An Extended TOPSIS Method for Multiple Attribute Decision Making based on Intuitionistic Uncertain Linguistic Variables

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Abstract—**The intuitionistic uncertain linguistic variables can easily express the fuzzy information in real world, and TOPSIS is a very effective decision making method and it has been achieved more and more extensive applications. In this paper, we will extend the TOPSIS method to deal with the intuitionistic uncertain linguistic information, and propose an extended TOPSIS method to solve the multiple attribute decision making problems in which the attribute values take the form of the intuitionistic uncertain linguistic variables and attribute weight is unknown. Firstly, the operational rules and properties for the intuitionistic uncertain linguistic variables are introduced. Then the distance between two intuitionistic uncertain linguistic variables is proposed and the attribute weight is calculated by the maximizing deviation method, and the closeness coefficients to the ideal solution for each alternative are used to rank the alternatives. Finally, an illustrative example is given to show the decision making steps and the effectiveness of the proposed method.**

Index Terms—**the intuitionistic uncertain linguistic variables, multiple attribute decision making, TOPSIS, maximizing deviation method**

I. Introduction

THE theory and methods of multiple attribute decision making (MADM) are an important part of modern decision science. Because of the complexity of the objective world, most decision-making problems are fuzzy. Since fuzzy set (FS) theory was proposed by Zadeh [1], it has achieved many applications [2-6]. However, FS can only have the membership function, and it cannot process some decision making problems such as voting and so on. Further, On the basis of FS, Atanassov [7, 8] proposed the intuitionistic fuzzy set (IFS) by adding a non-membership function, which can easily deal with such voting problems which cannot be handled by FS. Since then, the researches on the multiple attribute decision making methods based on IFS have made many achievements.

Firstly, in real decision making, it may be difficult to get the crisp numbers for the membership function and non-membership function in IFS because of complexity of real decision making problems, and they can be easily expressed by some fuzzy numbers, such as interval numbers, triangular fuzzy numbers, or trapezoidal fuzzy numbers. For example, Atanassov and Gargov [9], Atanassov [10] extended the membership function and non-membership function in IFS to interval numbers, and proposed the interval-valued intuitionistic fuzzy set (IVIFS); Zhang and Liu [11] extended them to triangular fuzzy numbers, and proposed triangular intuitionistic fuzzy numbers (TIFN).

Secondly, in real decision making problems, there is a great deal of qualitative information, such as the vehicle performance, customer satisfaction and etc. which are easily expressed by linguistic terms, such as “very good”, "good", “general”, "bad" and “very bad”, etc. However, the linguistic term can imply that its membership is one and its non-membership is zero. For example, we give the evaluation value for one vehicle performance as “good”, and it means our certainty degree is 100 percent. However, sometimes, we are not 100% sure, and maybe we have also partial negation. In order to assign the particular values to membership and non-membership of linguistic terms, we can combine the linguistic terms with intuitionistic fuzzy numbers, and present the intuitionistic linguistic variables. Obviously, intuitionistic linguistic variables are the generalization of the existing fuzzy set, intuitionistic fuzzy set, linguistic variables, and so on. The intuitionistic linguistic variables can easily express the fuzzy information, and have a wide range of applications in real decision making. Some research achievements about intuitionistic linguistic variables have been made. Wang and Li [12] proposed the intuitionistic linguistic sets, intuitionistic linguistic number, and developed some decision making methods with the intuitionistic linguistic numbers. Wang et al. [13] defined the score function and accuracy function of intuitionistic linguistic numbers, and comparison method between two intuitionistic linguistic numbers. Further, the intuitionistic linguistic ordered weighted averaging operator and the intuitionistic linguistic hybrid aggregation operator were developed, and a new multi-criteria group decision making method was proposed. Liu [14] developed the intuitionistic linguistic generalized dependent ordered weighted average operator and an intuitionistic linguistic generalized dependent hybrid weighted aggregation operator based on dependent operator, and discussed some desirable properties of these operators, such as idempotency, commutativity and monotonicity, etc., and some special cases of them are also presented. Further, a multiple attribute group decision making method with intuitionistic linguistic information is proposed. Liu and Wang [15] developed an intuitionistic linguistic power generalized weighted average operator and an intuitionistic

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linguistic power generalized ordered weighted average operator based on power operator, and discussed some special cases of these operators with respect to the generalized parameters. Then two multiple attribute group decision making methods with intuitionistic linguistic information are proposed. In order to deal with the more complex fuzzy information, Liu and Jin [16] further proposed the intuitionistic uncertain linguistic variables by extending linguistic variables to uncertain linguistic variables, and proposed the operational rules, expected value, score functions and accuracy functions of intuitionistic uncertain linguistic variables. Then the intuitionistic uncertain linguistic weighted geometric average operator, intuitionistic uncertain linguistic ordered weighted geometric operator, and intuitionistic uncertain linguistic hybrid geometric operator are developed, and two multiple attribute group decision making methods with intuitionistic uncertain linguistic information were proposed. Liu et al. [17] developed the intuitionistic uncertain linguistic arithmetic Heronian mean operator, intuitionistic uncertain linguistic weighted arithmetic Heronian mean operator, intuitionistic uncertain linguistic geometric Heronian mean operator, and intuitionistic uncertain linguistic weighted geometric Heronian mean operator, and some decision making methods based on the developed operators are proposed.

Obviously, the above decision making methods with intuitionistic linguistic information or intuitionistic uncertain linguistic information were proposed based on some aggregation operators for different purposes. These methods have the advantages of getting the comprehensive evaluation values for each alternative; however, these methods are too complex, especially for the multiple attribute decision making problems which need only rank the alternatives.

TOPSIS (Technique for Order Performance by Similarity to Ideal Solution), proposed by Hwang and Yoon [18], is a very simple and effective ranking method which is widely used to solve the multiple attribute decision making problems. Its basic principle is that the best alternative should be the shortest from the positive-ideal solution and the farthest distance from the negative-ideal solution. The traditional TOPSIS method is only used to solve the decision making problems where the attribute values take the form of crisp numbers, and many extended TOPSIS were proposed to deal with fuzzy information. Yue [19-21] extended TOPSIS to deal with interval numbers, Lee et al. [22] extended TOPSIS to deal with fuzzy numbers, Liu and Su [23], Wei and Liu [24] extended TOPSIS to linguistic information environments, Jin et al. [25], Ashtiani et al. [26], Boran et al. [27] and Li et al. [28] extended TOPSIS to intuitionistic fuzzy information, and Liu [29] extended TOPSIS to interval-valued intuitionistic fuzzy information.

Obviously, TOPSIS method has been extended to process the different fuzzy information. Now it has been not extended to deal with the intuitionistic uncertain linguistic information. So the purpose of this study is to extend TOPSIS to process the intuitionistic uncertain linguistic information and propose an extended TOPSIS method with respect to the MADM problems in which attribute values take the form of the intuitionistic uncertain linguistic information and attribute weight is unknown. In order to do so, the remainder of this paper is organized as follows. In Section 1, we give an introduction of the research background and research object. Section 2 briefly reviews some basic concepts and operational rules about the intuitionistic uncertain linguistic variables and traditional TOPSIS method. In Section 3, we develop an extended TOPSIS method for the intuitionistic uncertain linguistic variables. In Section 4, we give an application example to show the decision making steps. Section 5 ends this paper with some concluding remarks.

II. Preliminaries

A. *The intuitionistic fuzzy set*

Definition 1[7]. Let $X=\{x_1,x_2,\cdots,x_n\}$ be a universe of discourse, then an intuitionistic fuzzy set (IFS) A in X is given by

$$A=\{<x,u_A(x),v_A(x)>x\in X\} \tag{1}$$

where $u_A:X\to[0,1]$, $v_A:X\to[0,1]$ and $0\le u_A(x)+v_A(x)\le 1$, $\forall x\in X$. $u_A(x)$ and $v_A(x)$ are called the membership degree and non-membership degree of the element x to the set A, respectively.

For each IFS A in X , if $\forall x\in X$, then $\pi(x)=1-u_A(x)-v_A(x)$ is called the degree of indeterminacy of x to the set A [7, 8]. Obviously, it also meets that $0\le\pi(x)\le 1$, $\forall x\in X$.

Let $A=\{<x,u_A(x),v_A(x)>x\in X\}$ and $B=\{<x,u_B(x),v_B(x)>x\in X\}$ be two IFSs in the set X and $n\ge 0$, then the operational rules for IFSs are defined as follows [7, 30].

$$A+B=\{<x,u_A(x)+u_B(x)-u_A(x)u_B(x),v_A(x)v_B(x)>x\in X\} \tag{2}$$

$$AB=\{<x,u_A(x)u_B(x),v_A(x)+v_B(x)-v_A(x)v_B(x)>x\in X\} \tag{3}$$

$$nA=\{<x,1-(1-u_A(x))^n,(v_A(x))^n>x\in X\} \tag{4}$$

$$A^n=\{<x,(u_A(x))^n,1-(1-v_A(x))^n>x\in X\} \tag{5}$$

B. *The linguistic set and uncertain linguistic variables*

Let $S=(s_0,s_1,\cdots,s_{l-1})$ be a finite and totally ordered discrete term set, where l is an odd value, we can call S a linguistic set. In general, l can be signed to 5, 7 and 9 etc. For example, when l =7, the linguistic set S could be assigned as follows:

$S=(s_0,s_1,s_2,s_3,s_4,s_5,s_6)$ = {very poor, poor, slightly poor, fair, slightly good, good, very good}.

In order to preserve all the given information in the calculation process, the discrete linguistic set $S=(s_0,s_1,\cdots,s_{l-1})$ was extended to a continuous linguistic set $\bar{S}=\{s_\alpha\mid\alpha\in[0,q]\}$, where q is a sufficiently large number [31, 32]. If $s_\alpha\in S$, then s_α is called as an original term, otherwise s_α is called as an extended term. Generally, original terms can be used to give the evaluation results for decision making objects by decision makers, and the extended terms can only be used in the process of calculating and ranking [33].

Definition 2 [34]: suppose $\tilde{s}=[s_a,s_b], s_a,s_b \in \bar{S}$ and $a \le b$, s_a and s_b are the lower limit and upper limit of $\tilde{s}$, respectively, and then $\tilde{s}$ is called an uncertain linguistic variable.

For the sake of convenience, we suppose $\tilde{S}$ is a set of all uncertain linguistic variables. For any two uncertain linguistic variables $\tilde{s}_1=[s_{a1},s_{b1}]$ and $\tilde{s}_2=[s_{a2},s_{b2}]$, the operation rules are defined as follows [33, 34]:

(1) $\tilde{s}_1 \oplus \tilde{s}_2=[s_{a1},s_{b1}]\oplus[s_{a2},s_{b2}]=[s_{a1+a2},s_{b1+b2}]$ (6)

(2) $\tilde{s}_1 \otimes \tilde{s}_2=[s_{a1},s_{b1}]\otimes[s_{a2},s_{b2}]=[s_{a1\times a2},s_{b1\times b2}]$ (7)

(3) $\tilde{s}_1/\tilde{s}_2=[s_{a1},s_{b1}]/[s_{a2},s_{b2}]=[s_{a1/b2},s_{b1/a2}]$ $\quad if \;\; a2\ne 0, b2\ne 0$ (8)

(4) $\lambda\tilde{s}_1=\lambda[s_{a1},s_{b1}]=[s_{\lambda * a1},s_{\lambda * b1}] \;\; \lambda>0$ (9)

(5) $\lambda(\tilde{s}_1\oplus\tilde{s}_2)=\lambda\tilde{s}_1\oplus\lambda\tilde{s}_2 \;\; \lambda>0$ (10)

(6) $(\lambda_1+\lambda_2)\tilde{s}_1=\lambda_1\tilde{s}_1\oplus\lambda_2\tilde{s}_1 \quad \lambda_1,\lambda_2>0$ (11)

Definition 3 [24]. Suppose $\tilde{s}_1=[s_{a1},s_{b1}]$ and $\tilde{s}_2=[s_{a2},s_{b2}]$ are two uncertain linguistic variables, then the distance between $\tilde{s}_1$ and $\tilde{s}_2$ is defined as follows.

$$d(\tilde{s}_1,\tilde{s}_2)=\frac{1}{2(l-1)}\left(|a2-a1|+|b2-b1|\right) \tag{12}$$

C. The intuitionistic uncertain linguistic set (IULS)

Definition 4 [12]. An intuitionistic linguistic set A in X is given by

$$A=\left\{<x[s_{\theta(x)},(u_A(x),v_A(x))]>|\, x\in X\right\} \tag{13}$$

Where $s_{\theta(x)}\in\bar{S}$, $u_A: X\to[0,1]$, $v_A: X\to[0,1]$, and $0\le u_A(x)+v_A(x)\le 1$, $\forall x\in X$ .The numbers $u_A(x)$ and $v_A(x)$ represent the membership degree and non-membership degree of the element x to linguistic index $s_{\theta(x)}$, respectively.

For any intuitionistic linguistic set A in X, if $\forall x\in X$, then $\pi(x)=1-u_A(x)-v_A(x)$ is called the degree of indeterminacy of x to linguistic index $h_{\theta(x)}$. It is obvious that $0\le\pi(x)\le 1$, $\forall x\in X$. (7)

Definition 5 [16]. An intuitionistic uncertain linguistic set(IULS) A in X is given by

$$A=\left\{<x[[s_{\theta(x)},s_{\tau(x)}],(u_A(x),v_A(x))]>|\, x\in X\right\} \tag{14}$$

where $[s_{\theta(x)},s_{\tau(x)}]\in\tilde{S}$ $s_{\theta(x)},s_{\tau(x)}\in\bar{S}$, $u_A: X\to[0,1]$ and $v_A: X\to[0,1]$, with the condition $0\le u_A(x)+v_A(x)\le 1$, $\forall x\in X$. The numbers $u_A(x)$ and $v_A(x)$ represent the membership degree and non-membership degree of the element x to the uncertain linguistic variable $[s_{\theta(x)},s_{\tau(x)}]$, respectively.

For each IULS A in X, if $\forall x\in X$, then $\pi(x)=1-u_A(x)-v_A(x)$ is called the degree of indeterminacy of x to the uncertain linguistic variable $[s_{\theta(x)},s_{\tau(x)}]$. It is obvious that $0\le\pi(x)\le 1$, $\forall x\in X$.

Definition 6[16]. Let $A=\left\{<x[[s_{\theta(x)},s_{\tau(x)}],(u_A(x),v_A(x))]>|\, x\in X\right\}$ be intuitionistic uncertain linguistic set, the quaternion $<[s_{\theta(x)},s_{\tau(x)}],(u_A(x),v_A(x))>$ is called an intuitionistic uncertain linguistic variable (IULV), and A can be viewed as a collection of the all intuitionistic uncertain linguistic variables. In addition, $\pi_A(x)=1-u_A(x)-v_A(x)$ represents the degree of indeterminacy.

Let $\tilde{a}_1=<[s_{\theta(a_1)},s_{\tau(a_1)}],(u(a_1),v(a_1))>$ and $\tilde{a}_2=<[s_{\theta(a_2)},s_{\tau(a_2)}],(u(a_2),v(a_2))>$ be two IULVs and $\lambda\ge 0$, then the operation rules for the IULVs are given as follows **[16].**

$$\begin{aligned}\tilde{a}_1+\tilde{a}_2=&<[s_{\theta(a_1)+\theta(a_2)},s_{\tau(a_1)+\tau(a_2)}],\\ &(1-(1-u(a_1))(1-u(a_2)),v(a_1)v(a_2))>\end{aligned} \tag{15}$$

$$\begin{aligned}\tilde{a}_1\tilde{a}_2=&<[s_{\theta(a_1)\times\theta(a_2)},s_{\tau(a_1)\times\tau(a_2)}],\\ &(u(a_1)u(a_2),v(a_1)+v(a_2)-v(a_1)v(a_2))>\end{aligned} \tag{16}$$

$$\lambda\tilde{a}_1=<[s_{\lambda\times\theta(a_1)},s_{\lambda\times\tau(a_1)}],(1-(1-u(a_1))^\lambda,(v(a_1))^\lambda)> \tag{17}$$

$$\tilde{a}_1^{\lambda}=<[s_{(\theta(a_1))^\lambda},s_{(\tau(a_1))^\lambda}],((u(a_1))^\lambda,1-(1-v(a_1))^\lambda)> \tag{18}$$

Obviously, the above operational results are still intuitionistic uncertain linguistic variables.

Theorem 1 [16]: Suppose $\tilde{a}_1=<[s_{\theta(a_1)},s_{\tau(a_1)}],(u(a_1),v(a_1))>$ and $\tilde{a}_2=<[s_{\theta(a_2)},s_{\tau(a_2)}],(u(a_2),v(a_2))>$ are any two IULVs, it can easily be proved the calculation rules have the properties shown as follows

(1) $\tilde{a}_1+\tilde{a}_2=\tilde{a}_2+\tilde{a}_1$ (19)

(2) $\tilde{a}_1\tilde{a}_2=\tilde{a}_2\tilde{a}_1$ (20)

(3) $\lambda(\tilde{a}_1+\tilde{a}_2)=\lambda\tilde{a}_1+\lambda\tilde{a}_2, \lambda\ge 0$ (21)

(4) $\lambda_1\tilde{a}_1+\lambda_2\tilde{a}_1=(\lambda_1+\lambda_2)\tilde{a}_1,\; \lambda_1,\lambda_2\;\ge 0$ (22)

(5) $\tilde{a}_1^{\lambda_1}\tilde{a}_1^{\lambda_2}=(\tilde{a}_1)^{\lambda_1+\lambda_2},\; \lambda_1,\lambda_2\;\ge 0$ (23)

(6) $\tilde{a}_1^{\lambda_1}\tilde{a}_2^{\lambda_1}=(\tilde{a}_1\tilde{a}_2)^{\lambda_1},\; \lambda_1\ge 0$ (24)

D. The standard TOPSIS

For the multiple attribute decision making (MADM) problems with m alternatives $A=(a_1,a_2,\cdots,a_m)$ which are evaluated by n attributes $C=(c_1,c_2,\cdots,c_n)$, they can be summarized by the following decision matrix (Suppose w_j is the weight of attribute c_j, and meets the conditions $0\le w_j\le 1,\; \sum_{j=1}^{n}w_j=1$).

$$R=\begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{array}\begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \end{array} \tag{25}$$

where r_{ij} represents the evaluation value of the i th alternative A_i with respect to the j th attribute c_j.

The TOPSIS, which is proposed by Hwang & Yoon [18], is a useful tool to solve the MADM problems. It is based on the idea that the best alternative should have the shortest distance from the positive ideal solution, and have the farthest distance from the negative ideal solution. The decision making steps based on TOPSIS method are shown as follows [18]:

(1) Normalize the decision matrix.

In general, there exist benefit or cost type for the attributes. Suppose decision matrix $R=\left(r_{ij}\right)_{m\times n}$ can be normalized to matrix $X=\left(x_{ij}\right)_{m\times n}$, the normalization can be made by

(i) For benefit type,

$$x_{ij}=\frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^2}} \tag{26}$$

$(1\le i\le m, 1\le j\le n)$

(ii) For cost type,

$$x_{ij}=\frac{1/r_{ij}}{\sqrt{\sum_{i=1}^{m}(1/r_{ij})^2}}\,(x_{ij}\ne 0) \tag{27}$$

$(1\le i\le m, 1\le j\le n)$

(2) Construct the weighted normalized matrix

Suppose the weighted normalized matrix is $V=\left(v_{ij}\right)_{m\times n}$, then we can get

$$V=(v_{ij})_{m\times n}=\begin{bmatrix} w_1x_{11} & w_2x_{12} & \cdots & w_nx_{1n} \\ w_1x_{21} & w_2x_{22} & \cdots & w_nx_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ w_1x_{m1} & w_2x_{m2} & \cdots & w_nx_{mn} \end{bmatrix} \tag{28}$$

(3) Identify the sets of the positive ideal solution (PIS) $V^+=\left(v_1^+,v_2^+,\cdots,v_n^+\right)$ and the negative ideal solution (NIS) $V^-=\left(v_1^-,v_2^-,\cdots,v_n^-\right)$, then we can get

$$\begin{cases} v_j^+=\max\limits_i(v_{ij}) \\ v_j^-=\min\limits_i(v_{ij}) \end{cases} \quad j=1,2,\cdots,n \tag{29}$$

(4) Obtain the distances between each alternative and the positive ideal solution, and between each alternative and the negative ideal solution (NIS), then we can get

$$\begin{cases} d_i^+=\left[\sum_{j=1}^{n}(v_{ij}-v_j^+)^2\right]^{1/2} \\ d_i^-=\left[\sum_{j=1}^{n}(v_{ij}-v_j^-)^2\right]^{1/2} \end{cases} \quad i=1,2,\cdots,m \tag{30}$$

(5) Calculate the closeness coefficients of each alternative to the ideal solution, and then we can get

$$cc_i=\frac{d_i^+}{d_i^++d_i^-}\,(i=1,2,\cdots,m) \tag{31}$$

(6) Rank the alternatives

According to the closeness coefficients above, we can choose an alternative with minimum cc_i or rank alternatives according to cc_i in ascending order.

III. THE EXTENDED TOPSIS FOR THE INTUITIONISTIC UNCERTAIN LINGUISTIC VARIABLES

A. The description of decision making problems with intuitionistic uncertain linguistic information

For the MADM problems with intuitionistic uncertain linguistic variables, there are m alternatives $A=(A_1,A_2,\cdots,A_m)$ which can be evaluated by n attributes $C=(c_1,c_2,\cdots,c_n)$, and the weight of attribute c_j is w_j, and meets the conditions $0\le w_j\le 1$, $\sum_{j=1}^{n} w_j=1$. Suppose $z_{ij}(i=1,2,\cdots,m; j=1,2,\cdots,n)$ is the evaluation values of alternative a_i with respect to attribute c_j, and it can be expressed by intuitionistic uncertain linguistic variable $z_{ij}=\left\langle [x_{ij}^L,x_{ij}^U]\left(u_{ij},v_{ij}\right)\right\rangle$, where, $[x_{ij}^L,x_{ij}^U]$ is the uncertain linguistic variable, and $x_{ij}^L,x_{ij}^U\in S$, $S=(s_0,s_1,\cdots,s_{l-1})$; $u_{ij}\in[0,1]$, $v_{ij}\in[0,1]$ and $u_{ij}+v_{ij}\le 1$. Suppose attribute weight vector $W=(w_1,w_2,\ldots,w_n)$ is completely unknown, according to these conditions, we can rank the alternatives $(a_1,a_2,\cdots,a_m)$.

B. Obtain the attribute weight vector by the maximizing deviations

In order to obtain the attribute weight vector, we firstly define the distance between two intuitionistic uncertain linguistic variables.

Definition 7. Let $\tilde{s}_1=\left\langle [s_{a1},s_{b1}]\left(u_{a1},v_{b1}\right)\right\rangle$, $\tilde{s}_2=\left\langle [s_{a2},s_{b2}]\left(u_{a2},v_{b2}\right)\right\rangle$ and $\tilde{s}_3=\left\langle [s_{a3},s_{b3}]\left(u_{a3},v_{b3}\right)\right\rangle$ be any three intuitionistic uncertain linguistic variables, and $\hat{S}$ be the set of all intuitionistic uncertain linguistic variables, f is a map, and $f:\hat{S}\times\hat{S}\to R$. If $d\left(\hat{s}_1,\hat{s}_2\right)$ meets the following conditions

(1) $0\le d\left(\hat{s}_1,\hat{s}_2\right)\le 1$, $d\left(\hat{s}_1,\hat{s}_1\right)=0$

(2) $d\left(\hat{s}_1,\hat{s}_2\right)=d\left(\hat{s}_2,\hat{s}_1\right)$

(3) $d(\hat{s}_1,\hat{s}_2)+d(\hat{s}_2,\hat{s}_3)\geq d(\hat{s}_1,\hat{s}_3)$

then $d(\hat{s}_1,\hat{s}_2)$ is called the distance between two intuitionistic uncertain linguistic variables $\hat{s}_1,\hat{s}_2$.

Definition 8. Let $\tilde{s}_1=\langle[s_{a1},s_{b1}](u_{a1},v_{b1})\rangle$ and $\tilde{s}_2=\langle[s_{a2},s_{b2}](u_{a2},v_{b2})\rangle$ be any two intuitionistic uncertain linguistic variables, then the Hamming distance between $\tilde{s}_1$ and $\tilde{s}_2$ can be defined as follows.

$$d(\hat{s}_1,\hat{s}_2)=\frac{1}{4(l-1)}\Big(|a1\times u_{a1}-a2\times u_{a2}|+|a1\times(1-v_{a1})-a2\times(1-v_{a2})|+|b1\times u_{a1}-b2\times u_{a2}|+|b1\times(1-v_{a1})-b2\times(1-v_{a2})|\Big) \quad (32)$$

In order to illustrate the effectiveness of definition 8, the distance defined in (32) must meet the three conditions in definition 7.

Proof.

Obviously, the distance defined in (32) can meets the conditions (1) and (2) in definition 7.

In the following, we will prove that the distance defined in (32) can also meet the condition (3) in definition 7.

For any one intuitionistic uncertain linguistic variable $\tilde{s}_3=\langle[s_{a3},s_{b3}](u_{a3},v_{b3})\rangle$, we have

$$\begin{aligned}d(\hat{s}_1,\hat{s}_3)&=\frac{1}{4(l-1)}\Big(|a1\times u_{a1}-a3\times u_{a3}|+|a1\times(1-v_{a1})-a3\times(1-v_{a3})|+|b1\times u_{a1}-b3\times u_{a3}|\\&\quad+|b1\times(1-v_{a1})-b3\times(1-v_{a3})|\Big)\\&=\frac{1}{4(l-1)}\Big(|a1\times u_{a1}-a2\times u_{a2}+a2\times u_{a2}-a3\times u_{a3}|\\&\quad+|a1\times(1-v_{a1})-a2\times(1-v_{a2})+a2\times(1-v_{a2})-a3\times(1-v_{a3})|\\&\quad+|b1\times u_{a1}-b2\times u_{a2}+b2\times u_{a2}-b3\times u_{a3}|\\&\quad+|b1\times(1-v_{a1})-b2\times(1-v_{a2})+b2\times(1-v_{a2})-b3\times(1-v_{a3})|\Big)\end{aligned}$$

$$\begin{aligned}&\leq\frac{1}{4(l-1)}\Big(|a1\times u_{a1}-a2\times u_{a2}|+|a2\times u_{a2}-a3\times u_{a3}|\\&\quad+|a1\times(1-v_{a1})-a2\times(1-v_{a2})|+|a2\times(1-v_{a2})-a3\times(1-v_{a3})|\\&\quad+|b1\times u_{a1}-b2\times u_{a2}|+|b2\times u_{a2}-b3\times u_{a3}|\\&\quad+|b1\times(1-v_{a1})-b2\times(1-v_{a2})|+|b2\times(1-v_{a2})-b3\times(1-v_{a3})|\Big)\end{aligned}$$

and

$$\begin{aligned}&\frac{1}{4(l-1)}\Big(|a1\times u_{a1}-a2\times u_{a2}|+|a2\times u_{a2}-a3\times u_{a3}|\\&\quad+|a1\times(1-v_{a1})-a2\times(1-v_{a2})|+|a2\times(1-v_{a2})-a3\times(1-v_{a3})|\\&\quad+|b1\times u_{a1}-b2\times u_{a2}|+|b2\times u_{a2}-b3\times u_{a3}|\\&\quad+|b1\times(1-v_{a1})-b2\times(1-v_{a2})|+|b2\times(1-v_{a2})-b3\times(1-v_{a3})|\Big)\\&=\frac{1}{4(l-1)}\Big(|a1\times u_{a1}-a2\times u_{a2}|+|a1\times(1-v_{a1})-a2\times(1-v_{a2})|\\&\quad+|b1\times u_{a1}-b2\times u_{a2}|+|b1\times(1-v_{a1})-b2\times(1-v_{a2})|\\&\quad+|a2\times u_{a2}-a3\times u_{a3}|+|a2\times(1-v_{a2})-a3\times(1-v_{a3})|\\&\quad+|b2\times u_{a2}-b3\times u_{a3}|+|b2\times(1-v_{a2})-b3\times(1-v_{a3})|\Big)\\&=\frac{1}{4(l-1)}\Big(|a1\times u_{a1}-a2\times u_{a2}|+|a1\times(1-v_{a1})-a2\times(1-v_{a2})|\\&\quad+|b1\times u_{a1}-b2\times u_{a2}|+|b1\times(1-v_{a1})-b2\times(1-v_{a2})|\Big)\\&\quad+\frac{1}{4(l-1)}\Big(|a2\times u_{a2}-a3\times u_{a3}|+|a2\times(1-v_{a2})-a3\times(1-v_{a3})|\\&\quad+|b2\times u_{a2}-b3\times u_{a3}|+|b2\times(1-v_{a2})-b3\times(1-v_{a3})|\Big)\\&=d(\hat{s}_1,\hat{s}_2)+d(\hat{s}_2,\hat{s}_3)\end{aligned}$$

So, $d(\hat{s}_1,\hat{s}_2)+d(\hat{s}_2,\hat{s}_3)\geq d(\hat{s}_1,\hat{s}_3)$.

Especially, when $u_{a1}=u_{a2}=1, v_{a1}=v_{a2}=0$, the intuitionistic uncertain linguistic variables $\hat{s}_1$ and $\hat{s}_2$ can be reduced to uncertain linguistic variables, and distance in (32) can be reduced to (12). So the uncertain linguistic variables are the special case of the intuitionistic uncertain linguistic variables.

Because the attribute weight is fully unknown, we can obtain the attribute weight vector by the maximizing deviation method. Its main idea can be described as follows. If all attribute values $z_{ij}\,(j=1,2,\cdots,n)$ in the attribute c_j have a small difference for all alternatives, it shows that the attribute c_j has a small importance in ranking all alternatives, and it can be assigned a small attribute weight, especially, if all attribute values $z_{ij}\,(j=1,2,\cdots,n)$ in the attribute c_j are equal, then the attribute c_j has no effect on sorting, and we can set zero to the weight of attribute c_j. On the contrary, if all attribute values $z_{ij}\,(j=1,2,\cdots,n)$ in the attribute c_j have a big difference, the attribute c_j will have a big importance in ranking all alternatives, and its weight can be assigned a big value [35]. Based on these ideals, we can construct the weight model.

For the attribute c_j, we can use the distance $d(z_{ij},z_{kj})$ to represent the deviation between attribute values z_{ij} and z_{kj}, and $D_{ij}(w_j)=\sum_{k=1}^{m}d(z_{ij},z_{kj})w_j$ can present the weighted deviation sum for the alternative a_i to all alternatives, then $D_j(w_j)=\sum_{i=1}^{m}D_{ij}(w_j)=\sum_{i=1}^{m}\sum_{k=1}^{m}d(z_{ij},z_{kj})w_j$ presents the weighted deviation sum for all alternatives to all alternatives, $D(w_j)=\sum_{j=1}^{n}D_j(w_j)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}d(z_{ij},z_{kj})w_j$ presents total weighted deviations for all alternatives with respect to all attributes.

Then the optimization model of determining the attribute weights can be constructed shown as follows.

$$\begin{cases}\max D(w_j)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}d(z_{ij},z_{kj})w_j\\ s.t\sum_{j=1}^{n}w_j^2=1,w_j\in[0,1],\ j=1,2\ldots\ldots n\end{cases}\tag{33}$$

Then we can build Lagrange multiplier function, and get

$$L(w_j,\lambda)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}d(z_{ij},z_{kj})w_j+\lambda(\sum_{j=1}^{n}w_j^2-1)$$

Let $\begin{cases}\dfrac{\partial L(w_j,\lambda)}{\partial w_j}=\sum_{i=1}^{m}\sum_{k=1}^{m}d(z_{ij},z_{kj})+2\lambda w_j=0\\ \dfrac{\partial L(w_j,\lambda)}{\partial \lambda}=\sum_{j=1}^{n}w_j^2-1=0\end{cases}$

We can get

$$\begin{cases}2\lambda=\sqrt{\sum_{j=1}^{n}(\sum_{i=1}^{m}\sum_{k=1}^{m}d(z_{ij},z_{kj}))^2}\\ w_j=\dfrac{\sum_{i=1}^{m}\sum_{k=1}^{m}d(z_{ij},z_{kj})}{\sqrt{\sum_{j=1}^{n}(\sum_{i=1}^{m}\sum_{k=1}^{m}d(z_{ij},z_{kj}))^2}}\end{cases}\tag{34}$$

Then we can get the normalized attribute weight, and have

$$w_j=\frac{\sum_{i=1}^{m}\sum_{k=1}^{m}d(z_{ij},z_{kj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}d(z_{ij},z_{kj})}\tag{35}$$

C. The extended TOPSIS method for the intuitionistic uncertain linguistic information

The standard TOPSIS method can only process the real numbers, and cannot deal with the intuitionistic uncertain linguistic information. In the following, we will extend TOPSIS to process the intuitionistic uncertain linguistic variables. The steps are shown as follows.

(1) Normalize the decision matrix.

Considering the benefit or cost type of the attribute values, we can give the normalized matrix $R=\left(r_{ij}\right)_{m\times n}$, where $r_{ij}=\left\langle[r_{ij}^L,r_{ij}^U]\left(\dot{u}_{ij},\dot{v}_{ij}\right)\right\rangle$. The normalization can be made shown as follows

(i) For benefit type,

$$\begin{cases}r_{ij}^L=x_{ij}^L,r_{ij}^U=x_{ij}^U\\ \dot{u}_{ij}=u_{ij},\dot{v}_{ij}=v_{ij}\end{cases}\quad(1\le i\le m,1\le j\le n)\tag{36}$$

(ii) For cost type,

$$\begin{cases}r_{ij}^L=neg(x_{ij}^U),r_{ij}^U=neg(x_{ij}^L)\\ \dot{u}_{ij}=u_{ij},\dot{v}_{ij}=v_{ij}\end{cases}\quad(1\le i\le m,1\le j\le n)\tag{37}$$

(2) Construct the weighted normalized matrix

$$Y=[y_{ij}]_{m\times n}=\begin{bmatrix}\left\langle[y_{11}^L,y_{11}^U]\left(\ddot{u}_{11},\ddot{v}_{11}\right)\right\rangle & \cdots & \left\langle[y_{1n}^L,y_{1n}^U]\left(\ddot{u}_{1n},\ddot{v}_{1n}\right)\right\rangle\\ \left\langle[y_{21}^L,y_{21}^U]\left(\ddot{u}_{21},\ddot{v}_{21}\right)\right\rangle & \cdots & \left\langle[y_{2n}^L,y_{2n}^U]\left(\ddot{u}_{2n},\ddot{v}_{2n}\right)\right\rangle\\ \vdots & \cdots & \vdots\\ \left\langle[y_{m1}^L,y_{m1}^U]\left(\ddot{u}_{m1},\ddot{v}_{m1}\right)\right\rangle & \cdots & \left\langle[y_{mn}^L,y_{mn}^U]\left(\ddot{u}_{mn},\ddot{v}_{mn}\right)\right\rangle\end{bmatrix}$$

where

$$\begin{cases}y_{11}^L=w_jr_{ij}^L,y_{ij}^U=w_jr_{ij}^U\\ \ddot{u}_{ij}=1-(1-\dot{u}_{ij})^{w_j},\ddot{v}_{ij}=\dot{v}_{ij}^{w_j}\end{cases}\tag{38}$$

(3) Identify the sets of the positive ideal solution $Y^+=\left(y_1^+,y_2^+,\cdots,y_n^+\right)$ and the negative ideal solution $Y^-=\left(y_1^-,y_2^-,\cdots,y_n^-\right)$, then we can get

$$\begin{aligned}Y^+&=\left(y_1^+,y_2^+,\cdots,y_n^+\right)=\Big(\left\langle[y_1^{L+},y_1^{U+}](\ddot{u}_1^+,\ddot{v}_1^+)\right\rangle,\\&\left\langle[y_2^{L+},y_2^{U+}](\ddot{u}_2^+,\ddot{v}_2^+)\right\rangle,\cdots,\left\langle[y_n^{L+},y_n^{U+}](\ddot{u}_n^+,\ddot{v}_n^+)\right\rangle\Big)\end{aligned}\tag{39}$$

$$\begin{aligned}Y^-&=\left(y_1^-,y_2^-,\cdots,y_n^-\right)=\Big(\left\langle[y_1^{L-},y_1^{U-}](\ddot{u}_1^-,\ddot{v}_1^-)\right\rangle,\\&\left\langle[y_2^{L-},y_2^{U-}](\ddot{u}_2^-,\ddot{v}_2^-)\right\rangle,\cdots,\left\langle[y_n^{L-},y_n^{U-}](\ddot{u}_n^-,\ddot{v}_n^-)\right\rangle\Big)\end{aligned}\tag{40}$$

where

$$\begin{cases}y_j^{L+}=\max_i(y_{ij}^L),y_j^{U+}=\max_i(y_{ij}^U)\\ \ddot{u}_j^+=\max_i(\ddot{u}_{ij}),\ddot{v}_j^+=\min_i(\ddot{v}_{ij})\\ y_j^{L-}=\min_i(y_{ij}^L),y_j^{U-}=\min_i(y_{ij}^U)\\ \ddot{u}_j^-=\min_i(\ddot{u}_{ij}),\ddot{v}_j^+=\max_i(\ddot{v}_{ij})\end{cases}\tag{41}$$

(4) Obtain the distances between each alternative and the positive ideal solution, and between each alternative and the negative ideal solution, then we can get

$$\begin{aligned}D^+&=\left(d_1^+,d_2^+,\cdots,d_m^+\right)\\ D^-&=\left(d_1^-,d_2^-,\cdots,d_m^-\right)\end{aligned}\tag{42}$$

where,

$$\begin{cases}d_i^+=\left[\sum_{j=1}^{n}\left(d(y_{ij},y_j^+)\right)^2\right]^{1/2}\\ d_i^-=\left[\sum_{j=1}^{n}\left(d(y_{ij},y_j^-)\right)^2\right]^{1/2}\end{cases}\tag{43}$$

where, $d(y_{ij},y_j^+)$ is the distance between the intuitionistic uncertain linguistic variables y_{ij} and y_j^+, and $d(v_{ij},v_j^-)$ is the distance between the intuitionistic uncertain linguistic variables y_{ij} and y_j^- which can be calculated by (32).

(5) Obtain the closeness coefficients of each alternative to the ideal solution, and then we can get

$$cc_i = \frac{d_i^+}{d_i^+ + d_i^-} (i = 1, 2, \cdots, m) \quad (44)$$

(6) Rank the alternatives

According to the closeness coefficients above, we can choose an alternative with minimum cc_i or rank alternatives according to cc_i in ascending order.

IV. An illustrative example

In this part, we give an illustrative example for the extended TOPSIS method to multiple attribute decision making problems in which the attribute values are the intuitionistic uncertain linguistic variables, and compare with the existing methods.

Suppose that an investment company wants to invest a sum of money to one of four possible companies (alternatives) shown as follows: (1) A_1 is a car company; (2) A_2 is a computer company; (3) A_3 is a TV company; (4) A_4 is a food company. They are evaluated by the following four attributes: (1) C_1 is the risk analysis; (2) C_2 is the growth analysis; (3) C_3 is the social–political impact analysis; (4) C_4 is the environmental impact analysis. The evaluation values of alternatives A_i under the attribute C_j can be expressed by the intuitionistic uncertain linguistic variable $\tilde{x}_{ij} = \left\langle [x_{ij}^L, x_{ij}^U] \left(u_{ij}, v_{ij}\right) \right\rangle$ which is listed in Table I, where $x_{ij}^L, x_{ij}^U \in S$, and the linguistic term set $S = (s_0, s_1, s_2, s_3, s_4, s_5, s_6)$; $u_{ij}, v_{ij} \in [0,1]$ and $u_{ij} + v_{ij} \le 1$. The decision matrix $\tilde{X} = \left[\tilde{x}_{ij}\right]_{4\times4}$ and the attribute weights are fully unknown. Please give the best alternative.

A. Decision steps

To get the best alternative, the following steps are involved:

Step 1. Normalization

Because the Attributes C_1, C_2, C_3 and C_4 are all the benefit types, we don't need the normalization of the decision matrix $\tilde{X}$

Step 2. Determine the attribute weight vector W, by formula (35), we can get

$$w_1 = 0.292, w_2 = 0.234, w_3 = 0.263, w_4 = 0.211$$

Step 3. Construct the weighted normalized matrix, by formula (38), we can get

$$Y = \begin{bmatrix} \langle [s_{1.461}, s_{1.461}], (0.297, 0.510)\rangle & \langle [s_{0.702}, s_{0.702}], (0.246, 0.754)\rangle & \langle [s_{1.050}, s_{1.313}], (0.214, 0.655)\rangle & \langle [s_{0.845}, s_{1.056}], (0.224, 0.712)\rangle \\ \langle [s_{1.168}, s_{1.461}], (0.235, 0.625)\rangle & \langle [s_{1.171}, s_{1.171}], (0.193, 0.754)\rangle & \langle [s_{0.525}, s_{0.788}], (0.345, 0.546)\rangle & \langle [s_{0.633}, s_{0.845}], (0.176, 0.824)\rangle \\ \langle [s_{0.876}, s_{1.168}], (0.297, 0.625)\rangle & \langle [s_{0.937}, s_{1.171}], (0.193, 0.686)\rangle & \langle [s_{0.263}, s_{0.525}], (0.271, 0.546)\rangle & \langle [s_{0.422}, s_{0.633}], (0.224, 0.615)\rangle \\ \langle [s_{0.876}, s_{0.876}], (0.183, 0.625)\rangle & \langle [s_{0.468}, s_{0.702}], (0.246, 0.583)\rangle & \langle [s_{0.788}, s_{1.050}], (0.214, 0.729)\rangle & \langle [s_{0.845}, s_{0.845}], (0.136, 0.776)\rangle \end{bmatrix}$$

Step 4. Identify the sets of the positive ideal solution $Y^+ = \left(y_1^+, y_2^+, y_3^+, y_4^+\right)$ and the negative ideal solution $Y^- = \left(y_1^-, y_2^-, y_3^-, y_4^-\right)$, by formulas (39)-(41), we can get

$$Y^+ = \left(\langle [s_{1.461}, s_{1.461}], (0.297, 0.510)\rangle \ \langle [s_{1.171}, s_{1.171}], (0.246, 0.583)\rangle \ \langle [s_{1.050}, s_{1.313}], (0.345, 0.546)\rangle \ \langle [s_{0.845}, s_{1.056}], (0.224, 0.615)\rangle\right)$$

$$Y^- = \left(\langle [s_{0.876}, s_{0.876}], (0.183, 0.625)\rangle \ \langle [s_{0.468}, s_{0.702}], (0.193, 0.754)\rangle \ \langle [s_{0.263}, s_{0.525}], (0.214, 0.729)\rangle \ \langle [s_{0.422}, s_{0.633}], (0.136, 0.824)\rangle\right)$$

Step 5. Obtain the distances between each alternative and the positive ideal solution, and between each alternative and the negative ideal solution, by formulas (42)-(43), we can get

$$D^+ = \left(0.044, 0.057, 0.073, 0.079\right)$$
$$D^- = \left(0.073, 0.044, 0.032, 0.027\right)$$

Step 6. Calculate the closeness coefficients of each alternative to the ideal solution, by formula (44), we can get

$$cc_i = \left(0.374, 0.562, 0.694, 0.749\right)$$

Step 7. Rank the alternatives

According to the closeness coefficients above, we can choose an alternative with minimum cc_i or rank alternatives according to cc_i in ascending order. We can get

$$A_1 \succ A_2 \succ A_3 \succ A_4.$$

So, the most desirable alternative is A_1.

Table I

Decision matrix $\tilde{X}$

	C_1	C_2	C_3	C_4
A_1	$\langle [s_5, s_5], (0.7, 0.1)\rangle$	$\langle [s_3, s_3], (0.7, 0.3)\rangle$	$\langle [s_4, s_5], (0.6, 0.2)\rangle$	$\langle [s_4, s_5], (0.7, 0.2)\rangle$
A_2	$\langle [s_4, s_5], (0.6, 0.2)\rangle$	$\langle [s_5, s_5], (0.6, 0.3)\rangle$	$\langle [s_2, s_3], (0.8, 0.1)\rangle$	$\langle [s_3, s_4], (0.6, 0.4)\rangle$
A_3	$\langle [s_3, s_4], (0.7, 0.2)\rangle$	$\langle [s_4, s_5], (0.6, 0.2)\rangle$	$\langle [s_1, s_2], (0.7, 0.1)\rangle$	$\langle [s_2, s_3], (0.7, 0.1)\rangle$
A_4	$\langle [s_3, s_3], (0.5, 0.2)\rangle$	$\langle [s_2, s_3], (0.7, 0.1)\rangle$	$\langle [s_3, s_4], (0.6, 0.3)\rangle$	$\langle [s_4, s_4], (0.5, 0.3)\rangle$

B. Discussion

In order to verify the validity of the proposed method, we used the method proposed by Wang and Zhang [36] to verify this example. Because the method by Wang and Zhang only can solve the multiple attribute decision problems with intuitionistic trapezoidal fuzzy numbers, so, we firstly convert the intuitionistic uncertain linguistic variables to intuitionistic trapezoidal fuzzy numbers by transforming the uncertain linguistic values into trapezoidal fuzzy numbers [37]. Then we can utilize the method proposed by Wang and Zhang [36], and get the result shown as follows:

$$A_1 \succ A_2 \succ A_3 \succ A_4 .$$

Obviously, the same ranking results are produced by two methods; this verifies the validity of the proposed method in this paper.

In order to show the advantages of proposed method, we can compare with the existing methods.

(1) Compared with the method for the intuitionistic fuzzy numbers

We can compared with method proposed by Wang and Zhang [36], the proposed method in this paper is simple and easy to use, and can process the MADM problems with unknown weight, and the method by Wang and Zhang [36] cannot deal with the unknown weight. In addition, the proposed method can directly process the intuitionistic uncertain linguistic variables, and don't need to convert to intuitionistic trapezoidal fuzzy numbers while the method Wang and Zhang [36] can only process intuitionistic trapezoidal fuzzy numbers because we can give the linguistic information not trapezoidal fuzzy information in real decision making.

Obviously, compared with method proposed by Wang and Zhang [36], the method in this paper has the advantages, such as simple for calculation, practical with directly processing intuitionistic uncertain linguistic variables, and general for solving the MADM problems with unknown weight.

(2) Compared with the method for the intuitionistic uncertain linguistic variables

Compared with the methods proposed in [16, 17], the method in this paper can solve the MADM problems with unknown weight, and rank the alternatives by the closeness coefficients of the TOPSIS method. However, methods proposed in [16, 17] can solve the MAGDM problems by some aggregation operators, the advantages of these methods are that they can rank the alternatives by their comprehensive values. Obviously, the method in this paper is simple.

(3) Compared with the other extended TOPSIS method

Because the intuitionistic uncertain linguistic variables are the generalization of the interval numbers, fuzzy numbers, linguistic variables, uncertain linguistic variables, intuitionistic fuzzy sets, and so on. Obviously, the extended TOPSIS methods, proposed by Yue [19-21], Lee et al. [22], Liu and Su [23], Wei and Liu [24], Jin et al. [25], Ashtiani et al. [26], Boran et al. [27] and Li et al. [28], are the special cases of the proposed method in this paper.

In a word, the method proposed in this paper is more generalized. At the same time, it is also simple and easy to use.

I. Conclusion

In real decision making, there is a great deal of qualitative information which can be expressed by uncertain linguistic variables. However, the uncertain linguistic variables only mean the certainty degree is 100 percent, and cannot express the uncertainty. In this paper, we combine the uncertain linguistic variables and intuitionistic fuzzy set, and proposed the intuitionistic uncertain linguistic variables which could easily express the fuzzy information in real world. The TOPSIS method had been proved that it is a very effective decision making method and has been achieved more and more extensive applications. However, the standard TOPSIS can only process the real numbers. In this paper, we extended TOPSIS method to deal with the intuitionistic uncertain linguistic information, and proposed an extended TOPSIS method with respect to the MADM problems in which the attribute values take the form of the intuitionistic uncertain linguistic variables and attribute weight is unknown. Firstly, the operational rules and properties for the intuitionistic uncertain linguistic variables were presented. Then the distance between two intuitionistic uncertain linguistic variables was proposed and the attribute weight was calculated by the maximizing deviation method, and the closeness coefficients to the ideal solution for each alternative were used to rank the alternatives. Finally, an illustrative example was given to illustrate the decision making steps and the effectiveness of the proposed method. Compared with the other extended TOPSIS method, they are the special cases of the proposed method in this paper, so the method proposed in this paper is more generalized; Compared with the method for the intuitionistic uncertain linguistic variables, the method in this paper has the advantages, such as simple for calculation, practical with directly processing intuitionistic uncertain linguistic variables, and general for solving the MADM problems with unknown weight. So, the proposed method is the enrichment and development of the theory and methods for the multiple attribute decision making with the intuitionistic uncertain linguistic variables. In the future, we will further research the other decision making models and methods for the intuitionistic uncertain linguistic variables, for example, VIKOR, TODIM and etc.

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