_{tp})}{n} P s\right)^{-m_g}\right]^{n-k} \end{aligned} \tag{8}$$

Using $M_{P_{ub}}(s)$ given in (8), we can write the upper bound for the SER as [2, Eq. (9.27)],

$$\begin{aligned} P(e) = {} & \left(\frac{M-1}{M}\right) \left(1 + \frac{\Omega_f}{m_f} \alpha_{tp} P \omega\right)^{-m_f} \left(1 + \frac{\Omega_h}{m_h} \alpha_{tp} P \omega\right)^{-k m_h} \\ & \times \left(1 + \frac{\Omega_g}{m_g} \frac{(1 - \alpha_{tp})}{n} P \omega\right)^{-(n-k) m_g} \end{aligned} \tag{9}$$

where $\omega = \sin^2(\frac{\pi}{M})$. This is the generalized upper bound for the average symbol error rate (SER). The bound given in (9), however, is not asymptotically tight [2, p.326], a more tighter upper bound can be obtained as in [2, Eq. (9.30)].

To obtain the optimal value of power distribution ratio (α_{tp_o}), at which SER is minimum, we differentiated (9) with respect to α_{tp}. To get a closed form expression of α_{tp_o} we assume that all the links are identical, however the distribution parameters may take any value. We get

$$\alpha_{tp_o} = \left[\frac{\frac{(n+1)km}{\Omega P \omega} + (k+1)}{n+1}\right], \tag{10}$$

the validity of (10) is only in the region where $0 < \alpha_{tp_o} < 1$

IV. NUMERICAL RESULTS

We have assumed inter-independency of all the links and assumed binary phase shift keying (BPSK) i.e. $M = 2$, as the modulation scheme. Fig. 1 shows the comparison of proposed power distribution scheme with conventional power distribution scheme, in which we consider the equal distribution of power i.e. $P_s = P_{r_i} = \frac{P}{n+1}$. We can observe that the proposed power ratio based distribution of power is more efficient than equal distribution, even in case when very less power is transmitted in the first phase transmission.

The dependence of SER on number of cooperating relays can be observed from Fig. 2. Intuitively, the error performance is improving as number of relays are increasing. The value of power ratio say α_{tp_o}, at which average symbol error probability gets its minimum for a given number of relays n, is also decreasing as n is going higher i.e. for $n = 1, 2, 3$ the values of α_{tp_o} are $0.5, 0.35, 0.25$ respectively. It means, with more number of relays to cooperate, power broadcasted by source in first phase reduces. These results can easily be verified mathematically by obtaining the point of minima.

Fig. 3 shows the dependence of average SER on the channels conditions i.e. parameters m_f, m_h, and m_g. The results are intuitive in sense error performance is improving with the improvement in channel conditions. Furthermore the improvement in error performance is comparatively more dependent on m_h, i.e. on the channel conditions of source to relay link. In Fig. 4, SER is plotted with power ratio for different values of m_h, m_f, m_g. It can be observed that the improvement in the source to destination channel conditions increases the value of α_{tp_o}, which signifies the more power to be given to the first phase transmission. Similarly, improvement in the relays to destination channel conditions decreases the value of α_{tp_o}, which signifies the more power to be given to the second phase transmission.

V. CONCLUSIONS

In this letter we proposed an algorithm for distributing the total broadcasting power in orthogonal time slots of a cooperative relaying protocol. We considered that the total power to be constant. The flexibility with proposed scheme lies in the fact that the different power allocation schemes may be separately employed for the distribution of power in the second time slot. The proposed algorithm has been found to be more efficient than the conventional equal power distribution through its SER performance.

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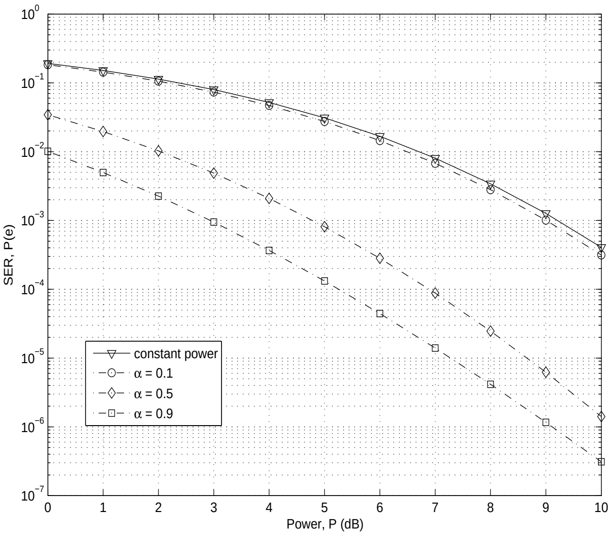

Fig. 1. Comparison of proposed power distribution scheme with equal power distribution scheme.

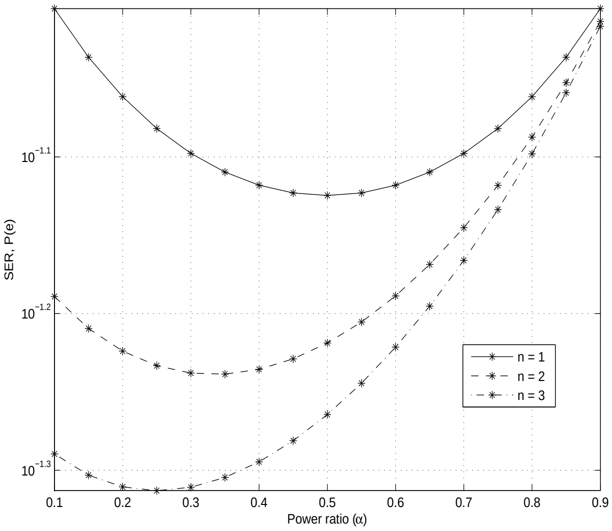

Fig. 2. Dependance of SER on number of relays, n.

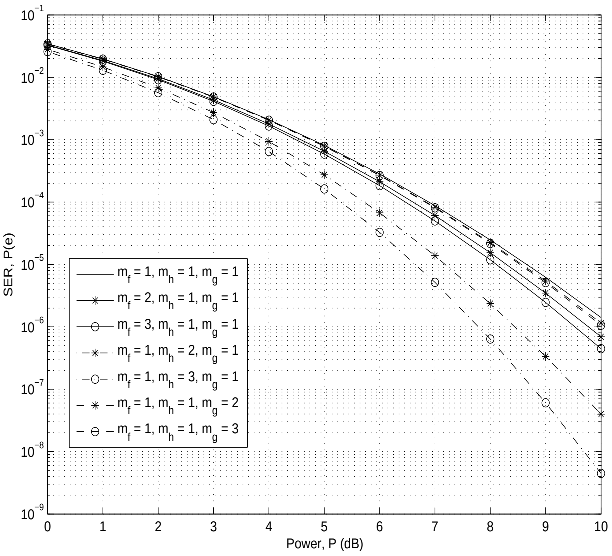

Fig. 3. Effect of channel statistics over error performance.

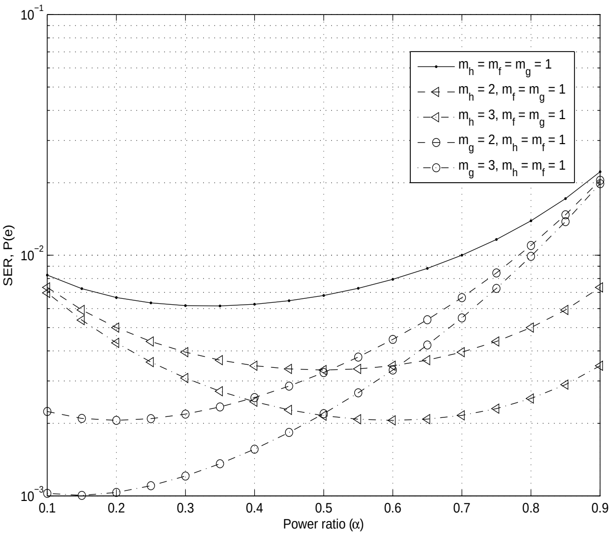

Fig. 4. Variation of SER with power ratio for different channel statistics.

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