Scheduling Operations in a Flow Network with Flexible Preventive Maintenance: A (max, +) Approach

Karla Quintero, Eric Niel, José Aguilar, and Laurent Piétrac

Abstract—**The following work proposes a (max,+) optimization model for scheduling operations on an oil seaport considering flexible maintenance activities on valves. The work is based on previous results for the same case study, where fixed maintenance was studied in the framework of scheduling oil transfer operations through a pipeline network. The case study is a Venezuelan seaport for oil export and real operational constraints and goals are modeled. Results corroborate the drawbacks that arise when considering fixed maintenance in the system. Moreover, the adjustments made to obtain a model considering maintenance relaxation are straightforward and intuitive. Some linear representations of the problem are also explored through prioritization of certain tasks.**

Index Terms—**algebraic modeling, schedule optimization, pipeline networks, (max,+) algebra.**

I. Introduction

THE following work proposes a (max, +) optimization model for scheduling transfer operations on a flow network while considering relaxation on predefined maintenance schedules. The case study is an oil seaport with an intricate pipeline network as the core of the physical system used to satisfy several oil transfer and maintenance operations. In a given time horizon, conflict phenomena due to resource assignment arises which can be intuitively modeled through (max,+) algebra.

A classic alternative to approach resource allocation conflicts are Petri Nets, specifically event graphs, where conflicts are previously solved through a routing policy, i.e. a criterion that allows the choice of one transition among a group of conflicting transitions demanding to be fired, see [1], [2], and [3] for an overview on common routing policies. Some heuristic approaches deal with conflicts directly within the resolution algorithm; for instance, [4] implements an ant colony optimization algorithm in which conflict is modeled as a probabilistic choice rule depending on the pheromone trail and a heuristic function.

In the first part of this work, a routing policy is not assumed and the optimization problem is modeled with the greatest degree of freedom so that the best possible schedule is determined for several oil transfer operations where maintenance activities are flexible, which allows to identify the advantages of maintenance relaxation. Further along, a prioritization indicator is considered in order to explore some linear representations of the model.

This work is based on a recently defined (max,+) optimization model for flow network operations (see [5]) and through maintenance relaxation determines the optimum schedule for oil transfer operations in the seaport that minimizes the total cost of penalties in the system for a given time horizon.

This work presents a framework for solving a scheduling problem through an industrial application of (max, +) algebra so that the system's algebraic optimization model intuitively synthesizes all constraints and objectives. Some examples on other approaches formulating similar problems are: [6], where an optimization model for flow-shop scheduling with setup times is formulated as sets of recursive constraints expressing the underlying dependency between completion times for jobs on machines, and [7] and [8], where classic resource conflict constraints are expressed through decision variables imposing a precedence and therefore forcing one machine operation to depend on the completion time of a conflicting one. These same principles constitute the base of the (max,+) approach but instead, with the proper algebraic structure (i.e. fundamental mathematical operators, decision variables based on the zero and/or identity element, and mathematical properties such as commutativity, idempotency, and distributivity, among others) formulations can be more intuitively constructed and additional and more intricate phenomena can be easily integrated.

To our knowledge, no similar work has been developed for this type of system; moreover, the results are extendable to applications to flow networks of a different nature. The developments in this work are part of a larger research scope aiming at optimizing operations in a more complex framework with direct industrial application in the oil sector.

Firstly, we present some preliminary notions on (max, +) algebra in section 2. Section 3 covers the system description, related previous work, and some operational aspects of a real seaport in Venezuela used further along in model validation. Section 4 presents the proposed (max, +) optimization model with flexible maintenance with validation results and comparisons with previous work in the fixed maintenance framework. Some linear representations of the model are ex-

Manuscript received January 27, 2014. This research has been financially supported by Thales Group France, and by the PCP (Post-graduate Cooperation Program) between Venezuela and France involving the collaboration between the academic institutions: ULA (in Spanish: *Universidad de Los Andes*) - research laboratory: CEMISID in Mérida, Venezuela and the INSA (in French: *Université de Lyon, INSA Lyon, Ampère* (UMR5005)) in Lyon, France; and the industrial partners Thales Group France and PDVSA (in Spanish: *Petróleos de Venezuela Sociedad Anónima*), the Venezuelan oil company. Industrial data for model validation has been granted by PDVSA.

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plored in section 5 and, finally, section 6 presents concluding remarks and future work.

II. (Max,+) Algebra Overview

The focus of this section is on a (max, +) theory overview allowing to understand the basis of this mathematical modeling technique with an envisaged application to the scheduling problem approached in the research.

(max, +) algebra is defined as a mathematical structure denoted as $\mathbb{R}_{max}$, constituted by the set $\mathbb{R}\bigcup\{-\infty\}$ and two binary operations $\oplus$ and $\otimes$, which correspond to maximization and addition, respectively. This algebraic structure is called an idempotent commutative semifield. As [9] states, a semifield $\mathcal{K}$ is a set endowed with two generic operations $\oplus$ and $\otimes$ complying with certain classic algebraic properties as follows:

Operation $\oplus$:

- is associative (e.g. $a \oplus (b \oplus c) = (a \oplus b) \oplus c$),
- is commutative (e.g. $a \oplus b = b \oplus a$),
- has a zero element ε (e.g. $a \oplus \varepsilon = a$),
- is idempotent (i.e. $a \oplus a = a$; $\forall a \in \mathcal{K}$).

Operation $\otimes$:

- is distributive with respect to $\oplus$ (e.g. $a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)$),
- is invertible. For example, in (max,+) algebra: if $2\otimes 3 = 5$ then $2 = 5 \oslash 3$ or in conventional notation: if $2+3 = 5$ then $2 = 5-3$ (here, operator $\oslash$ denotes the inverse of the $\oplus$ operation),
- has an identity element e which satisfies $\varepsilon \otimes e = e \otimes \varepsilon = \varepsilon$.

In (max,+) algebra, the zero element is $\varepsilon = -\infty$, and the identity element is $e = 0$. Considering that in this algebraic structure operators $\oplus$ and $\otimes$ correspond to maximization and addition, respectively, some basic examples on the use of these operators are:

$$\begin{array}{ll} 2 \oplus 3 = 3 & 2 \otimes 3 = 5 \\ 2 \oplus 2 = 2 & 2 \otimes 2 = 4 \\ 2 \oplus \varepsilon = 2 & 2 \otimes \varepsilon = \varepsilon \\ 2 \oplus e = 2 & 2 \otimes e = 2 \end{array}$$

(max, +) models aim at describing the system's main properties through two basic mathematical operations: maximization and addition. The best candidates to be modeled with this tool are systems exhibiting synchronization phenomena as their main feature. However, research in this field continues to explore further possibilities.

For the purposes of this research, the interest is in the application of the modeling technique to a system in which resource allocation conflicts constitute the main characteristic; i.e. valves can be allocated for maintenance operations as well as for several (and possibly conflicting) oil transfer operations.

The application of this theory to discrete event systems exhibiting synchronization phenomena leads to the formulation of very intuitive (max,+)-linear models formed by equations such as $x_3 = x_1 \otimes \tau_1 \oplus x_2 \otimes \tau_2$. In this equation, x_i is the start date of an event i, and τ_i is its duration. x_i is usually denoted as *'dater'* in the (max, +) context. In this example, the dater of event 3 is given by the maximum of the completion times of events 1 and 2; which can be interpreted as the synchronization of 2 tasks or 2 task sequences (e.g. a train that only departs when 2 other trains arrive at the station with connecting passengers).

With the principle shown in the former equation, a (max,+)-linear system model describing the interactions among all relevant tasks or processes can be obtained in the form of X=AX, where X is the variables vector (i.e. $X = [x_1\ x_2 \ldots\ x_n]^T$) and A is the matrix containing all time relations between the variables. Analogies with classic linear system theory would be applicable to this simple model by considering maximization and addition as basic operations, as well as all the aforementioned properties in the algebraic structure.

(Max,+) theory is a research field that has caught the attention of the scientific community for its intuitive modeling potential of discrete event systems' phenomena that would usually involve more intricate mathematical models. For further information on (max, +) algebra for production chains and transportation networks [10] can be consulted. [9] can be consulted for (max, +)-linear system theory, [11] for (max, +) theory applied to traffic control, [12] for an application to production scheduling in manufacturing systems, and [13] for maintenance modeling for a helicopter. Moreover, considerable effort has been dedicated to exploiting the potential of (max, +) algebra combined with automata theory, leading to the study of (max, +) automata which can also be applicable to schedule optimization problems; see [14], [15], and [16] for developments in this field. To our knowledge, no work has yet been developed to optimize pipeline networks' scheduling while integrating maintenance based on a (max, +) approach.

III. System Description

A. Case Study

The case study is a seaport for oil export, but the work is be extendable to flow networks of different nature. The system consists of a set of tanks, for oil storage, linked through an intricate pipeline network to a set of loading arms placed at the docks of the seaport where clients arrive to be served. It is considered that oil flows by gravity through the pipeline network.

1) Oil Transfer Aspects: A request in the system (i.e. an oil transfer operation) represents an oil tanker requesting a specific type and quantity of oil with a deadline to be respected as strictly as possible. If this deadline is exceeded, the oil company incurs into a penalty, which is considered to be related to the delay in the fulfillment of the request and also to the priority of the client. Each of these requests is fulfilled through the selection of an alignment (i.e. a path) in the oil pipeline network, which implies the opening of the valves included in this alignment and the closing of all adjacent valves to the alignment, in order to isolate it from the rest of the network since two types of oil must not mix[1]. From industrial data it is known that oil transfer operations

[1]Even though a specific case could correspond to the mixture of two identical types of oil, oil mixture is not allowed in any scenario since sharing a section of an alignment by two transfer operations could result in lower product flow rate and several aspects such as pumping power and pipeline dimensions would have to be considered and are not the focus of this work.

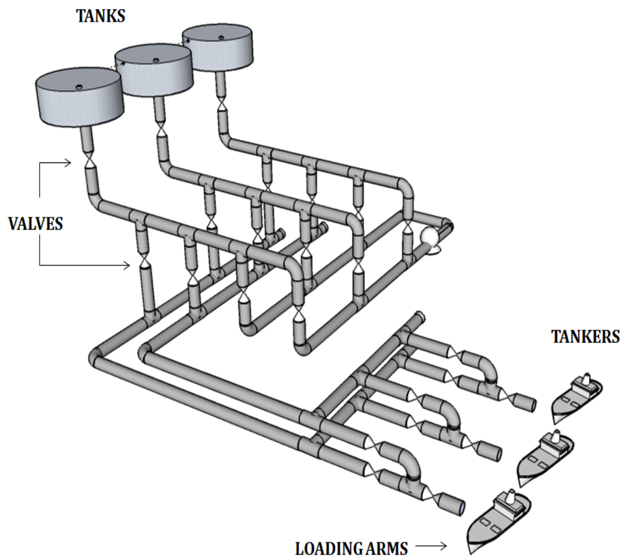

Fig. 1. Oil seaport example

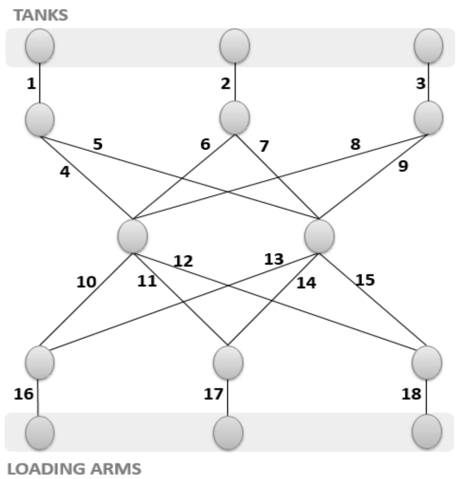

Fig. 2. Undirected graph model for the oil seaport example in Fig. 1

take hours, whereas commutations on valves are assumed to take seconds. In this work, it is considered that the alignment is previously established for each transfer operation.

Considerable effort has been devoted to optimizing other features of this type of system, most of the results being adaptable for flow networks of different nature. [17] can be consulted for generic alignment selection maximizing operative capacity (i.e. simultaneous disjoint alignments) in the network and [18] for generic alignment selection maximizing operative capacity while minimizing failure risk on valves. For illustration purposes on the system configuration, Fig. 1 shows an example of a simplified oil seaport and Fig. 2 shows the network model as an undirected graph where arcs represent the valves and the nodes represent the linked pipeline segments.

2) Maintenance Aspects: It is considered, that maintenance activities are to be executed on valves. In order to do so, all adjacent valves must be closed so that the valve in question can be maintained. We call these adjacent valves: isolating valves. A preventive maintenance schedule is considered to have been determined by the specialized personnel, and this maintenance information must be integrated with other optimization needs related to client satisfaction and possible costs for the overall system performance.

Maintenance schedules are considered to have been determined taking into account the reliability of the network's valves, according to the particular maintenance policies of the oil company. In any case, whether the company deals with preventive or predictive maintenance, the schedule of the maintenance activities on each valve is considered to be an input for the overall optimization model for the pipeline network.

The flexibility level of each and every one of the maintenance activities proposed in the schedule should be coherent with device reliability, as well as global costs for the seaport. Therefore, according to reliability information on devices, as well as possible estimated maintenance costs (i.e. the monetary consequences of applying vs not applying the specific maintenance task), and possible penalties due to delays in the service of a client, maintenance operations and oil transfer operations should be adjusted to optimize the overall network performance.

3) Conflicts between Oil Transfer Operations: As aforementioned, simultaneous alignments to be used for two or more requests must be disjoint since different oil batches are not allowed to mix. The work in [5] yields the following definition.

Definition 1. Two or more alignments (for oil transfers) are in conflict if they share at least one valve and if either the valve requires different states for different alignments or it requires being open for more than one alignment.

Fig. 3(a) (from [5]) shows two disjoint alignments to satisfy requests R_1 and R_2. Solid lines illustrate the valves to open and dotted lines (of the same color) the valves to close in order to isolate the alignment; e.g.: to enable the alignment for R_1 valves 1, 4, 10, and 16 must open and valves 5, 6, 8, 12, 11, and 13 must close. In Fig. 3(a), no conflict arises for any valve since the common resources (valves 5, 8, 12, and 13) are all valves to be closed, therefore they can enable both transfer operations simultaneously.

On Fig. 3(b), another request is added and conflicts arise for valves 10 and 16, since they should open for 2 transfer operations (therefore, mixing 2 types if oil), and for valves 4 and 6, since their required commutations are different for both transfer operations (which is physically impossible); therefore, R_1 and R_2 cannot be processed simultaneously.

It is crucial for the overall network performance to serve as many clients as possible in the shortest amount of time (which liberates resources to be used in future requests) which translates into simultaneous execution of operations whenever possible.

4) Conflicts between Maintenance and Oil Transfer Operations: Naturally, when a valve is being used within an alignment to satisfy a request (i.e. an oil transfer operation), it cannot be used to carry out a maintenance operation on it, and vice versa.

This type of conflict is very straightforward and an optimum schedule must determine the time intervals in which the valve will be used to satisfy a request and the time intervals in which it will be put out of service to carry out a maintenance operation so that these intervals do not overlap. Fig. 4(a) shows an example depicting only open valves in the

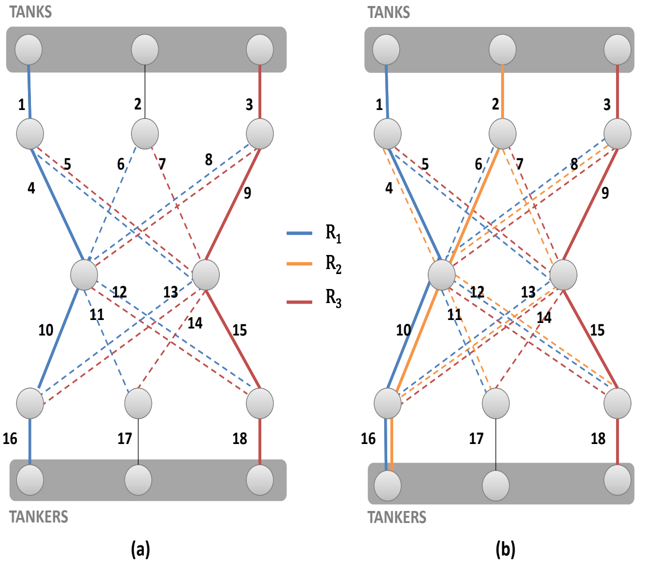

Fig. 3. Non-conflicting and conflicting alignments for oil transfer operations

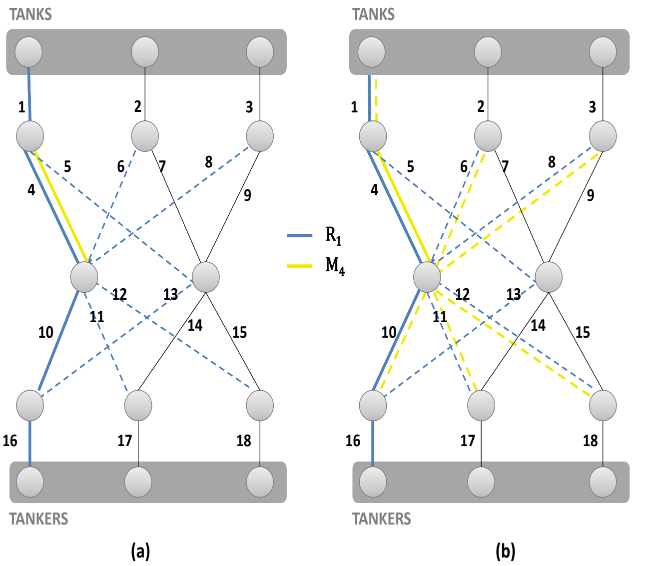

Fig. 4. Conflict between an oil transfer operation and a maintenance activity

alignment and the valve to be maintained. Fig. 4(b) illustrates the isolating valves for each operation.

The work in [5] has yielded the following definitions for conflicts between maintenance and oil transfer operations.

Definition 2. A valve can enable oil flow in an alignment or it can isolate the alignment, but it cannot simultaneously be subject to maintenance.

Definition 3. For a valve, the conflict between its request as an isolating valve for maintenance and as an open valve for oil transfer in an alignment will always generate a conflict between the valve in maintenance and an isolating valve for the alignment in question.

5) Scheduling Oil Transfer Operations on a Seaport - General Aspects: Schedules are determined mainly in terms of client deadline requirements and on network availability. Network availability translates into resource availability in order to enable alignments. In this work, the resources of interest are valves and their availability is determined by their maintenance activities (either preventive or corrective) and by their allocation by different alignments aiming at satisfying other oil transfer operations for other clients. Client requirements include deadlines for tanker loading, which in case of violation by the seaport imply monetary penalties. Hence, the seaport aims at minimizing global penalty costs, for a time horizon with '*nc*' clients to serve and minimizing the time invested to serve all clients in the time horizon (this way, following operations can be treated earlier and more clients can be served). In this work, operation scheduling refers to scheduling oil transfer operations as well as maintenance operations, all leading to different unavailability periods on devices.

Maintenance scheduling implies an entire research field. Typical aspects to consider are device reliability; repair, replacement and inspection costs; condition monitoring costs and results; and storage of spare parts as well as potential costs for not applying the proper maintenance operations, among others. Maintenance scheduling should then be an exhaustive study usually carried out in the seaport by a specialized maintenance department.

In this work, a preliminary maintenance schedule is assumed to have been properly generated by the specialized maintenance personnel and operations scheduling is studied as the integration of oil transfer operations and maintenance activities in order to optimize global network performance. Therefore, in certain cases, maintenance operations would be delayed under certain conditions in order to optimize the global optimization objective.

For each client, a negotiation occurs with the seaport. In this phase, the client imposes (within certain conditions not relevant to this work) for a specific tanker, the penalty to be paid by the seaport in case of delay (in thousands of dollars per hour) caused by the seaport. At the same time, the seaport imposes a time window of three days within which the tanker can arrive and be immediately docked and served. From the moment of arrival within this time window, the maximum service time for every tanker is 36 hours for loading and 4 hours for paperwork. Since the focus of this paper is on seaport transfer operations, the paperwork interval is discarded and the focus is on the maximum loading interval of 36 hours as the deadline for each tanker. From that point on, every extra hour invested in the service of the tanker will result in a penalty for the seaport, if the delay has been indeed caused by the seaport. Conversely, if the service of a tanker surpasses the 36 hours due to tanker's technical difficulties, then the client pays the seaport a penalty for dock over-occupation. Operations management on the seaport contributes to the general objective of profit maximization but client-payed-penalties do not represent in any way an optimization objective, i.e. they are unexpected events which the seaport does not aim at maximizing through operations' scheduling. If the tanker arrives after its time window, the seaport does not incur into any penalties for the waiting time for the tanker to be served. No further information has been granted concerning other arrival scenarios and possible consequences in the service.

IV. (MAX,+) OPTIMIZATION MODEL WITH FLEXIBLE MAINTENANCE

As it was proposed in [5], a (max, +) optimization model for scheduling operations can be formulated in order to minimize the TCP or Total Cost due to Penalties. In [5] the TCP was defined as the total cost in which the seaport incurs due to late service of a set of clients for a time horizon. The TCP is calculated as the sum of all delays (i.e. delays in the service for tankers arriving within their authorized time windows) each one multiplied by their respective costs (which depends on each client). One of the main set of constraints proposed in [5] was the one for resource sharing presented in (1) in conventional algebra and (2) in (max,+) algebra. In the following, only (max, +) notation will be used.

The aforementioned constraint determines the start date (x_{ikl}), also called *'dater'* in the (max, +) context, for a commutation l (open or close) on a valve k to satisfy a request i.

In (2) variables include:

- x_{ikl}: dater for a commutation l on valve k for an oil transfer operation, also called request i,
- xp_{hk}: dater for a maintenance operation h on the same valve k (therefore a conflicting operation for resource allocation),
- $x_{i'kl'}$: dater for a conflicting transfer operation i' requesting the same valve,
- $V_{ikl,hk}$: binary decision variable which ultimately solves the conflict (i.e. the precedence) between the use of the valve for the studied oil transfer operation i and the maintenance operation h,
- $V_{ikl,i'kl'}$: analogously, it defines the precedence between two conflicting oil transfer operations i and i',
- u_i: arrival date for the tanker for request i,
- $ztp_{hk}, zp_{i'}$, and $zc_{i'}$ represent, respectively, the possible unexpected delays in the maintenance operation, in the service of a client due to technical difficulties in the terminal and in the service of a client due to technical difficulties within the tanker.
- Parameters include: t_0, tp_{hk}, and $p_{i'}$ which respectively correspond to the start date of the scheduling time horizon, and the nominal durations for the maintenance activity, and the oil transfer operation.

In (2), $\mathcal{O}$ is the set of all possible commutations on valves to be executed in order to satisfy a set of nc requests. $\mathcal{M}$ is the set of all maintenance activities previously scheduled, and ISO_{hk} denotes the set of isolating valves for a maintenance operation hk. $\mathcal{M}$ denotes the set of maintenance operations hk.

$$x_{ikl} = max\Big(t_0 \ ; u_i \ ; max_{hk}\big(xp_{hk} + tp_{hk} + ztp_{hk} + \mathrm{V}_{ikl,hk}\big); max_{i'}\big(x_{i'kl'} + p_{i'} + zp_{i'} + zc_{i'} + \mathrm{V}_{ikl,i'kl'}\big)\Big) \quad \forall\ ikl, i'kl' \in \mathcal{O} | \ [i \neq i' \wedge (l \neq l' \vee l = l' = 1)], \ \forall hk \in \mathcal{M} \tag{1}$$

$$x_{ikl} = t \oplus u_i \oplus \Big(\bigoplus\nolimits_{hk}\big(xp_{hk} \otimes tp_{hk} \otimes ztp_{hk} \otimes \mathrm{V}_{ikl,hk}\big)\Big) \oplus \Big(\bigoplus\nolimits_{i'}\big(x_{i'kl'} \otimes p_{i'} \otimes zp_{i'} \otimes zc_{i'} \otimes \mathrm{V}_{ikl,i'kl'}\big)\Big) \quad \forall\ ikl, i'kl' \in \mathcal{O} | \ [i \neq i' \wedge (l \neq l' \vee l = l' = 1)], \ \forall hk \in \mathcal{M} \tag{2}$$

Equation (2) states that the start date for a commutation to satisfy an oil transfer operation will depend on the start date of the time horizon for scheduling, the arrival date of the tanker in the terminal, the maximum completion time of all conflicting maintenance operations which will precede the oil request, and the completion time of all conflicting oil transfer operations which also be executed before request i. Notice that for all conflicting operations interruption variables model the possible delays that could arrive in the execution of the set of operations. All decision variables are binary, taking the values $e = 0$ or $\varepsilon = -\infty$ in (max,+) theory. For instantiation purposes, values are $zero$ or B so that B is a very large negative real number.

Moreover, each decision variable has a complementary one (e.g. if $V_{ikl,i'kl'} = 0$, then $V_{i'kl',ikl} = B$ or vice versa). For example, in (2), when $V_{ikl,i'kl'} = B$ the entire fourth term of the global maximization is negligible which implies that the completion time of operation $x_{i'kl'}$ is not relevant to calculate x_{ikl}, indicating that request i will be executed before request i'. This value assignment would automatically generate the value assignment of the complementary decision variable (i.e. $V_{i'kl',ikl} = e$) which means that in the constraint to determine $x_{i'kl'}$ the completion time of operation ikl would indeed be taken into account.

This work is an extension of the results obtained in [5] (where oil transfer operations were scheduled within a fixed maintenance framework). Here, maintenance relaxation is considered in order to improve the overall system performance and diminish the TCP. Also, we explore some (max,+)-linear representations and solutions of the optimization model under certain assumptions. Maintenance relaxation is justified in this work given the time horizon in which we consider oil transfer operations to be scheduled and the time intervals in which maintenance is carried out in valves within an oil pipeline network. Oil transfers are scheduled for a time horizon of a week given the possible variability of the arrival dates for tankers in the seaport; however, valve maintenance operations could be executed in larger time intervals such as once per year.

This work does not cover a thorough research on maintenance optimization for valves in oil pipeline networks. The aim is at optimizing oil transfer operations through minimization of the TCP while providing the best integration with a pre-established flexible maintenance schedule. For a given application of the model, this maintenance schedule should stem from the behavior of each particular system, considering the maintenance policies and constraints through which devices are managed. Depending on the instrumentation in place to monitor the devices' condition, and consequently on the reliability of each device, as well as its criticality within the process, it could be determined if, and to which extent, the maintenance operation could be delayed or advanced in order to allow the optimal execution of oil transfer operations and minimize the TCP.

Equation (3) shows the proposal for the set of constraints determining the *'daters'* for flexible maintenance operations.

$$xp_{hk} = \Big(\bigoplus\nolimits_{i}\big(x_{ikl} \otimes p_i \otimes zp_i \otimes zc_i \otimes V_{hk,ikl}\big)\Big)$$

$$\oplus \left(\bigoplus_{h'k'} (xp_{h'k'} \otimes tp_{h'k'} \otimes ztp_{h'k'} \otimes V_{hk,h'k'}) \right)$$
$$\oplus xp_{fixed_{hk}} \otimes xp_{delay_{hk}}$$
$$\forall\ i|ikl \in \mathcal{O},\ \forall\ hk \in \mathcal{M},\ \forall\ h'k' \in ISO_{hk} \qquad (3)$$

Analogously to (2), in (3) the first maximization terms model potential conflicts of the maintenance operation hk with other oil transfer or maintenance operations. Decision variables behave analogously to the ones in (2). The maintenance schedule is relaxed through the last term of the maximization in (3): $xp_{fixed_{hk}} \otimes xp_{delay_{hk}}$, where $xp_{fixed_{hk}}$ denotes the start date for the maintenance operation hk obtained from the predefined schedule and $xp_{delay_{hk}}$ denotes the possible variation of the predefined date.

In [5], the optimal schedule for the instance presented in Fig. 5 and the input data in Table I is obtained; see Fig. 6. In that instance, 2 maintenance activities are scheduled on valves 13 and 15 at $t = 100$ and $t = 130$ with durations of 10 and 12 hours, respectively. In order to obtain the optimal schedule in the flexible maintenance framework, all other constraints in the (max,+)-optimization model proposed in [5] still hold. These constraints are (4-11) with the objective function of minimizing the TCP (see (14)).

$$x_{ikl} = x_{ik'l'} \qquad (4)$$

$$V_{ijkl,i'kl'} \otimes V_{i'kl',ijkl} = B \qquad (5)$$

$$V_{ijkl,i'kl'} \oplus V_{i'kl',ikl} = 0 \qquad (6)$$

Equation (4) states that daters for all valves involved in the same oil transfer operation i have the same value. Equations (5) and (6) restrict the values of the decision variables to be either $zero$ or B for potential conflicts between two transfer operations. Equations (7) and (8) restrict the values of decision variables for conflicts between a maintenance activity and a transfer operation, and finally (9) and (10) do the same for conflicts between two maintenance operations.

$$V_{ikl,hk} \otimes V_{hk,ikl} = B \qquad (7)$$

$$V_{ikl,hk} \oplus V_{hk,ikl} = 0 \qquad (8)$$

$$V_{hk,h'k'} \otimes V_{h'k',hk} = B \qquad (9)$$

$$V_{hk,h'k'} \oplus V_{h'k',hk} = 0 \qquad (10)$$

$$D_i = \begin{cases} u_i \otimes 36 & \forall\ i|u_i \in tw_i \\ x_{ikl} \otimes 36 & \forall\ i|u_i > utw_i \\ ltw_i \otimes 36 & \forall\ i|u_i < ltw_i \end{cases} \qquad (11)$$

$$dpr_i = (x_{ikl} \otimes p_i \otimes zp_i \otimes zc_i \oslash D_i) \oplus 0 \qquad \forall i|ikl \in \mathcal{O} \quad (12)$$

In (11) the deadline D_i for a request i is modeled where $tw_i = [ltw_i, utw_i]$ is the authorized time window of three days for the tanker's arrival.

The *delay per request* (dpr) is determined in (12) which is the difference between the completion time of a request (including the possible delays caused by the seaport and/or the client) and its deadline. For validation purposes, we rely on hypothesis 1 and, thereby, if both parties incur into delays of the same length, no penalty is paid by either party. However, if the delays are not equal, the party with the greatest delay pays the difference between both delays. Ultimately, the policy for handling shared delays can be adjusted to the characteristics of each operational framework. Within this context, (13) models the *penalized delay for the seaport* (pds) per request; i.e. the time interval (hours) for which the seaport will actually incur into penalties.

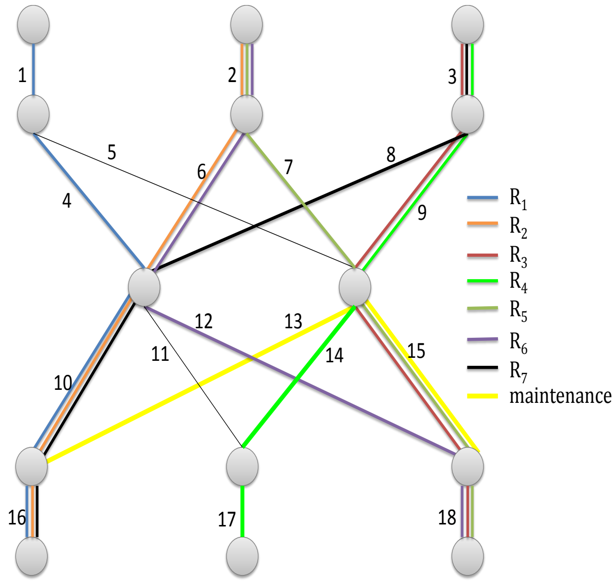

Fig. 5. Operations to be scheduled

TABLE I
INPUT DATA FOR INSTANCE IN FIG. 5

Request	Processing Time (hours)	Penalty (\$/hour)	Time Window for Arrival (days)
R_1	20	4000	[4,6]
R_2	25	2500	[2,4]
R_3	20	3000	[2,4]
R_4	15	2500	[1,3]
R_5	20	2500	[1,3]
R_6	15	3000	[2,4]
R_7	10	2000	[3,5]

Hypothesis 1. the dock over-occupation penalty per hour per client (paid by each client) is considered equal to the penalty per hour for that same client paid by the seaport in the case of delay caused by the seaport[2].

$$pds_i = \begin{cases} \oslash \left[(\oslash zu_i \oslash zp_i \otimes zc_i) \oplus (\oslash dpr_i) \right] \\ \qquad\qquad \forall (zu_i \otimes zp_i) > zc_i \\ 0 \qquad\quad otherwise \end{cases} \qquad (13)$$

$$Min\ TCP = \bigotimes_i \left(\bigotimes_{n=1}^{pds_i} c_i \right) \forall i \qquad (14)$$

Equation (14) computes the *Total Cost due to Penalties* (TCP) for all requests in the time horizon. It is the (max, +) algebra representation for the sum of the products of each penalized delay (in hours) and its corresponding penalty (in \$/hour).

To illustrate the advantages of maintenance relaxation, in the example shown in Fig. 5 and the input data in Table I maintenance could be relaxed in ± 10 hours from the

[2]For validation purposes only and can be adjusted according to each flow network

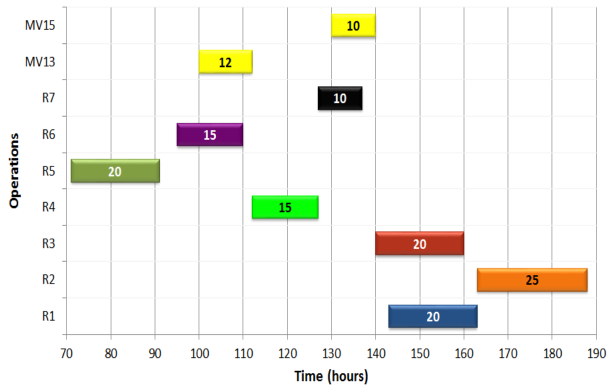

Fig. 6. Optimal schedule with fixed maintenance, [5]

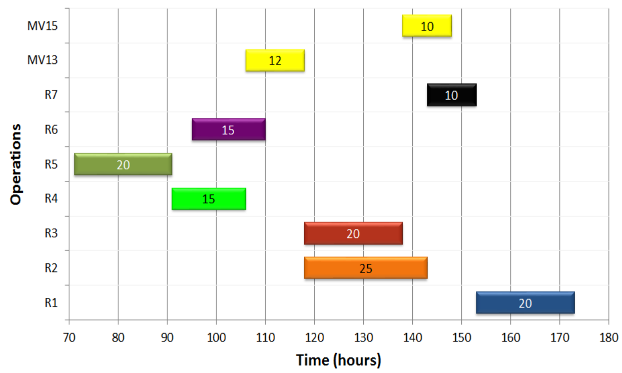

Fig. 7. Optimal schedule with flexible maintenance

fixed dates of $t = 100$ and $t = 130$, for valves 13 and 15, respectively. This is in no way restrictive; in fact, it is encouraged that this variability should be determined as a function of the reliability of each device, as well as costs related to the delay.

In the flexible maintenance framework, the optimal schedule is depicted in Fig. 7 with a TCP of \$21000, whereas in the fixed maintenance framework (see Fig. 6) the TCP is minimized to a value of \$137000.

As expected, a flexible maintenance management allowed processing certain clients which would have been delayed otherwise[3].

In both the fixed and the flexible maintenance framework it is assumed that all tankers arrive within their respective time windows (which generates considerable conflicts) at the last day at the last hour except for the client for R_2 which arrives at the 5th day at 10 am (i.e. after its authorized time window). It is also considered that at this point no unexpected interruption occurs while processing a request or a maintenance operation (i.e. $zp_i = zc_i = ztp_i = 0$) The objective function's value is due to a delay of 29 hours and 20 hours for requests R_3 and R_4. In other words, given a normalized time scale in hours, the tanker for R_3 arrives at the last day of its time window (i.e. day 4) at 10 am which yields and arrival time of $u_3 = 4days \times 24hours - 1hours = 95hours$. This yields a deadline of $D_3 = (95+36)hours = 131hours$ and as can be seen in the schedule in Fig. 6 the completion time for R_3 is at $t = 160$.

[3]model validation was done in the optimization software LINGO as in [5]

Given the cost of delay for this client (Table I) of \$3000, this yields a penalty of $[(160 - 131)hours] \times \$3000 = \$87000$. For R_4, applying the same analysis, the incurred penalty is of $20hours \times \$2500 = \$50000$, generating a TCP of \$137000.

In the flexible maintenance schedule, maintenance operations on valves 13 and 15 were delayed 6 and 8 hours, respectively, which allowed rearranging oil transfer operations so that the TCP could decrease.

V. Exploration of Linear Representations

In the results in [5] for fixed maintenance, as well as in the results for flexible maintenance, constraints in resource allocation are such that the optimal schedule is obtained through decision variables which establish the precedence among conflicting operations such as $V_{hk,ikl}, V_{ikl,hk}, V_{ikl,i'kl'}$, and $V_{i'kl',ikl}$, among others. In both cases, the model is instantiated with the software optimization tool LINGO which assigns values to the decision variables allowing to determine the dater for each operation and, consequently, the TCP (for more details on LINGO [19] can be consulted). In the (max,+) sense, this equation is linear when all decision variables take a value.

In order to explore some linear representations of the model, a routing policy to determine the precedence among conflicting operations has been considered. This routing policy may follow any precedence criterion or combined criteria that can be identified as crucial in any given system. Here, for validation purposes, we propose to consider an indicator of the Total Potential Penalty, in order to compare conflicting operations and determine the best precedence.

Here, the Total Potential Penalty, or TPP_i for a given request i, is the potential penalty in which the seaport would incur in case of starting client service right after its deadline.

For a request i, u_i is the tanker's arrival time to the seaport, p_i its processing time, c_i the penalty cost per time unit of delay, and tw_i the authorized time window for arrival. Assuming 2 conflicting clients i and $i^{'}$ arrive at the same time at the seaport and that $u_i \in tw_i$ and $u_{i'} \in tw_{i'}$, then (in conventional algebra notation) $TPP_i = p_i \times c_i$ and $TPP_{i'} = p_{i'} \times c_{i'}$. Request i precedes request $i^{'}$ iff $TPP_i \geq TPP_{i'}$. In other words, the potential risk of delaying request i is greater that the potential risk of delaying $i^{'}$.

Assuming precedence is known for conflicting operations, for example comparing the Total Potential Penalties (TPP) for conflicting clients, value assignment of the decision variables would generate a (max,+)-linear system. The proposal of the TPP as a differentiator for the precedence of conflicting operations is not restrictive and could be any other criteria that the modeler considers fit for the specific case study.

To illustrate the linearity of the problem for this application, for the example shown in Fig. 5, assuming only the seven oil transfer operations are to be scheduled, and also that constraints are simplified by formulating daters directly for requests instead of a dater for each valve commutation, then we would obtain seven conflict-related constraints for seven requests. Given the same constraint structure as in (2) and assuming the time horizon starts at $t = 0$, the constraint for dater x_1 of request $i = 1$ (see Fig. 5 to detail the alignment) would be: $x_1 = u_1 \oplus x_2 p_2 V_{1,2} \oplus x_6 p_6 V_{1,6} \oplus x_7 p_7 V_{1,7}$, since

TABLE II
INPUT DATA FOR LINEAR MODEL VALIDATION

Request	p_i (hours)	c_i ($/hour)	TPP_i
R_1	20	1000	20000
R_2	20	2000	40000
R_3	20	3000	60000
R_4	20	4000	80000
R_5	20	5000	100000
R_6	20	6000	120000
R_7	20	7000	140000

conflicting operations are requests 2, 6, and 7. Here, $V_{i,i'}$ denotes the decision variable determining the precedence between requests i and i'. The entire set of constraints for the proposed instance is as stated in (15-21).

$$x_1 = u_1 \oplus x_2 p_2 V_{1,2} \oplus x_6 p_6 V_{1,6} \oplus x_7 p_7 V_{1,7} \tag{15}$$

$$x_2 = u_2 \oplus x_1 p_1 V_{2,1} \oplus x_5 p_5 V_{2,5} \oplus x_6 p_6 V_{2,6} \oplus x_7 p_7 V_{2,7} \tag{16}$$

$$x_3 = u_3 \oplus x_4 p_4 V_{3,4} \oplus x_5 p_5 V_{3,5} \oplus x_6 p_6 V_{3,6} \oplus x_7 p_7 V_{3,7} \tag{17}$$

$$x_4 = u_4 \oplus x_3 p_3 V_{4,3} \oplus x_5 p_5 V_{4,5} \oplus x_7 p_7 V_{4,7} \tag{18}$$

$$x_5 = u_5 \oplus x_2 p_2 V_{5,2} \oplus x_3 p_3 V_{5,3} \oplus x_4 p_4 V_{5,4} \oplus x_6 p_6 V_{5,6} \tag{19}$$

$$\begin{aligned} x_6 = u_6 \oplus x_1 p_1 V_{6,1} \oplus x_2 p_2 V_{6,2} \oplus x_3 p_3 V_{6,3} \oplus \\ x_5 p_5 V_{6,5} \oplus x_7 p_7 V_{6,7} \end{aligned} \tag{20}$$

$$\begin{aligned} x_7 = u_7 \oplus x_1 p_1 V_{7,1} \oplus x_2 p_2 V_{7,2} \oplus x_3 p_3 V_{7,3} \oplus \\ x_4 p_4 V_{7,4} \oplus x_6 p_6 V_{7,6} \end{aligned} \tag{21}$$

Assuming the worst case scenario where all tankers arrive at the same time and within their time windows, e.g. $u_i = 0$ and $u_i \in tw_i \forall i$, and using the input data proposed in Table II for testing purposes, so that $\forall i | 0 < i < 7 \quad \Rightarrow TPP_i < TPP_{i+1}$, the value assignment process for decision variables is done based on the prioritization of requests with greatest TPP values and also on the maximum operative capacity of the network.

The maximum operative capacity represents the maximum set of disjoint alignments in the network. Since it is of paramount importance to carry out as many parallel operations as possible, in order to serve more clients in shorter time periods, operations with the greatest TPP with possible parallel execution must be the firsts to start.

Since it can be directly inferred that for the proposed topology in Fig. 1 the maximum operative capacity corresponds to 2 alignments, we start by considering the execution of R_7 (operation with the greatest TPP) in parallel with the next possible simultaneous operation with the highest TPP, which corresponds to R_5 (since R_6 cannot be simultaneously executed with R_7). The non-dependence of these 2 operations from other operations in the system translates into the assignment of the decision variables in (21) and (19) to $\varepsilon = -\infty$. This value assignment propagates into the assignment of other decision variables in the system to respect the complementarity constraint. For example, if in (19) $V_{5,6} = \varepsilon$, then in (20) $V_{6,5} = e$.

Moreover, all remaining variables are set to the proper values which must respect the TPP criterion; for example in (16) $V_{2,6} = e$ (which states that R_2 depends on the completion time of R_6) since $TPP_6 > TPP_2$ which also yields that $V_{6,2} = \varepsilon$ in (20). Substituting values, the equations set (15-21) transforms into (22-28).

$$x_1 = x_2 p_2 \oplus x_6 p_6 \oplus x_7 p_7 \oplus e \tag{22}$$

$$x_2 = x_5 p_5 \oplus x_6 p_6 \oplus x_7 p_7 \oplus e \tag{23}$$

$$x_3 = x_4 p_4 \oplus x_5 p_5 \oplus x_6 p_6 \oplus x_7 p_7 \oplus e \tag{24}$$

$$x_4 = x_5 p_5 \oplus x_7 p_7 \oplus e \tag{25}$$

$$x_5 = e \tag{26}$$

$$x_6 = x_5 p_5 \oplus x_7 p_7 \oplus e \tag{27}$$

$$x_7 = e \tag{28}$$

The matrix representation for this system is of the form $X = AX \oplus b$ as follows:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} . & p_2 & . & . & . & p_6 & p_7 \\ . & . & . & . & p_5 & p_6 & p_7 \\ . & . & . & p_4 & p_5 & p_6 & p_7 \\ . & . & . & . & p_5 & . & p_7 \\ . & . & . & . & . & . & . \\ . & . & . & . & p_5 & . & p_7 \\ . & . & . & . & . & . & . \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} \oplus \begin{pmatrix} e \\ e \\ e \\ e \\ e \\ e \\ e \end{pmatrix}$$

As stated in [20], $X = A^* b$ satisfies the equation $X = AX \oplus b$, where $A^* \stackrel{def}{=} \bigoplus_{n \in \mathbb{N}} A^n$.

(max,+) matrix product is defined as $(A \otimes B)_{ij} = \bigoplus_{k=1}^{n} A_{ik} \otimes B_{kj}$; for more details [9] can be consulted. Considering that for the proposed system $A^m = \varepsilon_{7\times7} \; \forall m > 3$ (where $\varepsilon_{7\times7}$ is a matrix with entries equal to ε), then $A^* = e_{7\times7} \oplus A \oplus A^2 \oplus A^3$ (where $e_{7\times7}$ is the identity matrix). The following results are obtained.

$$A^2 = \begin{pmatrix} . & . & . & . & 40 & 40 & 40 \\ . & . & . & . & 40 & . & 40 \\ . & . & . & 40 & 40 & . & 40 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \end{pmatrix}$$

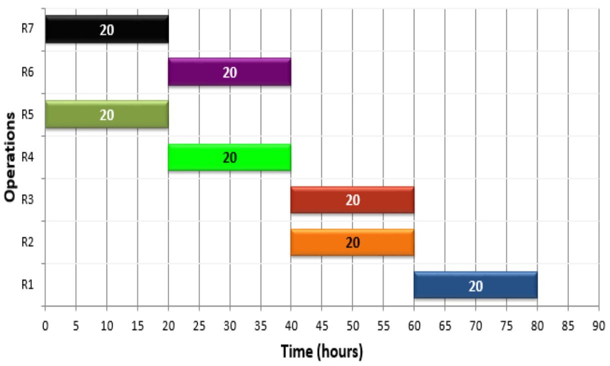

Fig. 8. Optimal schedule for oil transfer operations from the linear model

$$A^3 = \begin{pmatrix} . & . & . & . & 60 & . & 60 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \end{pmatrix}$$

$$A^* = \begin{pmatrix} e & 20 & . & . & 60 & 40 & 60 \\ . & e & . & . & 40 & 20 & 40 \\ . & . & e & 40 & 40 & 20 & 40 \\ . & . & . & e & 20 & . & 20 \\ . & . & . & . & e & . & . \\ . & . & . & . & 20 & e & 20 \\ . & . & . & . & . & . & e \end{pmatrix}$$

Therefore the linear system's solution obtained through $X = A^*b$ is the following, and corresponds to the schedule depicted in Fig. 8.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} 60 \\ 40 \\ 40 \\ 20 \\ e \\ 20 \\ e \end{pmatrix}$$

These are the daters for the seven oil transfer operations so that the TCP is minimized ($TCP = \$204000$ since all deadlines are $D_i = 36$) and operative capacity is fully exploited. For this trivial example the analysis can be done directly through Fig. 5 and the data in Table II. The simplicity of the instance allows to deduce that this is indeed the optimal schedule given the input data and topology of the network. In this case, the system is autonomous since a control variable is not required to delay or advance operations. All operations execute given pre-established conflict situations and the only initial premise is to maximize operative capacity and prioritize operations with greater potential penalties. For more complex systems, the procedure remains intuitive, i.e. formulation of the main constraints as simple and direct (max,+) equations, and finally the schedule for the autonomous system can be obtain directly through (max,+) matrix products.

VI. Conclusion

The proposed model exploits the benefits of (max,+) algebra in terms of intuitiveness and practicity. Maintenance relaxation was proposed with a fairly simple mathematical adjustment to a previously defined (max,+) model which has allowed to improve global results in the minimization of the TCP. The advantage of the approach is the concise and intuitive representation. Moreover, (max,+)-linear representations, for a flow network model, have been obtained under certain considerations in a scenario in which a set of given criteria can suggest a level of prioritization among the conflicting operations. Further work aims at approaching multi-objective optimization to incorporate additional supervision optimization criteria in the network.

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