

# Energy Efficiency and Spectral Efficiency Tradeoff in Cognitive Heterogeneous System with Economic Consideration

Baogang Li, Xuewei Wang, and Wei Zhao

**Abstract**—Rapid development of the wireless communication industry has led to a dramatic increase of energy consumption. Energy Efficiency (EE) for the wireless networks has received considerable attention. Unfortunately, the increasing EE performance often leads to decreasing Spectral Efficiency (SE) performance and vice versa. Hence, it is often urgent to build a tradeoff between EE and SE. In this paper, the tradeoff between EE and SE is considered under the smart grid and cognitive heterogeneous environment. The economic EE and SE are proposed based on the price factor of economics. Based on these definitions, the economic EE-SE relation is deduced, where the price factors can adjust the minimum economic EE. Then a new EE-SE tradeoff metric called the Economic Resource Efficiency (ERE) is built, which can be maximized by adjusting the weight factor and utilizing the fluctuation of the electricity price and the spectrum price. Furthermore, the optimization problem of ERE is presented, and the solution of ERE is proposed for the Orthogonal Frequency Division Multiplexing (OFDM) system. The simulation results validate that the proposed ERE metric is efficient to the tradeoff between EE and SE, and to utilize the energy and bandwidth resources.

**Index Terms**—Energy efficiency, spectral efficiency, resource efficiency, price

## I. INTRODUCTION

The traditional studies on the Spectral Efficiency (SE) have met the bandwidth requirement of emerging high data rate wireless applications, which also lead to a dramatic increase of energy consumption. Green communications, which aim at enhancing Energy Efficiency (EE) for the wireless networks, have received considerable attention. Unfortunately, the increasing EE performance often leads to decreasing SE performance and vice versa, i.e., the performance of EE and SE often could not reach maximization simultaneously [1]. Hence, it is often urgent to build a tradeoff between EE and SE.

Recently, green communications for the wireless communication have received considerable attention [2-5]. [2] formulates the resource allocation problem as a maximization of effective capacity based bits-per-joule

capacity under statistical Quality of Service (QoS) provisioning. [3] investigates the distributed power allocation for the multicell Orthogonal Frequency Division Multiple Access (OFDMA) networks by taking both the energy efficiency and the intercell interference mitigation into account. [4] develops an analytical framework for downlink performance evaluation of small cell networks, based on random spatial network model, where base stations and users are modeled as two independent spatial Poisson point processes. Flit transmission in the vertical direction of 3D Network-on-Chip -Bus mesh architecture just needs one hop and consumes less energy. To take advantage of that and to solve the defect of poor heat dissipation, [5] proposes a new traffic equilibrium and energy minimization mapping method for the architecture.

Especially, the relationship of EE and SE has become an important research topic. Firstly, the relationship of EE and SE can be characterized in various scenarios, such as the point-to-point link [1], and the heterogeneous networks [6]. [7] studies the energy efficient and spectral efficient designs for type-I ARQ (Automatic Repeat reQuest) systems operating in quasistatic Rayleigh fading channels, in which the optimum transmission energy and frame length for various design criteria have been identified. [8] develops a cooperative multicast technique to support the strong demand of mobile video multimedia services in future mobile communications systems, with high spectral and energy efficiency, which has been demonstrated that it outperforms traditional multicast with path loss gain, spatial diversity, and time diversity. [9] proposes a relay cooperation scheme for the downlink of multicell multiple-input-multiple-output cellular networks, and considers different relay station decoding strategies during the broadcast phase and joint relay transmission with different degrees of Channel State Information (CSI) sharing during the relay phase.

Then one metric is also maximized with the constraint of another metric [10, 11]. [10] sets up a general EE-SE Tradeoff (EST) framework, where the overall EE, SE and per-user quality-of-service are all considered, and prove that under this framework, EE is strictly quasiconcave in SE. Given the SE requirement and maximum power limit, [11] formulates a constrained optimization problem to maximize EE, which is first transformed into a simpler single objective optimization problem as the multicriteria optimization problem with high complexity.

Furthermore, the tradeoff frameworks of EE and SE are established based on the multi-objective optimization method

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[12, 13]. To overcome the limitations of the previous design criteria, [12] proposes a unified EST metric that can be used to optimize both EE and SE simultaneously, for which the Pareto optimal set is characterized and the weighted product scalarization method is used. These recent works have provided good insight into the joint EE–SE tradeoff with the assumption that the bandwidth was fully occupied regardless of the transmission requirements. [13] proposes a new paradigm of EE–SE tradeoff for OFDMA cellular network, with taking into consideration different transmission bandwidth requirements.

Under the smart grid environment, the wireless networks are powered by the conventional energy and renewable energy with the Real-Time Price (RTP), which is utilized to guarantee the balance of energy supply and demand. Under the cognitive heterogeneous environment, the small cell utilizes the spectrum licensed by the macrocell also with a spectrum price based on the idle spectrum quantity. Therefore, balancing the EE-SE relation through the resource quantity or cost under the smart grid and cognitive heterogeneous environment is important and meaningful, which is the focus of this paper.

The main contributions of this paper are as follows: (a) the economic EE and SE are proposed based on the resource prices; (b) a new EE-SE tradeoff metric is built to maximize the resource utilization efficiency; (c) the adjustment effect by the fluctuation of the electricity price and the spectrum price is analyzed; (d) the practical consideration and the problem solution in the OFDM system are presented.

The rest of this paper is organized as follows. In Section II, the system model is described. In Section III, a new EE-SE tradeoff metric is introduced. In Section IV, the resource solution of the OFDM system is proposed. Numerical results are provided in Section V. Finally, we summarize the paper with some concluding remarks in Section VI.

## II. SYSTEM MODEL AND PROBLEM PRESENTATION

Consider a point-to-point communication downlink of one small cell in the macrocell, where both the transmitter and receiver are equipped with only one antenna. The cognitive Small Cell Base Station (SCBS) is powered by the smart grid with RTP and utilizes the idle spectrum licensed by the macrocell with the spectrum price. Assume that the transmit power is  $P$ , the channel gain between the transmitter and the receiver is  $G$ , the channel bandwidth is  $W$ .  $N_0$  represents the power spectral density of the Additive White Gaussian Noise (AWGN).

The transmission rate can be expressed as  $R = W \log_2(1 + GP/WN_0)$ . Define SE as the ratio of the transmission rate to the bandwidth, EE as the transmission rate per unit of power consumption, which are given respectively as

$$\eta_{SE} = \frac{R}{W} = \log_2 \left( 1 + \frac{GP}{WN_0} \right) \quad (1)$$

$$\eta_{EE} = \frac{R}{P} = (W/P) \log_2 \left( 1 + \frac{GP}{WN_0} \right) \quad (2)$$

Considering the smart grid and cognitive radio environment, the available energy and bandwidth are fluctuating, which lead to the change of the energy cost and bandwidth cost. We introduce the price factor in the economics, denote the electricity price and the spectrum price as  $\alpha$  and  $\beta$ . Different from the real price,  $\alpha$  and  $\beta$  are mainly based on the available energy quantity  $P_{ava}$  and the idle spectrum quantity  $W_{ava}$ , i.e.  $\alpha \sim 1/P_{ava}$ ,  $\beta \sim 1/W_{ava}$ , which don't equal to the real value of the resource only for resource utilization and comparable each other. The units of  $\alpha$  and  $\beta$  are bits/s/W and bits/s/Hz respectively.

Therefore, define the economic EE as the transmission rate per unit of energy cost, which is as follows

$$\eta_{EE}^{EP} = \frac{R}{\alpha \cdot P} = \left( \frac{W}{\alpha \cdot P} \right) \log_2 \left( 1 + \frac{GP}{WN_0} \right) = \frac{\eta_{EE}}{\alpha} \quad (3)$$

Define the economic SE as the transmission rate per unit of bandwidth cost, which is shown as

$$\eta_{SE}^{SP} = \frac{R}{\beta \cdot W} = (1/\beta) \log_2 \left( 1 + \frac{GP}{WN_0} \right) = \frac{\eta_{SE}}{\beta} \quad (4)$$

According to (3), under a fixed bandwidth, the lower electricity price leads to the higher economic EE. With the more bandwidth consumption, the economic EE achieves improvement quickly, but the economic SE also reduces quickly as shown in Fig.1 (a). While the economic SE with the smaller spectrum price decreases slower than that with higher spectrum price. Similarly, the electricity price and the energy consumption have same effect as shown in Fig.1 (b). Therefore, the quantity or prices of two resources have more influence on adjusting the economic EE and the economic SE.

Based on  $\eta_{EE}^{EP}$  and  $\eta_{SE}^{SP}$ , the economic EE-SE relation can be written as

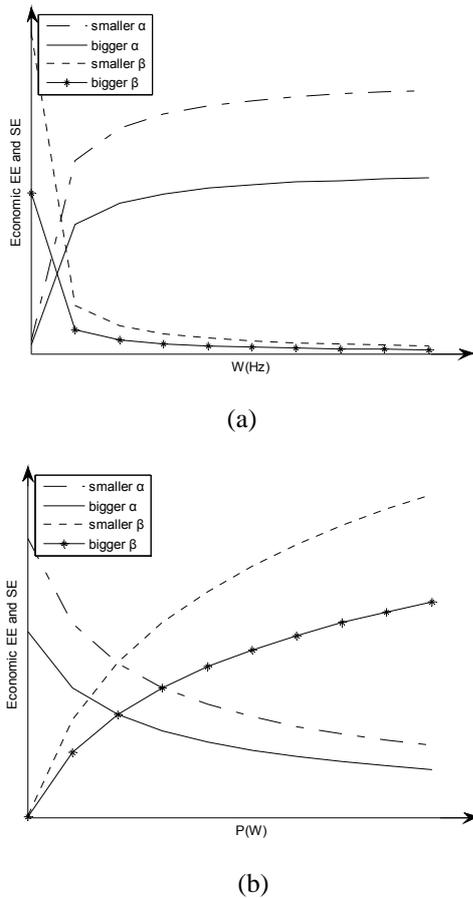
$$\eta_{EE}^{EP} = \left( \frac{\beta \cdot W}{\alpha \cdot P} \right) \eta_{SE}^{SP} = \frac{\beta \cdot \eta_{SE}^{SP} \cdot G}{\alpha \cdot (2^{\beta \cdot \eta_{SE}^{SP}} - 1) N_0} \quad (5)$$

Now consider the economic EE-SE relation (5), we have

$$\eta_{EE}^{EP} \rightarrow \begin{cases} \beta G / (\alpha N_0 \ln 2) & \eta_{SE}^{SP} \rightarrow 0 \\ 0 & \eta_{SE}^{SP} \rightarrow +\infty \end{cases}$$

The above expression is similar to the conventional EE-SE relation in [1], but the price factors appear when  $\eta_{SE}^{SP} \rightarrow 0$ . The limit and behavior of economic EE performance for a cognitive heterogeneous system can be predicted. For example, a minimum economic EE is guaranteed for the case of low economic SE, which can be adjusted through the electricity price  $\alpha$  and the spectrum price  $\beta$ . When considering the circuit power as the important part of power consumption in the SCBS, the monotonic relation of the

economic EE-SE tradeoff may be broken, which will be considered for future.



**Fig.1** (a) The economic EE and SE v.s. the bandwidth consumption; (b) The economic SE and EE v.s. the energy consumption

### III. EE-SE TRADEOFF AND A NEW TRADEOFF METRIC

In order to maximize EE and SE simultaneously, the multi-objective optimization is always introduced, which is as follows:

$$\max_P \{ \eta_{EE}, \eta_{SE} \} \quad (6)$$

The scalarization method is often applied to solve it by combining its multiple objectives into a single-objective scalar function. It is important to make all object functions comparable, but the unit for EE is bits/Joule while that for SE is bits/s/Hz.

Considering the proposed economic EE and SE with the price factor of economics, we introduce a new EE-SE tradeoff metric called economic resource efficiency (ERE), which is defined as

$$\xi_{ERE} = \gamma \eta_{EE}^{EP} + (1-\gamma) \eta_{SE}^{SP} \quad (7)$$

where  $\gamma \in [0, 1]$ , is a weight factor to control the balance of economic EE and SE. Now we prove the following property,

which can reveal the relation between this new economic EE-SE tradeoff and the original EE-SE tradeoff.

**Property I:** Economic resource efficiency is capable of exploiting the tradeoff between EE and SE, with the weight factor  $\gamma$ , the electricity price  $\alpha$  and the spectrum price  $\beta$ .

*Proof:* Firstly, substitute (3) and (4) into (7), we have

$$\begin{aligned} \xi_{ERE} &= \gamma \frac{R}{\alpha \cdot P} + (1-\gamma) \frac{R}{\beta \cdot W} \\ &= \eta_{EE}^{EP} \left[ \gamma + (1-\gamma) \frac{\alpha \cdot P}{\beta \cdot W} \right] \end{aligned} \quad (8)$$

$(\alpha \cdot P)$  and  $(\beta \cdot W)$  are comparable according to the definitions of price factors, so the economic EE and SE can be integrated in the metric. Meanwhile  $\gamma$  acts as the weight factor to control the balance of economic EE and economic SE. Then, (7) can be transformed to EE-SE tradeoff based on (3) and (4) as follows

$$\xi_{ERE} = \gamma \frac{\eta_{EE}}{\alpha} + (1-\gamma) \frac{\eta_{SE}}{\beta} \quad (9)$$

The price factors  $\alpha$  and  $\beta$  act as cost normalizer for EE and SE. Different from the conventional metric design, where EE or SE has limitation in efficient use of energy and spectrum resources, there the new economic resource efficiency can optimize both EE and SE through the adjustment of price factor  $\alpha$  and  $\beta$ .  $\square$

Furthermore, ERE optimize EE when  $\gamma = 1$  but optimize SE when  $\gamma = 0$ . On the other hand, from (9), when the idle bandwidth quantity increase, which means that the decreasing bandwidth cost lead to increasing spectrum consumption, ERE will emphasis more on SE to increase ERE. For this case, if ERE emphasize more on EE, the SCBS will occupy more bandwidth to maximize the EE as demonstrated in Fig.1 (a), which lower the bandwidth resource utilization. When the available energy quantity increases, ERE will emphasize more on EE with similar analysis.

Thus, SCBS can choose the spectrum quantity and the energy quantity to maximize its tradeoff between EE and SE based on the spectrum price and the electricity price. The multi-objective optimization problem (6) can be transformed to the following problem

$$\max_{P,W} \gamma \frac{\eta_{EE}}{\alpha} + (1-\gamma) \frac{\eta_{SE}}{\beta} \quad (10)$$

### IV. THE RESOURCE ALLOCATION SOLUTION OF OFDM SYSTEM

Consider an OFDM based point and point communication system, the small cell share the spectrum licensed by the macrocell. Assume that the small cell has perfect knowledge of channel state information between the small cell's transmitter and the user's receiver. We mainly focus on the resource allocation solution, so the interference between the

small cell and the macrocell is not considered. The transmission rate of the user can be multiple times higher than that of one subcarrier. For simplify, the total bandwidth consumption can be denoted as  $W_{tot}$ , which is equally divided into a certain amount of subcarriers, each with a bandwidth of  $W_o$ . Considering the idle spectrum quantity  $W_{ava}$ , denote the number of available subcarriers as  $N = W_{ava} / W_o$ , which generally is an integer for practice. In this case, denote the achievable upper transmission rate as  $r$ , that is

$$r = \sum_{n=1}^N \rho_n W_o \log_2 \left( 1 + \frac{G_n P_n}{W_o N_0} \right) \quad (11)$$

where  $P_n$  is the power allocated to the  $n$ th subcarrier.  $G_n$  is the channel power gain on subcarrier  $n$ .  $\rho_n$  can be either 1 or 0 informing whether the subcarrier  $n$  is occupied by the user.

Based on the definition (7) of economic resource efficiency, the economic resource efficiency of the OFDM system can be expressed as

$$\begin{aligned} \xi_{ERE} &= \frac{\sum_{n=1}^N \rho_n W_o \log_2 \left( 1 + \frac{G_n P_n}{W_o N_0} \right)}{\alpha \cdot \sum_{n=1}^N \rho_n P_n} \left[ \gamma + (1-\gamma) \frac{\alpha \cdot \sum_{n=1}^N \rho_n P_n}{\beta \cdot \sum_{n=1}^N \rho_n W_o} \right] \\ &= \frac{\sum_{n=1}^N \rho_n \log_2 \left( 1 + \frac{G_n P_n}{W_o N_0} \right)}{\alpha \beta \cdot \sum_{n=1}^N \rho_n P_n \cdot \sum_{n=1}^N \rho_n} \left[ \gamma \beta \cdot \sum_{n=1}^N \rho_n W_o + (1-\gamma) \alpha \cdot \sum_{n=1}^N \rho_n P_n \right] \end{aligned} \quad (12)$$

where  $P_{tot} = \sum_{n=1}^N \rho_n P_n$  is the total energy consumption for the small cell. To obtain the EE and SE adaptive tradeoff optimization model by maximizing the economic resource efficiency, we have the concerned problem in OFDM system as follows

$$\begin{aligned} \max_{P_n, \rho_n} & \frac{\sum_{n=1}^N \rho_n \log_2 \left( 1 + \frac{G_n P_n}{W_o N_0} \right)}{\alpha \beta \cdot \sum_{n=1}^N \rho_n P_n \cdot \sum_{n=1}^N \rho_n} \left[ \gamma \beta \cdot \sum_{n=1}^N \rho_n W_o + (1-\gamma) \alpha \cdot \sum_{n=1}^N \rho_n P_n \right] \\ \text{s.t. C1} & \quad r \geq R_{\min} \\ \text{C2} & \quad P_{tot} = \sum_{n=1}^N \rho_n P_n \leq P_{ava} \\ \text{C3} & \quad P_n \geq 0, n = 1, 2, \dots, N \\ \text{C4} & \quad W_{tot} = \sum_{n=1}^N \rho_n W_o \leq W_{ava} \\ \text{C5} & \quad \rho_n \in \{0, 1\}, n = 1, 2, \dots, N \end{aligned} \quad (13)$$

where C1 guarantees the minimal rate requirement of the user, C2 is the constraints on the transmission power consumption. C4 contains the constraints on the bandwidth consumption.

For the problem formulation (13), it is a Mixed Integer Programming (MIP) problem, where the consumed quantity of subcarrier and energy are both uncertain, which are difficult to solve. The main difficulty of MIP lies in the integer constraint  $\rho_n$  for subcarrier consumption quantity. Generally, if  $\rho_n$  can be also allowed as fraction, it can be regarded as partly use of the subcarrier in time domain [14, 15]. To obtain an optimal solution, an exhaustive search is needed for all the feasible combination of subcarrier and energy, which is different from [14, 15]. Firstly, the power is allocated under a certain given consumed subcarrier quantity; secondly, compare the ERE for each of the possible quantity of subcarrier consumption.

Given any subcarrier assignment set  $\psi$  with the given number of subcarriers  $M = W_{tot} / W_o$ , the concerned problem (13) is transformed to

$$\begin{aligned} \max_{P_n} & \frac{\sum_{n \in \psi} \log_2 \left( 1 + \frac{G_n P_n}{W_o N_0} \right)}{\alpha \beta M \cdot \sum_{n \in \psi} P_n} \left[ \gamma \beta \cdot M W_o + (1-\gamma) \alpha \cdot \sum_{n \in \psi} P_n \right] \\ \text{s.t. C1} & \quad r \geq R_{\min} \\ \text{C2} & \quad P_{tot} = \sum_{n \in \psi} P_n \leq P_{ava} \\ \text{C3} & \quad P_n \geq 0, n \in \psi \\ \text{C4} & \quad W_{tot} = M W_o \leq W_{ava} \end{aligned} \quad (14)$$

After the binary variables are given, the objective function (14) is also a nonlinear fractional programming about  $P_n$ . We describe an ingenious optimal power allocation strategy based on the Dinkelbach algorithm [16]. For a given subcarrier consumption quantity, denote  $\mathbf{p} = (P_1, P_2, \dots, P_M)$  for the set  $\psi$  of  $P_n$ . For later convenience of analysis, denote

$$f(\mathbf{p}) = \left[ \gamma \beta \cdot M W_o + (1-\gamma) \alpha \cdot \sum_{n \in \psi} P_n \right] \sum_{n \in \psi} \log_2 \left( 1 + \frac{G_n P_n}{W_o N_0} \right)$$

$$g(\mathbf{p}) = \alpha \beta M \cdot \sum_{n \in \psi} P_n$$

Denote  $F(\mathbf{p}, \mu) = f(\mathbf{p}) - \mu \cdot g(\mathbf{p})$ , where  $\mu$  is a positive parameter factor. Based on the optimization problem (14), a new optimization problem is formulated as follows

$$\max_{\mathbf{p}} F(\mathbf{p}, \mu)$$

$$\text{s.t.} \quad \text{C1} \sim \text{C4} \quad \text{in (14)}. \quad (15)$$

According to the theorem given in [16], it is easy to prove that, by realizing  $F(\mu) = 0$ , the optimization problem (15) is equivalent to (14). Denote  $S$  is the definitional domain of  $\mathbf{p}$  in (14) and (15). The typical Dinkelbach\_type algorithm is described as follows

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**Algorithm 1: Dinkelbach\_type Algorithm**


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- Step 1.** Take  $\mathbf{p}_0 \in S$ , compute  $\mu_1 = \max f(\mathbf{p}_0)/g(\mathbf{p}_0)$  and let  $l:=1$ ;
- Step 2.** Determine  $\mathbf{p}_l := \arg \min_{\mathbf{p} \in S} \{ \max \{ f(\mathbf{p}) - \mu_l g(\mathbf{p}) \} \}$ ;
- Step 3.** If  $F(\mu_l) = 0$   
 Then  $\mathbf{p}_l$  is an optimal solution of (14) with value  $\mu_l$  and Stop;  
 Else GoTo Step 4;
- Step 4.** Let  $\mu_{l+1} := \max f(\mathbf{p}_l)/g(\mathbf{p}_l)$ ; Let  $l := l+1$ , and GoTo Step 2.
- 

The key of the above algorithm is to obtain the power allocation  $\mathbf{p}_l$  in step 2, so the problem is converted into how to find the optimal solution of subproblem (15) under the given  $\mu$ . The barrier method [17] can be employed to solve it, where the objective problem is converted into a sequence of unconstrained minimization problems. The barrier function of (15) is

$$\begin{aligned} \varphi(\mathbf{p}) = & -\log_2(r - R_{\min}) - \log_2 \left( P_{ava} - \sum_{n \in \Psi} P_n \right) \\ & - \sum_{n \in \Psi} \log_2 P_n - \log_2(W_{ava} - MW_o) \end{aligned} \quad (16)$$

The optimal solution of (15) can be approximated by solving the following unconstrained minimization problem [17]

$$\min \phi_v(\mathbf{p}) = -vF(\mathbf{p}, \mu) + \varphi(\mathbf{p}) \quad (17)$$

where  $v > 0$ , which decides the accuracy of the approximation. Particularly, each unconstrained minimization problem (17) with the given  $v$  can be solved by Newton method. The approximation becomes more and more accurate as  $v$  increases. Denote the Hessian matrix and the gradient of  $\chi_v(\mathbf{p})$  respectively as follows

$$\begin{aligned} H &= \nabla^2 \chi_v(\mathbf{p}) \\ \mathbf{g} &= \nabla \chi_v(\mathbf{p}) \end{aligned}$$

Further to reduce the computational difficulty of the Hessian matrix, the BFGS algorithm based on the Armijo search can be applied to solve the unconstrained minimization problem. BFGS correction for Hessian matrix approximation has faster convergence and superlinear convergence rate. The update rule of approximate matrix is

relatively simple, which usually adopts a matrix with rank 1 or 2. The efficient joint of the barrier method and the BFGS algorithm is denoted as B\_BFGS algorithm. The B\_BFGS algorithm is described in Algorithm 2.

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**Algorithm 2: The B\_BFGS algorithm**


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**0. The barrier method part:**
**1. Initialization**

2. Find feasible point  $\mathbf{p}_0$ ,  $v := v^{(0)} > 0$ , tolerance  $\epsilon > 0$ ,  $\rho > 1$

**3. Outer loop**

4. Centering step: compute  $\mathbf{p}^*(v)$  derived by problem (17)

**5. The BFGS algorithm part:**
**6. Initialization**

7. Starting point  $\mathbf{p}_0$ ,  $\delta \in (0,1)$ ,  $\sigma \in (0,0.5)$ ,  $A_0 = H(\mathbf{p}_0)$ ,

8. termination error value  $0 \leq \epsilon \leq 1$ ,  $j := 0$

**9. Inner loop**

10. Compute  $\mathbf{g}_j$ , quit if  $\|\mathbf{g}_j\| \leq \epsilon$ , output  $\mathbf{p}_j$ ;

11. else compute  $A_j \mathbf{d} = -\mathbf{g}_j$ , get  $\mathbf{d}_j$ ;

12. Denote minimum nonnegative integer  $m_j$  meet

$$13. \chi_v(\mathbf{p}_j + \delta^m \mathbf{d}_j) \leq \chi_v(\mathbf{p}_j) + \sigma \delta^m \mathbf{g}_j^T \mathbf{d}_j,$$

14. Denote  $\mathbf{a}_j = \delta^{m_j}$ ,  $\mathbf{p}_{j+1} = \mathbf{p}_j + \mathbf{a}_j \mathbf{d}_j$ ;

15. Compute  $A_{j+1}$  by (18),  $j := j+1$ ;

16. Update:  $\mathbf{p}^*(v) = \mathbf{p}_j$ .

17. Stopping criterion:  $(M+3)/v < \epsilon$ .

18. Increase:  $v := \rho v$

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During the iteration, assume offset  $s_j = \mathbf{p}_{j+1} - \mathbf{p}_j$ , gradient difference  $\mathbf{y}_j = \mathbf{g}_{j+1} - \mathbf{g}_j$ , Hessian matrix can be approximated by the symmetric positive definite matrix as

$$A_{j+1} = \begin{cases} A_j, & \mathbf{y}_j^T s_j \leq 0, \\ A_j - \frac{A_j s_j s_j^T A_j}{s_j^T A_j s_j} + \frac{\mathbf{y}_j \mathbf{y}_j^T}{\mathbf{y}_j^T s_j}, & \mathbf{y}_j^T s_j > 0 \end{cases} \quad (18)$$

The optimal solution to (13) can be obtained by applying the Dinkelbach\_type algorithm and barrier method to every feasible subcarrier quantity and then choose the one with the maximum ERE. In addition, the complexity depends on the number of optimizing variables.

## V. NUMERICAL RESULTS

In this section, we firstly demonstrate the effectiveness of ERE in a simple scenario. Consider a static channel gain  $G = 1$ , the transmit power constraint is assumed to be  $2W$ . The AWGN is with zero mean and unit variance. These simulation parameters are chosen to demonstrate the effectiveness of ERE for simplicity, which can be modified easily to other values for different scenarios.

Fig. 2 shows the impact of  $\gamma$  to the ERE, the economic EE and SE with  $W=1$ Hz. Under the optimal transmit power and

bandwidth, the economic EE increases while the economic SE decreases with increasing  $\gamma$ . This is because increasing  $\gamma$  leads more weight putting on EE and keeping ERE over a certain level, until  $\gamma=1$ , ERE reaches maximization again. The ERE is always above 1.28, which is about 85% of the upper value.

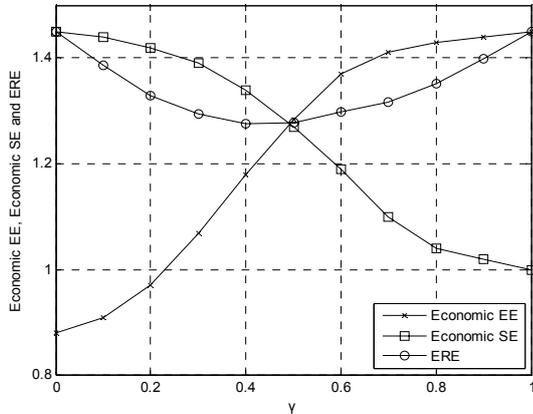


Fig. 2 The ERE, economic EE and SE under the impact of  $\gamma$

The interaction of the three lines with  $\gamma=0.5$  under the impact of spectrum quantity is shown in Fig. 3. The economic EE increases while the economic SE decreases with the increasing bandwidth consumption, but the ERE changes less. That's because with increasing idle bandwidth resource, the decreasing spectrum price makes more bandwidth be utilized and slows down the decreasing SE, meanwhile lead to the decreasing energy consumption and the increasing economic EE. The influence analysis of energy quantity to the three lines is similar to the spectrum quantity.

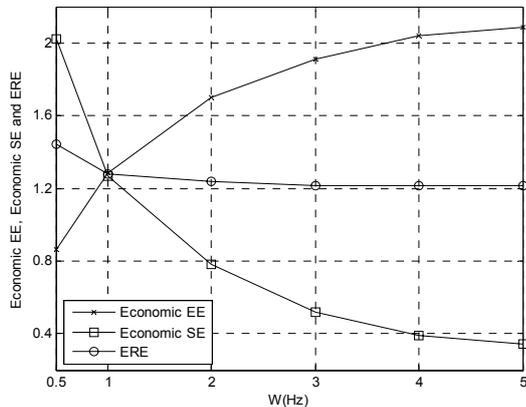


Fig. 3 The ERE, economic EE and SE under the impact of spectrum

Then for the OFDM system scenario, assume that the BS's maximum transmission power is 1.25W with one user. There is a certain available bandwidth, which is divided into OFDM subcarriers. The channel gain is modeled as independent, identically distributed Rayleigh random variables with an average of 0 dB. Suppose the noise power is  $10^{-13}W$ ,  $R_{min} = 6\text{Mbps/s}$ ,  $\gamma=0.5$ .

Fig.4 depicts the energy consumption versus the idle spectrum quantity with the OFDM system. Under a certain available energy quantity, the curve of the energy

consumption decreases with the increasing idle spectrum quantity. More the idle spectrum quantity means that more the spectrum can be used to increase the transmission capacity, which is adjusted by the lower spectrum price. So the effect of the price adjustment can be realized, and less energy is used which saves the energy. By comparing the two curves with different energy quantity, we can see that more energy is used for the higher energy quantity under the same spectrum quantity. Higher energy quantity means lower energy price, which will balance the energy consumption and spectrum consumption.

Similarly, Fig.5 depicts the spectrum consumption versus the available energy quantity with the OFDM system. Under a certain idle spectrum quantity, the curve of the spectrum consumption decreases with the increasing available energy quantity. By comparing the two curves with different idle spectrum quantity, we can see that more spectrums are used for the higher spectrum quantity under the same energy quantity. Higher spectrum quantity means lower spectrum price, which will balance the energy consumption and spectrum consumption.

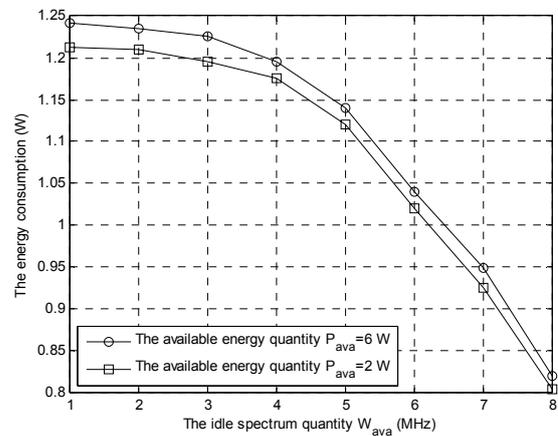


Fig. 4 The energy consumption versus the idle spectrum quantity

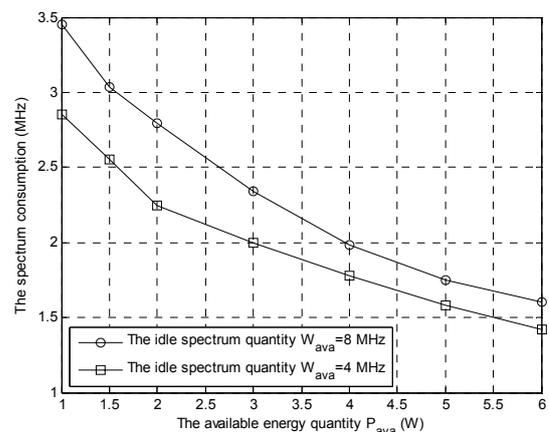
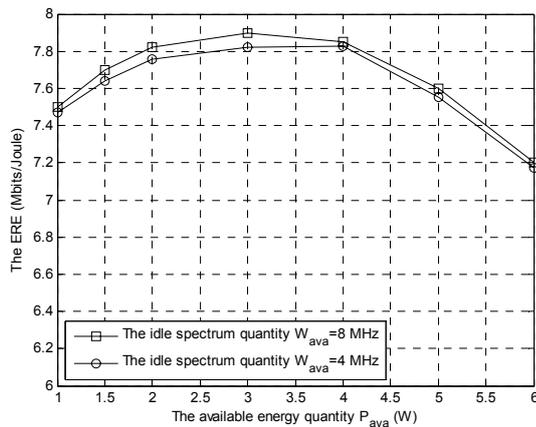


Fig. 5 The spectrum consumption versus the achievable energy quantity

We also verify the achievable maximum ERE versus the available energy quantity for the different idle spectrum quantity. In Fig. 6, when the available energy quantity  $P_{ava}$  is less at the beginning, the ERE curve increases quickly with

the increasing  $P_{ava}$ . But when  $P_{ava}$  is comparably high, there has been sufficient energy with lower energy price, more energy will be consumed, so that the ERE curve decreases gradually. The turning point of ERE curve is about  $P_{ava}=3W$  for  $W_{ava}=8\text{Mbits/s}$ , and  $P_{ava}=4W$  for  $W_{ava}=4\text{Mbits/s}$ . But the ERE of  $W_{ava}=8\text{Mbits/s}$  is higher than that of  $W_{ava}=4\text{Mbits/s}$ , for the  $W_{ava}=8\text{Mbits/s}$  solution can reach the system optimization easily with the adjustment of available energy and idle spectrum among two resources. That is because the higher available spectrum quantity needs less the energy consumption, and the effect of the price adjustment can be realized easily, which results in the higher achievable maximum ERE. It is very important for green cellular networks, as we can save much energy by supplying the idle spectrum properly.



**Fig. 6** The achievable maximum ERE versus available energy quantity

## VI. CONCLUSION

This paper introduces two new definitions of economic EE and economic SE, which are based on the resource cost related to the available energy quantity and the idle spectrum quantity. Different from the classical methods, the ERE is built to utilize the energy and bandwidth efficiently under the smart grid and cognitive environment. Then in the OFDM system, the ERE optimization problem is formed and solved effectively. Simulation results show that, through adjusting consumed energy and occupied bandwidth, the ERE can realize the tradeoff between EE and SE based on economic cost according to respective resource prices.

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