Coordinating a Three Stage Supply Chain with Fuzzy Demand

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Abstract—In a three stage supply chain composed of one manufacturer, one distributor and one retailer, the supply chain coordination mechanism is studied under a fuzzy demand environment. The market demand is treated as a positive trapezoidal fuzzy number, and the models of centralized decision-making system and spanning revenue sharing (SRS) contract are proposed by the method of the fuzzy set theory. The optimal solutions of the fuzzy models are also proposed. Finally, numerical examples are given to illustrate and validate the models and conclusions. It is shown that the optimal order quantity of the retailer fluctuates at the range of the most possible values of the fuzzy demand, and increases with the raise of the retail price. The coordination of the supply chain members can be achieved by changing the values of contract parameters. In addition, the optimal fuzzy expected profits of the supply chain members will increase with the increasing of the minimum and maximum possible values of the fuzzy demand.

***Index Terms*—three-stage supply chain, spanning revenue sharing contract, fuzzy demand, trapezoidal fuzzy number**

I. Introduction

In recent years, coordination mechanism of supply chain contracts has become one of the most challenging problems faced by both practitioners and scholars. In order to coordinate the supply chain, various kinds of contract mechanisms are used, including return contracts [1-6], quantity discount contracts [7-12], revenue sharing contracts, and so on.

Revenue sharing contract has been applied in the video cassette rental and movie industry with much success. Giannoccaro and Pontrandolfo [13] showed that revenue sharing contract could coordinate members in the newsboy channel with three stages: supplier, manufacturer and retailer. Cachon and Lariviere [14] intensively discussed a revenue sharing contract between a single supplier and a single retailer in a single period newsboy problem. Gupta and Weerawat [15] designed a revenue-sharing contract to maximize the centralized revenue by selecting an appropriate inventory level. Yao et al. [16] investigated a revenue-sharing contract for coordinating a supply chain comprising one manufacturer and two competing retailers. Linh and Hong [17] studied a revenue sharing contract in a two-period newsboy problem. Rhee et al. [18] proposed a revenue sharing mechanism in multi-echelon supply chains. Ouardighi and Kim [19] considered a single supplier collaborates with two manufacturers on design quality improvements for their respective products under a revenue sharing contract. Krishnan and Winter [20] studied the role of revenue sharing contracts in supply chains and established a foundation in aligning incentives. Sheu [21] explored revenue sharing contracts under price promotion to end-customers with three types of promotional demand patterns. Zhang et al. [22] investigated a revenue sharing contract with demand disruptions in a supply chain comprising one manufacturer and two competing retailers. Palsule-Desai [23] studied revenue-dependent contracts and revenue-independent contracts in a two-period model, they showed that both types of revenue sharing contracts could coordinate the supply chain; however, there existed situations in which revenue-dependent contracts outperformed revenue-independent contracts. Huang and Huang [24] studied the price coordination problem in a three-echelon supply chain and considered three types of channel structures, namely, the decentralized, the semi-integrated, and the integrated. Koide and Sandoh [25] analyzed a discount pricing problem in two periods and revealed that a profit function in two periods was also concave if target consumers were loss-neutral.

The conventional studies have focused on the cases that the demands are probabilistic. That is to say, the demands follow certain distribution function. However, in practice, especially for some new products, the probabilities are not known due to lack of history data. Thus, the uncertain theory, rather than the traditional probability theory is well suited to the supply chain models problem. Recently, more and more researchers have applied the fuzzy set theory and technique to develop and solve the supply chain models problem [26-30].

In this paper, the demands are approximately estimated by experts, and regarded as fuzzy numbers. The centralized decision-making system and spanning revenue sharing contract in a three-stage supply chain under a fuzzy demand environment will be discussed and the impact of the retail price, the values of contract parameters, and the fuzzy degree of the demand on the models will be also analyzed.

The rest of paper is organized as follows. In section II, the fuzzy set theory and the problem descriptions in our models are described. Section III develops the centralized decision marking with fuzzy demand. Section IV develops a spanning revenue sharing contract with fuzzy demand. Section V provides numerical examples to illustrate the result of the proposed models. The last section summarizes

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the work done in this paper and further research areas.

II. PRELIMINARIES

A. Fuzzy Set Theory

Definition 1. The fuzzy set $\tilde{A}=(a_1,a_2,a_3,a_4)$, where $a_1<a_2<a_3<a_4$ and defined on R, is called the trapezoidal fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & if\ a_1\le x\le a_2,\\ 1, & if\ a_2\le x\le a_3,\\ \dfrac{a_4-x}{a_4-a_3}, & if\ a_3\le x\le a_4,\\ 0, & if\ x<a_1, or\ x>a_4.\end{cases} \tag{1}$$

where a_1 and a_4 are the lower limit and upper limit of the trapezoidal fuzzy number $\tilde{A}$.

Definition 2. The trapezoidal fuzzy number $\tilde{A}$ is called the positive trapezoidal fuzzy number if $a_1>0$.

Definition 3. The set $\tilde{A}_\alpha=\{x\mid \mu_{\tilde{A}}(x)\ge\alpha\}$ is called the α cut set of $\tilde{A}$, for any $\alpha\in[0,1]$. $\tilde{A}_\alpha$ is a non-empty bounded closed interval contained in the set of real numbers, and it can be denoted by $\tilde{A}_\alpha=\left[\tilde{A}_\alpha^L,\tilde{A}_\alpha^R\right]$. Where, $\tilde{A}_\alpha^L$ and $\tilde{A}_\alpha^R$ are respectively the left and right boundary of $\tilde{A}_\alpha$, with

$$\tilde{A}_\alpha^L=\inf\{x\in R:\mu_{\tilde{A}}(x)\ge\alpha\},$$
$$\tilde{A}_\alpha^R=\sup\{x\in R:\mu_{\tilde{A}}(x)\ge\alpha\} \tag{2}$$

For any $\alpha\in[0,1]$, the α cut set of a trapezoidal fuzzy number $\tilde{A}=(a_1,a_2,a_3,a_4)$ is

$$\tilde{A}_\alpha^L=a_1+(a_2-a_1)\alpha,$$
$$\tilde{A}_\alpha^R=a_4-(a_4-a_3)\alpha \tag{3}$$

Based on the extension principle in fuzzy sets, we have the following propositions 1 and 2.

Proposition 1. For any $\alpha\in[0,1]$, let $\tilde{A}$ be a positive trapezoidal fuzzy number and k be a non-zero real number, then

$$k\tilde{A}_\alpha=\begin{cases}\left[k\tilde{A}_\alpha^L,k\tilde{A}_\alpha^R\right], & if\ k\in R^+,\\ \left[k\tilde{A}_\alpha^R,k\tilde{A}_\alpha^L\right], & if\ k\in R^-.\end{cases} \tag{4}$$

Proposition 2. For any $\alpha\in[0,1]$, let $\tilde{B}_\alpha=\left[\tilde{B}_\alpha^L,\tilde{B}_\alpha^R\right]$ and $\tilde{C}_\alpha=\left[\tilde{C}_\alpha^L,\tilde{C}_\alpha^R\right]$ respectively be the α cut set of the positive trapezoidal fuzzy numbers $\tilde{B}$ and $\tilde{C}$, then

$$\tilde{B}_\alpha+\tilde{C}_\alpha=\left[\tilde{B}_\alpha^L+\tilde{C}_\alpha^L,\tilde{B}_\alpha^R+\tilde{C}_\alpha^R\right],$$
$$\tilde{B}_\alpha-\tilde{C}_\alpha=\left[\tilde{B}_\alpha^L-\tilde{C}_\alpha^R,\tilde{B}_\alpha^R-\tilde{C}_\alpha^L\right],$$
$$\tilde{B}_\alpha\times\tilde{C}_\alpha=\left[\tilde{B}_\alpha^L\times\tilde{C}_\alpha^L,\tilde{B}_\alpha^R\times\tilde{C}_\alpha^R\right] \tag{5}$$

Proposition 3 (Dubois and Prade [31])**.** Let $\tilde{A}$ be a positive trapezoidal, the fuzzy expected value of $\tilde{A}$ is

$$E\left[\tilde{A}\right]=\frac{1}{2}\int_0^1\left(\tilde{A}_\alpha^L+\tilde{A}_\alpha^R\right)\mathrm{d}\alpha \tag{6}$$

B. Problem Description

Consider a three stage supply chain with one manufacturer, one distributor and one retailer. The manufacturer wholesales his short life and new products, such as personal computers, consumer electronics or fashion items to the distributor, who in turn wholesales them to the retailer. Then, the retailer sells his order short life products to the customers with a high uncertain demand.

The uncertain demand faced by the retailer is assumed to be a positive trapezoidal fuzzy number $\tilde{D}=(l,a,b,m)$ with the range of the most possible values $[a,b]$, where $0<l<a<b<m$. The fuzzy demand $\tilde{D}$ means the most possible value of demand locates in $[a,b]$. l and m are the lower limit and upper limit respectively of the fuzzy demand $\tilde{D}$ and described by the following membership function $\mu_{\tilde{D}}(x)$:

$$\mu_{\tilde{D}}(x)=\begin{cases}L(x), & if\ l\le x<a,\\ 1, & if\ a\le x\le b,\\ R(x), & if\ b<x\le m,\\ 0, & if\ x<l, or\ x>m.\end{cases} \tag{7}$$

For $l\le x<a$, the left membership function $L(x)=\dfrac{x-l}{a-l}$ is an increase function of x. For $b<x\le m$, the right membership function $R(x)=\dfrac{m-x}{m-b}$ is a decrease function of x.

The following notations are used to formulate the supply chain models discussed in this paper

p: the retail price.

q: the order quantity.

w_m: the wholesale price per unit product offered by the manufacturer.

w_d: the wholesale price per unit product offered by the distributor.

c_m: the per unit manufacturing cost incurred to the manufacturer.

c_d: the per unit operational cost incurred to the distributor.

c_r: the per unit operational cost incurred to the retailer.

$\tilde{\Pi}_M$: the fuzzy profit of the manufacturer.

$\tilde{\Pi}_D$: the fuzzy profit of the distributor.

$\tilde{\Pi}_R$: the fuzzy profit of the retailer.

$\tilde{\Pi}_{SC}$: the fuzzy profit of the supply chain system.

The manufacturer, the distributor and the retailer are assumed to be risk neutral and pursue maximization their fuzzy expected profits.

III. CENTRALIZED DECISION-MAKING WITH FUZZY DEMAND

Consider a supply chain occupied by an integrated-actor, which can also be regarded as the manufacturer, the distributor and the retailer making cooperation. The fuzzy profit of the three stage supply chain can by expressed as

$$\tilde{\Pi}_{SC} = p\min\{q,\tilde{D}\} - (c_m + c_d + c_r)q \quad (8)$$

The integrated-actor tries to maximize his fuzzy expected profit $E\left[\tilde{\Pi}_{SC}\right]$ by selecting his optimal order quantity q, which solves the following model:

$$\text{Max}_q\ E\left[\tilde{\Pi}_{SC}\right] = E\left[p\min\{q,\tilde{D}\} - (c_m + c_d + c_r)q\right]$$
$$\text{s.t.}\ l \le q \le m \quad (9)$$

Since the fuzzy demand $\tilde{D} = (l,a,b,m)$ in (9) is a positive trapezoidal fuzzy number, we know that the order quantity q has three cases, i.e., $l \le q < a$, $a \le q \le b$ and $b < q \le m$.

We will discuss this optimization problem by the following three cases.

Case1: $l \le q < a$.

In this case, the α cut set of $\min\{q,\tilde{D}\}$ is

$$\left(\min\{q,\tilde{D}\}\right)_\alpha = \begin{cases} \left[L^{-1}(\alpha), q\right], & 0 < \alpha \le L(q), \\ [q,q], & L(q) < \alpha < 1. \end{cases} \quad (10)$$

Then, the α cut set of the supply chain's fuzzy profit is

$$\left(\tilde{\Pi}_{SC}\right)_\alpha = \begin{cases} \left[pL^{-1}(\alpha) - (c_m + c_d + c_r)q,\right. \\ \quad \left. pq - (c_m + c_d + c_r)q\right], & 0 < \alpha \le L(q), \\ \left[pq - (c_m + c_d + c_r)q,\right. \\ \quad \left. pq - (c_m + c_d + c_r)q\right], & L(q) < \alpha < 1. \end{cases} \quad (11)$$

By (6), the fuzzy expected profit $E\left[\tilde{\Pi}_{SC}\right]$ can be obtained as

$$E\left[\tilde{\Pi}_{SC}\right] = \frac{1}{2}\left(\int_0^{L(q)} \left(\tilde{\Pi}_{SC}\right)_\alpha \mathrm{d}\alpha + \int_{L(q)}^1 \left(\tilde{\Pi}_{SC}\right)_\alpha \mathrm{d}\alpha\right)$$
$$= \frac{1}{2}p\left(2q + \int_0^{L(q)} L^{-1}(\alpha)\mathrm{d}\alpha - qL(q)\right) - (c_m + c_d + c_r)q \quad (12)$$

From (12), we can derive the first and second order derivatives of $E\left[\tilde{\Pi}_{SC}\right]$ with respect to q as follows

$$\frac{\mathrm{d}E\left[\tilde{\Pi}_{SC}\right]}{\mathrm{d}q} = p - \frac{1}{2}pL(q) - (c_m + c_d + c_r) \quad (13)$$

$$\frac{\mathrm{d}^2 E\left[\tilde{\Pi}_{SC}\right]}{\mathrm{d}q^2} = -\frac{1}{2}pL'(q) \quad (14)$$

Since $L(q)$ is an increasing function with $L'(q) > 0$ and $p > 0$, therefore $\frac{\mathrm{d}^2 E\left[\tilde{\Pi}_{SC}\right]}{\mathrm{d}q^2}$ is negative and $E\left[\tilde{\Pi}_{SC}\right]$ is concave in q. Hence, the optimal quantity of the retailer can be obtained by solving the first-order condition as follows

$$p - \tfrac{1}{2}pL(q) - (c_m + c_d + c_r) = 0 \quad (15)$$

Solving (15), we can get

$$L(q^*) = \frac{2(p - c_m - c_d - c_r)}{p} \quad (16)$$

If $\frac{2(p - c_m - c_d - c_r)}{p} < 1$, namely, $p < 2(c_m + c_d + c_r)$, then

$$q^* = L^{-1}\left(\frac{2(p - c_m - c_d - c_r)}{p}\right) \quad (17)$$

Therefore, the optimal fuzzy expected profit of the supply chain system in this case is

$$E\left[\tilde{\Pi}_{\mathrm{SC}}\right]^* = \frac{1}{2}p\int_0^{\frac{2(p - c_m - c_d - c_r)}{p}} L^{-1}(\alpha)\mathrm{d}\alpha \quad (18)$$

Case2: $a \le q \le b$.

In this case, the α cut set of $\min\{q,\tilde{D}\}$ is

$$\left(\min\{q,\tilde{D}\}\right)_\alpha = \left[L^{-1}(\alpha), q\right] \quad (19)$$

By (6), the fuzzy expected profit $E\left[\tilde{\Pi}_{SC}\right]$ can be obtained as

$$E\left[\tilde{\Pi}_{SC}\right] = \tfrac{1}{2}\int_0^1 \left(\tilde{\Pi}_{SC}\right)_\alpha \mathrm{d}\alpha$$
$$= \tfrac{1}{2}p\left(q + \int_0^1 L^{-1}(\alpha)\mathrm{d}\alpha\right) - (c_m + c_d + c_r)q \quad (20)$$

From (20), we can derive the first order derivative of $E\left[\tilde{\Pi}_{SC}\right]$ with respect to q as follows

$$\frac{\mathrm{d}E\left[\tilde{\Pi}_{SC}\right]}{\mathrm{d}q} = \tfrac{1}{2}p - (c_m + c_d + c_r) \quad (21)$$

If $p > 2(c_m + c_d + c_r)$, then the supply chain system gets his optimal fuzzy expected profit at a ; if $p < 2(c_m + c_d + c_r)$, then the supply chain system gets his optimal fuzzy expected profit at b ; if $p = 2(c_m + c_d + c_r)$, then the supply chain system obtains his optimal fuzzy expected profit for any $q^* \in [a,b]$. We can conclude that in this case

$$q^* \in \begin{cases} \{a\}, & p > 2(c_m + c_d + c_r), \\ [a,b], & p = 2(c_m + c_d + c_r), \\ \{b\}, & p < 2(c_m + c_d + c_r). \end{cases} \quad (22)$$

Case3: $b < q \le m$.

In this case, the α cut set of $\min\{q,\tilde{D}\}$ is

$$\left(\min\{q,\tilde{D}\}\right)_\alpha = \begin{cases} \left[L^{-1}(\alpha), q\right], & 0 < \alpha \le R(q), \\ \left[L^{-1}(\alpha), R^{-1}(\alpha)\right], & R(q) < \alpha < 1. \end{cases} \quad (23)$$

Then, the α cut set of the supply chain's fuzzy profit is

$$\left(\tilde{\Pi}_{SC}\right)_\alpha = \begin{cases} \left[pL^{-1}(\alpha) - (c_m + c_d + c_r)q,\right. \\ \quad \left. pq - (c_m + c_d + c_r)q\right], & 0 < \alpha \le R(q), \\ \left[pL^{-1}(\alpha) - (c_m + c_d + c_r)q,\right. \\ \quad \left. pR^{-1}(\alpha) - (c_m + c_d + c_r)q\right], & R(q) < \alpha < 1. \end{cases} \quad (24)$$

By (6), the fuzzy expected profit $E\left[\tilde{\Pi}_{SC}\right]$ can be obtained as

$$E\left[\tilde{\Pi}_{SC}\right]=\frac{1}{2}\left(\int_0^{R(q)}\left(\tilde{\Pi}_{SC}\right)_\alpha \mathrm{d}\alpha+\int_{R(q)}^1\left(\tilde{\Pi}_{SC}\right)_\alpha \mathrm{d}\alpha\right)$$
$$=\frac{1}{2}p\left(\int_0^1 L^{-1}(\alpha)\mathrm{d}\alpha+qR(q)+\int_{R(q)}^1 R^{-1}(\alpha)\mathrm{d}\alpha\right)-\left(c_m+c_d+c_r\right)q \quad (25)$$

From (25), we can derive the first and second order derivatives of $E\left[\tilde{\Pi}_{SC}\right]$ with respect to q as follows

$$\frac{\mathrm{d}E\left[\tilde{\Pi}_{SC}\right]}{\mathrm{d}q}=\frac{1}{2}pR(q)-\left(c_m+c_d+c_r\right) \quad (26)$$

$$\frac{\mathrm{d}^2E\left[\tilde{\Pi}_{SC}\right]}{\mathrm{d}q^2}=\frac{1}{2}pR'(q) \quad (27)$$

Since $R(q)$ is a decreasing function with $R'(q)<0$ and $p>0$, therefore $\frac{\mathrm{d}^2E\left[\tilde{\Pi}_{SC}\right]}{\mathrm{d}q^2}$ is negative and $E\left[\tilde{\Pi}_{SC}\right]$ is concave in q. Hence, the optimal quantity of the retailer can be obtained by solving the first-order condition as below

$$\frac{1}{2}pR(q)-\left(c_m+c_d+c_r\right)=0 \quad (28)$$

Solving (28), we can get

$$R\left(q^*\right)=\frac{2\left(c_m+c_d+c_r\right)}{p} \quad (29)$$

If $\frac{2\left(c_m+c_d+c_r\right)}{p}<1$, namely, $p>2\left(c_m+c_d+c_r\right)$, then

$$q^*=R^{-1}\left(\frac{2\left(c_m+c_d+c_r\right)}{p}\right) \quad (30)$$

Therefore, the optimal fuzzy expected profit of the supply chain system in this case is

$$E\left[\tilde{\Pi}_{SC}\right]^*=\frac{1}{2}p\left(\int_0^1 L^{-1}(\alpha)\mathrm{d}\alpha+\int_{\frac{2(c_m+c_d+c_r)}{p}}^1 R^{-1}(\alpha)\mathrm{d}\alpha\right) (31)$$

Combining the three cases, we have

$E\left[\tilde{\Pi}_{SC}\left(q^*\right)\right]^*\geq E\left[\tilde{\Pi}_{SC}(a)\right]$, for $p<2\left(c_m+c_d+c_r\right)$,

and

$E\left[\tilde{\Pi}_{SC}(b)\right]\leq E\left[\tilde{\Pi}_{SC}\left(q^*\right)\right]^*$, for $p>2\left(c_m+c_d+c_r\right)$,

which lead to the following theorem.

Theorem 1. The optimal order quantity of the retailer and the fuzzy expected profit of the supply chain system in centralized decision-making system are

$$q^*=\begin{cases}L^{-1}\left(\frac{2\left(p-c_m-c_d-c_r\right)}{p}\right), & p<2\left(c_m+c_d+c_r\right),\\ [a,b], & p=2\left(c_m+c_d+c_r\right),\\ R^{-1}\left(\frac{2\left(c_m+c_d+c_r\right)}{p}\right), & p>2\left(c_m+c_d+c_r\right).\end{cases} (32)$$

and

$$E\left[\tilde{\Pi}_{SC}\right]^*=\begin{cases}\frac{1}{2}p\int_0^{\frac{2(p-c_m-c_d-c_r)}{p}}L^{-1}(\alpha)\mathrm{d}\alpha, & p<2\left(c_m+c_d+c_r\right),\\ \frac{1}{2}p\int_0^1 L^{-1}(\alpha)\mathrm{d}\alpha, & p=2\left(c_m+c_d+c_r\right),\\ \frac{1}{2}p\left(\int_0^1 L^{-1}(\alpha)\mathrm{d}\alpha+\right.\\ \left.\int_{\frac{2(c_m+c_d+c_r)}{p}}^1 R^{-1}(\alpha)\mathrm{d}\alpha\right), & p>2\left(c_m+c_d+c_r\right).\end{cases} \quad (33)$$

Remark 1. If $a=b$, then the trapezoidal fuzzy number $\tilde{D}$ degenerates into the triangular fuzzy number, the main results in Theorem 1 can degenerate into

$$q^*=\begin{cases}L^{-1}\left(\frac{2\left(p-c_m-c_d-c_r\right)}{p}\right), & p\leq 2\left(c_m+c_d+c_r\right),\\ R^{-1}\left(\frac{2\left(c_m+c_d+c_r\right)}{p}\right), & p>2\left(c_m+c_d+c_r\right).\end{cases} \quad (34)$$

and

$$E\left[\tilde{\Pi}_{SC}\right]^*=\begin{cases}\frac{1}{2}p\int_0^{\frac{2(p-c_m-c_d-c_r)}{p}}L^{-1}(\alpha)\mathrm{d}\alpha, & p\leq 2\left(c_m+c_d+c_r\right),\\ \frac{1}{2}p\left(\int_0^1 L^{-1}(\alpha)\mathrm{d}\alpha+\right.\\ \left.\int_{\frac{2(c_m+c_d+c_r)}{p}}^1 R^{-1}(\alpha)\mathrm{d}\alpha\right), & p>2\left(c_m+c_d+c_r\right).\end{cases} \quad (35)$$

IV. Spanning Revenue Sharing Contract with Fuzzy Demand

The spanning revenue sharing (SRS) contract indicates that the retailer simultaneously shares his fuzzy profit with all supply chain members. In the SRS contract, the retailer shares his fuzzy profit with the distributor and the manufacturer, and the portions are $\Phi_1\left(0<\Phi_1<1\right)$ and $\Phi_2\left(0<\Phi_2<1\right)$ respectively. Thus, the fuzzy profits of the retailer, the distributor and the manufacturer in the SRS contract can be expressed as follows

$$\tilde{\Pi}_R=\left(1-\Phi_1-\Phi_2\right)p\min\left\{q,\tilde{D}\right\}-\left(w_d+c_r\right)q \quad (36)$$

$$\tilde{\Pi}_D=\Phi_1 p\min\left\{q,\tilde{D}\right\}+\left(w_d-w_m-c_d\right)q \quad (37)$$

$$\tilde{\Pi}_M=\Phi_2 p\min\left\{q,\tilde{D}\right\}+\left(w_m-c_s\right)q \quad (38)$$

The retailer tries to maximize his fuzzy expected profit $E[\tilde{\Pi}_R]$ in SRS contract by choosing the optimal order quantity q, which solves the following model

$$\mathrm{Max}_q\ E\left[\tilde{\Pi}_R\right]=E\left[\left(1-\Phi_1-\Phi_2\right)p\min\left\{q,\tilde{D}\right\}-\left(w_d+c_r\right)q\right]$$
$$\mathrm{s.t.}\ l\leq q\leq m \quad (39)$$

The distributor tries to solve the following model

$$\mathrm{Max}_q\ E\left[\tilde{\Pi}_D\right]=E\left[\Phi_1 p\min\left\{q,\tilde{D}\right\}+\left(w_d-w_m-c_d\right)q\right]$$
$$\mathrm{s.t.}\ l\leq q\leq m \quad (40)$$

Theorem 2. For any $\Phi_2<\frac{c_m}{c_m+c_d+c_r}$ and $\Phi_1+\Phi_2<$

$\frac{c_m + c_d}{c_m + c_d + c_r}$, the optimal wholesale prices w_d^* and w_m^* in SRS contract are

$$w_d^* = c_m + c_d - (\Phi_1 + \Phi_2)(c_m + c_d + c_r) \tag{41}$$

$$w_m^* = c_m - \Phi_2 (c_m + c_d + c_r) \tag{42}$$

Proof. Case 1: $l \le q < a$.

Similar to the discussions in above section, we have the fuzzy expected profit of the retailer in SRS contract as

$$E\left[\tilde{\Pi}_R\right] = \frac{1}{2}(1 - \Phi_1 - \Phi_2)\, p\left(2q + \int_0^{L(q)} L^{-1}(\alpha)\,\mathrm{d}\alpha - qL(q)\right) - (w_d + c_r)q \tag{43}$$

From (43), we can derive the first and second order derivatives of $E\left[\tilde{\Pi}_R\right]$ with respect to q as follows

$$\frac{\mathrm{d}E\left[\tilde{\Pi}_R\right]}{\mathrm{d}q} = \frac{1}{2}(1-\Phi_1-\Phi_2)\,p\left(2-L(q)\right)-(w_d+c_r) \tag{44}$$

$$\frac{\mathrm{d}^2 E\left[\tilde{\Pi}_R\right]}{\mathrm{d}q^2} = -\frac{1}{2}(1-\Phi_1-\Phi_2)\,pL'(q) \tag{45}$$

Since $L(q)$ is an increasing function with $L'(q) > 0$, $p > 0$ and $\Phi_1 + \Phi_2 < 1$, therefore $\frac{\mathrm{d}^2 E\left[\tilde{\Pi}_R\right]}{\mathrm{d}q^2}$ is negative and $E\left[\tilde{\Pi}_R\right]$ is concave in q. Hence, the optimal quantity of the retailer in SRS contract can be obtained by solving the first-order condition as below

$$\frac{1}{2}(1-\Phi_1-\Phi_2)\,p\left(2-L(q)\right)-w_d-c_r=0 \tag{46}$$

Solving (46), we can get

$$L\left(q^{**}\right) = \frac{2\left((1-\Phi_1-\Phi_2)\,p-w_d-c_r\right)}{(1-\Phi_1-\Phi_2)\,p} \tag{47}$$

In order to coordinate the supply chain, we require $L\left(q^{**}\right) = L\left(q^*\right)$. From (16) and (47), we can obtain

$$w_d^* = c_m + c_d - (\Phi_1 + \Phi_2)(c_m + c_d + c_r) \tag{48}$$

Since $w_d^* > 0$, we have $\Phi_1 + \Phi_2 < \frac{c_m + c_d}{c_m + c_d + c_r}$.

The fuzzy expected profit of the distributor in SRS contract can be expressed as

$$E\left[\tilde{\Pi}_D\right] = \frac{1}{2}\Phi_1 p\left(2q + \int_0^{L(q)} L^{-1}(\alpha)\,\mathrm{d}\alpha - qL(q)\right) + (w_d - w_m - c_d)q \tag{49}$$

From (49), we can derive the first and second order derivatives of $E\left[\tilde{\Pi}_D\right]$ with respect to q as follows

$$\frac{\mathrm{d}E\left[\tilde{\Pi}_D\right]}{\mathrm{d}q} = \frac{1}{2}\Phi_1 p\left(2-L(q)\right)+(w_d-w_m-c_d) \tag{50}$$

$$\frac{\mathrm{d}^2 E\left[\tilde{\Pi}_D\right]}{\mathrm{d}q^2} = -\frac{1}{2}\Phi_1 pL'(q) \tag{51}$$

Since $L(q)$ is an increasing function with $L'(q) > 0$, $p > 0$ and $0 < \Phi_1 < 1$, therefore $\frac{\mathrm{d}^2 E\left[\tilde{\Pi}_D\right]}{\mathrm{d}q^2}$ is negative and $E\left[\tilde{\Pi}_D\right]$ is concave in q. Hence, the optimal quantity of the distributor in SRS contract can be obtained by solving the first-order condition as below

$$\frac{1}{2}\Phi_1 p\left(2-L(q)\right)+w_d-w_m-c_d=0 \tag{52}$$

Solving (52), we can get

$$L\left(q^{**}\right) = \frac{2\left(\Phi_1 p + w_d - w_m - c_d\right)}{\Phi_1 p} \tag{53}$$

In order to coordinate the supply chain, we require $L\left(q^{**}\right) = L\left(q^*\right)$. From (16) and (53), we can obtain

$$w_m^* = w_d^* + \Phi_1(c_m + c_d + c_r) - c_d \tag{54}$$

Substituting w_d^* in (48) into (54), we can get

$$w_m^* = c_m - \Phi_2(c_m + c_d + c_r) \tag{55}$$

Since $w_m^* > 0$, we have $\Phi_2 < \frac{c_m}{c_m + c_d + c_r}$.

Case2: $a \le q \le b$.

If $a \le q \le b$, we have the fuzzy expected profit of the retailer in SRS contract as

$$E\left[\tilde{\Pi}_R\right] = \frac{1}{2}(1-\Phi_1-\Phi_2)\,p\left(q + \int_0^1 L^{-1}(\alpha)\,\mathrm{d}\alpha\right) - (w_d + c_r)q \tag{56}$$

From (56), we can get the first-order derivative of $E\left[\tilde{\Pi}_R\right]$ with respect to q as

$$\frac{\mathrm{d}E\left[\tilde{\Pi}_R\right]}{\mathrm{d}q} = \frac{1}{2}(1-\Phi_1-\Phi_2)\,p-(w_d+c_r) \tag{57}$$

If $(1-\Phi_1-\Phi_2)\,p > 2(w_d + c_m)$, then the retailer gets his optimal fuzzy expected profit at a; if $(1-\Phi_1-\Phi_2)\,p < 2(w_d + c_m)$, then the retailer gets his optimal fuzzy expected profit at b; if $(1-\Phi_1-\Phi_2)\,p = 2(w_d + c_m)$, then the retailer obtains his optimal fuzzy expected profit for any $q^{**} \in [a,b]$.

In order to coordinate the supply chain, we require $q^{**} = q^*$. That is

$$\frac{2(w_d + c_r)}{(1-\Phi_1-\Phi_2)} = 2(c_m + c_d + c_r) \tag{58}$$

From (58), we can get

$$w_d^* = c_m + c_d - (\Phi_1 + \Phi_2)(c_m + c_d + c_r) \tag{59}$$

Since $w_d^* > 0$, we have $\Phi_1 + \Phi_2 < \frac{c_m + c_d}{c_m + c_d + c_r}$.

If $a \le q \le b$, we have the fuzzy expected profit of the distributor in SRS contract as

$$E\left[\tilde{\Pi}_D\right] = \frac{1}{2}\Phi_1 p\left(q + \int_0^1 L^{-1}(\alpha)\,\mathrm{d}\alpha\right) + (w_d - w_m - c_d)q \tag{60}$$

From (60), we can get the first order derivative of $E\left[\tilde{\Pi}_D\right]$ with respect to q as follows

$$\frac{\mathrm{d}E\left[\tilde{\Pi}_R\right]}{\mathrm{d}q} = \frac{1}{2}\Phi_1 p - (w_m - w_d + c_d) \tag{61}$$

If $\Phi_1 p > 2(w_m - w_d + c_d)$, then the distributor gets his optimal fuzzy expected profit at a; if $\Phi_1 p < 2\left(w_m - w_d + c_d\right)$, then the distributor gets his optimal fuzzy expected profit at b; if $\Phi_1 p = 2\left(w_m - w_d + c_d\right)$, then the distributor obtains his optimal fuzzy expected profit for any $q^{**} \in [a,b]$.

In order to coordinate the supply chain, we require $q^{**} = q^{*}$. That is

$$\frac{2\left(w_m - w_d + c_d\right)}{\Phi_1} = 2\left(c_m + c_d + c_r\right) \tag{62}$$

From (62), we can get

$$w_m^{*} = w_d^{*} + \Phi_1\left(c_m + c_d + c_r\right) - c_d \tag{63}$$

Substituting w_d^{*} in (59) into (63), we can get

$$w_m^{*} = c_m - \Phi_2\left(c_m + c_d + c_r\right) \tag{64}$$

Since $w_m^{*} > 0$, we have $\Phi_2 < \dfrac{c_m}{c_m + c_d + c_r}$.

Case3: $b < q \le m$.

In this case, we have the fuzzy expected profit of the retailer in SRS contract as

$$E\left[\tilde{\Pi}_R\right] = \frac{1}{2}\left(1 - \Phi_1 - \Phi_2\right) p\left(\int_0^1 L^{-1}(\alpha)\,\mathrm{d}\alpha + qR(q) + \int_{R(q)}^1 R^{-1}(\alpha)\,\mathrm{d}\alpha\right) - \left(w_d + c_r\right) q \tag{65}$$

From (65), we can get the first-order and second-order derivatives of $E\left[\tilde{\Pi}_R\right]$ with respect to q as follows

$$\frac{\mathrm{d}E\left[\tilde{\Pi}_R\right]}{\mathrm{d}q} = \frac{1}{2}\left(1 - \Phi_1 - \Phi_2\right) pR(q) - \left(w_d + c_r\right) \tag{66}$$

$$\frac{\mathrm{d}^2 E\left[\tilde{\Pi}_R\right]}{\mathrm{d}q^2} = \frac{1}{2}\left(1 - \Phi_1 - \Phi_2\right) pR'(q) \tag{67}$$

Since $R(q)$ is a decreasing function with $R'(q) < 0$, $p > 0$ and $\Phi_1 + \Phi_2 < 1$, therefore $\dfrac{\mathrm{d}^2 E\left[\tilde{\Pi}_R\right]}{\mathrm{d}q^2}$ is negative and $E\left[\tilde{\Pi}_R\right]$ is concave in q. Hence, the optimal quantity of the retailer can be obtained by solving the first order condition as follows

$$\frac{1}{2}\left(1 - \Phi_1 - \Phi_2\right) pR(q) - w_d - c_r = 0 \tag{68}$$

Solving (68), we can get

$$R\left(q^{**}\right) = \frac{2\left(w_d + c_r\right)}{\left(1 - \Phi_1 - \Phi_2\right) p} \tag{69}$$

In order to coordinate the supply chain, we require $R\left(q^{**}\right) = R\left(q^{*}\right)$. From (69) and (29), we can obtain

$$w_d^{*} = c_m + c_d - \left(\Phi_1 + \Phi_2\right)\left(c_m + c_d + c_r\right) \tag{70}$$

Since $w_d^{*} > 0$, we have $\Phi_1 + \Phi_2 < \dfrac{c_m + c_d}{c_m + c_d + c_r}$.

In this case, we have the fuzzy expected profit of the distributor in SRS contract as

$$E\left[\tilde{\Pi}_D\right] = \frac{1}{2}\Phi_1 p\left(\int_0^1 L^{-1}(\alpha)\,\mathrm{d}\alpha + qR(q) + \int_{R(q)}^1 R^{-1}(\alpha)\,\mathrm{d}\alpha\right) + \left(w_d - w_m - c_d\right) q \tag{71}$$

From (71), we can get the first-order and second-order derivatives of $E\left[\tilde{\Pi}_D\right]$ with respect to q as follows

$$\frac{\mathrm{d}E\left[\tilde{\Pi}_D\right]}{\mathrm{d}q} = \frac{1}{2}\Phi_1 pR(q) + \left(w_d - w_m - c_d\right) \tag{72}$$

$$\frac{\mathrm{d}^2 E\left[\tilde{\Pi}_D\right]}{\mathrm{d}q^2} = \frac{1}{2}\Phi_1 pR'(q) \tag{73}$$

Since $R(q)$ is a decreasing function with $R'(q) < 0$, $p > 0$ and $0 < \Phi_1 < 1$, therefore $\dfrac{\mathrm{d}^2 E\left[\tilde{\Pi}_D\right]}{\mathrm{d}q^2}$ is negative and $E\left[\tilde{\Pi}_D\right]$ is concave in q. Hence, the optimal quantity of the distributor can be obtained by solving the first order condition as follows

$$\frac{1}{2}\Phi_1 pR(q) + w_d - w_m - c_d = 0 \tag{74}$$

Solving (74), we can get

$$R\left(q^{**}\right) = \frac{2\left(w_m - w_d + c_d\right)}{\Phi_1 p} \tag{75}$$

In order to coordinate the supply chain, we require $R\left(q^{**}\right) = R\left(q^{*}\right)$. From (75) and (29), we can obtain

$$w_m^{*} = w_d^{*} + \Phi_1\left(c_m + c_d + c_r\right) - c_d \tag{76}$$

Substituting w_d^{*} in (70) into (76), we can get

$$w_m^{*} = c_m - \Phi_2\left(c_m + c_d + c_r\right) \tag{77}$$

Since $w_m^{*} > 0$, we have $\Phi_2 < \dfrac{c_m}{c_m + c_d + c_r}$.

The poof of Theorem 2 is completed.

Theorem 3. For any $\Phi_2 < \dfrac{c_m}{c_m + c_d + c_r}$ and $\Phi_1 + \Phi_2 < \dfrac{c_m + c_d}{c_m + c_d + c_r}$, the retailer, the distributor and the manufacturer obtain their optimal fuzzy expected profits at w_d^{*} and w_m^{*} in SRS contract as follows

$$E\left[\tilde{\Pi}_R\right]^{*} = \left(1 - \Phi_1 - \Phi_2\right) E\left[\tilde{\Pi}_{SC}\right]^{*},$$

$$E\left[\tilde{\Pi}_D\right]^{*} = \Phi_1 E\left[\tilde{\Pi}_{SC}\right]^{*},$$

$$E\left[\tilde{\Pi}_M\right]^{*} = \Phi_2 E\left[\tilde{\Pi}_{SC}\right]^{*} \tag{78}$$

Proof. Case 1: $l \le q < a$.

Substituting w_d^{*} and w_m^{*} in (48) and (55) into (43) and (49), the fuzzy expected profits of the retailer and the distributor are given as

$$E\left[\tilde{\Pi}_R\right]^{*} = \frac{1}{2}\left(1 - \Phi_1 - \Phi_2\right) p\int_0^{\frac{2\left(p - c_m - c_d - c_r\right)}{p}} L^{-1}(\alpha)\,\mathrm{d}\alpha$$

$$=\left(1-\Phi_1-\Phi_2\right)E\left[\tilde{\Pi}_{SC}\right]^* \quad (79)$$

$$E\left[\tilde{\Pi}_D\right]^*=\frac{1}{2}\Phi_1 p\int_0^{\frac{2(p-c_m-c_d-c_r)}{p}}L^{-1}(\alpha)\,\mathrm{d}\alpha$$

$$=\Phi_1 E\left[\tilde{\Pi}_{SC}\right]^* \quad (80)$$

Then, the fuzzy expected profit of the manufacturer is given as

$$E\left[\tilde{\Pi}_M\right]^*=E\left[\tilde{\Pi}_{SC}\right]^*-E\left[\tilde{\Pi}_R\right]^*-E\left[\tilde{\Pi}_D\right]^*$$

$$=\Phi_2 E\left[\tilde{\Pi}_{SC}\right]^* \quad (81)$$

Case2: $a\le q\le b$.

Substituting w_d^* and w_m^* in (59) and (64) into (56) and (60), the fuzzy expected profits of the retailer and the distributor are given as follows

$$E\left[\tilde{\Pi}_R\right]^*=\frac{1}{2}\left(1-\Phi_1-\Phi_2\right)p\int_0^1 L^{-1}(\alpha)\,\mathrm{d}\alpha$$

$$=\left(1-\Phi_1-\Phi_2\right)E\left[\tilde{\Pi}_{SC}\right]^* \quad (92)$$

$$E\left[\tilde{\Pi}_D\right]^*=\frac{1}{2}\Phi_1 p\int_0^1 L^{-1}(\alpha)\,\mathrm{d}\alpha$$

$$=\Phi_1 E\left[\tilde{\Pi}_{SC}\right]^* \quad (93)$$

Then, the fuzzy expected profit of the manufacturer is given as

$$E\left[\tilde{\Pi}_M\right]^*=E\left[\tilde{\Pi}_{SC}\right]^*-E\left[\tilde{\Pi}_R\right]^*-E\left[\tilde{\Pi}_D\right]^*$$

$$=\Phi_2 E\left[\tilde{\Pi}_{SC}\right]^* \quad (94)$$

Case3: $b<q\le m$.

Substituting w_d^* and w_m^* in (70) and (77) into (65) and (71), the fuzzy expected profits of the retailer and the distributor are given as follows

$$E\left[\tilde{\Pi}_R\right]^*=\frac{1}{2}\left(1-\Phi_1-\Phi_2\right)p\left(\int_0^1 L^{-1}(\alpha)\,\mathrm{d}\alpha+\int_{\frac{2(c_m+c_d+c_r)}{p}}^1 R^{-1}(\alpha)\,\mathrm{d}\alpha\right)$$

$$=\left(1-\Phi_1-\Phi_2\right)E\left[\tilde{\Pi}_{SC}\right]^* \quad (95)$$

$$E\left[\tilde{\Pi}_D\right]^*=\frac{1}{2}\Phi_1 p\left(\int_0^1 L^{-1}(\alpha)\,\mathrm{d}\alpha+\int_{\frac{2(c_m+c_d+c_r)}{p}}^1 R^{-1}(\alpha)\,\mathrm{d}\alpha\right)$$

$$=\Phi_1 E\left[\tilde{\Pi}_{SC}\right]^* \quad (96)$$

Then, the fuzzy expected profit of the manufacturer is given as

$$E\left[\tilde{\Pi}_M\right]^*=E\left[\tilde{\Pi}_{SC}\right]^*-E\left[\tilde{\Pi}_R\right]^*-E\left[\tilde{\Pi}_D\right]^*$$

$$=\Phi_2 E\left[\tilde{\Pi}_{SC}\right]^* \quad (97)$$

The poof of Theorem 3 is completed.

The values of contract parameters Φ_1 and Φ_2 depend on the bargaining power of the retailer, the distributor and the manufacturer. The total optimal fuzzy expected profit of supply chain system in the centralized decision marking system can be allocated with specified ratios among the retailer, the distributor and the manufacturer in SRS contract.

V. Numerical Examples

In this section, we tend to further elucidate the proposed fuzzy models with numerical examples. We will analyze that the effective of the retail price *p*, the values of contract parameters Φ_1 and Φ_2, and the fuzzy degree of the fuzzy demand $\tilde{D}$ on the other parameters.

Discussion A

Firstly, we consider that the most possible value of demand located in $[200,250]$, the maximum and minimum possible values of the demand are respectively, $l=100$ and $m=300$, that is to say the fuzzy demand is $\tilde{D}=(100,200,250,300)$. Let $c_m=12$, $c_d=5$ and $c_r=3$.

From Theorem 2, we can get the range of the contract parameters Φ_1 and Φ_2 as

$$0<\Phi_2<0.6 \text{ and } 0<\Phi_1+\Phi_2<0.85$$

The optimal order quantity q^*, wholesale prices w_d^* and w_m^*, and the fuzzy expected profit of the retailer, the distributor and the manufacturer in SRS contract can be listed in Table I.

TABLE I
EQUILIBRIUM VALUE OF THE PARAMETERS FOR DIFFERENT p IN SRS CONTRACT ($\Phi_1=0.35$, $\Phi_2=0.25$)

p	q^*	w_d^*	w_m^*	$E[\tilde{\Pi}_R]$	$E[\tilde{\Pi}_D]$	$E[\tilde{\Pi}_M]$
25	140.00	5.00	7.00	240.00	210.00	150.00
30	166.67	5.00	7.00	533.33	466.67	333.33
35	185.71	5.00	7.00	857.14	750.00	535.71
40	[200, 250]	5.00	7.00	1200.00	1050.00	750.00
45	255.56	5.00	7.00	1602.78	1402.43	1001.74
50	260.00	5.00	7.00	2010.00	1758.75	1256.25
55	263.64	5.00	7.00	2420.46	2117.90	1512.78

From Table I, we analyze the influence of parameter *p* on the optimal equilibrium values as follows:

(a) It is obviously from the Table I that the optimal order quantity q^* will increase along with the raise of the retail price *p* when the other parameters are fixed. This is because increases in retail price have some incentives for the retailer. Especially, in this numerical example, the optimal order quantity locates in the range of the most possible values of fuzzy demand when $p=40$. When $p<40$ and $p>40$, the optimal order quantity of the retailer locates at the left and right of the most possible value of fuzzy demand $\tilde{D}$, respectively.

(b) From Table I, it can be noted that when the other parameters are fixed in SRS contract, the optimal fuzzy expected profits of the retailer, the distributor and the manufacturer will all increase along with the raise of the retail price *p*, and the different *p* does not affect the optimal wholesale prices w_d^* and w_m^* . It indicates that once the feasible values of Φ_1 and Φ_2 are determined, the optimal wholesale prices proposed by the distributor and the manufacturer do not vary.

Discussion B

Secondly, we analyze the effect of the values of contract parameters Φ_1 and Φ_2 on the fuzzy supply chain models. The other parameters are the same as the values in

Discussion A. The results obtained are given in Table II.

TABLE II
EQUILIBRIUM VALUE OF THE PARAMETERS FOR DIFFERENT Φ_1 AND Φ_2 IN SRS CONTRACT ($p = 50$)

(Φ_1,Φ_2)	w_d^*	w_m^*	$E[\tilde{\Pi}_R]$	$E[\tilde{\Pi}_D]$	$E[\tilde{\Pi}_M]$
(0.40,0.25)	4.00	7.00	1916.25	2190.00	1368.75
(0.40,0.30)	3.00	6.00	1642.50	2190.00	1642.50
(0.40,0.35)	2.00	5.00	1368.75	2190.00	1916.25
(0.40,0.40)	1.00	4.00	1095.00	2190.00	2190.00
(0.45,0.25)	3.00	7.00	1642.50	2463.75	1368.75
(0.50,0.25)	2.00	7.00	1368.75	2737.50	1368.75
(0.55,0.25)	1.00	7.00	1095.00	3011.25	1368.75

(c) The optimal wholesale prices w_d^* and w_m^* will decrease with the increasing of Φ_2 when Φ_1 is fixed. The optimal wholesale price w_d^* will also decrease with the increasing of Φ_1 when Φ_2 is fixed. The different of Φ_1 does not affect the optimal wholesale price w_m^*, when Φ_2 does not vary.

(d) The optimal fuzzy expected profit of the distributor will increase with the increasing of Φ_1. When Φ_2 decreases, the manufacturer's optimal fuzzy expected profit will decrease. The retailer's optimal fuzzy expected profit will decrease with the increasing of the sum of Φ_1 and Φ_2. Moreover, the fuzzy expected profit of the distributor is equal to that of the manufacturer when $\Phi_1 = \Phi_2$. Therefore, we conclude that the spanning revenue sharing contract is flexible in coordinating all supply chain actors since Φ_1 and Φ_2 can be reasonably settled through negotiation between the retailer, the distributor and the manufacturer without sacrificing the fuzzy expected maximum channel profit.

Discussion C

Thirdly, we analyze the effect of the fuzzy degree of the demand $\tilde{D}$ on the optimal order quantity and the optimal fuzzy expected profits for supply chain members. Let $\Phi_1 = 0.35$ and $\Phi_2 = 0.25$ .The other parameters are the same as the values in Discussion A. The results obtained are given in Tables III and IV.

TABLE III
EQUILIBRIUM VALUE OF THE PARAMETERS FOR DIFFERENT l IN SRS CONTRACT ($p = 30$)

l	q^*	$E[\tilde{\Pi}_R]$	$E[\tilde{\Pi}_D]$	$E[\tilde{\Pi}_M]$
100	166.67	533.33	466.67	333.33
110	170.00	560.00	490.00	350.00
120	173.33	586.67	513.33	366.67
130	176.67	613.33	536.67	383.33
140	180.00	640.00	560.00	400.00

TABLE IV
EQUILIBRIUM VALUE OF THE PARAMETERS FOR DIFFERENT m IN SRS CONTRACT ($p = 50$)

m	q^*	$E[\tilde{\Pi}_R]$	$E[\tilde{\Pi}_D]$	$E[\tilde{\Pi}_M]$
300	260.00	2010.00	1758.75	1256.25
290	258.00	2008.00	1757.00	1255.00
280	256.00	2006.00	1755.25	1253.75
270	254.00	2004.00	1753.50	1252.50
260	252.00	2002.00	1751.75	1251.25

(e) It is obviously from the discussion A that the optimal order quantity q^* locates at the left of the most possible value of demand $\tilde{D}$ when $p = 30$. The fuzziness of the demand $\tilde{D}$ will decrease with the increasing of minimum possible value of the demand l. The optimal order quantity q^* and the optimal fuzzy expected profits of the supply chain actors will increase, when the fuzzy degree of the demand $\tilde{D}$ decreases. Thus, the supply chain actors should pursue as low fuzzy degree of the demand $\tilde{D}$ as possible in this case.

(f) The optimal order quantity q^* locates at the right of the most possible value of demand $\tilde{D}$ when $p = 50$. The fuzziness of the demand $\tilde{D}$ will decrease with the decreasing of the maximum possible value of the demand m. The optimal order quantity q^* and the optimal fuzzy expected profits of the supply chain actors will decrease, when the fuzzy degree of the demand $\tilde{D}$ decreases. Thus, the supply chain actors should pursue as high fuzzy degree of the demand $\tilde{D}$ as possible in this case.

VI. CONCLUSION

In this study, we formulate fuzzy three stage supply chain models based on the fuzzy set theory, where the supply chain members adopt the spanning revenue sharing contract mechanism. In order to examine models performance in fuzzy demand, we use fuzzy cut sets method to solve this problem. Thus, fuzzy sets theory is the most appropriate tool when the uncertain parameters cannot be described by probability distributions.

The main contribution of this paper is that the spanning revenue sharing contract has been formulated under a three stage supply chain facing fuzzy demand. The limitation of our model is our assumption that the supply chain has only one manufacturer, one distributor and one retailer. Further work is desirable to test whether our conclusions extend to the models with multiple competitive manufacturers and retailers in a fuzzy demand environment. The revenue sharing contract with fuzzy demand and imperfect quality in a multi-echelon supply chain environment will be also considered.

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