Optimal Strategy for the Three-echelon Inventory System with Defective Product, Rework and Fuzzy Demand under Credit Period

M.-F. Yang, M.-C. Lo, and W.-H. Chen, *Member, IAENG*

Abstract—**Multi-echelon transaction is a common situation in the supply chain nowadays. This research is to formulate a three-echelon integrated inventory model under defective products, reworking and credit period consideration. In order to deal with the uncertainty of demand in real life, we consider fuzzy demand in the integrated model by using fuzzy theory. An algorithm and numerical analysis are used to observe the effect of fuzzy demand to the inventory policy and total profit.**

Index Terms — **Inventory model, Defective products, Reworking, Credit period, Fuzzy theory**

I. Introduction

Recently, in a competitive market, how to satisfy customers' demand is one of critical issues for companies. In addition to constant and high quality, enough stock is an important fundamental factor to affect the level of customer satisfaction. Enterprisers should frame appropriate inventory policies to perform inventory management well. Inventory policy describes how to stock inventory and when to replenish. It determines: (1) How much product is stored at a site, (2) when replenishment orders are generated, and (3) what quantity is replenished [1]. Started from Harris's [2] economic order quantity (EOQ) model, the researchers as well as practitioners are interested in optimal inventory policy. Harris [2] focused on inventory decisions of an individual firm, yet from supply chain management's (SCM) point of view, collaborating closely with the members of supply chain is certainly necessary. In the network (supply chain), each node's (the member in the supply chain) position is corresponding to its relative position in reality. These nodes serve external demand which generates orders to the down-stream echelon. Meanwhile, they are served by external supply which responds to the orders of the up-stream echelon [3]. Ben-Daya et al. [4] pointed out that the reason to collaborate with the other members of supply chain is to remain competitive. Better collaboration with customers and suppliers will not only provide a better service to satisfy customer's demand but reduce the total cost of the whole supply.

In 1950s, Arrow et al. [5] have been focused on multi-echelon inventory problem. Burns and Sivazlian [6] investigated the dynamic response of a multi-echelon supply chain to various demands placed upon the system by a final consumer. Van der Heijden [7] determined a simple inventory control rule for multi-echelon distribution systems under periodic review without lot sizing. Pal et al. [8] developed a three-layer integrated production-inventory model considering out-of-control quality occurs in supplier and manufacturer stage. The defective products are reworked at a cost after the regular production time. Chung et al. [9] combined deteriorating items with two levels of trade credit under three-layer condition in supply chain system. A new economic production quantity (EPQ) inventory is proposed to minimize the total cost.

Yield rate is an important factor in manufacturing industry. In practice, imperfect production could be the result of insufficient process control, wrongly planned maintenance, inadequate work instructions, or damages that occur during handling process [10]. The manufacturer may face production interruptions, such as: machine breakdown, raw material shortage or any other type of system failure. In an imperfect production system, it is expected that a certain percentage of products will be defective [11]. High defective rate will not only waste production costs but also pay more inspecting costs and repair costs, even cause the shortage. In early researches, defective product was rarely considered in economic ordering quantity (EOQ) model; however, defective production is a common condition in real life. Schwaller [12] added fixed defective rate and inspecting costs to the traditional EOQ model. Salameh and Jaber [13] pointed that all products should be divided into good products and defective products. They also found EOQ will increase if defective products increase. Lin [14] assumed a random number of defective goods in buyer's arriving order lot with partial lost sales for the mixtures of distributions of the controllable lead time demand to accommodate more practical features of the real inventory systems. Pal et al. [15] introduced a two echelon imperfect production inventory model over two cycles. The imperfect rate followed a probability distribution. The retailer sold the good products in the first cycle while the defective items are remanufactured after the regular production and sold with discount price in the second cycle. Their objective is to find out the optimal inventory lot-size and selling price to maximize the total profit.

Credit period is a common business strategy between vendors and buyers. It will bring additional interest or opportunity cost to each other, hence delayed period is a critical issue that researchers should consider when

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Ming-Feng Yang is with Department of Transportation Science, National Taiwan Ocean University, No. 2, Beining Rd., Jhongjheng District, Keelung City 202, Taiwan, R.O.C. (phone: +886-2-24622192#7011; fax: +886-2-24633745; e-mail: yang60429@mail.ntou.edu.tw)

Ming-Cheng Lo is with Department of Business Administration, Chien Hsin University of Science and Technology, No. 229, Jianxing Rd., Zhongli City, Taoyuan County 32097, Taiwan, R.O.C. (e-mail: lmc@uch.edu.tw)

Wei-Hao Chen is with Department of Shipping & Transportation Management, National Taiwan Ocean University, No. 2, Beining Rd., Jhongjheng District, Keelung City 202, Taiwan, R.O.C. (e-mail: chaos800317@gmail.com.)

developing inventory models. In traditional EOQ assumptions, the buyer has to pay immediately when the vendor delivers products to the buyer; however, in real business transactions, the vendor usually gives a fixed delayed period to reduce the stress of capital. During the period, the buyer can keep selling products without paying the vendor; they can also earn extra interest from sales. Goyal [16] developed an EOQ model with delay in payments. Two situations were discussed in the research; time interval between successive orders was longer than or equal to permissible delay in settling accounts, or time interval between successive orders was shorter than permissible delay in settling accounts. Sarkar et al. [17] derived an EOQ model for various types of time-dependent demand when delay in payment and price discount are permitted by suppliers to retailers. Yang and Tseng [18] proposed a three-echelon inventory model with permissible delay in payments under controllable lead time and backorder consideration to find out the suitable inventory policy to enhance profit of the supply chain. Furthermore, the purpose of this paper is to maximize the joint expect total profit on inventory model and attempt to discuss the inventory policy under different conditions. Yang et al. [19] made a series of analysis to observe which coordination policies would bring the most effective performance to the supply chain. And then three inventory models are developed with corresponding policies: credit period policy, centralized supply chain, and quantity discount policy.

In industry, the costs of holding, ordering and backordering are always likely to vary from one cycle to another. Demand may also vary from time to time. Absence of historical data makes it difficult to estimate the probability distribution of these variables. Thus, fuzzy set theory, rather than the traditional probability theory, is better suited for analysis of inventory. The investigators had carried out a study for fixed demand, treating holding cost, ordering cost and backordering cost (if any) to be fuzzy in nature (Samal and Pratihar [20]). Many researchers have being applied fuzzy theory and techniques to develop and solve inventory problem. Park [21][20] considered fuzzy inventory costs by using arithmetic operations of the Extension Principle. Chen and Wang [22][21] fuzzified the demand, ordering cost, inventory cost, and backorder cost into trapezoidal fuzzy numbers in an EOQ model with backorder consideration. Kazemi et al. [23] extended an existing EOQ inventory model with backorders in which both demand and lead times are fuzzified. The assumption of constant fuzziness is relaxed by incorporating the concept of learning in fuzziness into the model considering that the degree of fuzziness reduces over the planning horizon. Das et al. [24] developed a inventory model which consists a single-supplier single-manufacturer multi-markets. A manufacturer receives the deteriorating raw materials from a supplier who offers a credit period (which is fuzzy in nature) to settle his/her account. Their purpose is to maximize the total profit in the business.

Building upon the work of Yang et al. [25], this paper proposes the model incorporates the fuzziness of annual demand. For the model, Yao and Wu's ranking method [26] for fuzzy number is employed to find the estimation of the joint total expected profit in the fuzzy sense, and the corresponding order quantity of the purchaser is derived accordingly. We expended the integrated inventory model by using fuzzy theory in Section II and showed numerical examples in Section III. In the end, we summarized the conclusions in Section IV.

II. Model Formulation

To develop a three-echelon inventory model with defective rate and permissible delay in payments, we divided the expected joint total annual profit of the model into three parts which are the annual profit of the supplier, the manufacturer, and the retailer, The following notations and assumptions below are used to develop the model:

A. Notations

To establish the mathematical model, the following notations and assumptions are used. The notations are shown as below.

Decision variable

Q = Economic delivery quantity
n = The number of lots delivered in a production cycle from the manufacturer to the retailer, a positive integer

Supplier side

P_s = Supplier's purchasing cost per unit
A_s = Supplier's ordering cost per order
h_s = Supplier's annual holding cost per unit
I_{sp} = Supplier's opportunity cost per dollar per year

Manufacturer side

P = Manufacturer's production rate
X = Manufacturer's permissible delay period
P_m = Manufacturer's purchasing cost per unit
A_m = Manufacturer's ordering cost per order
Z = Defective rate in production process
W = Manufacturer's inspecting cost per unit
G = Manufacturer's repair cost per unit
t_m = Defective products' reworking time
F_m = Manufacturer's transportation cost per shipment
h_m = Manufacturer's annual holding cost per unit
I_{mp} = Manufacturer's opportunity cost per dollar per year
I_{me} = Manufacturer's interest earned per dollar per year

Retailer side

D = Average annual demand per unit time
Y = Retailer's permissible delay period
P_c = Retailer's selling price per unit
P_r = Retailer's purchasing cost per unit
Ar = Retailer's ordering cost per order
F_r = Retailer's transportation cost per shipment
h_r = Retailer's annual holding cost per unit
I_{rp} = Retailer's opportunity cost per dollar per year
I_{re} = Retailer's interest earned per dollar per year
TP_s = Supplier's total annual profit
TP_m = Manufacturer's total annual profit
TP_r = Retailer's total annual profit
$EJTP_i$ = The expected joint total annual profit, i = 1, 2, 3, 4*

*"i" represents four different cases due to the relationship of replenishment time and permissible payment period of manufacturer and the relationship of replenishment time and permissible payment period of retailer.

B. Assumptions

(i) This supply chain system consists of a single supplier, a single manufacturer, and a single retailer for a single product.
(ii) Economic delivery quantity multiplies by the number of delivery per production run is economic order quantity (EOQ).
(iii) Shortages are not allowed.

(iv) The sale price must not be less than the purchasing cost at any echelon, $P_c > P_r > P_m > P_s$.
(v) Defective products only occur in the production process.
(vi) The inspecting time is ignored and defective products can be inspected immediately.
(vii) Defective products are repaired after the production process is end.
(viii) The time horizon is infinite.

C. *Basic model*

In this study, a model proposed by Yang et al. [25] will be considered, which posits a inventory system with a single supplier, a single manufacturer, and a single retailer for a single product. Our purpose is to find the maximum profit in the inventory system.

The supplier's profit includes sales revenue, purchasing cost, ordering cost, holding cost, and opportunity cost. The manufacturer's profit includes sales revenue, purchasing cost, ordering cost, holding cost, transportation cost, inspecting cost, repair cost, interest income, and opportunity cost. The retailer's profit includes sales revenue, purchasing cost, ordering cost, holding cost, transportation cost, interest income, and opportunity cost.

The length of payment period will affect the amount of interest income and opportunity cost. As the items are sold out before the deadline of the payment period, the manufacturer earns interest by sales revenue (see Fig. I). Contrarily, as the items are sold out after the deadline of the payment period, the manufacturer still earns interest by sales revenue during the replenishment time, yet the items in stock result in opportunity cost (see Fig. II).

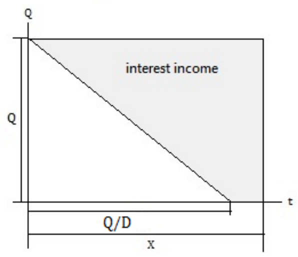

Fig. I Q/D < X

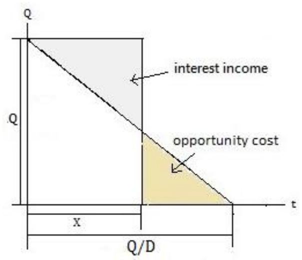

Fig. II Q/D ≥ X

According to the four different conditions, the expected joint total annual profit function, $EJTP_i$ (Q, n), can be expressed as

$$J_i = \begin{cases} EJTP_1 = TP_s + TP_{m1} + TP_{r1} & \text{if } Q/D < X, Q/D < Y \\ EJTP_2 = TP_s + TP_{m2} + TP_{r1} & \text{if } Q/D \geq X, Q/D < Y \\ EJTP_3 = TP_s + TP_{m1} + TP_{r2} & \text{if } Q/D < X, Q/D \geq Y \\ EJTP_4 = TP_s + TP_{m2} + TP_{r2} & \text{if } Q/D \geq X, Q/D \geq Y \end{cases}$$

where

$$J_1(n,Q) = D(P_c - P_s - h_m t_m Z - W - GZ) - \frac{D}{nQ}(A_s + A_m + F_m + A_r + F_r n) - \frac{Q}{2}\left[\frac{h_s Dn + h_m(2-n)}{P} + h_m(n - 1 - 2t_m Z^2 nD) + h_r\right] - P_m I_{sp} DX - P_r I_{mp} DY + P_r I_{me}\left(DX - \frac{Q}{2}\right) + P_c I_{re}\left(DY - \frac{Q}{2}\right) \quad (1)$$

$$J_2(n,Q) = D(P_c - P_s - h_m t_m Z - W - GZ) - \frac{D}{nQ}(A_s + A_m + F_m + A_r + F_r n) - \frac{Q}{2}\left[\frac{h_s Dn + h_m(2-n)}{P} + h_m(n - 1 - 2t_m Z^2 nD) + h_r\right] - P_m I_{sp} DX - P_r I_{mp} DY + \frac{P_r I_{me}(DX)^2}{2Q} - \frac{P_m I_{mp}(Q-DX)^2}{2Q} + P_c I_{re}\left(DY - \frac{Q}{2}\right) \quad (2)$$

$$J_3(n,Q) = D(P_c - P_s - h_m t_m Z - W - GZ) - \frac{D}{nQ}(A_s + A_m + F_m + A_r + F_r n) - \frac{Q}{2}\left[\frac{h_s Dn + h_m(2-n)}{P} + h_m(n - 1 - 2t_m Z^2 nD) + h_r\right] - P_m I_{sp} DX - P_r I_{mp} DY + P_r I_{me}\left(DX - \frac{Q}{2}\right) + \frac{P_c I_{re}(DY)^2}{2Q} - \frac{P_r I_{rp}(Q-DY)^2}{2Q} \quad (3)$$

$$J_4(n,Q) = D(P - P_s - h_m t_m Z - W - GZ) - \frac{D}{nQ}(A_s + A_m + F_m + A_r + F_r n) - \frac{Q}{2}\left[\frac{h_s Dn + h_m(2-n)}{P} + h_m(n - 1 - 2t_m Z^2 nD) + h_r\right] - P_m I_{sp} DX - P_r I_{mp} DY + \frac{P_r I_{me}(DX)^2}{2Q} - \frac{P_m I_{mp}(Q-DX)^2}{2Q} + \frac{P_c I_{re}(DY)^2}{2Q} - \frac{P_r I_{rp}(Q-DY)^2 2Q}{2Q} \quad (4)$$

D. *Fuzzy integrated inventory model*

Consider the model fuzzy D to triangular fuzzy number $\tilde{D}$, where $\tilde{D} = (D - \Delta_1, D, D + \Delta_2)$, $0 < \Delta_1 < D$, $0 < \Delta_2$ and Δ_1, Δ_2 are both determined by decision-makers. Modify the model of Yang et al. (2015), the expected joint total profit can be expressed as

$$\tilde{J}_1(n,Q) = \tilde{D}(P_c - P_s - h_m t_m Z - W - GZ) - \frac{\tilde{D}}{nQ}(A_s + A_m + F_m + A_r + F_r n) - \frac{Q}{2}\left[\frac{h_s \tilde{D} n + h_m(2-n)}{P} + h_m(n - 1 - 2t_m Z^2 n\tilde{D}) + h_r\right] - P_m I_{sp} \tilde{D}X - P_r I_{mp} \tilde{D}Y + P_r I_{me}\left(\tilde{D}X - \frac{Q}{2}\right) + P_c I_{re}\left(\tilde{D}Y - \frac{Q}{2}\right) \quad (5)$$

$$\tilde{J}_2(n,Q) = \tilde{D}(P_c - P_s - h_m t_m Z - W - GZ) - \frac{\tilde{D}}{nQ}(A_s + A_m + F_m + A_r + F_r n) - \frac{Q}{2}\left[\frac{h_s \tilde{D} n + h_m(2-n)}{P} + h_m(n - 1 - 2t_m Z^2 n\tilde{D}) + h_r\right] - P_m I_{sp} \tilde{D}X - P_r I_{mp} \tilde{D}Y + \frac{P_r I_{me}(\tilde{D}X)^2}{2Q} - \frac{P_m I_{mp}(Q-\tilde{D}X)^2}{2Q} + P_c I_{re}\left(\tilde{D}Y - \frac{Q}{2}\right) \quad (6)$$

$$\tilde{J}_3(n,Q) = \tilde{D}(P_c - P_s - h_m t_m Z - W - GZ) - \frac{D\tilde{D}}{nQ}(A_s + A_m + F_m + A_r + F_r n) - \frac{Q}{2}\left[\frac{h_s \tilde{D} n + h_m(2-n)}{P} + h_m(n - 1 - 2t_m Z^2 n\tilde{D}) + h_r\right] - P_m I_{sp} \tilde{D}X - P_r I_{mp} \tilde{D}Y + P_r I_{me}\left(\tilde{D}X - \frac{Q}{2}\right) + \frac{P_c I_{re}(\tilde{D}Y)^2}{2Q} - \frac{P_r I_{rp}(Q-\tilde{D}Y)^2}{2Q} \quad (7)$$

$$\tilde{J}_4(n,Q) = \tilde{D}(P - P_s - h_m t_m Z - W - GZ) - \frac{\tilde{D}}{nQ}(A_s + A_m + F_m + A_r + F_r n) - \frac{Q}{2}\left[\frac{h_s \tilde{D} n + h_m(2-n)}{P} + h_m(n - 1 - 2t_m Z^2 n\tilde{D}) + h_r\right] - P_m I_{sp} \tilde{D}X - P_r I_{mp} \tilde{D}Y + \frac{P_r I_{me}(\tilde{D}X)^2}{2Q} - \frac{P_m I_{mp}(Q-\tilde{D}X)^2}{2Q} + \frac{P_c I_{re}(\tilde{D}Y)^2}{2Q} - \frac{P_r I_{rp}(Q-\tilde{D}Y)^2}{2Q} \quad (8)$$

The objective of this problem is to determine the optimal order quantity and the optimal integer number of lots in which the items are delivered from the manufacturer to the retailer such that $\tilde{J}_i(Q, n)$ achieves its maximum value. In order to maximize $\tilde{J}_i(Q, n)$, we set $[\partial \tilde{J}_i(Q, n)/\partial Q] = 0$ and obtain the economic value of $Q = Q_1^*$, Q_2^*, Q_3^*, and Q_4^*. To prevent the equations are too long to read, we set $[2(A_s + A_m + F_m + A_r + F_r n)] = U$, $h_m(2 - n + Pn - P)/P + h_r = K$, $P_m I_{mp} - P_r I_{me} = S$, and $P_r I_{rp} - P_c I_{re} = T$.

$$Q_1^* = \left(\frac{\tilde{D}U}{n\left\{\left[\frac{h_s n\tilde{D}}{P} - 2h_m t_m Z^2 n\tilde{D} + K\right] + P_r I_{me} + P_c I_{re}\right\}}\right)^{0.5} \tag{9}$$

$$Q_2^* = \left(\frac{\tilde{D}U + nS(\tilde{D}X)^2}{n\left\{\left[\frac{h_s n\tilde{D}}{P} - 2h_m t_m Z^2 n\tilde{D} + K\right] + P_m I_{mp} + P_c I_{re}\right\}}\right)^{0.5} \tag{10}$$

$$Q_3^* = \left(\frac{\tilde{D}U + nT(\tilde{D}Y)^2}{n\left\{\left[\frac{h_s n\tilde{D}}{P} - 2h_m t_m Z^2 n\tilde{D} + K\right] + P_r I_{me} + P_r I_{rp}\right\}}\right)^{0.5} \tag{11}$$

$$Q_4^* = \left(\frac{\tilde{D}U + n\left[S(\tilde{D}X)^2 + T(\tilde{D}Y)^2\right]}{n\left\{\left[\frac{h_s n\tilde{D}}{P} - 2h_m t_m Z^2 n\tilde{D} + K\right] + P_m I_{mp} + P_r I_{rp}\right\}}\right)^{0.5} \tag{12}$$

Definition: From Yao and Wu [26], Kaufmann and Gupta [27], and Zimmermann [28], for any a and 0 ∊ R, define the signed distance from a to 0 as $d_0(a, 0) = a$.

If a > 0, a is on the right hand side of origin 0; and the distance from a to 0 is $d_0(a, 0) = a$. If a < 0, a is on the left hand side of origin 0; and the distance from a to 0 is $d_0(a, 0) = -a$. This is the reason why $d_0(a, 0) = a$ is called the signed distance from a to 0.

Let Ω be the family of all fuzzy sets $\tilde{A}$ defined on R, the α-cut of $\tilde{A}$ is $A(\alpha) = [A_L(\alpha), A_U(\alpha)]$, $0 \le \alpha \le 1$, and both $A_L(\alpha)$ and $A_U(\alpha)$ are continuous functions on α ∊ [0, 1]. Then, for any $\tilde{A}$ ∊ Ω, we have

$$\tilde{A} = \bigcup_{0\le\alpha\le1}[A_L(\alpha)_\alpha, A_U(\alpha)_\alpha]\ . \tag{13}$$

Besides, for every α ∊ [0, 1], the α-level fuzzy interval $[A_L(\alpha)_\alpha, A_U(\alpha)_\alpha]$ has a one-to-one correspondence with the crisp interval $[A_L(\alpha), A_U(\alpha)]$, that is, $[A_L(\alpha)_\alpha, A_U(\alpha)_\alpha] \leftrightarrow [A_L(\alpha), A_U(\alpha)]$ is one-to-one mapping. The signed distance of two end points, $A_L(\alpha)$ and $A_U(\alpha)$ to 0 are $d_0(A_L(\alpha), 0) = A_L(\alpha)$ and $d_0(A_U(\alpha), 0) = A_U(\alpha)$, respectively.

Hence, the signed distance of interval $[A_L(\alpha), A_U(\alpha)]$ to 0 can be represented by their average, $(A_L(\alpha) + A_U(\alpha))/2$. Therefore, the signed distance of interval $[A_L(\alpha), A_U(\alpha)]$ to 0 can be represented as

$$d_0([A_L(\alpha), A_U(\alpha)], 0) = [d_0(A_L(\alpha), 0) + d_0(A_U(\alpha), 0)]/2 = \left(A_L(\alpha) + A_U(\alpha)\right)/2. \tag{14}$$

Further, because of the 1-level fuzzy point $\tilde{0}_1$ is mapping to the real number 0, the signed distance of $[A_L(\alpha)_\alpha, A_U(\alpha)_\alpha]$ to $\tilde{0}_1$ can be defined as

$$d_0\left([A_L(\alpha)_\alpha, A_U(\alpha)_\alpha], \tilde{0}_1\right) = d_0([A_L(\alpha), A_U(\alpha)], 0) = \left(A_L(\alpha) + A_U(\alpha)\right)/2. \tag{15}$$

Thus, from (13) and (15), since the above function is continuous on $0 \le \alpha \le 1$ for $\tilde{A}$ ∊ Ω, we can use the following equation to define the signed distance of $\tilde{A}$ to $\tilde{0}_1$ as follows.

Proof: For a fuzzy set $\tilde{A}$ ∊ Ω and α ∊ [0, 1], the α-cut of the fuzzy set $\tilde{A}$ is $A(\alpha) = \{x \in \Omega | \mu_A(x) \ge \alpha\} = [A_L(\alpha), A_U(\alpha)]$, where $A_L(\alpha) = a + \alpha(b - a)$ and $A_U(\alpha) = c - \alpha(c - b)$. From the definition, we can obtain the following equation. The signed distance of $\tilde{A}$ to $\tilde{0}_1$ is defined as

$$d(\tilde{A}, \tilde{0}_1) = \int_0^1 d\left([A_L(\alpha)_\alpha, A_U(\alpha)_\alpha], \tilde{0}_1\right) d\alpha = \frac{1}{2}\int_0^1 \left(A_L(\alpha), A_U(\alpha)\right) d\alpha.$$

So this equation is

$$d(\tilde{A}, \tilde{0}_1) = \frac{1}{2}\int_0^1 [A_L(\alpha), A_U(\alpha)]\, d\alpha = \frac{1}{4}(2b + a + c). \tag{16}$$

Substituting the result of Eq. (16) into Eq. (5), (6), (7), (8), (9), (10), (11), and (12). We also set $(P_c - P_s - h_m t_m Z - W - GZ) = V$. we have

$$\tilde{J}_1(n, Q) = V\left(D + \frac{\Delta_2 - \Delta_1}{4}\right) - \frac{U\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)}{2nQ} - \frac{Q}{2}\left[\frac{h_s n\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)}{P} - 2h_m t_m Z^2 n\left(D + \frac{\Delta_2 - \Delta_1}{4}\right) + K\right] - \left(P_m I_{sp} X + P_r I_{mp} Y\right)\left(D + \frac{\Delta_2 - \Delta_1}{4}\right) + P_r I_{me}\left[\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)X - \frac{Q}{2}\right] + P_c I_{re}\left[\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)Y - \frac{Q}{2}\right] \tag{17}$$

$$\tilde{J}_2(n, Q) = V\left(D + \frac{\Delta_2 - \Delta_1}{4}\right) - \frac{U\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)}{2nQ} - \frac{Q}{2}\left[\frac{h_s n\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)}{P} - 2h_m t_m Z^2 n\left(D + \frac{\Delta_2 - \Delta_1}{4}\right) + K\right] - \left(P_m I_{sp} X + P_r I_{mp} Y\right)\left(D + \frac{\Delta_2 - \Delta_1}{4}\right) + \frac{P_r I_{me}\left[\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)X\right]^2}{2Q} - \frac{P_m I_{mp}\left[Q - \left(D + \frac{\Delta_2 - \Delta_1}{4}\right)X\right]^2}{2Q} + P_c I_{re}\left[\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)Y - \frac{Q}{2}\right] \tag{18}$$

$$\tilde{J}_3(n, Q) = V\left(D + \frac{\Delta_2 - \Delta_1}{4}\right) - \frac{U\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)}{2nQ} - \frac{Q}{2}\left[\frac{h_s n\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)}{P} - 2h_m t_m Z^2 n\left(D + \frac{\Delta_2 - \Delta_1}{4}\right) + K\right] - \left(P_m I_{sp} X + P_r I_{mp} Y\right)\left(D + \frac{\Delta_2 - \Delta_1}{4}\right) + P_r I_{me}\left[\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)X - \frac{Q}{2}\right] + \frac{P_c I_{re}\left[\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)Y\right]^2}{2Q} - \frac{P_r I_{rp}\left[Q - \left(D + \frac{\Delta_2 - \Delta_1}{4}\right)Y\right]^2}{2Q} \tag{19}$$

$$\tilde{J}_4(n, Q) = V\left(D + \frac{\Delta_2 - \Delta_1}{4}\right) - \frac{U\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)}{2nQ} - \frac{Q}{2}\left[\frac{h_s n\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)}{P} - 2h_m t_m Z^2 n\left(D + \frac{\Delta_2 - \Delta_1}{4}\right) + K\right] - \left(P_m I_{sp} X + P_r I_{mp} Y\right)\left(D + \frac{\Delta_2 - \Delta_1}{4}\right) + \frac{P_r I_{me}\left[\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)X\right]^2}{2Q} - \frac{P_m I_{mp}\left[Q - \left(D + \frac{\Delta_2 - \Delta_1}{4}\right)X\right]^2}{2Q} + \frac{P_c I_{re}\left[\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)Y\right]^2}{2Q} - \frac{P_r I_{rp}\left[Q - \left(D + \frac{\Delta_2 - \Delta_1}{4}\right)Y\right]^2}{2Q} \tag{20}$$

and

$$Q_1^* = \left(\frac{\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)U}{n\left\{\left[\frac{h_s n\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)}{P} - 2h_m t_m Z^2 n\left(D + \frac{\Delta_2 - \Delta_1}{4}\right) + K\right] + P_r I_{me} + P_c I_{re}\right\}}\right)^{0.5} \tag{21}$$

$$Q_2^* = \left(\frac{\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)U + nS\left(\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)X\right)^2}{n\left\{\left[\frac{h_s n\left(D + \frac{\Delta_2 - \Delta_1}{4}\right)}{P} - 2h_m t_m Z^2 n\left(D + \frac{\Delta_2 - \Delta_1}{4}\right) + K\right] + P_m I_{mp} + P_c I_{re}\right\}}\right)^{0.5} \tag{22}$$

$$Q_3^* = \left(\frac{\left(D+\frac{\Delta_2-\Delta_1}{4}\right)U+nT\left(\left(D+\frac{\Delta_2-\Delta_1}{4}\right)Y\right)^2}{n\left\{\left[\frac{h_s n\left(D+\frac{\Delta_2-\Delta_1}{4}\right)}{P}-2h_m t_m Z^2 n\left(D+\frac{\Delta_2-\Delta_1}{4}\right)+K\right]+P_r I_{me}+P_r I_{rp}\right\}}\right)^{0.5} \quad (23)$$

$$Q_4^* = \left(\frac{\left(D+\frac{\Delta_2-\Delta_1}{4}\right)U+n\left[S\left(\left(D+\frac{\Delta_2-\Delta_1}{4}\right)X\right)^2+T\left(\left(D+\frac{\Delta_2-\Delta_1}{4}\right)Y\right)^2\right]}{n\left\{\left[\frac{h_s n\left(D+\frac{\Delta_2-\Delta_1}{4}\right)}{P}-2h_m t_m Z^2 n\left(D+\frac{\Delta_2-\Delta_1}{4}\right)+K\right]+P_m I_{mp}+P_r I_{rp}\right\}}\right)^{0.5} \quad (24)$$

E. Algorithm

In order to obtain the optimal values of $EJTP_i$ (Q, n), follow these steps:

Step 1. Obtain Δ_1 and Δ_2 from the decision makers.

Step 2. Set $n = n_i = 1$ and substitute into Eq. (21), (22), (23), and (24) to obtain Q_1, Q_2, Q_3, and Q_4.

Step 3. Find $\tilde{J}_i$ by substituting n_i and Q_i into Eq. (17), (18), (19), and (20).

Step 4. Let $n_i = n_i + 1$ and repeat step 1 to step 2 until $\tilde{J}_i\ (n_i) > \tilde{J}_i\ (n_i + 1)$. The optimal $n_i^* = n_i$; $Q_i^* = Q(n_i^*)$, $\forall_i = 1, 2, 3$, and 4.

Step 5. Compute the replenishment time and compare with payment period. Examine the relationship whether is conform to the situation and select the most expected joint total profit.

III. Numerical Example

A numerical example is used to demonstrate the proposed models in this section.

Given D = 1000 units/year, P_s = 20 \$/per unit, A_s = 50 \$/per order, h_s = 2 \$/per unit, I_{sp} = 0.02 \$/year, P = 2000 units/year, X = 0.205479 year (75 days), P_m = 35 \$/per unit, A_m = 70 \$/per order, h_m = 3 \$/per unit, F_m = 50 \$/per shipment, Z = 0.1, W = 0.5 \$/per unit, G = 1 \$/per unit, t_m = 0.000274 year/per unit (0.1 day), I_{mp} = 0.035 \$/year, I_{me} = 0.03 \$/year, Y = 0.041096 year (15 days), P_r = 50 \$/per unit, P_c = 70 \$/per unit, A_r = 100 \$/per order, F_r = 65 \$/per shipment, h_r = 5 \$/per order, I_{rp} = 0.04 \$/year, I_{re} = 0.035 \$/year

To solve for the optimal order quantity and find the optimal expected joint total profit in the fuzzy sense for various given sets of (Δ_1, Δ_2). Note that in practical situation, Δ_1 and Δ_2 determined by the decision makers due to the uncertainty of the problem. The result is summarized in Table I.

Table I. Optimal solution for the fuzzy three-echelon inventory model

Δ_1	Δ_2	$\tilde{D}$	n	Q	J(Q, n)
200	350	1037.5	2	175	48934.17
200	300	1025	2	174	48331.66
200	250	1012.5	2	174	47728.42
200	200	1000	2	173	47123.06
200	150	987.5	2	172	46523.73
200	100	975	2	171	45921.49
200	50	962.5	2	170	45319.31

IV. Conclusion

From the viewpoint of Yang et al. [25], we know that the length of credit period and imperfect production are two important factors to affect the joint total profit. Longer credit period can release the pressure of the down-stream firm's capital using. If the down-stream firm uses the sale revenue well, it will lead additional interest to increase the profit of the whole supply chain. Imperfect production causes additional time and cost on purchasing and production. Lower quality products repel the down-stream firm to trade with the vendor.

However, in the numerical example of Yang et al. [25], the values of parameters are fixed, while in practice, it is difficult to estimate or collect some data because of the uncertainty and the lack of historical data. In this paper, we extended the model of Yang et al. [25] by considering fuzzy annual demand. We proposed a fuzzy model for the three-echelon integrated inventory model with defective products and rework under credit period.

We used a method of defuzzification which is called signed distance to find the estimation of annual demand in the fuzzy sense, and then the corresponding optimal *n* and *Q* are derived to maximize the total profit. In addition, the proposed fuzzy model can solve the crisp problem. The optimal order quantity retailer in the fuzzy sense is decreased. Still, there are other fuzzy theories which can be used to discuss the fuzzy model. Although we didn't compare the affection of different fuzzy theories, the fuzzy approach shows its advantage to deal with the uncertainty.

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