Investigations for the Evolution Behavior of Cos-Gauss Pulse in Dispersion Dominant Regime of Single Mode Optical Fiber

M. Y. Hamza, Sumaira Saeed, N. Sarwar, and S. Yang

***Abstract*—We present the results of the investigations for the evolution behavior of Cos-Gauss pulse by propagating it through a single mode optical fiber in the regime where fiber dispersion is dominant. We consider both un-chirped and chirped pulses. For this purpose, nonlinear Schrödinger equation of single mode optical fiber communication is implemented using split-step Fourier method. Results clearly show that, by controlling certain parameters of Cos-Gauss pulse, even the compression of the pulse can be achieved rather than its broadening. These results also show interesting behavior that for positive values of input chirp, cos-Gauss pulse only goes to broadening. While for negative values of input chirp, the pulse initially experiences compression after which it broadens. These explorations are quite helpful to have a deeper view and understanding about the optical fiber response to such types of pulses, and they can eventually lead to achieve higher performance of optical fiber communication systems.**

***Index Terms*— chirped pulses, cos-Gauss pulse, dispersion dominant regime, single mode fiber**

I. INTRODUCTION

Optical fiber, due to its numerous benefits including lower loss and higher bandwidth [1]-[3], in comparison to other communication media, is in commercial use for communication across the globe. As the contents of internet are becoming more and more video concentric, the demand for more bandwidth at ultra-high data rate is increasing at much higher rate. Optical fiber, inspite of its very attractive benefits, is not a perfect media, and has many limitations. One such limitation is degradation of optical pulse that propagates through fiber. Such degradation eventually causes the communication errors. The major factors that degrade optical pulse propagating through single mode fiber (SMF) are fiber attenuation, group velocity dispersion (GVD) and nonlinearities [4]. Fiber loss attenuates the signal power and hence puts the limit on the maximum repeater-less distance between the transmitter and the receiver. Group velocity dispersion broadens the pulse leading to inter symbol interference and hence limits the transmission rate of optical communication system. Fiber nonlinearities also degrade the optical pulse. More is the fiber degradation effects on propagating optical pulse due to GVD and nonlinearities, the lower would be the performance of optical communication system. Hence, it is of prime interest to investigate the evolution patterns of different types of pulses by propagating these through SMF. Several researchers have made effort in this direction by propagating various types of optical pulses through fibers [5]-[9].

In this paper, we consider Cos-Gauss (CG) optical pulse and investigate its various effects by propagating it through SMF in dispersion dominant regime. For this purpose, we implemented nonlinear Schrödinger equation (NLSE) using split-step Fourier method. We report our results both for un-chirped and chirped Cos-Gauss pulses. These results are quite helpful to understand the response of SMF to such types of optical pulses which may ultimately lead to achieve higher performance of optical communication systems at higher data rates.

II. THEORY

A. Cos-Gauss Pulse

Cos-Gauss optical pulse is generated by the interference of two initially chirped complex conjugate Gaussian pulses. Mathematically, its spectral distribution is represented in accordance with reference [10] (with minor modifications in the terminology) as

$$\tilde{U}(0,\omega) = \tilde{A}e^{-\omega^2 T_0^2}\cos\left(T_0^2\omega^2\tan\phi_0 - \phi\right), \quad (1)$$

and the temporal distribution is given by

$$U(0,T) = A\, e^{-\frac{T^2\cos^2\phi_0}{4T_0^2} + i\frac{cT^2}{2}}\cos\left(\frac{\sin(2\phi_0)}{8T_0^2}T^2 + \left(\phi - \frac{\phi_0}{2}\right)\right), \quad (2)$$

here $U(0,T)$ describes the input field, A is the amplitude of the field, T_0 represents initial root-mean-squared width of each of the Gaussian pulses, c is the initial frequency chirp, ϕ_0 is the phase difference between the two Gaussian pulses and ϕ is the phase of the CG pulse. The two free parameters, ϕ_0 and ϕ provide the flexibility to change the shape of CG

Manuscript received January 08, 2014, revised February 06, 2014.

M. Yousaf Hamza acknowledges Higher Education Commission (HEC) Pakistan for providing financial support to present this paper at 2014 IAENG International Conference on Electrical Engineering, Hong Kong.

Muhammad Yousaf Hamza is with Pakistan Institute of Engineering and Applied Sciences, Islamabad, Pakistan (Corresponding Author, Phone: +92-51-2207381; Fax: +92-51-2208070; email: myhamza@pieas.edu.pk).

Sumaira Saeed is with Pakistan Institute of Engineering and Applied Sciences, Islamabad, Pakistan.

Nadeem Sarwar is with National Institute of Lasers and Optronics, Islamabad, Pakistan (e-mail: nadeemsarwar_pk@hotmail.com).

Shiquan Yang was with University of Ottawa, Canada, during this research work. He is now with Archcom Tech. Inc., Azusa, CA, USA, (email: eyang@archcomtech.com).

pulse. When ϕ_0 is set to zero, the CG pulse becomes a pure Gaussian pulse. A positive value of the chirp means up-chirp, whereas a negative value of the chirp mean down chirp.

For the analysis of various aspects of CG pulse, we propagate CG pulse through SMF using NLSE. To implement NLSE, we use split-Step Fourier method as described below.

B. Split Step Fourier Model

In single mode optical fiber, the effects of attenuation, group-velocity dispersion and nonlinearities are investigated through the nonlinear Schrödinger equation [4], as provided below,

$$\frac{\partial A}{\partial z}+\frac{\alpha}{2}A+\frac{i}{2}\beta_2\frac{\partial^2 A}{\partial T^2}=i\gamma\left[\,|A|^2\,A\,\right], \quad (3)$$

where *A* represents the slowly varying amplitude of the envelope, α, β_2, and γ are fiber attenuation, group velocity dispersion parameter and nonlinear co-efficient, respectively. Unfortunately, NLSE can't be solved analytically. Split-step Fourier method is an efficient technique to solve this equation numerically [4],[11]-[13]. The procedure to implement SSFM is briefly described below.

Eq. (3) can be represented in the following form

$$\frac{\partial A}{\partial z}=\left(\hat{D}+\hat{N}\right)A, \quad (4)$$

where

$$\hat{D}=-\frac{i}{2}\beta_2\frac{\partial^2}{\partial T^2}-\frac{\alpha}{2}, \quad (5)$$

and

$$\hat{N}=i\gamma|A|^2, \quad (6)$$

where the operator $\hat{D}$ is used to investigate the effects of loss and GVD, whereas the nonlinear operator $\hat{N}$ is used to include the nonlinear effect.

To implement NLSE using SSFM, the effects of fiber loss, dispersion and nonlinearity are included in two steps using these two operators. As a procedure, the optical fiber is conceptually divided into small pieces, each of length *h* meters. The optical pulse is propagated through each segment from *z* to *z* + *h*, where *z* is used as a running variable for distance along the fiber. In first step, the fiber loss and GVD effects are included using $\hat{D}$ operator over distance *h*. In the send step, the output obtained in the first step is propagated using $\hat{N}$ operator through the same segment of fiber of length *h* meters. The procedure of these two steps is represented mathematically as [4],

$$A(z+h,T)=e^{h\hat{D}}e^{h\hat{N}}A(z,T), \quad (7)$$

where the factor $e^{h\hat{D}}$ is executed in the Fourier domain using the mathematical formula as

$$e^{h\hat{D}}B(z,T)=\left\{F^{-1}e^{h\hat{D}(i\omega)}F\right\}B(z,T), \quad (8)$$

where *F* denotes the Fourier-transformation, and $\frac{\partial}{\partial T}$ in Eq. (3) is replaced with ($i\omega$) in accordance of the basic rules of Fourier transformation.

Following this procedure, the optical pulse is propagated from one end to the other end. The output obtained at the receiving end can be used to investigate various interesting phenomena in optical fiber transmission. Following this procedure, we implemented SSFM software model. In our model, we used step size *h* as 40 meter to be on very safe limit to meet the condition of $\gamma.P.h$ << 0.1 [4], where *P* represents the peak power of the pulse at the input of each segment.

Our developed model can be used to investigate various effects of optical fiber on the propagating pulses. Using this model, we reported very interesting and useful results in [9],[13]-[15].

III. RESULTS AND DISCUSSIONS

For the investigations presented in this paper, we consider initial width of CG pulse as 15 ps and it is propagated through a single mode fiber span of length 80 km. The attenuation coefficient and GVD parameter of SMF are considered as 0.2 dB/km and 16 ps/(nm km), respectively. The fiber loss was compensated using optical amplifier at the receiving end of the fiber. The amplifier was considered as noiseless because in this work we are interested to investigate the effects of GVD alone. The launch power in this work is considered too low so that nonlinearity would not be significant and hence can be ignored. For simulation results presented in this paper, we set ϕ = ¾ ϕ_0 [10]. Here first we consider un-chirped (c = 0) CG pulse.

A. Propagation of Un-chirped Cos-Gauss Pulse

First of all, we show the effect of changing the phase difference ϕ_0 between two interfering Gaussian pulses on input CG pulse, as in Fig. 1. It illustrates that as ϕ_0 goes on increasing, the side lobes tends to appear.

Response of fiber on CG pulse at various distances for different values of ϕ_0 can be observed by plotting output pulse peak power versus distance. Fig. 2 shows these results, in which peak power of the pulse is normalized to one at the input (z = 0 km).

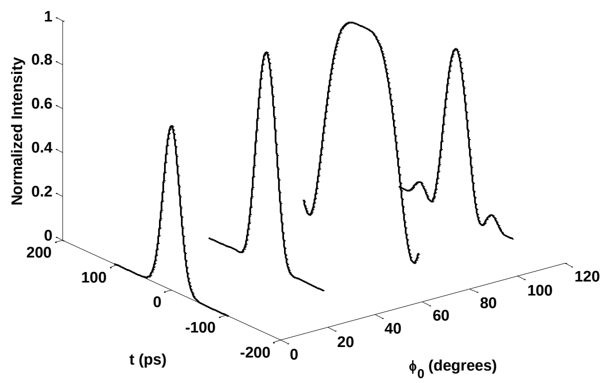

Fig. 1. Effect of changing ϕ_0 on the shape of CG Pulse

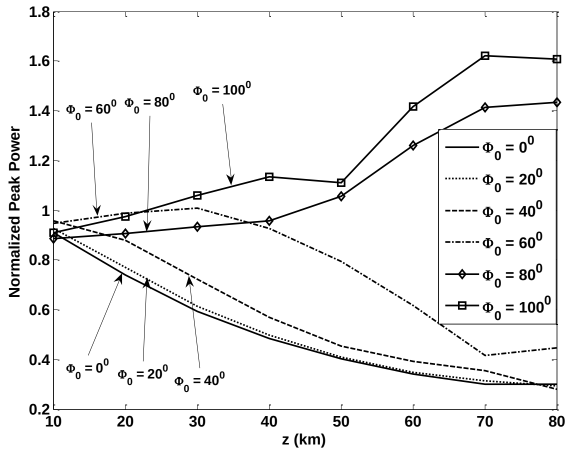

Fig. 2. Variations in CG pulse peak power versus distance for various values of ϕ_0

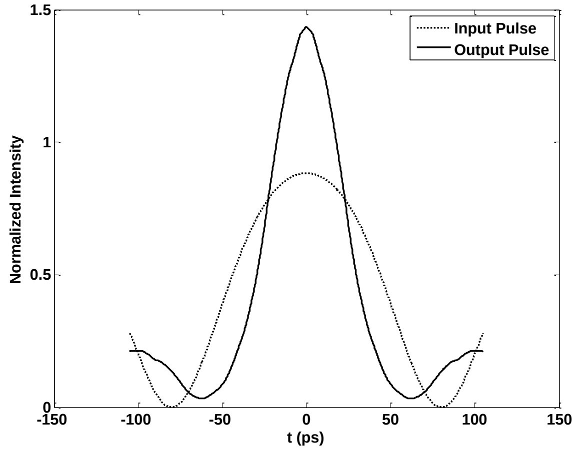

Fig. 3. Comparison of input and output pulses at a distance of 80 km with ϕ_0 = 80°

It may also be recalled that as the fiber loss has been compensated using optical amplifier, so the net energy (i.e. areas under the curves) of input pulse and output pulse are same. In this view, in Fig. 2, decrease in peak power with distance means broadening of the pulse while increase in peak power with distance means compression of the pulse. It can easily be observed that the peak power of the pulse varies for different values of ϕ_0. It happens due to the fact that by varying the values of ϕ_0, the shape of the pulse changes. These pulses having different shapes experience different GVD effect in the same SMF, therefore, it leads to different evolution behavior which ultimately appears in terms of different peak power values at various distances through SMF. This figure also illustrates another important fact that for the case of ϕ_0 = 0°, the pulse becomes pure Gaussian and its peak power decreases enormously with distance which is in well accordance to already reported results [4]. This figure also shows an interesting observation that as the value of ϕ_0 goes on increasing, the peak power also increases. For ϕ_0 = 80°, the peak power of CG pulse first goes on decreasing and then becomes equal to 1 at ~ 45 km propagated distance, after which it goes on increasing.

From this behavior, it is obvious that at ~ 45 km distance, the output pulse is identical to the input pulse. It means by having a suitable value of ϕ_0, the input pulse can be achieved in its original shape at certain propagated distance. After ~ 45 km distance, increase in peak power in fact shows compression (rather than broadening) of the pulse. For more clarity, this effect is shown in Fig. 3, in which output pulse is compared with the input pulse. It clearly shows higher peak power and hence more compression of the output pulse as compared with the input pulse. The recovery of the input pulse at certain distance and compression phenomenon becomes possible only due to phase difference between two interfering Gaussian pulses that form CG pulse.

The evolution of CG pulse through SMF at different distances is shown in Fig. 4, as a 3-D plot of output pulse power versus time and distance. It clearly shows that CG pulse at ϕ_0 = 80° first goes on broadening and then it experiences compression on increasing the propagated distance.

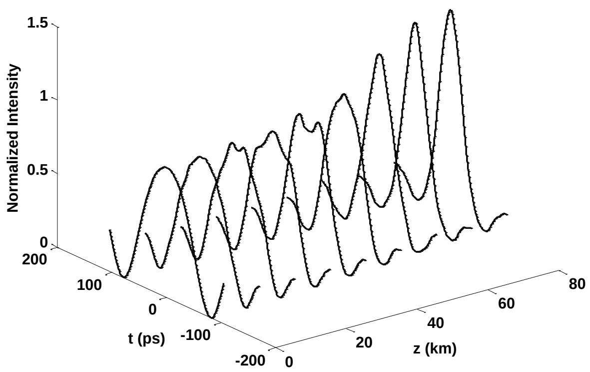

Fig. 4. Output pulses at different distances up to 80 km with ϕ_0 = 80°

B. Propagation of Chirped Cos-Gauss Pulse

The above investigations were carried for initially un-chirped CG pulse. In this section, we investigate and present the results by launching chirped-CG pulse at the input of the fiber. For this purpose, first of all the effects of positive chirp on the pulse evolution is observed. The results are shown in Fig. 5. It is observed that by increasing positive value of chirp, the output pulse broadens at a faster rate. By comparing the results of Figs. 4 and 5, it can be observed that by introducing positive chirp in the input CG pulse, the pulse goes on more and more broadening as the propagated distance increases. Also, no compression is observed in the positive chirped case. Further, the broadening rate is much more for positively chirped-CG pulse as compared to un-chirped CG pulse.

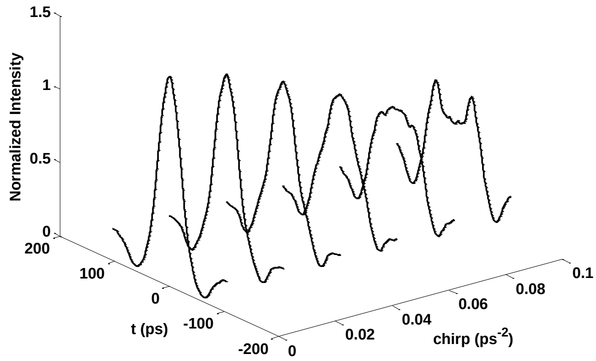

Fig. 5. Effect of positive frequency chirp on output pulse

As a next step, the effect of negatively introduced chirp in the input CG pulse is explored. The results are shown in Fig. 6. Here it can be observed that output pulse first compresses and then broadens as the value of chirp becomes more and more negative. It is important to note that only negative external chirp values can lead to an output pulse which is similar to input pulse because pulse compression and broadening both are observed with negative chirp and using these two phenomena we can obtain an output pulse which is similar to the input pulse.

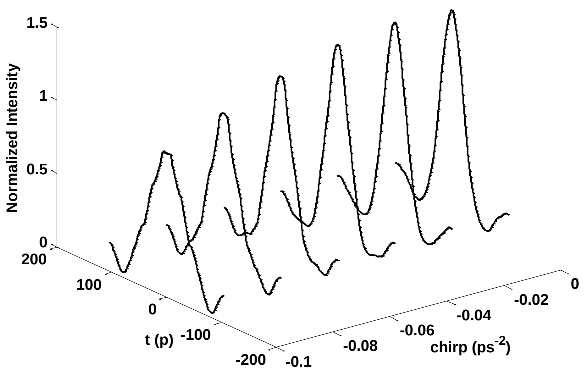

Fig. 6. Effect of negative frequency chirp on output pulse

IV. Conclusion

We have investigated and presented the results for evolution behavior of Cos-Gauss pulse by its propagation through single mode optical fiber in the regime where fiber dispersion is dominant. We considered both un-chirped and chirped pulses. For this purpose, we implemented nonlinear Schrödinger equation for single mode optical fiber, using split-step Fourier technique. These results showed interesting phenomena that although GVD leads to broadening of the pulse, however by controlling certain parameters of Cos-Gauss pulse, even the compression of the pulse can be achieved, rather than its broadening.

Results also illustrated that by inducing positive chirp in input pulse, Cos-Gauss pulse only experiences broadening. However, by introducing negative values of the chirp, the pulse first experiences compression by propagating it through SMF, then its shape becomes identical to the input pulse at certain distance, after which it goes on broadening. These results show the deeper insight about the response of optical fiber to such types of optical pulses, and their possible use for enhancing the performance of optical fiber transmission systems.

References

[1] J. M. Senior, *Optical Fiber Communications*, Pearson, Dorling Kindersley, 2011.

[2] J. C. Palais, *Fiber Optic Communications*, 5th ed., Prentice Hall, Upper Saddle River, New Jersey, 2004.

[3] G. Keiser, *Optical Fiber Communications*, 4th ed., McGraw-Hill Education, 2011.

[4] G. P. Agrawal, *Nonlinear Fiber Optics*, 5th ed., Academic Press, UK, 2013.

[5] A. V. Ramprasad, and M. Meenakshi, "Study on the propagation characteristics of pulses in optical fiber communication systems," *IEEE Workshop on Signal Propagation on Interconnects,* Germany, 2006.

[6] Soumendu Jana, and S. Konar, "Tunable spectral switching in the far field with a chirped cosh-Gaussian pulse," *Optics Communications* vol. 267, pp. 24–31, 2006.

[7] Z. Hong-Jun, and L. Shan-Liang, "Effects of initial frequency chirp on the linear propagation characteristics of the exponential pulse," *Chinese Physics*, vol. 15, pp. 1831-1837, 2006.

[8] Y. J. Wen , and A. Nirmalathas, "Impact of optical pulse shape on the performance of long-haul high capacity DWDM systems," *Optics Communications*, vol. 234, pp. 217–227, 2004.

[9] M. Y. Hamza, N. Sarwar, F. Khan, and S. Yang, "Chirped super-exponential pulse propagation behavior in dispersion dominant regime of single mode fiber," *Proceedings of 33rd International Conference on Telecommunications and Signal processing TSP – 2010*, Baden, Near Vienna, Austria, 17-20 August, 2010.

[10] A. Gajadharsingh, P. A. Belanger, "Linear and nonlinear propagation properties of Cosine-Gauss(CG) pulses in optical fibers," *Optics Communications*, vol. 256, pp. 451-469, 2005.

[11] O. V. Sinkin, R. Holzlöhner, J. Zweck, and C. R. Menyuk, "Optimization of the split-step Fourier method in modeling optical-fiber communications systems," *Journal of Lightwave Technology*, vol. 21, pp. 61-68, January 2003.

[12] G. M. Muslu, and H. A. Erbay, "Higher-order split-step Fourier schemes for the generalized nonlinear Schrödinger equation," *Mathematics and Computers in Simulation,* vol. 67, pp. 581–595, 2005.

[13] M. Y. Hamza, S. Tariq, and L. Chen, "Dispersion compensation in the presence of nonlinearity in optical fiber communications," *Proceedings of 10th IEEE Singapore International Conference on Communication Systems (ICCS),* Singapore, October 2006.

[14] M. Y. Hamza, S. Tariq, M. M. Awais, and S. Yang, "Mitigation of the effects of self phase modulation and group-velocity dispersion in fiber optic communications: dispersion- and power-map co-optimization using the genetic algorithm," *Optical Engg.* vol. 47, no.7, USA, July 2008.

[15] M. Y. Hamza, N. Akhtar, N. Sarwar, and S. Yang, "Evolution behavior of chirped tan-hyperbolic pulse through single mode fiber in the simultaneous presence of fiber loss, dispersion and self-phase modulation," *Telecommun. Syst*, Springer Science + Business Media, New York, USA, August 2013.