Optimisation of Stochastic Multi-item Manufacturing for Shareholder Wealth Maximisation

X. J. Wang, S. H. Choi

Abstract—**Lot-sizing is pivotal to manufacturing, especially in stochastic multi-item batch production. Although its criticality has been generally acknowledged in operation management, most optimisation approaches advanced in recent years are often rendered impractical and sometimes misleading. This is because little attention has been paid to the real corporate capital structure and the overall business goal of maximising the shareholder wealth. In this paper, we attempt to optimise stochastic multi-item lot-sizing for make-to-order manufacturing in a complex yet realistic capital structure, with an aim to realize the overall business objective of maximising the shareholder wealth. Our study takes account of a series of interconnected economical parameters pertinent to maximisation of the shareholder wealth, and also examines the impacts of corporate capital structures on shareholder wealth advancement. Computational studies are presented to numerically and analytically demonstrate the important implications of our proposed manufacturing model on gaining corporate wealth in a practical capital structure.**

Index Terms — **stochastic, lot sizing, queuing, shareholder wealth, capital structure**

I. Introduction

Timely provision of quality products at the lowest prices possible has become the utmost competitive edge being pursued by virtually all businesses. Firms endeavour to speed up their manufacturing and delivery of goods or provision of services to end customers. It was, however, estimated that only less than 15% of workshop time is spent on the actual processing of a job, while over 85% is wasted in work-in-process (WIP) and queuing delays [1]. As a result, manufacturing time optimisations have been one of the key research mainstreams in this area [2, 3].

Despite the wide popularity of the manufacturing time minimisation in operation management, the optimising results are often unrealistic, leading to many technical difficulties in applying them to real businesses, because their economic factors and financial positions have not been duly considered in operation optimisation. Some researchers seek to address this problem by choosing to optimise certain economic objectives, instead of the operational ones. Most of these economic optimisation objectives are targeted at optimising some accounting costs or profits. Ref. [4], for example, developed a cost minimisation model with several relevant costs taken into account. Ref. [5] chose to maximise the accounting profit in a multi-product capacity-constrained lot sizing environment.

However, either minimisation of costs or maximisation of profits, in general, may not necessarily represent the long-term full interest of owners of business, especially in some adverse economic situations, such as unexpected inflations and recessions in a business cycle. Indeed, the core mindset of modern corporate governance of most businesses in competitive markets is to maximise the interest of their stakeholders, especially their equity holders [6, 7]. It is the long-term sustainable interests of firms' owners, well-known as the shareholder wealth, that have currently become the top priority of most enterprises [8-11].

Thus, it can be seen that most production optimisation models either overlook a firm's economic conditions and financial position, or optimise some short-term and local objective functions without considering the overall business goal of maximising its shareholders' long-term sustainable interests. Moreover, some key macroeconomic factors, such as impacts of inflation and business cycle on optimisation, have not been taken into account. In this paper, we attempt to address these problems by setting up a stochastic queuing network for the concerned multi-item make-to-order manufacturing, with an aim to maximise the shareholder wealth. Our proposed approach is characterised in the following aspects.

A. Long-term Sustainable Profitability

In this paper, we aim to optimise the long-term sustainable profitability in terms of the purchasing power, instead of the traditional short-term or local optimisation objectives, for better representation of the full interests of the firm's shareholders.

As stated previously, the objective functions of most current optimisation approaches are short-term or local, rather than of the long-term, global perspective. For example, Ref. [12] chose to minimise the weighted expected lead time for a stochastic multi-operation, multi-item job shop under a make-to-order environment; Ref. [13] focused on an M/G/1 lot sizing model with an aim to minimise a weighted average of the queuing time, setup cost and inventory cost of finished goods; and Ref. [14] examined the cost minimisation

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problem with the research focus on the cycle time and product volume.

Although these works may be somewhat useful for operation management, the optimisation objectives do not necessarily align with the overall business goal of pursuing the maximum long-term sustainable profitability by maximising the shareholder wealth [9, 10, 15]. In some cases, improper choices of objective functions may even lead to undesirable consequences. This type of discrepancy between practical requirements and academic research has recently received considerable attentions. Indeed, the research focus, to some extent, has recently been shifted to the long-term interests of shareholders from the original short-term or local optimisation objectives. For instance, Ref. [16] has attempted to minimise the annualised capital investment plus cash/material inventory minus the benefit to shareholders for an integral production plant model, taking the relevant financial decisions into account. Ref. [17] derived a holistic model for the short-term supply chain management (SCM) for optimising the change in equity. A seemingly better metric, economic value added (EVA), has recently been adopted to optimise an integrated financial-operational lot sizing queuing model for single-item, single-server cases [18].

Despite its conceptual completeness and the increasing attention attracted, the shareholder wealth has seldom been adopted in research work. Among its implementation complexities, it is vital to design an appropriate financial measure that may best reflect the interests of shareholders. In practice, some financial performance metrics, such as net present value (NPV), return on investment (ROI), EVA, and cash flow return on investment (CFROI), are available to evaluate the shareholder wealth.

NPV is one of the key financial ratios for valuation of the capital budgeting projects. It is widely used to evaluate the priorities of projects across the business world. However, NPV is mostly based on book values, emphasizing more on accounting profits than on cash flows, even by excluding the cost of capital in its discount rate [19]. These shortcomings, to a great extent, limit its uses for measuring the shareholder wealth.

Additionally, ROI was developed by DuPont Power Company in early 1900s to help manage vertically integrated enterprises with the intent to evaluate a firm's performance by comparing its operating income to its invested capital. Morse et al. [20] stated that the primary limitation of ROI is that it can readily bring about the principal-agent problem. In other words, management tends to make decisions based on their own interests instead of on the best interests of their shareholders.

In contrast, both EVA and CFROI examine to what extent the shareholder wealth would change as a result of management decisions made and implemented [21]. EVA takes into account the total cost of capitals, and it is not constrained by generally accepted accounting principles (GAAP) [9]. However, Kramer et al. [22] mentioned four application limitations of EVA, encompassing the size difference, the financial orientation, the short-term orientation, and the result orientation. De Villiers [23] argued that inflation can distort EVA, and suggested that EVA is not suitable for evaluating the shareholder wealth without adjustment under inflationary conditions. Moreover, some researchers found little relationship between EVA and the shareholder wealth [22, 24].

In comparison with these three financial metrics, CFROI is defined as the sustainable cash flow a firm generates in a given year as a percentage of the outlay invested in its assets [7]. Instead of being a measure of economic profit, CFROI calculates the internal rate of return (IRR), in terms of real purchasing power of capital, to provide a consistent basis for evaluation of a firm's performance, regardless of its size [25]. As such, CFROI eliminates the adverse distorting impingement of both inflationary and deflationary conditions on a firm's performance. These superior merits of CFROI to other financial measures persuade us to adopt it as the financial performance metric for the shareholder wealth in this paper.

B. Operating, Investing, and Financing Activities

In addition to the objective functions, another limitation of current optimisation approaches is that they focus merely on operating activities, with little consideration of the other two important corporate activities—the financing and investing activities. In fact, these two corporate activities are as important as the operating activities, and they have significant impacts on a firm's capital structure and its shareholder wealth.

Practically, financing activities are essential for a manufacturing firm to fund its daily operations through a variety of financial instruments, while investing excessive cash in financial markets may help shareholders gain additional return. The rapidly developing financial markets today further highlight the importance of these two activities. However, their economic impacts on the corporate wealth of a manufacturing firm have yet to be well examined. Thus, it is apparent that conventional operation optimisation, which tends to focus merely on operating activities without due consideration of financing and investing activities, might be managerially misleading and practically unrealistic.

Therefore, in order to improve the preciseness of our proposed model, this paper takes operating activities into consideration and examines the influences of corporate financing and investing activities on the shareholder wealth improvement. In our proposed model, we suppose that the manufacturer is allowed to invest its excessive cash flow in short-term financial investing instruments, such as stocks, bills, and index futures to earn additional profit. On the other hand, when the firm is in need of cash flow, resulting from some business distresses, it can finance its daily operations and future developments by a series of financing instruments, such as taking loans from banks, issuing short-term corporate bonds, or even private equity. As such, in addition to the traditional manufacturing operating activities, when estimating the total return for the manufacturer, we need to consider another two additional return sources—the financing and investing activities.

C. Stochastic Make-to-order Manufacturing

Another critical concern in this paper is the mathematical formulation of the proposed stochastic multi-item make-to-order manufacturing circumstance. We adopt the multi-item lot sizing queuing model to represent the proposed make-to-order manufacturing environment, because of its widespread applications and acceptance in academia and industry. For example, Ref. [26] formulated two different types of flow time queuing problems, including the item-flow formulation and batch-flow formulation, for the design of new systems. Not only is the lot sizing policy involved in their models, but the capacity issue is also examined. Ref. [27] demonstrated the significant implications of the lot sizing policy on SCM. Ref. [28] explored a multi-item capacitated lot sizing problem with setup times, safety stock and demand shortages, where demand cannot be backlogged, but can be totally or partially lost. Ref. [29] examined another multi-product dynamic lot sizing issue, allowing inventories to be replenished jointly with the same quantity whenever a production occurs. Ref. [30] introduced a GI/G/1 multi-item, single-server queuing model. In addition to the cost structure used in the economic order quantity (EOQ) model, the WIP holding cost, setup cost and inventory cost of finished goods are incorporated into this model to determine fixed lot sizes for each product type that can minimise the weighted sum of all costs or can maximise profits.

In spite of the wide applications of the queuing models in the manufacturing optimisation, a main issue in modelling stochastic manufacturing is the unrealistic assumptions on random variables involved, often leading to impractical optimisation results. In current research, it is common to assume that the interarrival time follows a Poisson process, and that the processing time is negative-exponentially distributed. Some studies even perceived certain stochastic parameters as deterministic, in order to simplify the model derivation or to achieve a closed form solution.

These theoretical assumptions may sometimes be misleading for a great number of real manufacturing systems [31]. Ref. [32], for example, reasoned that an Erlang distribution was much better than the Poisson interarrival process for manufacturing when individual orders stuck to a negative exponentially distribution and demand came from a small number of independent sources. Ref. [33] showed the effects of auto-correlation among batch interarrival times to highlight the inappropriateness of assuming the independent Poisson interarrival process. Ref. [34] argued that these factitious assumptions were extremely restrictive and thus not realistic.

Thus, in order to improve the generality as well as the exactness of our proposed model, we choose to characterize random variables by their two statistic merits—expected values (or rates) and standard deviations, rather than making any assumptions on their specific theoretical distributions.

To summarize, we propose a shareholder wealth maximisation mechanism for the stochastic multi-item make-to-order manufacturing environment, with a primary concern of the sustainable long-term profitability in terms of the real purchasing power, measured by CFROI. Our proposed model does not only consider the operational activities, but also keeps tabs on the impacts of the financing and investing activities in the real capital structure, in order to exploit their synergy to advance the shareholder wealth. The uncertain multi-item manufacturing environment is formulated as a stochastic lot sizing queuing network without any impractical distribution assumptions on the random variables.

II. Mathematical Formulation

A. Manufacturing Environment Description

Fig 1 shows the workflow of a make-to-order manufacturing scenario with stochastic multi-item lot sizing, in which the sales department is responsible for gathering individual customer orders for products. When these individual orders accumulate to a batch of the lot size Q_i, where $i = 1,2,\ldots,N$ denotes the product type, they are collected and transferred in a batch order for product i to the manufacturing department to queue for batch setup and subsequent processing on an individual basis. Afterwards, the batch of finished products leaves the manufacturing department for temporary storage in the warehouse, where the batch is subsequently broken down for deliveries of individual products to end customers.

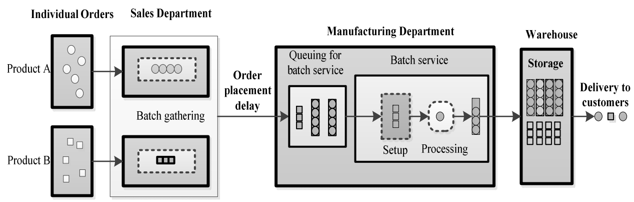

Fig 1 Workflow of make-to-order manufacturing

To formulate the proposed model, the market demands for each type of products are assumed mutually independent on each other. In the case of competition for capacitated resources, all types of orders would be served in accordance with the first-come-first-served (FCFS) queuing principle. Without loss of generality, we further assume that each individual order contains only one product item, and that the manufacturer is a price taker in either the perfect or the monopolistic competition environment.

B. Manufacturing Time Derivation

Since we focus on the manufacturing optimisation, as illustrated in Fig 1, the lead time W_i for an individual customer order of product i is defined as the time that elapses after it arrives at the sales department and immediately before being delivered to the customer, as in:

$$E(W_i)=E\left(W_i^c\right)+E\left(W_i^d\right)+E\left(W_i^{WIP}\right)+E\left(W_i^{FG}\right)+E\left(W_i^t\right) \quad (1)$$

where

- W_i^c = queuing time that an order takes during the batch gathering stage for product *i;*
- W_i^d = order placement delay time from sales department to manufacturing department for an order of product i ;
- W_i^{WIP} = WIPs holding time for an order of product *i;*
- W_i^{FG} = inventory holding time for a finished product *i;*
- W_i^t = shipping time of a finished product i to customer;

and the symbol $E(\cdot)$ represents the expected value function that can be expected of the specified random variable in the bracket. $E(W_i)$, for example, is used to denote the average lead time that a customer order can be expected to take during the entire stochastic manufacturing work flow for all customer orders of product *i*.

For a specific individual order with the j^{th} arrival sequence in a batch of lot size Q_i, we use the symbol $W_{i_j}^c$ to denote the waiting time that this order needs to take during the batch gathering stage for product type *i*, and X_{i_j} to represent the interarrival time of this customer order. Based on the proposed manufacturing procedures, $W_{i_j}^c$ may be readily formulated as:

$$W_{i_j}^c = X_{i_{(j+1)}} + X_{i_{(j+2)}} + \cdots + X_{i_{Q_i}} \quad (2)$$

Further, the expected average waiting time that a specific individual order spends during the batch gathering stage for product *i* can be estimated by taking expectations on both sides of the above equation, that is,

$$E(W_{i_j}^c) = E(X_{i_{(j+1)}}) + E(X_{i_{(j+2)}}) + \cdots + E(X_{i_{Q_i}}) \quad (3)$$

Since the distributions of the interarrival times for all individual customer orders are independent and identical, we can define the following:

$$E(X_{i_{(j+1)}}) = E(X_{i_{(j+2)}}) = \cdots = E(X_{i_{Q_i}}) \overset{def}{=} \frac{1}{\lambda_{ai}} \quad (4)$$

where λ_{ai} means the expected interarrival rate of an individual customer order for product *i*, that is, the expected average number of the arriving orders per unit time period. Hence, we can simplify (3) into

$$E(W_{i_j}^c) = \frac{Q_i - j}{\lambda_{ai}}, \quad (5)$$

which implies that $E(W_i^c)$ may be perceived as a discrete random variable in terms of j with the following distribution law:

$$p_j = P\left\{E(W_{i_j}^c) = \frac{Q_i - j}{\lambda_{ai}}\right\} = \frac{1}{Q_i} \quad (6)$$

In this equation, $P\{\cdot\}$ denotes the probability that the events encompassed in the brace occurs. The corresponding probability is denoted using the symbol p_j .

Finally, it follows from the above theoretical analysis and derivation that

$$E\left(W_i^c\right) = \sum_{j=1}^{Q_i} p_j E\left(W_{i_j}^c\right) = \frac{Q_i - 1}{2\lambda_{ai}} \quad (7)$$

In our formulation, instead of dealing with the setup stage and the processing stage separately, we intentionally combine these two stages together and treat them as one integral part, named the batch service stage, to represent the total work-in-progress (WIP) involved manufacturing stages. So for the product type *i*, the expected mean time of WIP may be computed as the sum of the waiting time in a queue for the batch service and the corresponding batch service time, as follows:

$$E\left(W_i^{WIP}\right) = E\left(W_{qs}\right) + E\left(W_i^s\right). \quad (8)$$

The queuing time for the batch service can be estimated using an approximation relationship [31], which has been proved to work very well and popularly adopted by a number of researchers and practitioners [5, 14], as in:

$$E(W_{qs}) = E\left(Y_{bs}\right)\frac{\left(c_{ba}^2 + c_{bs}^2\right)}{2}\frac{\rho}{1-\rho} \quad (9)$$

On the basis of the stochastic manufacturing procedure described in Fig 1 and the probability theory, we can derive the following equations:

$$E\left(Y_{bs}\right) = \frac{\sum_{i=1}^{N}\frac{\lambda_{ai}}{Q_i}\left(\tau_{si} + Q_i\tau_{pi}\right)}{\sum_{i=1}^{N}\frac{\lambda_{ai}}{Q_i}} \quad (10)$$

$$c_{ba}^2 = \frac{\left(\sum_{i=1}^{N}\frac{\lambda_{ai}}{Q_i}\right)^3}{\left(N^2\sum_{i=1}^{N}\frac{Q_i}{\lambda_{ai}}\right)} - 1 \quad (11)$$

$$c_{bs}^2 = \frac{\sum_{i=1}^{N}\left(\frac{\lambda_{ai}}{Q_i}\times\left(\tau_{si} + Q_i\tau_{pi}\right)^2\right)\sum_{i=1}^{N}\frac{\lambda_{ai}}{Q_i}}{\left(\sum_{i=1}^{N}\frac{\lambda_{ai}}{Q_i}\left(\tau_{si} + Q_i\tau_{pi}\right)\right)^2} - 1 \quad (12)$$

$$\rho = \sum_{i=1}^{N}\frac{\lambda_{ai}}{Q_i}\left(\tau_{si} + Q_i\tau_{pi}\right) \quad (13)$$

where

- $E\left(W_{qs}\right)$ = expected waiting time for batch service for all types of products ;
- $E(Y_{bs})$ = weighted mean batch service time of all types of products;
- c_{ba} = coefficient of variation of batch inter-arrivals for all types of products;
- c_{bs} = coefficient of variation of batch service time for all types of products;
- ρ = traffic intensity;
- τ_{si} = expected batch setup time for product *i*;
- τ_{pi} = expected processing time of each order for product *i.*

Thus, it can be seen that $E\left(W_{qs}\right)$ may be formulated as

$$E\left(W_{qs}\right) = E(Y_{bs})\frac{c_{ba}^2 + c_{bs}^2}{2}\frac{\rho}{1-\rho} \quad (14)$$

In addition, it is easy to estimate

$$E\left(W_i^s\right)=\tau_{si}+Q_i\tau_{pi} \tag{15}$$

Thus,

$$\begin{aligned}E\left(W_i^{WIP}\right)&=E\left(W_{qs}\right)+E\left(W_i^s\right)\\&=E(Y_{bs})\frac{c_{ba}^2+c_{bs}^2}{2}\frac{\rho}{1-\rho}+\tau_{si}+Q_i\tau_{pi}\end{aligned} \tag{16}$$

Moreover, in a similar fashion to solving for $E\left(W_i^c\right)$, we can also derive that

$$E\left(W_i^{FG}\right)=\frac{(Q_i-1)\tau_{ri}}{2} \tag{17}$$

Further considering

$$E\left(W_i^d\right)=\tau_{di} \tag{18}$$

$$E\left(W_i^t\right)=\tau_{ti} \tag{19}$$

and the total lead time for the product type i is finally computed as follows:

$$\begin{aligned}E\left(W_i\right)=&\frac{Q_i-1}{2\lambda_{ai}}+\tau_{di}+E(Y_{bs})\frac{c_{ba}^2+c_{bs}^2}{2}\frac{\rho}{1-\rho}+\tau_{si}+Q_i\tau_{pi}\\&+\frac{(Q_i-1)\tau_{ri}}{2}+\tau_{ti}\end{aligned} \tag{20}$$

where

- τ_{ri} = expected value of the inter-delivery time for the finished products of type *i*;
- τ_{di} = expected value of the random variable W_i^d;
- τ_{ti} = expected value of the random variable W_i^t.

C. *Product Pricing*

In practice, there exists a close relationship between the sales prices and the delivery times. To some extent, changes in delivery times may affect the sales prices. For example, intuitively, customers are willing to pay more for relatively shorter delivery times; conversely, they are inclined to pay less for products with longer delivery times, or simply choose substitutes.

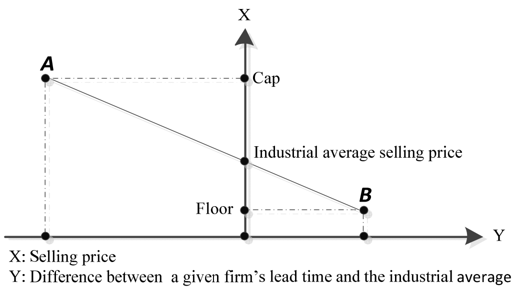

Fig 2 Pricing illustration of finished products

Therefore, a firm may choose to control the sales price of a product, to some degree, by adjusting its lead time, which constitutes a main part of its delivery time. Although the firm cannot arbitrarily alter the selling price of a product because of some production restrictions and the industrial average pricing, there exist a floor (the minimum selling price) and a cap (the maximum selling price), between which the selling price of the product can be adjusted by varying the lead time.

As shown in Fig 2, the cap price of a product is limited by a firm's manufacturing capacity, while its floor price depends on the gross unit cost, as illustrated by points A and B, respectively. By shortening the lead time, the firm may seek to increase the selling price from the floor up to the cap, at which the lead time cannot be reduced any further because of the limitation of the manufacturing capacity. On the other hand, the floor cannot be lowered to enhance price competitiveness by prolonging the lead time, because the minimum selling price is constrained by the gross unit cost of production, below which the product may become unprofitable.

In addition to the practical analysis, this close interconnection between lead times and sales prices are also demonstrated by a great majority of literatures scholars. Ref. [35], for example, stated that an appropriate price premium is allowed for a relatively short delivery time. More and more industry practices suggest that customers are willing to pay a price premium for relatively shorter delivery times than the industrial average [36-38], and conversely for products with longer delivery times, customers are inclined to pay less or would simply go for substitutes.

Based on the above analysis and the relevant pioneering research work, we assume an inverse linear relationship for the cap price, the industrial average sales price, and the floor price of a product. Using this relationship, the sales price of the product type i can be related to its lead time as follows:

$$p_i=-\kappa_i\left(E(W_i)-E(W_i)_{AVG}\right)+p_i^{AVG} \tag{21}$$

where κ_i indicates the level of customer sensitivity to the lead time of product *i*. A large κ_i means that customers have a strong desire to acquire the product soon. Since it is difficult to determine κ_i theoretically, we set it heuristically between the range of 0 and 100. $E(W_i)_{AVG}$ and p_i^{AVG} respectively represent the industrial average lead time and the industrial average sales price for product *i*.

D. *Shareholder Wealth Representation*

As mentioned in the previous section, we adopt CFROI to represent the shareholders' interests, for it is considered a better financial metric of sustainable long-term interests of a firm's shareholders. The first key input to CFROI is the real periodic cash flow, composed of operating, financing, and investing cash flows. According to [15], the operating cash flow OCF_t is estimated as the net income NI_t plus noncash expenses NC_t. NI_t equals the sales revenue minus the variable and fixed costs, denoted by VC_t and TC_t respectively, that is,

$$OCF_t=NI_t+NC_t=\lambda_i p_i-VC_t-FC_t+NC_t \tag{22}$$

where VC_t can be estimated as

$$VC_t=\sum_{i=1}^{N}\left(\omega_{i,t}+\frac{s_{i,t}}{Q_i}+E(W_i^{WIP})h_{i,t}^{WIP}+E(W_i^{FG})h_{i,t}^{FG}+v_{i,t}+\xi_{i,t}\right)\lambda_i \tag{23}$$

Fig 3 illustrates the total costs as a linear relationship between various costs involved in manufacturing, and their changing trends with the increase in the number of finished products or services produced.

In (22) and (23), the terms $\omega_{i,t}$, $s_{i,t}$, $\upsilon_{i,t}$ and $\xi_{i,t}$ represents respectively the unit raw material cost, unit setup cost, unit sales price, and unit tax cost for product *i* at period *t*. $h_{i,t}^{WIP}$ and $h_{i,t}^{FG}$ are the unit inventory cost, respectively corresponding to the WIPs and finished products.

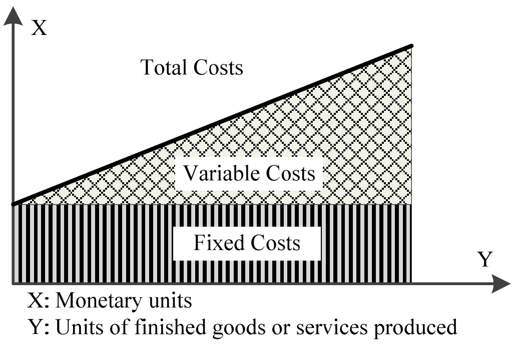

Fig 3 Total costs in manufacturing

Like the operating cash flow, the financing and investing cash flows are equally important to equity holders. In our proposed model, the manufacturer is allowed to adopt a policy of rolling over the excessive operating cash flows through the short-term financial instruments, which results in the following investing cash return ICF_t:

$$ICF_t = IR_t \sum_{k=1}^{t} \max\left(OCF_t, 0\right) \ \left(t = 1, 2, \cdots, T\right) \tag{24}$$

where $\max\left(OCF_t, 0\right)$ denotes the excessive cash revenue from the operating activities at period t. IR_t stands for the dominant annualized investing rate of return in that year.

Similarly, the financing cash flow FCF_t is

$$FCF_t = FR_t \sum_{k=1}^{t} \min\left(OCF_t, 0\right) \ \left(t = 1, 2, \cdots, T\right) \tag{25}$$

where $\min\left(OCF_t, 0\right)$ represents the capital quantity in shortage at period t. FR_t is the financing cost of capital at that time.

Then, we can get the total nominal cash flow NCF_t by summing up (22), (24), and (25), as in

$$NCF_t = OCF_t + ICF_t + FCF_t \tag{26}$$

Afterwards, we adjust NCF_t for the inflation rate r to obtain the periodic real cash flow RCF_t:

$$RCF_t = \frac{NCF_t}{\left(1+r\right)^t} \tag{27}$$

Finally, the conception of IRR and discounted cash flow (DCF) can be adopted to calculate CFROI [15], as follows:

$$TA = \sum_{t=1}^{L} \frac{RCF_t}{(1+CFROI)^t} + \frac{NA}{(1+CFROI)^t} \tag{28}$$

where TA and NA respectively denotes the initial outlays invested in the total assets and the non-depreciating assets.

E. Constraint Conditions

Since our proposed model incorporates both the operational and economic parameters, we need to take the operational and the financial constraints into consideration.

The operational constraints involve the lot size and the traffic intensity. In any case, the lot size should be larger than or equal to one and traffic intensity less than 100% has to be assumed for realistic queuing models. The operational constraints, therefore, can be summarized as follows:

$$\begin{cases} Q \geq 1; \\ \rho < 100\%; \end{cases} \tag{29}$$

In addition, we have stated previously that a firm's manufacturing capacity and production factors impose restrictions on the sales price of a product. The sales price should lie between these limitations, that is, $F_i \leq p_i \leq C_i$. As a result, the relevant operational and the financial constraints conditions on (28) can be summarized as follows:

$$\begin{cases} Q \geq 1; \\ \rho < 100\%; \\ F_i \leq p_i \leq C_i; \end{cases} \tag{30}$$

III. Model Demonstration

As stated in the previous section, our proposed research model attempts to optimise the multi-item lot sizing for a make-to-order manufacturing circumstance under uncertainty, taking the operating, financing and investing activities into consideration, aimed at maximisation of the shareholder wealth.

The proposed model incorporates some real industrial practices. In manufacturing of specialised bicycles, for example, orders for bicycles arrive on an individual basis and are gathered by the sales department, and then some operations, such as electroplating, are conducted on a batch basis. Subsequently, a setup procedure is triggered contingent on the type of bicycles to be produced. Finally, components are assembled into finished bicycles one by one for immediate delivery to end customers. Another typical example is in the metal industry, where metal workpieces arrive individually at furnaces for heat treatment. As soon as a given number of metal workpieces are batched, they are loaded as a whole for heat treatment. Subsequently, they are sandblasted on an individual basis before delivery. It can be seen, thus, that our proposed model has wide applications in the real industrial manufacturing.

To further test the proposed model, three independent numerical experiments are conducted. The first numerical experiment compares the proposed shareholder wealth maximisation model to the traditional operation optimisation. The second one explores the impacts of financing and investing activities on corporate wealth. In the last experiment, we examine the effects of various risks on the proposed approach by risk analysis to provide managerial insights into how possible and at what level these risks affect the full interests of investors, especially equity holders. For simplicity but without less of generality, we assume that there are only two types of products for all numerical experiments, although our model can deal with any number of types of products.

A. Optimisation Comparison

To optimise the shareholder wealth, which is represented by (28) subject to the constraint conditions (30), we firstly need to determine the optimal combination of lot sizes that can maximise CFROI. The trend of CFROI in relation to the combination of lot sizes is graphically illustrated in Fig 4. It can be seen that the optimal combination of lot sizes of Q_1 =334.0470 and Q_2 =397.1531 corresponds to a maximum CFROI of 82.44%.

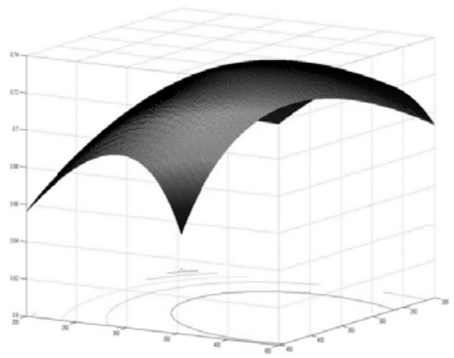

Fig 4 Shareholder wealth as a function of lot size

In order to compare the above optimisation result with its traditional operation optimisation approach, then we find out the optimal combination of lot sizes based on (20). Obviously, the optimal combination is supposed to minimise the total expected lead time, which can be achieved in the case of Q_1 =202.3934 and Q_2 =306.7601 with a shareholder wealth of 78.26%. The comparative information, as listed in Table I and illustrated in Fig 5, demonstrates that the lead time minimisation does not necessarily align with the shareholder wealth maximisation since it omits the shareholders' interests by approximately 4.18%.

Table I Comparison between lead time minimisation and shareholder wealth maximisation

Optimisation objective	Optimal lot sizes Q_1	Optimal lot sizes Q_2	CFROI
Lead time minimisation	202.3934	306.7601	78.26%
Corporate wealth maximisation	334.0470	397.1531	82.44%.
Difference	131.6536	90.3930	4.18%

B. Impacts of Financing and Investing Activities

From the first numerical experiment, we find that the maximum shareholder wealth of 82.44% can be reached at Q_1 =334.0470, Q_2 =397.1531 when both the financing and investing activities are considered.

To further illustrate the implications of the financing and investing activities on the shareholder wealth, we set both IR_t and FR_t to zero, which means that both financing and investing activities will be neglected in the newly designed optimisation method. We rerun our algorithm and get the new optimal combination of Q_1 =334.0470 and Q_2 =397.1531, yielding a maximum shareholder wealth of 73.31%.

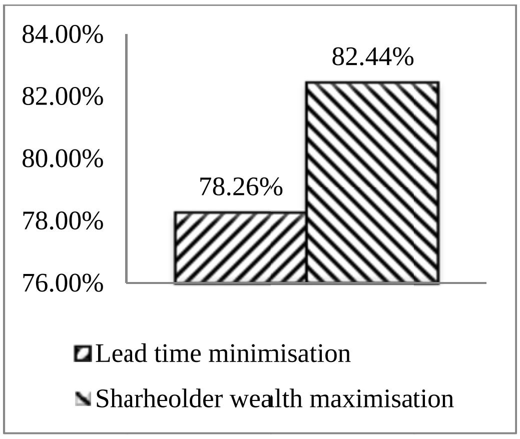

Fig 5 Comparison between the shareholder wealth maximisation and the traditional operation optimisation

Table II Shareholder wealth maximisation with and without the financing and investing activities

Shareholder wealth optimisation	Optimal lot sizes Q_1	Optimal lot sizes Q_2	CFROI
With financing and investing activities	334.0470	397.1531	82.44%.
Without financing and investing activities	334.0470	397.1531	73.31%
Difference	0	0	-9.13%

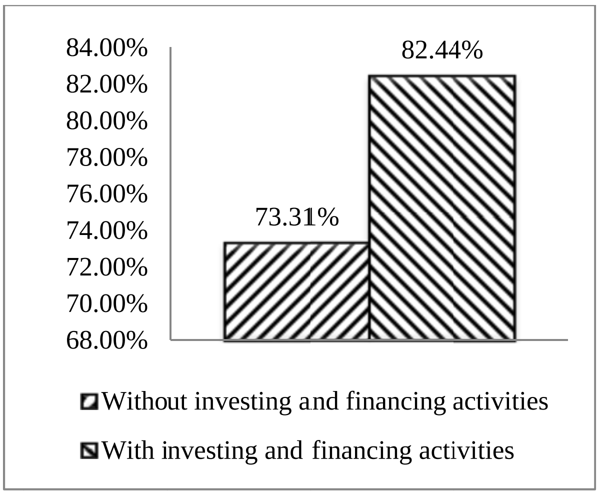

Fig 6 CFROI of optimisation with and without the financing and investing activities

As shown in Table II and Fig 6, the shareholder wealth would drop by 9.13% when the financing and investing activities were not considered, even though the optimal combination of lot sizes remains unchanged. To further demonstrate the relevance of these two activities to a firm, we provide a graphical illustration on how the shareholder wealth changes with the lot size of each product type in Fig 7 and Fig 8. Both figures clearly reveal the importance of the financing and investing activities in manufacturing optimisation.

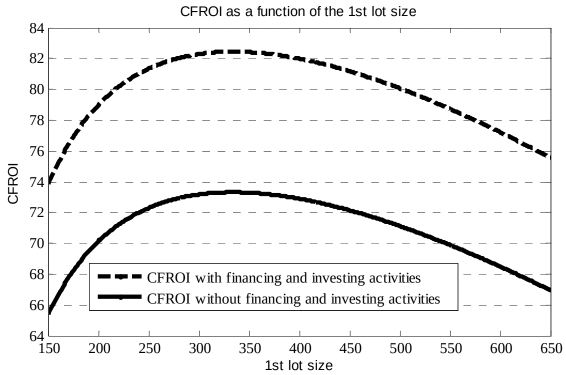

Fig 7 Shareholder wealth as a function of Q_1

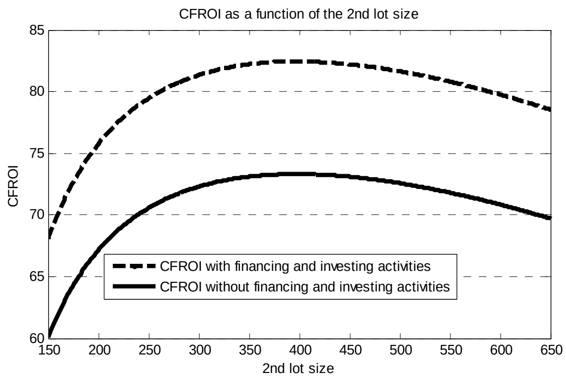

Fig 8 Shareholder wealth as a function of Q_2

C. Sensitivity Testing

In addition to the absolute return of CFROI, its dispersion from the optimal expected value, well-known as risk in finance, is another critical concern, especially in times of dramatic market swings. Here, the riskiness facing the manufacturer is measured by the dispersion of CFROI. The popular risk analysis method, i.e., sensitivity analysis, is used to provide managerial guidance on how to respond to a risky environment. We therefore use sensitivity analysis to determine the effect on the shareholder wealth by changing one single input variable at a time.

We first determine which variables to change and by how much. Since we are more concerned about k_i, $E(W_i)_{AVG}$, p_i^{AVG}, FR_t, IR_t, and NA due to their large influences on the shareholder wealth and their susceptibility to market changes. Table III lists the base values for these key parameters on which the sensitivity analysis will be performed. The CFROI is recalculated by changing one variable from its base value either to a higher value (10% higher than base case) or a lower case value (10% less than base case).

The results of the sensitivity analysis are listed in Table IV and compared in Fig 9. It can be seen that the firm's shareholder wealth is most sensitive to changes in the industrial average price. Changes in the investing rate of return have also a substantial effect, but not as much as the changes in the industrial average price.

Table III Changes in key parameters for sensitivity analysis

Inputs	Base value	Low value	High value
k_i	1	0.90	1.10
$E(W_i)_{AVG}$	0.9	0.81	0.99
p_i^{AVG}	100	90	110
FR_t	10%	9%	11%
IR_t	10%	9%	11%
NA	3000	2700	3300

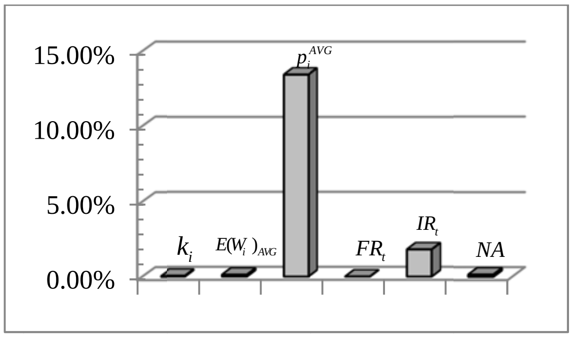

Fig 9 Comparison of Influence of risk factors

Table IV Sensitivity of CFROI to changes in risk factors

Inputs	Base value	Low value	High value	Changing Range
k_i	82.44%	82.47%	82.42%	0.05%
$E(W_i)_{AVG}$	82.44%	82.38%	82.50%	0.12%
p_i^{AVG}	82.44%	75.68%	89.14%	13.46%
FR_t	82.44%	82.44%	82.44%	0
IR_t	82.44%	81.54%	83.35%	1.81%
NA	82.44%	82.37%	82.52%	0.15%

In addition, the shareholder wealth is less susceptible to the changes in the delivery sensitivity, the industrial average lead time and the investment portion of the non-depreciating assets in total assets. In the risk analysis, the financing rate of return has no effect on the shareholder wealth since the optimisation nature of our model excludes the possibility of financing using external capital.

IV. CONCLUSION

In this paper, we propose a multi-item stochastic lot sizing optimisation model for enhancing the sustainable long-term performance of a make-to-order manufacturing firm under uncertainties.

The proposed model is characterised by taking the operating activities as well as financing and investing activities into consideration. It adopts general distributions for stochastic variables involved, instead of the traditional theoretical distributions such as the Poison process on the interarrival of customer orders, to improve its generality and extensibility for dealing with multi-item lot sizing in more realistic manufacturing scenarios. Most importantly, the model optimises the sustainable long-term profitability of a firm in terms of CFROI, which is considered a relevant financial metric that can better reflect the firm's overall business goal and hence the full interest of its equity holders. Moreover, the proposed model eliminates distorting impacts of inflation, such that the optimisation results are projected in real purchasing power, rather than in nominal terms.

Numerical experiments reveal that there is considerable spread of optimisation between the traditional operation optimisation approach and the proposed shareholder wealth maximisation model. This highlights the importance of taking financial and economic factors into account for manufacturing optimisation. It is found that the financing and investing activities are as important as the operating activities in promoting the shareholder wealth. Hence, in addition to the traditional short-term operational objectives, a firm should put more attentions on the interest of its equity holders—the global long-term business goal. This provides a practical guidance on the use of cash flows from operations, and highlights the importance of cash reinvestment in advancing the firm's performances.

Risk analyses are performed to test the susceptibility of a firm's shareholder wealth to a variety of microeconomic and macroeconomic market swings. This numerical experiment is designed to address a real management concern that a firm should care not only about how to maximise its prospective shareholder wealth, but also about its capability to hedge various risks to keep a stable performance improvement. The result shows that the shareholder wealth is most sensitive to the industrial average price of a product, followed by investing rate of return, while the impacts of other key factors seem negligible.

Currently, the proposed model has some limitations which may be addressed in future work. For example, the lot sizing model may be extended to cope with a multi-item, multi-machine stochastic manufacturing environment; the specific linkage between the lead time and sales prices should be investigated in depth; furthermore, a multi-stage stochastic programming may be adopted as a more practical tool in line with periodic accounting purposes. Moreover, some potential extensions to the proposed model are being considered. For example, we are trying to examine the influences of the carbon footprint management on the shareholder wealth, aimed to optimise the long-term full interests of shareholders, as well as to reduce emission of carbon dioxides (CO_2) and other greenhouse gasses (GHGs).

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