Multi Permanent Magnet Synchronous Motor Synchronization Control based on Variable Universe Fuzzy PI Method

Zhongda Tian, Shujiang Li, Yanhong Wang, Quan Zhang

Abstract—This paper describes a variable universe fuzzy PI control method for multi permanent magnet synchronous motors master-slave synchronization control system. The speed loop modulator and current loop modulator is determined by the mathematical model of permanent magnet synchronous motor. The vector control and voltage space vector pulse width modulation (SVPWM) control is applied to the closed-loop control system. Considering the defects of the traditional speed regulator, fuzzy control and PI method is combined to solve the multi-motor synchronous system speed control problem. Meanwhile, in order to improve the control effect, the interpolation variable universe fuzzy PI method is used to change the rules to eliminate the multi motor synchronous control steady-state error. The simulation model for multi synchronous permanent magnet synchronous motor control system is built. Compared with fuzzy PI control method, simulation results verify the proposed method can improve the robustness against external changed interference coefficient added to the system, change the dynamic characteristics, reduce the stable error and overshoot, and has better control effect.

Index Terms—permanent magnet synchronous motor, multi motor synchronization, variable universe, fuzzy PI

I. INTRODUCTION

In recent years, with the development of industrial technology, the performance and quality of various mechanical products operation gradually increased, only for control a motor on many occasions has been unable to meet the requirement of scientific and technological development, which need to control multi motors, make them better coordinated operation[1]. There are two ways in the multi motor synchronous operation: mechanical and electrically. Since the mechanical structure is simple and widely used, but it has a high mechanical wear rate, control accuracy is not high, and the range of transmission distance is limited. The electrically motor synchronous control mode is not basically affected by the above conditions, usage is also very flexible[2]. The master-slave synchronization control for permanent magnet synchronous motor is an important problem since it is often used in manufacturing and production processes. Some applications require strict synchronous of motor drive shaft, such as door type crane, sluice gate, lift bridge, etc. While others require rotation speed ratio of several motors continuously changed according to a certain rule, such as the paper machine. In these systems, synchronization quality between multi motors directly affects the reliability and control accuracy of the whole system. Therefore, the study of multi motor synchronous control has a very important practical significance[3].

At present, the traditional PID control method can not meet the requirements of multi motor speed synchronous control accuracy. So there are many synchronization control methods based on modern control theory, such as neural networks[4]-[5], fuzzy control[6]-[7], sliding mode variable structure control[8]-[9], $H_{\infty}$ control[10], etc. These control methods display good simulation results. However, due to the complex algorithms, there are some difficulties in real applications. So this paper choose fuzzy PI as control method which can use field work experience. At the same time, fuzzy PI control process can not change the established rules. It is difficult to further improve the control effect, so the interpolation variable universe fuzzy PI control method is introduced according to the idea of universe expansion factor. The permanent magnet synchronous motor system is established through some modules including PMSM module, coordinate transformation module, current and speed loop, the SVPWM module and inverter module. Through simulation experiments, this method have good control effect and precision in multi permanent magnet synchronous motors synchronous control system.

II. PRELIMINARIES

In a multi or master-slave motor synchronization control system, a motor is selected as master, the rest motors as slave motors, the master motor output speed as the speed reference value for the slave motors. So it was concluded that any speed command or load disturbance added to the master motor will be reflected and followed from the slave motor, but any disturbance from slave motor can not feedback to the master motor or any other slave motors. Multi permanent magnet synchronous motor synchronization control system is

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is moment of inertia.

\(R\) is d-axis flux and \(eT\) is quadrature axis flux generated by permanent magnet rotor winding. \(L_s\) is direct axis inductance of stator winding. \(L_q\) is quadrature axis inductance of stator winding. \(\psi_f\) is flux generated by permanent magnet rotor winding. \(\omega\) is rotor velocity. \(\omega_r\) is motor mechanical angular velocity (\(\omega_r = \frac{\omega}{p_n}\)). \(p_n\) is motor poles. \(T_e\) is electromagnetic torque. \(T_L\) is load torque. \(J\) is moment of inertia. \(R_o\) is damping or friction coefficient.

The permanent magnet synchronous motor control model can be established through mathematical method in a d-q-axis coordinate system. Control model is shown in Fig.2.

### III. PERMANENT MAGNET SYNCHRONOUS MOTOR MODELLING

#### A. Control Strategies

The control strategies of permanent magnet synchronous motor include constant proportion of voltage to frequency control, vector control, direct torque control, etc. Vector control can achieve strict simulation for DC motor. The stator current can be divided into excitation component and torque component. These two parts are controlled respectively through the rotor magnetic orientation and we can obtain good control effects. So this paper use vector control as permanent magnet synchronous motor control strategy. At present, the vector control of permanent magnet synchronous motor include four kinds of control methods: \(i_d=0\) control strategy, \(\cos \theta=1\) control strategy, constant flux control and flux-weakening control. The \(i_d=0\) control strategy is used in this paper, the advantage of this control strategy can obtain the maximum output torque which can be maintained with minimum current magnitude, small torque ripple, and wide speed range.

#### B. Mathematical Model of PMSM

Permanent magnet synchronous motor is a multi variable, nonlinear, strong coupling system. In order to achieve linear torque control, vector control method is used for torque control parameters decoupling. The stator three-phase current is converted to two-phase synchronous rotating d-q coordinate system by the method of the coordinate transformation.

Through rotor field oriented principle, vector control transform characteristics of three-phase AC motor to the characteristics of the two-phase DC motor. The rotor magnetic orientation control put d-q coordinate system established on the rotor magnetic, that is d-axis coincides with the rotor flux. The q-axis advanced rotor flux 90 electrical degrees, at this time, the rotor flux only has d-axis component and q-axis component is equal to 0. The advantage of rotor vector control is achieved completely decoupled flux current and torque current component, without adding decoupling device.

Mathematical model of permanent magnet synchronous motor is as followed [11]:

\[
\begin{align*}
u_d &= -\omega \psi_q - \omega L_q i_q \\
u_q &= R i_q + \omega \psi_f + L_q \frac{di_q}{dt}
\end{align*}
\]  

(1)

Magnetic chain equation is:

\[
\begin{align*}
\psi_d &= \psi_f \\
\psi_q &= L_q i_q
\end{align*}
\]  

(2)

Electromagnetic torque equation is:

\[
T_e = p_n \psi_f i_q
\]  

(3)

Mechanical motion equation is:

\[
T_e - T_L = J \frac{d \omega}{dt} + R \omega
\]  

(4)

wherein, \(u_d\) is d-axis voltage of the stator winding and \(u_q\) is d-axis voltage of the stator winding. \(\psi_d\) is d-axis flux and \(\psi_q\) is q-axis flux. \(i_d\) is d-axis current of the stator winding. \(i_q\) is q-axis current of the stator winding. \(L_s\) is direct axis inductance of stator winding. \(L_q\) is quadrature axis inductance of stator winding. \(\psi_f\) is flux generated by permanent magnet rotor winding. \(\omega\) is rotor velocity. \(\omega_r\) is motor mechanical angular velocity (\(\omega_r = \frac{\omega}{p_n}\)). \(p_n\) is motor poles. \(T_e\) is electromagnetic torque. \(T_L\) is load torque. \(J\) is moment of inertia. \(R_o\) is damping or friction coefficient.

The permanent magnet synchronous motor control model can be established through mathematical method in a d-q-axis coordinate system. Control model is shown in Fig.2.

### C. The Design of Current Loop

The current loop is inner regulator, its main function is to make the current \(i_d\) and \(i_q\) track the quantitative value, that is output of the speed regulator. And suppress disturbance of internal current loop, so the dynamic response does not appear excessive current overshoot. When the load changed, the overshoot is small. PI controller regulation law of current loop is:

\[
u(t) = k_p[e(t) + \frac{1}{T_i} \int_0^t e(t) dt]
\]  

(5)

That is

\[
\frac{U(s)}{E(s)} = k_p (1 + \frac{1}{T_i}) = k_p \frac{TS + 1}{TS}
\]  

(6)

The current loop generally only changed with the PWM inverter and the motor parameters. It is not affected by the change of external load, so the current loop has the fixed internal structure [12], as shown in Fig.3.
PWM inverter can be seen as an inertia link, its open-loop transfer function is:

\[ W(S) = \frac{k_p k_{pwm} k_i (T_S S + 1)}{T_S (T_S S + 1)(T_S S + 1)} \]  \hspace{1cm} (7)

Wherein, \( T_L \gg T_c, T_L \gg T_s \). \( k_{pwm} \) is amplification of the inverter, \( T_c \) is switching cycles, \( R_s \) is motor armature circuit resistance, \( L_s \) is inductance which can be seen as one order inertial link, \( T_s \) is inductor time constant. And \( T_L = L_s/R_s \), \( k_K = 1/R_s \), they reflect proportional relationship of the voltage and current under the d-q coordinates. \( T_F \) is filter time constant of current feedback channel, \( k_f \) is amplification factor. When SVPWM control is used, the inverter output voltage is equal to a given voltage, that is \( k_{pwm} = 1 \).

The current feedback value represents the actual current value, here \( k_f = 1 \).

According to the engineering design applications, the integral time constant of current controller is selected as:

\[ T_i = T_L = \frac{L_s}{R_s} \]

So the open-loop transfer function can be expressed as:

\[ W(S) = \frac{k_f}{T_S (T_S S + 1)} \]  \hspace{1cm} (8)

The above equations can be expressed as a typical I type system:

\[ W(S) = \frac{k_f}{S(T_S S + 1)} = \frac{k_f}{1 + TS} \]  \hspace{1cm} (9)

\[ T_p = T_s + T_r, \quad k = \frac{k_f T_r}{T_s} \]  \hspace{1cm} (10)

The closed-loop transfer function corresponding to this open-loop transfer function is a typical second-order system:

\[ G(S) = \frac{W(S)}{1 + W(S)} = \frac{k_f}{S^2 + \frac{k}{T_p} S + \frac{k}{T_p}} \]  \hspace{1cm} (11)

According to the engineering experiences, damping ratio is desirable as \( \xi = 0.707 \) when overshoot \( \sigma = 5\% \).

From the above equations, \( kT_p = 1/2 \), so current loop controller proportional coefficient and integral coefficients is:

\[ k_p = \frac{R_s T_s}{2T_p}, \quad k_i = \frac{R_s}{2T_p} \]  \hspace{1cm} (12)

\[ k_f = \frac{R_s}{2T_p} \]  \hspace{1cm} (13)

### D. The Design of Speed Loop

Speed control system can follow the input changes with a certain response speed and control precision. The main role of control system is to solve the problem of dynamic response speed. Speed loop can suppress load disturbance and speed fluctuation. It can track the given speed with small error. Through the permanent magnet synchronous motor vector control method, speed loop output is the q-axis current \( i_q \), therefore the design of speed loop is first to get the closed-loop transfer function of \( i_q \), which is used as a part of the speed loop.

Since the cut-off frequency of speed loop is generally low, so the current loop transfer function can reduce system order through removing the high-order term, that is:

\[ G(S) = \frac{W(S)}{1 + W(S)} = \frac{k_f}{S^2 + \frac{k}{T_p} S + \frac{k}{T_p}} \]

\[ = \frac{1}{T_c S^2 + \frac{S}{K} + 1} \approx \frac{1}{S + 1} = \frac{1}{2T_c S + 1} \]  \hspace{1cm} (14)

Through the permanent magnet synchronous motor model, the closed loop transfer function of \( i_q \) is known. The speed loop control system structure is shown in Fig.4.

![Fig. 4. Speed loop chart](image-url)

The transfer function of speed loop PI controller is:

\[ G'(s) = K_s \frac{\tau_n S + 1}{\tau_n S} \]  \hspace{1cm} (15)

\( K_s \) is speed loop proportional coefficient, \( \tau_n \) is speed loop regulator integral time constant.

System open loop transfer function is:

\[ W'(S) = \frac{k_p p S (\tau_n S + 1)}{T_S S (2T_p S + 1)(JS + R_n)} \]  \hspace{1cm} (16)

According to the design of typical II type system, parameter equations are:

\[ \tau_n = h \times 2T_p \]  \hspace{1cm} (17)

\[ K_s = \frac{h + 1}{2h} \times \frac{J}{2T_p K_i} \]  \hspace{1cm} (18)

In the above equations, \( h \) is intermediate frequency bandwidth, the bigger the \( h \), the overshoot is smaller. The
adjusting time is the shortest and dynamic response is the fastest when $h = 5$. $K_i$ is the torque parameters, that is:

$$K_i = \frac{T_p}{I_n}$$  \hspace{1cm} (19)$$

Substituted into $h$ and $K_i$ can be obtained speed loop proportional coefficient [13], integral coefficients are:

$$k'_p = \frac{h + 1}{2h} \frac{J}{2T_p K_i} = \frac{5 + 1}{2 \times 5} \frac{J}{2T_p I_n} = \frac{3J}{10T_p I_n}$$  \hspace{1cm} (20)$$

$$k_i = \frac{K_i}{\tau_n} = \frac{h + 1}{2h} \frac{J}{h \times 2T_p} = \frac{3J}{100T_p I_n}$$  \hspace{1cm} (21)$$

IV. THE DESIGN OF VARIABLE UNIVERSE FUZZY PI CONTROLLER

The traditional permanent magnet synchronous motor vector control system generally uses PI controller as the speed regulator, and its control scheme is simple and easy to be realized. However, the permanent magnet synchronous motor system has strong coupling and nonlinear characteristics. It is difficult to establish exact mathematical model of the system, so the fast response, anti-interference ability and the robustness to parameter fluctuations of the system are not ideal enough. Self-tuning fuzzy PID controller take deviation $e$ and deviation change rate $\dot{e}$ as input. It can meet deviation $e$ and deviation change rate $\dot{e}$ on PID parameter self-tuning requirements at different moments. However, the differential block is too sensitive to suppress disturbance. Therefore, this paper presents the controller with variable universe fuzzy PI structure.

A. Fuzzed Process

After the structure of fuzzy controller is determined, the input is needed for sampled, quantized and fuzzed [14]-[16]. Here select variable numbers as 7, that is [NB, NM, NS, O, PS, PM, PB]. As shown in Fig.5, the continuous variation between [-6,6] is divided into 7 parts, each part corresponds to a fuzzy set, the fuzzy variable membership assignment table is as shown in Table I.

TABLE I

<table>
<thead>
<tr>
<th>Membership</th>
<th>Quantization levels</th>
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<tbody>
<tr>
<td></td>
<td>-6</td>
</tr>
<tr>
<td>PB</td>
<td>0</td>
</tr>
<tr>
<td>PM</td>
<td>0</td>
</tr>
<tr>
<td>PS</td>
<td>0</td>
</tr>
<tr>
<td>The linguistic variable</td>
<td>O</td>
</tr>
<tr>
<td>NS</td>
<td>0</td>
</tr>
<tr>
<td>NM</td>
<td>0.2</td>
</tr>
<tr>
<td>NB</td>
<td>1</td>
</tr>
</tbody>
</table>

B. Fuzzy Rules

The self-tuning rule in this paper as follows.

1. When the error $|e|$ is large, it was indicated that the absolute value of the error is large, in order to enhance rapid response, $\Delta k_p$ should take larger. In order to prevent the instantaneous $|\dot{e}|$ is too large, and control overshoot, $\Delta k_i$ should be small, usually take $\Delta k_i = 0$.

2. When the error $|e|$ is medium, in order to ensure that the system response speed and overshoot control, $\Delta k_p$ should be smaller, $\Delta k_i$ should be increased.

3. When the error $|e|$ is small, in order to ensure the system has a good steady-state characteristics, $\Delta k_p$ and $\Delta k_i$ should be increased.

The linguistic variable of error $e$ and error change rate $\dot{e}$ is selected as 7 fuzzy values, that is {NB, NM, NS, O, PS, PM, PB}. The output is chosen as $k_p$ and $k_i$, the linguistic variable of output is also selected as {NB, NM, NS, O, PS, PM, PB}. The fuzzy control rules table of $k_p$ and $k_i$ as shown in Table II according to self tuning rules of PID parameters.
According to the fuzzy control rules table, $k_p$ and $k_i$ can be dynamically tuning, $k_p$ and $k_i$ is pre-set value by using the conventional tuning method, $\Delta k_p$ and $\Delta k_i$ is the incremental value of $k_p$ and $k_i$, then:

$$\begin{align*}
&\Delta k_p = k_p + \Delta k_p \\
&\Delta k_i = k_i + \Delta k_i
\end{align*}$$

(22)

C. The Universe Expansion Factor

Fuzzy PID does not change universe when control process changed. In order to improve the control precision under the premise of not increasing the rules, the variable universe interpolation idea is used to change the fuzzy PID rules, eliminate the steady state error and chatter phenomenon of multi motor synchronous control. Li Hongxing [17] first proposed the variable universe fuzzy control theory. Under the rules remain unchanged, discourse error of universe compression will become small. The universe compression means that adding rules, interpolation nodes in densely, so as to improve the control precision. Assume the initial universe of $e$ is $[-E, E]$, $E$ is the real number, it has 7 rules. The initial universe is changed to $[-a(x)E, a(x)E]$ through the universe expansion factor $a(x)$, $a(x)$ is continuous function on error change $x$. Assuming the universe expansion factor of input variable $E$ and $EC$ is $a_1$ and $a_2$, the fuzzy rules of input universe expansion factor as shown in Table III.

<table>
<thead>
<tr>
<th>$e$</th>
<th>$ec$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>PB/NB</td>
</tr>
<tr>
<td>NM</td>
<td>NB/NM</td>
</tr>
<tr>
<td>NS</td>
<td>PM/NM</td>
</tr>
<tr>
<td>O</td>
<td>PS/NS</td>
</tr>
<tr>
<td>PS</td>
<td>O/O</td>
</tr>
<tr>
<td>PM</td>
<td>O/O</td>
</tr>
<tr>
<td>PB</td>
<td>O/O</td>
</tr>
</tbody>
</table>

TABLE III

THE FUZZY RULES OF INPUT UNIVERSE EXPANSION FACTOR

<table>
<thead>
<tr>
<th>$e$</th>
<th>$ec$</th>
</tr>
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<tbody>
<tr>
<td>NB</td>
<td>VB</td>
</tr>
<tr>
<td>NM</td>
<td>VB</td>
</tr>
<tr>
<td>NS</td>
<td>MB</td>
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<td>O</td>
<td>S</td>
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<td>PS</td>
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<td>PM</td>
<td>S</td>
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<tr>
<td>PB</td>
<td>O</td>
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</table>

D. The Defuzzification Process

The output of fuzzy controller is a fuzzy set. It contains a variety of information of control value, but the controlled object can only accept a precise control value, which requires that the fuzzy decision, fuzzy quantity into the accurate quantity, also known as defuzzification process. This paper adopts the weighted average method to defuzzification, the output can be expressed as:

$$u' = \frac{\sum_{i=1}^{n} \phi_{a}(x_i) x_i}{\sum_{i=1}^{n} \phi_{a}(x_i)}$$

(23)

Here it is:

$$u = \frac{0}{-7} + \frac{0}{-6} + \frac{0}{-5} + \frac{0}{-4} + \frac{0.5}{-3} + \frac{0.7}{-2} + \frac{0}{-1}$$

$$+ \frac{0}{1} + \frac{0.3}{2} + \frac{0.5}{3} + \frac{0.7}{4} + \frac{1}{5} + \frac{0.7}{6} + \frac{0.2}{7}$$

Through the weighted average method, there is:

$$u' = \frac{\sum_{i=1}^{n} \phi_{a}(x_i) x_i}{\sum_{i=1}^{n} \phi_{a}(x_i)}$$

(24)

For the output universe expansion factor $\beta$, it should be determined by $e$ and $ec$ together. The expand and reduced degree should be determined according to $e$ and $ec$ values reflected in the current system response state[18], thus the fuzzy rules of the output universe expansion factor is shown in Table IV.

<table>
<thead>
<tr>
<th>$e$</th>
<th>$ec$</th>
</tr>
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<tbody>
<tr>
<td>NB</td>
<td>VB</td>
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<tr>
<td>NM</td>
<td>VB</td>
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<td>NS</td>
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<td>PS</td>
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<td>PM</td>
<td>S</td>
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<td>PB</td>
<td>O</td>
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</table>

TABLE IV

THE FUZZY RULES OF OUTPUT UNIVERSE EXPANSION FACTOR

In conclusion, the single motor variable universe fuzzy PI controller structure in this paper is as shown in Fig.6.
The major parameters in the PMSM module includes the stator resistance $R_s = 2.875\,\Omega$, the quadrature and direct axis inductance $L_d = L_q = 8.5e^{-3}\,\text{H}$, the rotor magnetic flux $\Psi_r = 0.067\,\text{Wb}$, the moment of inertia $J = 8.7e^{-4}\,\text{kg}\cdot\text{m}^2$, the viscous friction coefficient $B = 1.2\,\text{N}\cdot\text{S}/\text{m}$, number of pole pairs $P = 1$, the rated speed is $3000\,\text{r/min}$. Permanent magnet synchronous motor system includes the PMSM ontology module, coordinate transformation module, current and speed regulator module, SVPWM and inverter module, as shown in Fig.7.

V. SIMULATION

In order to study the effect of permanent magnet synchronous motor synchronous control system, a system include 3 synchronous motors is built for simulation. A motor is selected as the master motor, the remaining two motors as slave motors, multi motor synchronization system as shown in Fig.8.

When the speed is $1000\,\text{r/min}$, the DC bus voltage is $300\,\text{V}$, PWM period is $0.0002\,\text{S}$, the simulation time is from $0$ to $0.1\,\text{S}$. The load torque is changed from $1\,\text{N}\cdot\text{m}$ to $3\,\text{N}\cdot\text{m}$ at $0.04\,\text{S}$. In order to illustrate the control effect, the control method in this paper is compared to fuzzy PI control method, the speed response of master and slave motor is shown in Fig.9 and Fig.10, the master and slave motor torque response is shown as in Fig.11 and Fig.12.

The simulation results show that the variable universe fuzzy PI controller is simple, flexible and practical with on-line parameter self-tuning capability. Compared with conventional fuzzy PI controller, it greatly improved the system robustness suppress external interference coefficient and reduced overshoot. It changed the dynamic characteristics and reduced the stability error, improved the stability of the equilibrium point. The proposed control method can make the multi motor synchronous operation well at a certain speed.
Fig. 9. The multi motor speed response with fuzzy PI control

Fig. 10. The multi motor speed response with variable universe fuzzy PI control

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Fig. 11. The multi motor torque response with fuzzy PI control

Fig. 12. The multi motor torque response with variable universe fuzzy PI control

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VI. CONCLUSION

In this paper, a universe fuzzy PI control method is proposed according to multi permanent magnet synchronous motor synchronous control problem. The variable universe fuzzy PI control method has the advantages of fuzzy control and linear control, which make the design of the system achieves good dynamic and static performance. This control method can improve the accuracy of fuzzy control without increasing the rules. The system has strong adaptability while load parameters are changed. Through the simulation of three motor synchronous control system, the effectiveness of the proposed method is verified.

REFERENCES


