# Bulk Flow Behaviour of a Two-Stage Impedance Pump

V. C.-C. Lee, H.-S. Gan, Y. A. Abakr, and K.-C. Woo, *Member, IAENG*

***Abstract*— Impedance pump is a type of valveless pumping mechanism, in which the design consists only elastic tubing joined with tubes of different impedances. By inducing a periodic asymmetrical compression on the elastic tube, thus will eventually produce a unidirectional flow within the system. Numerous works, computational and experimental, had been performed in past decades to explore this phenomenon and to further understand its characteristics. However, not many works were found in multi-stage valveless impedance pumping. The aim of this work is to have an insight on the bulk flow behaviour of an open-loop two-stage impedance pumping system at resonance. This paper emphasizes on the experimental study of the bulk flow behaviour with a partial of numerical study. Characteristics of a two-stage impedance pumping system with respect to its governing parameters were evaluated and results obtained were very significant.**

***Index Terms*—bulk-flow, two-stage, valveless, resonance**

## I. Introduction

Impedance pump is a type of valveless pump formed by joining flexible tubing to a rigid one and by asymmetrical compression at a single location of the fluid-filled elastic tube, thus will eventually result in unidirectional flow due to the mismatch in fluid impedance [1–8]. The pumping mechanism is shown to be highly sensitive towards the impedances in the tube, the location and frequency of excitation [3,4,6,7,9]. The first demonstration of valveless pumping through an impedance pump, was demonstrated by Gerhart Liebau in 1954, using an elastic tube connected to reservoirs at different heights [10, 11]. Hickerson [4] conducted a comprehensive experimental study on impedance pump performance demonstrating their intrinsic behaviours. Bringley et al [5] performed experimental study and through a simple mathematical model, which can be described by ordinary differential equations, a physical explanation of valveless pumping and essential pumping mechanisms were identified. Another study of Hickerson [12] showed that the flows are highly sensitive to duty cycle and excitation frequency and demonstrated that an open-loop system can create and sustain a pressure head, and that an elastic material is not a necessary condition for impedance driven flow.

Loumes [9] in 2007 introduced the concept of multilayer impedance pump, a pumping mechanism inspired by the embryonic heart structure in which the flow output and inner wall motion are found to be maximal when the pump is excited at the resonant frequency while only a small excitation is needed to produce a significant flow. Occurrence of valveless pumping in a fluid-filled system consisting of two open reservoirs connected by an elastic tube was investigated by studying the relationships among wave propagation velocity, tube length, and resonant frequencies associated with shifts in the pumping direction via numerical simulations [7]. The study showed that the eigenfrequencies of the system constitute the resonant frequencies and the horizontal slope frequencies.

Jung in 2007 developed a lumped model of impedance pump [13]. In her model, the system is governed by ordinary differential equations for pressure and flow, with time-dependent elasticity, viscosity and inertia. Her studies involved numerous parameters including the excitation frequency, tube wall thickness and tube stiffness which showed that the phenomena of impedance pumping can be modelled using a simple lumped model which simplify the mathematical formulation of an impedance pump. Her work is however focused only on a closed-loop system. A multi-pincher impedance pump was studied using a one-dimensional numerical simulation [14]. The results indicated that flow rate can significantly increase when using a sequential array of compression mechanisms operating in resonant frequency with the appropriate phase between them. However, the results were not compared with experimental data.

Computational Fluid Dynamics (CFD) study of a two-stage system, integrated from two single-stage systems [15] using commercial software, Fluent, was performed and showed that the pumping work of two-stage system could reach doubling amplitude that of the single system. These results were however not validated with any experimental work back then. A number of experimental studies of open-loop two-stage impedance pumping system [16-18] were carried and results obtained were hypothetically in agreement with the simulation results [15]. Since then, no more work was found studying the pumping mechanism of valveless

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impedance pump in a multi-stage system. This paper describes the bulk flow behaviour for an open-loop two-stage system at its resonance through a series of numerical and experimental investigations. The bulk flow behaviour of a two-stage system with respect to varying excitation frequencies will be evaluated experimentally. Experimental observation at resonance will be compared with the simulated numerical results as well to verify the model's validity. Performance of a two-stage system in reference to a single-stage at resonance will be evaluated as well.

## II. EQUATION OF MOTIONS

In accordance to Refs [4,7,12], it is discovered that the highest efficiency of pumping work to be at the resonant frequency of the system. The resonant frequency can be expressed theoretically according to Ref [7] based on equation (1) as shown,

$$f_r = 0.17c \tag{1}$$

The resonant frequency, $f_r$ is expressed as a function of the wave propagation, $c$, which is entirely dependent on the physical properties of the tube. Substituting the wave propagation equation into equation (1), the resonant frequency hence becomes

$$f_r = 0.17\sqrt{\frac{Eh}{\rho d}} \tag{2}$$

where $E$ is the Young's Modulus of the elastic tube, $h$ is the thickness of the tube, $d$ is the diameter of the tube and $\rho$ is the density of fluid, which in this case is the water density. For the equation of motions in a valveless impedance pump, the factor of impedance is playing a more important role than the fluid viscosity. This is due to the physical nature of the system in which elastic tubing is used rather than rigid tubing. With elastic tubing in the system, the flow is resisted not only by wall shear stress but also obstruction of wave propagation due to the reactance, combination effects of tube elasticity and fluid inertia. With this additional obstruction in flow, the fluid viscosity term is hence substituted with the impedance term, which incorporates of viscosity and reactance.

Having the driving pressure difference excited in a sinusoidal waveform, the fluid motion can be expressed in terms of equations of motion as introduced in Refs [13,16,18,19] in second order as

$$\overline{L}\ddot{Q} + \overline{\Delta Z}\dot{Q} + \frac{1}{\overline{C}}Q = \omega \Delta P \cos \omega t \tag{3}$$

where

$$\omega = 2\pi f_r \tag{4}$$

and

$$\Delta P = P_e - P_t \tag{5}$$

for $\overline{L}$ is the fluid inertia, $\overline{\Delta Z}$ is the impedance difference within the tube, $\overline{C}$ is the tube elasticity, $\omega$ is the angular frequency of oscillation, and $\Delta P$ is the driving pressure difference. The driving pressure difference term consists of two parameters; $P_e$ is the excitation pressure applied onto the tube while $P_t$ is the transmural pressure in the tube. The left hand side of the equation is the equation of motion for flow, $Q$ and the right hand side of the equation is the driving pressure difference term of the system. Impedance difference is used here rather than the standard impedance term, as the flow induced is relative to the impedance differences across the system. This way, the fluid behaviour with reference to the change in the system physicality can relate in a more concise manner. A single-stage system will be studied in the beginning of this numerical study, in accordance to equations (3) to (5) and this will serve as a comparison benchmark for the two-stage system.

Figure 1 shows the physical model of a two-stage impedance pump illustrating how the flow is channelled in a two-stage system. Typically, the system will be filled with certain volume of water ($V_1$ to $V_5$) where each subscript denotes different part of the tube. As the excitation pressure is applied on the elastic tube, momentum is created in the system denoted as fluid inertia ($m\dot{u}_1$ to $m\dot{u}_5$) with respect to the impedance difference created within the system.

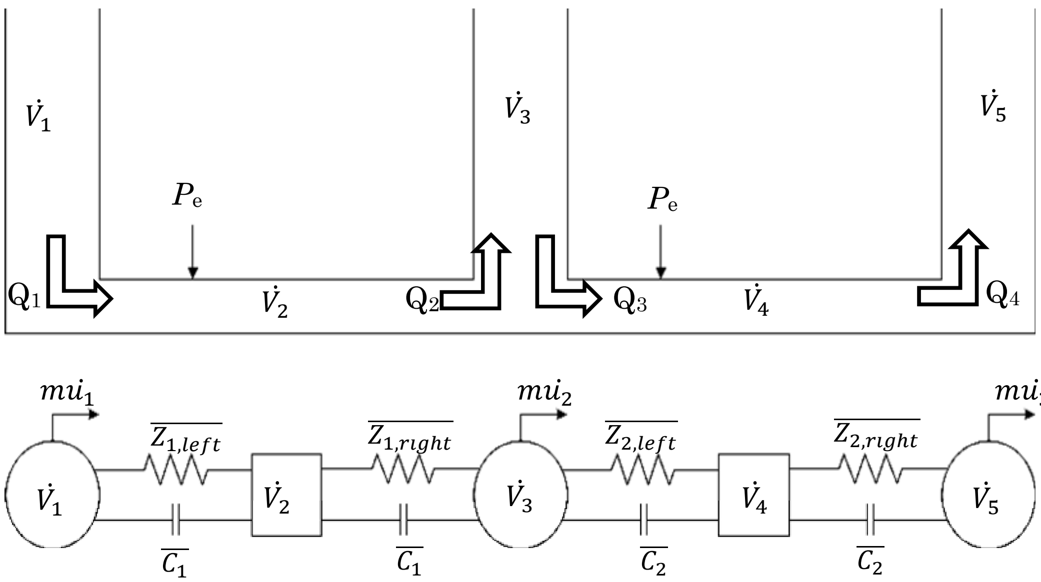


Fig. 1. Physical model of a two-stage system.

The fluid inertia hence induces flows ($Q_1$ to $Q_4$) in the system and caused a change in volumes ($\dot{V}_1$ to $\dot{V}_5$) creating pressure head difference in each individual reservoir. Maximum pressure head difference will be reached at zero flow when the system is under steady-state condition.

In a two-stage system as illustrated in Figure 1, there will be two driving pressure difference terms which will sum up to be the total driving pressure difference. Similarly, due to two compression mechanisms and two none-connected elastic tubes; the impedance and elasticity can be taken individually in this case as well. Hence, the equation of motion will then be

$$\left(\overline{L_1}+\overline{L_2}\right)\ddot{Q}+\left(\overline{\Delta Z_1}+\overline{\Delta Z_2}\right)\dot{Q}+\frac{\overline{C_1}+\overline{C_2}}{C_1 C_2}Q=\omega\Delta P_{total}\cos(\omega t+\phi) \quad (6)$$

where $\phi$ is the phase difference due to second frequency of oscillation.

As the compression mechanism originated from the same source and elastic tube is of the same material properties throughout the investigation, the fluid inertia and tube elasticity term can be taken as constant; leaving the only variable to be the impedance difference. The impedance differences can be expressed individually where $\Delta Z_1$ is due to the compression location along the tube between left reservoir and middle reservoir determined by the compression location, $x/L$ (a normalized representation in which $x$ is the instantaneous location of compression and $L$ is the total length of the elastic tube); $\Delta Z_2$ is due to the compression location along the tube between the middle reservoir and right reservoir. The subscript, 1 and 2, denotes the first and second half of the system. The impedance difference in both none connected tube may be expressed as follows

$$\overline{\Delta Z_1}=\overline{Z_{1,left}}-\overline{Z_{1,right}} \quad (7)$$

$$\overline{\Delta Z_2}=\overline{Z_{2,left}}-\overline{Z_{2,right}} \quad (8)$$

In accordance to literature, the direction of flow can be predicted based on the location of compression such that shall the impedance on the left is larger than the impedance on the right, the flow shall goes right and vice versa.

## III. NUMERICAL SIMULATIONS OF A SINGLE- AND TWO-STAGE SYSTEM

As the focus of this paper is on the resonance of the system, the simulations are centred on the pump's performance at its resonant frequency. In accordance to the formulations established in Section II, numerical simulations are performed for the single- and two-stage system. The two-stage will be studied in accordance to equation (6) and its results will be compared against the single-stage system. The analysis of results will be evaluated in its non-dimensional form where the pumping rate for a two-stage system will be normalized to the highest flow in a single-stage system. The compression location will be normalized such that the instantaneous location, $x$ over the total length of tube, $L$. A summary of the simulation parameters of a single-stage system is tabulated in Table I.

TABLE I
PARAMETERS OF INVESTIGATION FOR SINGLE-STAGE SYSTEM

| Parameters | Range | Step size |
|---|---|---|
| Compression location, $x/L$ (%) | 10-50 | 10 |
| Constants | | |
| Fluid viscosity, $\mu$ (kg m$^{-1}$ s$^{-1}$) | | 0.001 |
| Fluid density, $\rho$ (kg m$^{-3}$) | | 998 |
| Young's Modulus, $E$ (MPa) | | 12 |
| Tube inner diameter, $d$ (mm) | | 30 |
| Tube wall thickness, $h$ (mm) | | 2 |
| Tube length, L (mm) | | 500 |
| Compression width, $x_2 - x_1$(mm) | | 50 |
| Amplitude of driving pressure, $P_o$ (Pa) | | 101325 |
| Angular frequency of oscillation, $\varpi$ | | 30.16 |

Numerical simulations are performed for a single-stage system in accordance to equations (3) to (5) and results obtained are presented in Figure 2. Results obtained are normalized with respect to highest pumping capacity of a single-stage open-loop impedance pump. Pumping capacity of the pump, also referred to as the pumping rate will be analysed and discussed in the latter text. The angular frequency of oscillation is obtained theoretically based on equation (4). Pumping is predicted to be at its maximum at resonance, hence the numerical study emphasizes strongly on the resonance region. Therefore, excitation frequency of 4.8 Hz, a calculation based on equation (2), is used for the excitation pressure term. Results presented are half the total tube length as at symmetrical location of 0.5, zero flow will be observed due to the existence of similar impedance which will not induce any flow. This had been widely discussed and validated both numerically and experimentally in the prior studies with reference to Refs [4, 7, 12]. Pumping rate on the next half section will be a reflection of the first half; such that in the first half section, fluid will flow from left to right while in second half section, fluid will flow from right to left, which had been extensively discussed in previous literature.

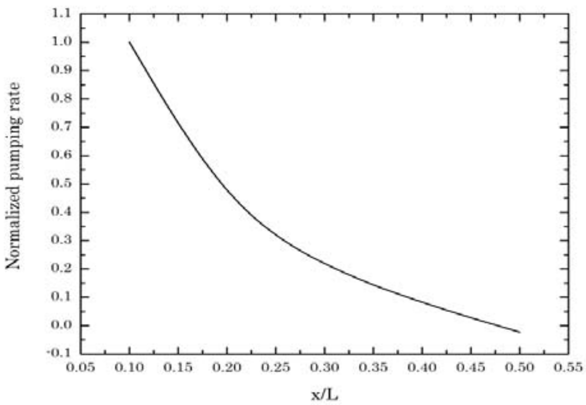


Fig. 2. Numerical simulated results for single-stage system's pumping rate with respect to varying compression locations.

In Figure 2, it is observed that as $x/L$ increases, the pumping rate decreases. Such phenomenon is due to the change in impedance difference between sections of the tube when compressed. In this study, for the case of $x/L$=0.1, impedance on the left end tube (10% of the total length) and right end tube (90% of the total length) is at its maximum difference in impedance. Impedance on the left end tube will give larger hindrance to the fluid flow as compared to the right end tube and hence fluid will flow from left to right. Comparing with case of $x/L$ =0.2, impedance on the left end tube (20% of the total length) and right end tube (80% of the total length), a lower impedance difference as compared to $x/L$ =0.1 is yield, hence reduces the fluid transported to the left. This explained the curve as $x/L$ approaches the symmetrical location, pumping rate reduces and eventually becomes zero flow at the symmetrical location, $x/L$ of 0.5. With reference to the above figure, the compression location $x/L$ of 0.2 has demonstrated to be very essential in determining the systematic impedance. With $x/L$ of 0.2 as the point of reference, it is shown that the decrease in pumping rate before this location ($x/L$ =0.1 – 0.2) is of a higher magnitude represented with the more drastic decrease in its trend. As the compression of elastic tube passes $x/L$ of 0.2, the decrease in pumping rate is shown to be of a rather gradual manner. Comparing its gradient, the gradient of decrease in pumping rate before and after $x/L$ of 0.2 is approximated at 3:1, having to say the rate of decrease in pumping rate is 3 times higher from $x/L$ =0.1 – 0.2 as compared to $x/L$ =0.2 – 0.5. Results shown in Figure 2 will serve as a benchmark for the study of two-stage system.

Based on equation (6), numerical study is performed to investigate the phenomenon of a two-stage system. Numerical results of pumping rate with respect to varying compression locations $x_1/L_1$ synchronized with varying $x_2/L_2$ are obtained and shown in Figure 3. A summary of the parameters of investigation for a two-stage system is tabulated and presented in Table II.

TABLE II
PARAMETERS OF INVESTIGATION FOR TWO-STAGE SYSTEM

| Parameters | Range | Step size |
|---|---|---|
| Compression location, $x_1/L_1$ (%) | 10-50 | 10 |
| Compression location, $x_2/L_2$ (%) | 10-50 | 10 |
| Constants | | |
| Fluid viscosity, $\mu$ (kg m$^{-1}$ s$^{-1}$) | | 0.001 |
| Fluid density, $\rho$ (kg m$^{-3}$) | | 998 |
| Young's Modulus, $E$ (MPa) | | 12 |
| Tube inner diameter, $d$ (mm) | | 30 |
| Tube wall thickness, $h$ (mm) | | 2 |
| Tube length, L (mm) | | 500 |
| Compression width, $x_2 - x_1$(mm) | | 50 |
| Amplitude of driving pressure, $P_o$ (Pa) | | 50663 |
| Angular frequency of oscillation, $\varpi$ | | 30.16 |

The pumping rate is normalized with respect to the single-stage system as shown in Figure 2. With reference to Figure 3, as the compression locations approach the symmetrical location of $x/L$= 0.5, the pumping rate reduces and reaches zero rate at $x_1/L_1 : x_2/L_2$ of 0.5:0.5. Here $x/L$ is a global representation of compression location(s), while $x_1/L_1$ and $x_2/L_2$ is the individual representation of compression location at respectively tubes. $x_1/L_1$ represents the compression location at the first section of the tube whereas $x_2/L_2$ denotes the compression location at the second section of the tube.

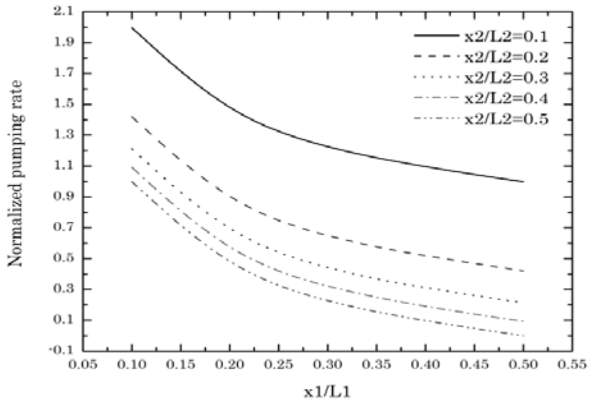


Fig. 3. Numerical simulated results for two-stage system's pumping rate with respect to varying compression locations $x_1/L_1$ synchronized with varying $x_2/L_2$.

Figure 3 shows a compilation of numerical results obtained for the synchronization of compression locations $x_1/L_1 : x_2/L_2$ under identical excitation frequency at resonance of 4.8 Hz. Identical excitation frequency is imposed, as the resonant frequency is believed to be dependent on the wave speed which indicates the tube length being the main factor of resonance. As the tubes are separated by middle reservoir, therefore it is assumed that the resonant frequency will based on individual tube and not a series of connected tube. This assumption is in accordance to the equation (2) which showed that the resonant frequency is dependent on the wave speed in which determined by the physical properties of the elastic tube.

Numerical results showed that the impedance difference is at its highest at $x/L$ = 0.1. As this value increases, indicating a shorter distance from the symmetrical location ($x/L$ = 0.5), decrease in the impedance difference results in a higher hindrance towards the flow and hence creates a lower pumping rate. This phenomenon is due to the shift of impedance difference in the system that causes the hindrance to flow differs which can be describe using equation (7) and (8). This is similar to the single-stage system observations. For two-stage system, the *RLC* (Resistance-Inductance-Capacitance) is however a more complex parameters as compared to the single-stage system.

As discussed in Figure 2, the compression location along the elastic tube was demonstrated to possess certain critical criteria where it largely affects the pumping rate. In this simulation, it is assumed that for impedance pumps in a series-arrangement, the characterized parameters are summed up as how it is in an electrical analogy of series connection. The curves are all normalized with respect to a single-stage system with maximum benchmark pumping rate of 1 at its optimum compression location of 0.1 and resonant frequency of 4.8 Hz. Resonant frequencies for both systems are assumed to be of the same as suggested by equation (2) which is shown to be dependent on the physical properties of the tube, which in both cases are identical.

The obtained numerical results show that with an increase in number of pumping stages, a higher rate can be obtained. Looking into the case of constant $x_1/L_1$ of 0.1 with $x_2/L_2$ of 0.1, doubling normalized pumping rate of 2.0 is achieved. Pumping rate of 1.4 is obtained for $x_2/L_2$ of 0.2, followed by 1.2, 1.1 and 1.0 for $x_2/L_2$ of 0.3, 0.4 and 0.5 respectively. It was also observed that the trend of pumping rate changes with respect to compression locations follows a drastic decrease from $x/L$ = 0.1 – 0.2. As the designated compression location passes the critical reference $x/L$ of 0.2, the trend is rather a gradual decrease. Drastic decrease in pumping rate between $x/L$ of 0.1 and 0.2 shows that the shifts of impedance difference is much higher in 0.1 and 0.2 as compared to 0.2 and 0.3 and so on. Trend of pumping rates as shown in Figure 3 clearly shows that the compression location of $x/L$ = 0.2 plays a very important role in determining the systematic impedance hence its associated pumping rate. As resonant pumping also dependent on the compression locations, it may then be theoretically said that for compression locations $x/L$ of 0.1 is the optimum prerequisite for high resonant pumping to occur in the system. In addition, compression location $x/L$ of 0.2 has also demonstrated to be playing a very important role in a two-stage impedance pumping system. This would however require further experimental study to verify its validity, which will be described explicitly in Section V.

## IV. EXPERIMENTAL METHODS AND MATERIALS

Experimental investigations were further conducted to investigate the deliverable heads and pumping rates of a two-stage impedance pumping system in order to validate and compare with the numerical model discussed in the previous section. The schematic diagram of the system is shown in Figure 4. The pump is made of AlphaSil silicon rubber tube of hardness 60 Shore A, 500 mm in length, 30 mm in diameter and 2 mm in wall thickness, held horizontally. The tube ends were fixed and connected to two reservoirs at each end of the test section. Each reservoir has a 134 mm inner diameter and height of 400 mm. These reservoirs serve as storage for the working fluid. Water with standard properties at 25°C was used as the working fluid. The mechanism to power the excitation and compression was built using an electromechanical vibro-impact machine with a total power of 720 Watts, enabling compression amplitude of 34 mm with compression width of 50 mm. Two linear variable displacement transducers (LVDTs) were placed at the middle and deliverable reservoirs for data collection as shown in Figure 4. The transducers were then connected to the data acquisition board (DAQ) which is attached to the computer for data logging.

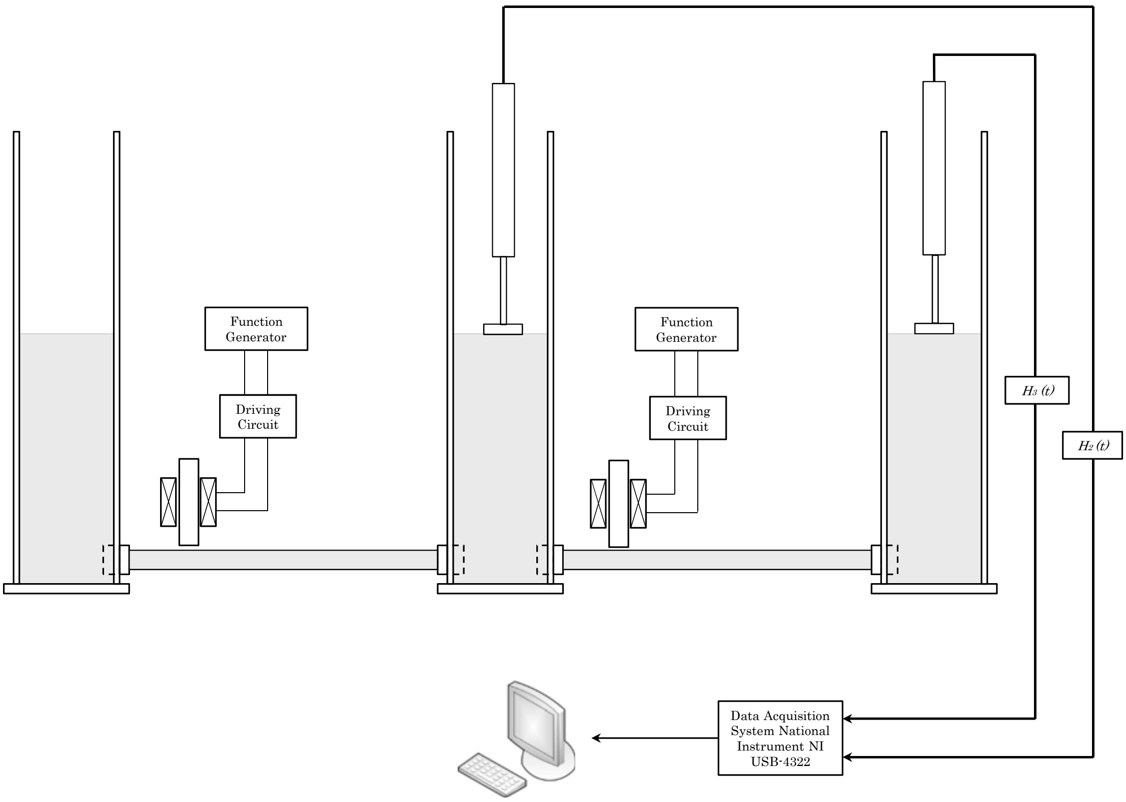


Fig. 4. Schematic diagram of a two-stage impedance pumping system.

The excitation will be generated from the electromechanical vibro-impact machines. Input signals such as the voltage, operating frequency, signal phase, duty cycle of the machine are controlled by hardware, connected to a circuit board as illustrated in Figure 4. A voltage of 120 V with sinusoidal waveform was used as input parameters throughout the whole experiment. Operating frequency is however made variable in this investigation. Connected at the other end of the circuit board is the electromechanical vibro-impact machine where the input signals are sent to the machine and this will operate the machine to compress the tube with an amplitude depth of 34 mm which exhibits a full compression onto the tube. The mechanisms are placed at two separate locations as illustrated in Figure 4.

In the analysis, the location of compression will be characterized in its non-dimensional form for the ease of comparison. $x$ will be used to denote the instantaneous location of the mechanism while $L$ represents the total length of the elastic tube. Subscripts of 1 and 2 are used to represent two different tube; 1 being the first half and 2 being the second half. In this study, both $L_1$ and $L_2$ are 500 mm in length; $x_1$ and $x_2$ will be ranging from 50 mm to 250 mm. In its non-dimensional form, both mechanism $x_1/L_1$ and $x_2/L_2$ will be ranging from 0.1 to 0.5. The mechanism is not connected to the outer surface of the tube. In-phase compression is excited in this experiment and the contact between the mechanism and tube only happens during the compression.

The analysis of pumping rate on the other hand, similar to the numerical section, will be normalized against a single-stage system of the same configuration, of pumping rate 7.64 L/min and the excitation frequency will be normalized as Womersley number similar to Ref [18].

## V. RESULTS AND DISCUSSION

The pumping rates of a two-stage system of different compression locations were measured and analysed. Figure 5 shows the normalized deliverable pumping rates with respect to varying compression locations $x_1/L_1 : x_2/L_2$ and Womersley numbers. Three distinguished peaks are observed in the figure at Womersley numbers of 59.4, 79.7 and 102.9 (2.5 Hz, 4.5 Hz and 7.5 Hz). For $x_2/L_2$ =0.2 however, the first peak is at 65.07 (3 Hz) and third peak is at 99.4 (7 Hz), indicating for the synchronized locations of 0.1:0.2, the pumping rate is shifted closer to the highest peak. With respect to the peaks, it is clearly shown that there are three resonance points in the system with the highest at Womersley number 79.7 (4.5 Hz). With reference to Figure 5, for compression location $x_1/L_1$ of 0.1 at its resonance, the pumping rate is observed to have a drastic decrease with an approximation of 42% in pumping rate over the span of compressed region. Minor decrease in pumping rate is observed from $x_2/L_2$ of 0.1- 0.2, whereas a drastic drop is observed from 0.2 – 0.3. $x_2/L_2$ of 0.3 – 0.5 on the other hand shows minor changes in the pumping rate. This resonant pumping condition is theoretically in a good agreement with the theoretical prediction by Timmermann [7] to be the optimum operating condition. A pumping rate increment of 69% is measured for a two-stage system and normalized with respect to single-stage system.

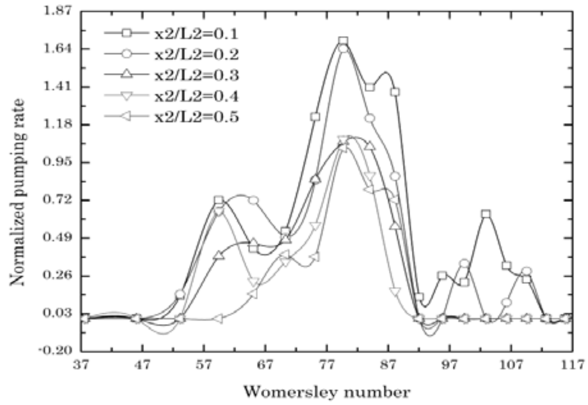


Fig. 5. $x_1/L_1$ = 0.1 with varying $x_2/L_2$.

Figure 6 shows the normalized deliverable pumping rates for $x_1/L_1$ = 0.2 synchronized with varying $x_2/L_2$. Similar case of three peaks is observed in the figure with highest pumping rate at Womersley number 79.7 (4.5 Hz). The pumping rate is however shown to be of a lower value as compared to $x_1/L_1$ = 0.1 which only contribute to an increment of 42%. Similar to Figure 5, the trend of pumping rates show a major drop from $x_2/L_2$ of 0.2 – 0.3. Based on Figure 5 and 6, $x_1/L_1$ of 0.1 and 0.2 are in good synchronization condition with $x_2/L_2$ of 0.1 and 0.2 in which the combinations induce a very close increment in pumping rate. For $x_2/L_2$ of 0.3 onwards, the pumping rate is lower. Having insights at the values, the performance is indeed lower than a single-stage system.

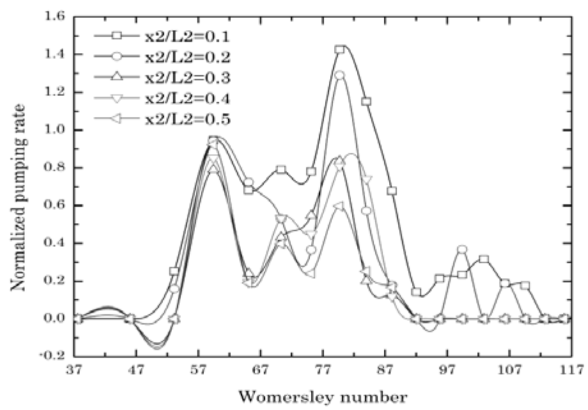


Fig. 6. $x_1/L_1$ = 0.2 with varying $x_2/L_2$.

Figure 7 shows the normalized pumping rates for the case of $x_1/L_1$ = 0.3. As observed in the figure, only at the compression locations of 0.3:0.1, improvement in the pumping rate is observed with a percentage increment of 13%. It is however noted that the optimum resonant pumping now shifted from 79.9 (4.5 Hz) to 84 (5 Hz). Meanwhile for the remaining compression location $x_2/L_2$, the pumping rates remained peak at 79.9 (4.5 Hz).

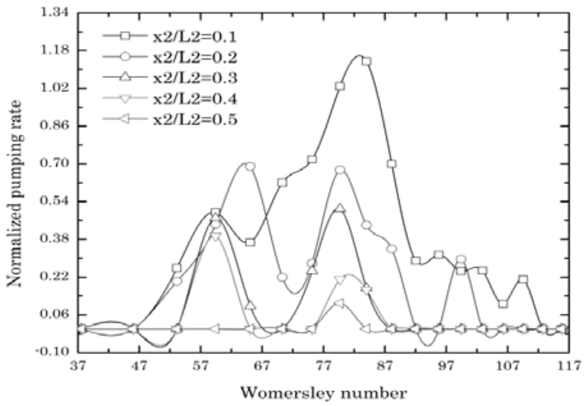


Fig. 7. $x_1/L_1$ = 0.3 with varying $x_2/L_2$.

Based on Figure 5 – 7, an obvious decreasing trend of pumping rate is observed. For compression location of $x_1/L_1$ = 0.3 however, the gradient in $x_2/L_2$ is observed to be rather similar to the numerical simulated result where the gradient of decrease is high at $x_2/L_2$ of 0.1 – 0.2, and gradual at 0.2 to 0.5. For $x_2/L_2$ of 0.5, it is observed that the pumping only works at resonance while zero pumping work is done at other excitation frequencies. It is also shown that the pumping rate for $x_2/L_2$ of 0.2 is more efficient than 0.1 at Womersley number of 65.07 (3 Hz).

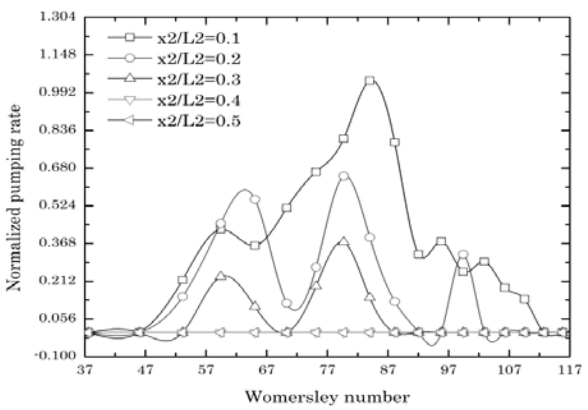


Fig. 8. $x_1/L_1$ = 0.4 with varying $x_2/L_2$.

Normalized pumping rates induced for compression locations synchronization of $x_1/L_1$ = 0.4 is presented in Figure 8. Minimal increment of 4% is observed at the synchronization of 0.4:0.1 while for $x_2/L_2$ of 0.4 and 0.5, zero pumping work is observed. Similar case is observed here where the pumping rate for $x_2/L_2$ of 0.2 is higher than 0.1 at Womersley number 65.07 (3 Hz) as compared to Figure 7. These trends of two-stage system pumping have shown to be at the pump's end boundary where the effects of multi-excitation are observed to be lessen and comparable to a single-stage system.

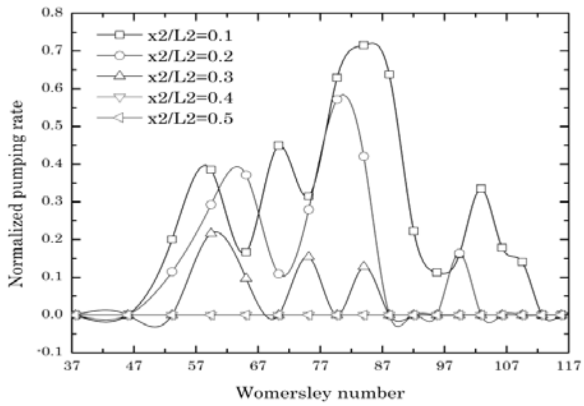


Fig. 9. $x_1/L_1$ = 0.5 synchronized with varying $x_2/L_2$.

Normalized pumping rates induced for compression locations synchronization of $x_1/L_1$ = 0.5 is shown in Figure 9. Similar case of pumping rate for $x_2/L_2$ of 0.2 is higher than 0.1 is shown at Womersley number of 65.07 (3 Hz). It is observed in Figure 9 that the all the pumping rates induced are beyond the benchmark rate of 1.0, indicating that the system has shown to be less efficient than a single-stage system. This shows that the pumping capacity is only up to $x_1/L_1$ of 0.4 with condition where $x_2/L_2$ has to be at 0.1.

Typically in a single-stage valveless impedance pump, the effects of compression location $x/L$ of 0.4 would be insignificant as compared to 0.1 to 0.3 and 0.5 would induce zero flow due to its symmetrical nature. Based on Figure 5 to 9, it is clearly shown that the effects of complex impedance are very essential in the transportation of fluid in a two-stage system. It is observed that for $x_1/L_1$ varying from 0.1 to the symmetrical location of 0.5, transportation of fluid is still achievable with condition that the compression locations $x_2/L_2$ ranges 0.1 to 0.3. Hence, it may be deduced that for a two-stage system, the second compression plays a dominant role in generating a flow. Resonant pumping is also found to be most efficient at Womersley number of 79.7 (4.5 Hz). Pumping of fluid at the resonance point is however found to be best at

Womersley number 79.7 for low $x_1/L_1$ of 0.1 and 0.2. A shift in Womersley number to 84 (5 Hz) was observed for high $x_1/L_1$ of 0.3 – 0.5.

Based on the experimental resuls, it is shown that the systematic impedance in the middle reservoir plays a crucial role in transportation of fluid. Similar to impedance difference in tube, the systematic impedance is between the complex parameters from both tube ends in a reservoir. Interference of two incoming waves from both ends will affect the systematic impedance in the middle reservoir. Similarly, as $x_2/L_2$ approaches its symmetrical location, the same phenomenon is observed. It is however shown that the compression location of 0.2 is a point of interest where it is able to transport fluid from the middle reservoir under low Womersley number of 65.07, even at $x_1/L_1$ of 0.3 to 0.5.

It is observed that for $x_1/L_1$, a sinusoidal-like curvature in the middle, indicating that systematic impedance is high at low excitation frequencies. As the excitation frequencies increase to the resonance region, pumping became more efficient and fluid is able to be transported from the middle reservoir. Phenomenon such as high turbulence is observed in the middle reservoir as well when the compression location synchronization of 0.2 is imposed; suggesting that systematic impedance in the middle reservoir is oscillating due to the two comparable incoming waves from both ends. From $x_1/L_1$ of 0.3 onwards, the systematic impedance in the middle reservoir is observed to be high where most fluid is stored and pumping is less efficient. For $x_2/L_2$ of 0.2 however, pumping is viable with low Womersley number of 65.07.

Based on these observations, it may be deduced that for low synchronization of compression location $x_1/L_1$ of 0.1, pumping work in the middle reservoir is efficient at the resonance region. For higher synchronization of compression location of 0.3 onwards, pumping in middle reservoir is only efficient at $x_2/L_2$ of 0.2 with low Womersley number of 65.07. Hence, the compression location of 0.2 serves as a threshold of pumping in the middle reservoir where the systematic impedance is found to be comparable.

Impedance difference plays a major role in the pump as it determines the direction and amplitude of the system. This is of course with the synchronization of excitation frequencies as well, especially for an efficient pumping at the resonance region. As the compression location shifts toward the symmetrical location of 0.5, the smaller the impedance difference will be. Hence, in order to induce a large pumping rate, the impedance difference had to be at its maximum value which is experimentally found to be $x_1/L_1 : x_2/L_2$ : 0.1:0.1.

The effective excitation frequency at resonance was found to be at 4.5 Hz, similar to the single-stage system [18]. This shows that the resonant frequency is not dependent on the number of stages added, but rather the length of the individual tube in the system. It is also found as well that at high $x_1/L_1$ of 0.3 – 0.5, the resonance is at excitation frequency of 5 Hz. The resonant frequencies found are however within ±6.3% range of the theoretical resonant frequency which in this case still considered valid within the resonance region with the theoretical approximation of 4.8 Hz.

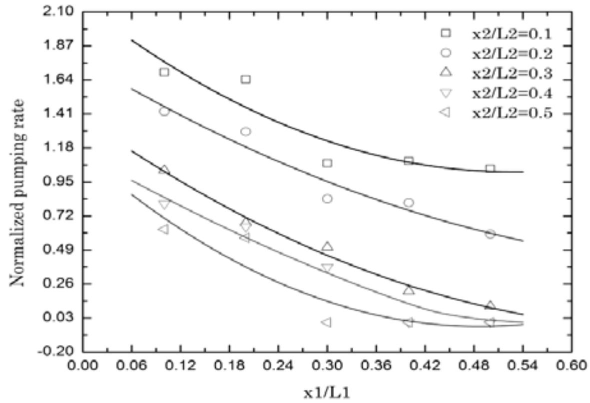


Fig. 10. Experimental results for two-stage system's pumping rate with respect to varying compression locations at Womersley number 79.7 (4.5 Hz).

With reference to Figure 3, Figure 10 is the compilation of experimental results of $x_1/L_1 : x_2/L_2$ at excitation frequency of 4.5 Hz, recognized as the experimental resonant frequency. Although resonance is also shown at excitation frequency of 5 Hz, this frequency however works only at high combination of $x_1/L_1 : x_2/L_2$ and pumping is less efficient as compared to 4.5 Hz. Comparing both figures, significant trend of decrease in pumping rate are observed. The gradient of the decrease due to $x_2/L_2$ is however different. In the numerically simulated results, gradient of decrement in pumping rate is observed to be higher at $x_2/L_2$ of 0.1 – 0.2 and gradually drop from 0.2 – 0.5. Experimental data however shows otherwise where the gradient of $x_2/L_2$ of 0.1 – 0.2 and 0.3 – 0.5 are shown to be in gradual decrement, highest drop in pumping rate is demonstrated to be at $x_2/L_2$ of 0.2 – 0.3. Despite the discrepancy, both results have shown that the compression location of 0.2 indeed plays a crucial role in determining its pumping efficiency in a two-stage impedance pump.

TABLE III
DISCREPANCY BETWEEN NUMERICAL AND EXPERIMENTAL METHOD

| ${x_1}/{L_1} : {x_2}/{L_2}$ | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** |
|---|---|---|---|---|---|
| **0.1** | 0.12 | 0.03 | 0.10 | 0.24 | 0.34 |
| **0.2** | 0.11 | 0.49 | 0.03 | 0.22 | 0.05 |
| **0.3** | 0.12 | 0.36 | 0.09 | 0.27 | 0.25 |
| **0.4** | 0.05 | 0.30 | 0.26 | 0.25 | 0.70 |
| **0.5** | 0.12 | 0.22 | 0.20 | 0.70 | 0.00 |

Table 3 above shows the discrepancy between both methods in this study. This research paper started with numerical modelling of single- and two-stage impedance pumping system, followed by experimental investigations. Based on the figures in Table 3, it can be concluded that the numerical model constructed is only viable for studying the optimum location of compression as the discrepancy is shown to be within 0.12. For studies outside of this operating condition, different types of assumptions may be required for the model in order to achieve better approximation. This could probably due to the turbulence generated in the middle reservoir; as such considerations should be included in the model in order to obtain a more reliable solution.

## VI. Conclusion

In this study, both numerical and experimental approaches were performed to investigate the bulk flow behaviour of a two-stage open-loop impedance. Mathematical expressions of both single-stage and two-stage system were modelled and simulated. Theoretical simulations shown that a two-stage system would exhibits pumping rate double of a single-stage system, while maintaining the flow characteristics of a single-stage system. Experimental study was further conducted to verify the validity of the mathematical model in which the pump's bulk behaviour for a two-stage system is observed to be similar to the single-stage system. The numerical model was shown to produce good approximation with discrepancy of 0.12 with the experimental results. The model is however valid only with compression location of 0.1.

The resonant frequencies are shown to be within ±6.3% range of the theoretical resonant frequency which in this case considered valid within the resonance region with the theoretical approximation. The resonant excitation frequency was also demonstrated to be independent of the number of stages whereas for both single- and two-stage system, maximum pumping rate is observed to be at the excitation frequency of 4.5 Hz or Womersley number 79.7; resonant pumping is also observed at excitation frequency of 5 Hz or Womersley number 84 for ${x_1}/{L_1}$ of 0.3 – 0.5; suggesting that as the compression location approaches the symmetrical location, the resonant frequency is shifted as well.

In multi-stage impedance pumping, systematic impedance has shown to be essential in transportation of fluid which is due to the synchronization of compression locations. The placement of compression locations alters the impedance differences in tube between reservoirs. Having the magnitude of impedance differences in the same direction will induce a unidirectional flow while having the magnitude to be opposite of each other will induce two unidirectional flow. The synchronization of compression locations was demonstrated to be essential in the amplitude of net flow where the highest pumping rate achieved in a two-stage system based on the analysis is doubling the pumping rate of a single-stage system at the optimum compression locations at its resonance.

The compression location ${x}/{L}$ of 0.2 was observed to be of critical parameter in multi-stage open-loop valveless impedance pumping. It is predicted that high turbulence activities occurs in the middle reservoir during the synchronization of two in-bound flows from both ends. This is however only a prediction, a more detailed study, be it numerical or experimental, is required to further understand the phenomenon. In addition, the numerical model constructed in this paper would require further optimization such that the model could incorporate some turbulence components generated in the middle reservoir in order to obtain a more reliable solution.

## Recommendation for Future Work

In this study, it is observed that the second compression location of 0.2 is of point of interest in which efficient pumping occurs. This point has caught the author's attention for which its unique behaviour may be utilized in multi-stage open-loop impedance pumping system of low excitation frequencies. This however requires further in-depth study on this compression location and its associate parameters. It is therefore suggested that additional work could be conducted to further understand the system characteristics at this compression location of 0.2.

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Manuscript modified October 27, 2014. Equation (6) is amended due to incorrect notation in its previous form.