# Numerical Simulation of Grazing Flow over a Self-excited Helmholtz Resonator

F. Ghanadi, M. Arjomandi, A. C. Zander and B. S. Cazzolato

***Abstract*— Self-sustained oscillations of the grazing flow along the orifice of a Helmholtz resonator were considered numerically. These fluctuations are driven by hydrodynamic instabilities inside the shear layer and the resonant acoustical field. Quantitative prediction of this process requires accurate calculations of grazing flow characteristics over the three dimensional resonator. In this paper flow excitation phenomenon assumed to be associated with external pressure fluctuations within the turbulent boundary layer of grazing flow and acoustic response of the cavity. To this end a Large Eddy Simulation (LES) of the three dimensional shear flow over the orifice carried out at a low Mach number to allow predictions of the amplitude and frequency of the pressure and velocity fluctuations. For validation propose, for pressure fluctuations inside the cavity, a good quantitative agreement with published data was obtained. Therefore the simulations provide an ability to predict the resonating frequency, pressure and velocity field for different inlet conditions.**

***Index Terms*—Helmholtz resonator, large eddy simulation, pressure fluctuations, self- excitation**

## I. Introduction

A resonator exposed to an external flow field can produce a flow excited resonance over a specific range of flow conditions. This process is common phenomenon in many engineering applications such as flow over open sunroof or window of an automobile, aircraft landing gear and pipelines with closed side branches. All of these systems can produce high amplitude pressure fluctuations at different Mach number. There is immense body of literature about velocity and pressure fluctuations within the Helmholtz resonator which was excited by external force. Most of these studies carried out experimentally using loudspeakers to apply external force to excite air flow inside the cavity. Moreover an appropriate quantitative explanation of self excitation phenomena requires extremely accurate prediction of shear flow properties, compressibility and flow inlet effects, etc. The motivation of the current study was derived from the excitation of a resonator caused by grazing flow to predict the flow characteristics in and outside the resonator. This paper presents a model to simulate a flow-excited resonator to investigate pressure and velocity fluctuations in and outside of the cavity. This simulation can provide an accurate quantitative description for capability of a Helmholtz resonator to reduce instinct disturbances within the turbulent boundary layer.

The Helmholtz resonator as an alternative type of side branch resonator is formed by a cavity with a fix volume of compressible flow connected to the surrounding through a single small opening [1, 2]. As can be seen in Fig. 1, a schematic of the Helmholtz resonator and the behaviour of a Helmholtz resonator can be described by a second-order mass spring or, equivalently, with a capacitor and an inductor. In this mass spring system, the mass comprises of the mass of fluid in the neck whereas compressibility of the fluid in the cavity acts as a spring. Owing to viscous effects of flow and acoustic radiation at the orifice, the mass spring system has also damping parameter which in calculations typically be considered.

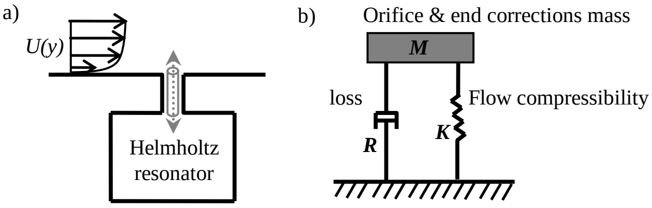

Fig. 1. a) Schematic of a Helmholtz resonator b) Mass-spring analogy

The quasi-periodic vortex shedding by an incompressible low Mach number grazing flow past the orifice and excitation of acoustic resonances in the cavity can produce intense energy exchange. When the frequency of the flow oscillations is far from that of natural frequency of the resonator, the two processes are weakly coupled. When the frequency of flow fluctuations over the orifice is near the natural frequency of the resonator, it is excited and then a small pressure fluctuation can produce a relatively large magnitude velocity fluctuation around the neck, which is accompanied by large pressure fluctuations inside the cavity. When wall boundary layer of grazing flow reaches the upstream edge of the orifice, it suddenly separates resulting in the formation of vortices, which, in turn, form a shear layer in this area. As a result, excitation is associated with the periodical shedding of these compact vortices from a separation edge of the resonator mouth to the downstream edge, and consequently an intense energy exchange occurs between the driving flow oscillations and the fluid inside the

F. Ghanadi is with the School of Mechanical Engineering, University of Adelaide, SA 5005, Australia (corresponding author to provide phone: (+61 8) 83136385; e-mail: farzin.ghanadi@adelaide.edu.au).

M. Arjomandi is with the School of Mechanical Engineering, University of Adelaide, SA 5005, Australia (email: maziar.arjomandi@adelaide.edu.au)

A. C. Zander is with the School of Mechanical Engineering, University of Adelaide, SA 5005, Australia (email: benjamin.cazzolato@adelaide.edu.au)

B. S. C Cazzolato is with the School of Mechanical Engineering, University of Adelaide, SA 5005, Australia (email: anthony.zander@adelaide.edu.au)

resonator [3]. Therefore, to predict self excitation condition of a Helmholtz resonator, it is very important to determine the natural frequency of such a resonator. In general using lumped element analysis, the following expression can be used for the calculation of the natural frequency of the resonator.

$$f_r = \frac{c}{2\pi}\sqrt{\frac{S}{l_e V}} \tag{1}$$

where $c$ is the speed of sound, $S$ and $V$ are the cavity volume and the cross sectional area of the orifice respectively. Orifice flow, of course, cannot fluctuate separated from grazing flow. The quasi-periodic vortex shedding by an incompressible low mach number grazing flow past the orifice and excitation of acoustic resonances in the cavity can produce intense energy exchange. When the frequency of the flow oscillations is far from that of natural frequency of the resonator, the two processes are weakly coupled. When the frequency of flow fluctuations over the orifice is near the natural frequency of the resonator, it is excited and then a small pressure fluctuation can produce a relatively large magnitude velocity fluctuation around the neck, which is accompanied by large pressure fluctuations inside the cavity. When wall boundary layer of grazing flow reaches the upstream edge of the orifice, it suddenly separates resulting in the formation of vortices, which, in turn, form a shear layer in this area. On of the earliest experimental investigation for flow excited resonance carried out by Rossiter (1964) who examined shallow cavities at subsonic and transonic flow regimes. It was demonstrated that the convection of discrete vortices within grazing flow over the resonator opening generates the frequency spectrum of the sound. A semi-empirical correlation was suggested to predict the frequency of natural hydrodynamic instability of low Mach number ($0.4 \le M \le 1.2$) grazing flow over the opening as

$$\frac{fl}{U} = \frac{n-\alpha}{Ma + 1/k}, \tag{2}$$

in which $n$ is the mode number ($n$ =1, 2, 3,….), $Ma$ is Mach number and $\alpha$ and $k$ are constant values

Prediction of the resonator pressure oscillations has been extensively studied both experimentally and analytically. The hydrodynamic force of this vortex shedding is also analysed by describing two forward and backward functions which illustrate oscillatory flow in the orifice and its effect or by implementing vortex sound theory. [4, 5]. The Helmholtz resonator has also been used for flow separation control in two different test cases of a diffuser and a wing [6]. They observed that the vortices ejected from the orifice can shift the momentum in the near-wall region, and therefore reattach the downstream flow. The most recent researchers numerically investigate to the subject of flow-excited Helmholtz resonator. For example Inagaki et al. [7] who derived new equations to predict numerically the flow oscillations over a Helmholtz resonator at low Mach number. They predicted frequency of pressure fluctuations within flow over a resonator orifice with high aspect ratio at low Mach number using compressible Navier-stokes equations. It was concluded that the pressure fluctuations have frequency very close to the natural frequency of resonator in a wide range of velocity. Mallick et al. [8] later applied a modified lattice– Boltzmann equation to calculate the pressure fluctuations spectrum within the grazing flow-excited over a Helmholtz resonator. Using $k-\varepsilon$ RNG turbulent model they demonstrated that to validate their results with Nelson et al. [3] experimental work some parameters required scaling. For example if velocity of grazing flow multiple by 0.7 predicted pressure fluctuation inside the cavity is very close to the experimental data. The reason of this scaling was described as the effects of boundary layer thickness on convection velocity of the vortex within the shear layer over the orifice. A recent experimental attempt to explore flow characteristics in and outside the resonator also carried out by Ma et al. [9] who tested a self-excited Helmholtz resonator with various free stream velocity. They concluded that the magnitude of external force which induced by grazing flow is in a specific range over a large variety of free stream speed and boundary layer thickness.

The objective of this paper is to simulate the effects of a self-excited Helmholtz resonator on the turbulent grazing flow. The simulations were performed in a wide range of Reynolds numbers and turbulent intensity to analyse the inlet flow properties on excitation condition. The outline of the remainder of the paper is as followed. The next sections are organized as follows. Necessity of using LES model and the basic equations related to that are described in §2. In §3 results of the proposed model are compared with experimental published data to validate the model. Then the effects of flow conditions on pressure and velocity fields are discussed. In the last section conclusions and summary of this study are presented.

## II. NUMERICAL SET UP AND PROCEDURE

In order to calculate the fluctuation parameters of shear flow over the orifice and flow field inside the resonator a sophisticated modelling tool is required to simulate adequately the three dimensional complex flow. The primary requirement of this model is that it must handle the complex geometries with high order of accuracy. The model also must have a very low dissipation part to predict the fluctuations accurately in shear flow over the orifice and acoustic field. It has been shown that Reynolds-averaged Navier-Stokes (RANS) methods have been typically are too dissipative and then the models under-predict velocity and pressure fluctuations in the flow field [10, 11]. The accuracy of simulations strongly depends on the spatial and temporal resolution employed. RANS models were not considered suitable since cannot resolve the time dependant parameters within the grazing flow. These methods are adequate for steady flows, or, unsteady flows with very weak non-resonant coupling. It has been observed that these models are inadequate for highly unsteady flow and involve strong vortex acoustic coupling and resonance like phenomena [12]. This is the main reason why the RANS method was useless for the flow calculations for Helmholtz resonator. Among turbulence models Large Eddy Simulation (LES) as an unsteady simulation can accurately simulate the characteristics of the flow in the presence of acoustic field [13]. LES is also capable of handling the flows for which

RANS models work and also for flows involving strong vortex-acoustic coupling, including resonating pressure disturbances. LES represents a three dimensional and time dependant solution of the Navier-Stokes equations. This kind of simulation is a method intermediate between the Direct Numerical Simulation (DNS) and Reynolds-average solutions.

In this paper LES was used as an invaluable tool for capturing the fluctuations which come from coherent structures in the turbulent boundary layer and perturbations which within separation flow within the resonator opening. This model, using “sub-grid” scale for the small-scale structures, can accurately predict flow fluctuations in the wall bounded regions [14]. LES model separates the small and large scale structures by using filtering and resolves the large scale motion so that the large scale structures are computed explicitly. On the other hand, this method, can model the small scale motion accurately [15]. Therefore, pressure fluctuations in acoustic feedback field can be modelled with adequate accuracy using this technique. Moreover, an appropriate prediction for the velocity perturbations generated by the grazing flow passing over the resonator can be calculated. On the following the detail of the numerical method and its governing equations are presented.

### *A. Governing Equations*

Owing to compressibility of the air flow inside the cavity the filtered compressible Navier–Stokes equations for a Newtonian fluid were used. To simplify the equations a mass-weighted change of variable defined as

$$\tilde{\phi} = \frac{\overline{\rho\phi}}{\overline{\rho}} \tag{3}$$

where $\overline{\rho}$ a space-filtered variable. Therefore, conversation and Navier-Sockes equations are then given by

$$\frac{\partial\overline{\rho}}{\partial t} + \frac{\partial(\overline{\rho}\tilde{u}_i)}{\partial x_i} = 0 \tag{4}$$

$$\frac{\partial(\overline{\rho}\tilde{u}_i)}{\partial t} + \frac{\partial(\overline{\rho}\tilde{u}_i\tilde{u}j)}{\partial x_j} + \frac{\partial}{\partial x_i} - \frac{\partial(\tilde{\delta}_{ij})}{\partial x_j} =$$
$$-\frac{\partial}{\partial x_j}\left[\underbrace{\overline{\rho}(\widetilde{u_i u_j} - \tilde{u}_i\tilde{u}j)}_{\tau_{ij}}\right] + \frac{\partial}{\partial x_j}(\overline{\sigma}_{ij} - \tilde{\delta}_{ij}) \tag{5}$$

where $\overline{\sigma}_{ij}$ is the filtered stress tensor and $\tilde{\delta}_{ij}$ the computable stress tensor. The filtered Navier-Stokes equations govern the evolution of the large scales of motion. The effect of the small structures comes out through a subgrid scale stress term ( $\tau_{ij}$ ) which is given by

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = 2\nu_t\overline{S}_{ij} \tag{6}$$

where $\delta_{ij}$ is Kronecker delta and $\overline{S}_{ij}$ is the deformation tensor of the subgrid field,

$$\overline{S}_{ij} = \left(\frac{\partial\overline{u}_i}{\partial x_j} + \frac{\partial\overline{u}_j}{\partial x_i}\right) \tag{7}$$

The most popular subgrid model is the Smagorinsky model, but this model is of order $O(1)$ and is just related to the strain rate of the turbulent structure. In this project the *Wale* subgrid model has been used which can accurately predict flow behaviour of the subgrid viscosity at the wall in zero pressure gradient incompressible boundary layer, and is of order $O(3)$ [16],

$$\nu_t = (C_w\Delta x)^2 \frac{(S_{ij}^d S_{ij}^d)^{3/2}}{(S_{ij}S_{ij})^{5/2} + (S_{ij}^d S_{ij}^d)^{5/2}}, \tag{8}$$

where $C_w$ is 0.5 and

$$S_{ij}^d = \frac{1}{2}\left(\left(\frac{\partial\overline{u}_i}{\partial x_j}\right)^2 + \left(\frac{\partial\overline{u}_j}{\partial x_i}\right)^2\right) - \frac{1}{3}\delta_{ij}\left(\frac{\partial\overline{u}_k}{\partial x_k}\right)^2, \tag{9}$$

In next section the characteristics of the simulations which include study of the effects of the mesh grid and of the nature of the inflow condition are described.

### *B. Simulation Characteristics*

The geometry studied here corresponds to that examined experimentally and analytically by Ma et al. [9]. The cavity is 59 cm long with square cross section of 50 cm × 50 cm. The 12.5 cm × 12.5 cm orifice is cut from a 0.16 cm thick plate. Various free stream velocity settings, corresponding to resonant and non-resonant conditions were studied by Ma et al. [9]. The free stream speed was in the range of 10-30 m/s with a boundary layer thickness of 0.5 cm to 2 cm at the orifice upstream lip. Moreover, as part of their experimental work, microphone measurements yielding pressure fluctuations inside the cavity. Therefore to validate the model it is necessary to simulate the resonator under these inlet flow conditions and calculate the pressure fluctuations within the cavity. In Ma et al. studies there were not any hot-wire measurements to evaluate the velocity profile inside the shear layer over the orifice. In the present paper mean velocity profile and velocity fluctuations within the grazing flow were calculated. Since in LES only small turbulence structures are modelled and the large and energy carrying are computed, requires high grid resolution in near wall region in streamwise and crosswise Wagner et al. [17]. Therefore, in the first part of this task a development of accurate physical geometries with a suitable mesh for the computational domain is required. As illustrated in Fig. 2, in order to capture the behaviour of the near-wall flow, a sufficiently fine mesh has been used in the present work. An accumulated mesh structure has been utilized in the region near the resonator orifice to ensure that there are at least a few cells in the buffer layer of the turbulent boundary layer. Using this kind of grid allows a fine mesh within the turbulent boundary layer and fits the plate shape very well. In order to calculate the characteristics of flow in outer region the number of grid points (Cells) should be proportional to $(Re)^{0.4}$ and in viscous region this is increased to $(Re)^{1.8}$. Although these cell sizes result in fairly fine meshes, in comparison with DNS method this approach allows to reduce the computational costs. A computational mesh containing 2.5 million cells (1 million vertices).

In order to evaluate near-wall fluctuations the $(Y)^+$ parameter should be less than 1, which in this project for all case studies this non-dimensional parameter was around 0.2.

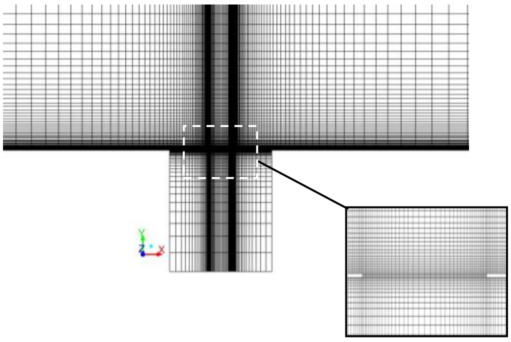

Fig 2. Volumetric mesh in the orifice of the resonator. The resolution is highest in this region.

The highest resolution needs to be applied over the surface to capture the characteristics of the boundary layer. In order to analyse the sensitivity of the results to the domain size three domain sizes equal (Cases 1, 2, 3) from the trailing edge were tested.

TABLE I

DOMAIN SIZES TESTED FOR SENSITIVITY

| Case | Channel volume (m^3) | Total number of nodes |
|---|---|---|
| 1 | 2.4 | 1,253,871 |
| *2* | 4.6 | 2,542,756 |
| *3* | 7.4 | 4,705,748 |

The results of the three runs with the same boundary conditions and surface grid parameters were compared. Mesh quality assessment was performed using definition of Maximum y+ for the cases.

## III. RESULTS AND DISCUSSION

Based on equation (1) the Helmholtz resonator frequency was determined to be 45 Hz. The primary non-dimensional parameter of interest was defined based on free stream velocity; $U^* = U / Lf_r$ .where $U$ is free stream velocity. The inlet velocity was varied from 5 to 30 m/s with turbulent intensity of 0.3 % and boundary layer thickness over the orifice was found to be 2 cm or, $\delta / L = 0.16$ . By use of the turbulence model and boundary conditions described above, velocity and pressure distributions inside and over the resonator were calculated.

### *A. Pressure fluctuations*

The resonator pressure fluctuations within the flow excited Helmholtz resonator are explained in this section. In order to show the results in terms of frequency, another variable based on natural frequency of the resonator was defined; $f^* = f / f_r$. where $f$ is the natural hydrodynamic instability frequency. Fig. 3 shows the calculated sound pressure level (SPL) as a function of $f^*$ when $U^* = 3$. It can be seen that one mode was dominant and it is just very close to the resonance frequency of the Helmholtz resonator.

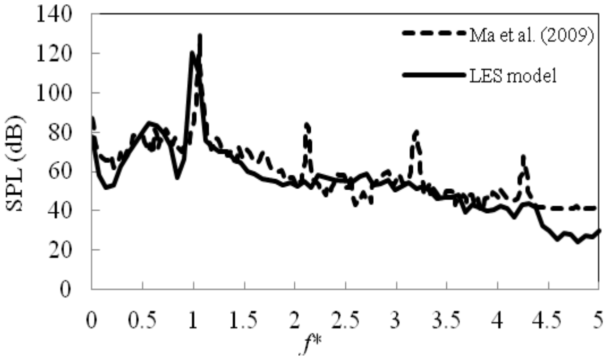

Fig 3. Sound pressure level of the resonator at different inlet flow velocity

Therefore it seems that just in specific range of free stream velocity the strong pick happen and out of this span there is no clear sign of resonance. It can be see that in the figure, in the number of mesh and domain size cannot affect on the results. The model also calculated the pressure fluctuations inside the resonator and it was examined the ability of the LES to match the frequency and amplitude of the fluctuations which have been described in the literature. The pressure fluctuations in the literature were normalized by integrating the spectral densities in the region of peak resonance which is given by

$$p^*_{res} = \frac{\sqrt{\int_{0.8f_p}^{1.2f_p} G_{pp}(f)\,df}}{0.5\rho U^2}, \qquad (10)$$

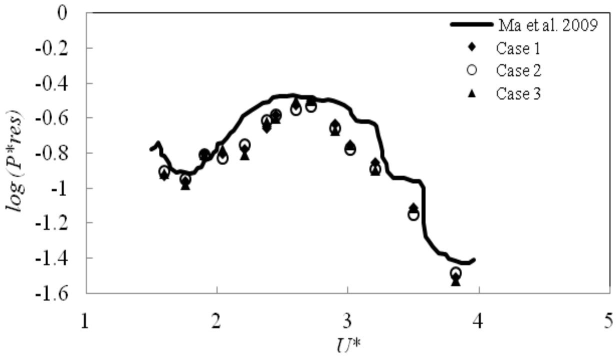

Fig 4. The resonator pressure amplitude for various free stream velocities

### *B. Velocity fluctuations*

The mean and fluctuation flow velocity was determined using the LES model over the resonator opening. In order to develop the determine behaviour of the shear flow over the orifice length, velocity profiles was calculated within the shear layer from leading edge to trailing edge. in first step the mean velocity profiles at the tips and middle points of the orifice was considered. As shown in the Fig. 5, the shear layer thickness at the three stations ( $x=-6.25\,cm, x=0$ and $x=6.25\,cm$ are corresponding to the upstream edge, midpoint, and the downstream edge of the resonator opening, respectively) are $\delta=1.25\,cm, \delta=2.4\,cm$ and $\delta=3.3\,cm$ .

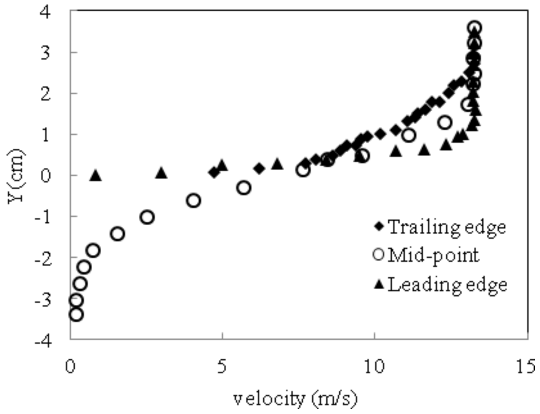

Fig 5. Mean velocity profiles, in resonator opening at three stations

The velocity of vortices inside the shear layer plays an important role to generate high pressure fluctuations within the resonator and exchange energy between boundary layer and cavity flow. Vortices in a thick turbulent boundary layer convect slower compared to a thin boundary layer. In the literature flow visualizations reported that this speed is a function of the boundary layer thickness and free stream velocity [4, 18]. Therefore the convection velocity is given by

$$U_c=\frac{U}{2}\left(\frac{0.05\,d}{\delta}\right)^{1/7} \tag{11}$$

Using the mean velocity profiles convection velocity of the vortex within the shear layer was found. As can be seen in the Fig 6. in the mid point at the middle of orifice height velocity of the flow is around 7 m/s. Considering turbulent boundary layer thickness in this area was about 1.5 cm, the convection velocity using Equation (11) is around 0.5$U$ which is match with simulation result.

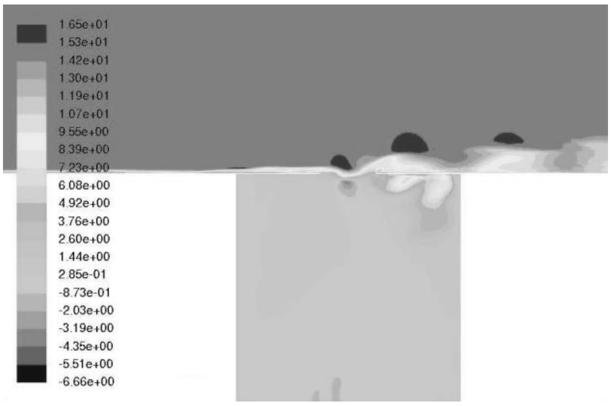

Fig 6. stream-wise velocity contour over the resonator

## IV. CONCLUSIONS

Owing to difficulties and time constraints of experimental tests, CFD investigations have been conducted to find the various parameters affecting the Helmholtz resonator performance. To this end the three-dimensional boundary layer within a channel with a grazing flow-excited Helmholtz resonator were simulated and time dependent pressures and velocity fluctuation were calculated. Since the RANS models cannot accurately model the fluctuation parameters of the turbulent flows the LES model was utilized which its dissipation part has minimum effects on sub-gird scales. The resonance characteristics of the resonator were described using LES for three different mesh cases.

The calculated sound pressure level of acoustic response of the resonator was compared with experimental results and concluded that in specific free stream speed the strong excitation of the resonator happen. Characteristics of inlet flow states were changed to evaluate their effects on excitation condition. It was observed that when $U^*=3$ frequency of instabilities was very close to the natural frequency of the resonator and then high pressure amplitude can be achieved (around 111 dB). It was also concluded from time series of pressure fluctuations inside the cavity that around this inlet flow velocity, maximum pressure takes place within the resonator and by increasing the Reynolds number the pressure dramatically dropped. It was also shown that increasing mesh numbers and domain sizes cannot affects the results so much.

In the next step to check capability of the model to calculate velocity fluctuations over the orifice, velocity distribution for a wide range of Reynolds numbers was tested. Mean velocity profile over the whole distance of the resonator orifice showed that the computed convection velocity of the vortices inside the shear layer was very close to the experimental observations. Pressure fluctuations in vertical direction were also calculated that to find the maximum amplitude of the shear flow velocity perpendicular to the shear flow. This value can help us to find the energy exchange between air flow inside the resonator and shear flow over the orifice.

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