Predicting Position of a Functional Target from an External Marker in Radiotherapy

Rūta Užupytė, Ka Lok Man, Tomas Krilavičius, Indrė Žliobaitė, and Henrikas Simonavičius

***Abstract*—The main goal of radiotherapy is to destroy the tumor while minimizing harm to nearby healthy tissue. Advances in the digital control have enabled planning and performing accurate treatments. However, todays technology is unable to compensate respiration induced motion, and therefore, ensure sufficient precision. One of the tasks in compensating respiratory motion is predicting position of the functional target (tumor) from an external marker during fraction. Performance of techniques, such as Pearson correlation, Gaussian filters, Fourries transformation, cross correlation, linear interpolation and partial-least squares, still leave plenty space for the improvement. We reports results of work in progress, i.e. experiments of applying different types of regressions to predict motion of functional target from different external markers. Results seem to be promising in most of the cases.**

***Index Terms*—respiratory motion compensation in radiotherapy, respiratory tumor motion, regression.**

I. Introduction

RADIOTHERAPY aims at focused emission of radiation dose to the target volume of tissue, while minimizing exposure to radiation for the surrounding healthy tissue. Progress of computer-based control allows accurate planning and treatment [1], [2]. However, respiration induced motion still remains unsolved problem, i.e. current techniques do not provide sufficient precision [3], [4], see [5], [6], [7] for an overview of recent results in predicting respiration induced motion of tumor.

In this paper we present a work–in–progress, which aims at proposing a technique for predicting a tumor position from the position of an external marker during a treatment session. There have been several attempts to analyze internal/external correlation [8], [9], where Pearson correlation and Gaussian filters, Fourier transformation and cross-correlation, simple linear regression [10] and models with firstorder autoregressive errors [11] are used. The results are promising, all techniques produce similar results, but there is a lot of space for improvement as well.

The paper is organized as follows. In section II problem formulation, and computational methods are presented. Section III discusses data collection. Section IV discusses experimental results. Section V presents concluding remarks, and discusses future research.

Manuscript received January 08, 2015; revised February 10, 2015. Research was partially financed by REPOZ PILOT (VP2-1.3-ŁM-02-K-03-031) and Employment of High Qualification Specialist in Baltic Institute of Advanced Technology to Perform Research (VP1-3.2-MM-01-K-02-002)

T. Krilavičius, R. Užupytė are with the Baltic Institute of Advanced Technology and Vytautas Magnus University, Lithuania , e-mail: t.krilavicius, ruta.uzupyte@bpti.lt

I. Žliobaitė is with Aalto University, Finland, e-mail:zliobaite@gmail.com

Ka Lok Man is with Xi'an Jiaotong-Liverpool University, China, e-mail:ka.man@xjtlu.edu.cn

H. Simonavičius is with UAB Rubedo Systems, Lithuania

II. Proposed Techniques

A. Problem Formulation

Suppose, we have two signals $M = o_1, o_2, \ldots, o_n$ and $T = p_1, p_2, \ldots, p_n$ consisting of n observations, where $o_i = (x_i^m, y_i^m)$ is a two–dimensional vector indicating the position of an external marker at time i while $p_i = (x_i^t, y_i^t)$ is a two–dimensional vector indicating the position of the target at time i. Our goal is to find an expression of functional relationship $T = \mathrm{F}(M)$ between the signals, separately for each component:

$$x^t = \mathrm{F}_1(x^m), \tag{1}$$

$$y^t = \mathrm{F}_2(y^m), \tag{2}$$

where the relations F_1 and F_2 are assumed to have the same functional form, but different values of the parameters.

B. Evaluation of performance

Suppose, we have a testing dataset consisting of n observations, where $p_i = (x_i^t, y_i^t)$ is the true position of the tumor at time i, and $\hat{p}_i = (\hat{x}_i^t, \hat{y}_i^t)$ is our prediction for the same time i. To evaluate the performance of prediction we use two different measures:

1) the mean absolute error, i.e. the average distance from the predicted position to the true position of the tumor:

$$MAE = \frac{\sum_{i=1}^{n} \sqrt{(\hat{x}_i^t - x_i^t)^2 + (\hat{y}_i^t - y_i^t)^2}}{n}. \tag{3}$$

2) the root mean square error, i.e. the sample standard deviation of the differences between predicted and observed tumor position:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (\hat{x}_i^t - x_i^t)^2 + (\hat{y}_i^t - y_i^t)^2}{n}}. \tag{4}$$

C. The periodogram

Periodogram [12] allows estimating the periodic tendencies in the observed time series. The periodogram of a time series $\{Y_n\}, n = 1, 2, , N$ is defined as

$$I_N(w_j) = \frac{1}{N} |Y(w_j)|^2, \tag{5}$$

where $Y(w_j)$ is the discrete Fourier transform of $\{Y_n\}$:

$$Y(w_j) = \sum_{n=0}^{N-1} Y_n \mathrm{e}^{-2\pi i w_j n}, \tag{6}$$

$w_j = j/N$ is a set of possible frequencies for $j = 1, 2, \ldots, (N-1)/2$ and i is imaginary unit.

In order to stabilize the estimate of the spectrum it is necessary to smooth the periodogram. It is recommended

(Advance online publication: 17 November 2015)

TABLE I
CORRELATION RESULTS INTERPRETATION

r_{xy}	Strength of relationship
$0.5 < \|r_{xy}\| < 1$	Strong
$0.3 < \|r_{xy}\| < 0.5$	Medium
$0.1 < \|r_{xy}\| < 0.3$	Weak
$0 < \|r_{xy}\| < 0.1$	None or very weak

[12] to use the Daniell window as a smoothing filter for generating an estimated spectrum from the periodogram. The modified Daniell window of length m is defined as

$$g_i = \begin{cases} \frac{1}{2(m-1)}, & i = 1 \text{ or } i = m \\ \frac{1}{m-1}, & i \text{ otherwise} \end{cases} \quad (7)$$

where m is the number of weights of the filter, g_i is the i^{th} weight of the filter and $i = 1, \ldots, m$ is an index.

Using periodogram plots it is easy to identify frequency (frequencies) w_j which corresponds to dominant spike (spikes) of the periodogram and then calculate dominant period (periods) using formula $T = 1/w_j$.

D. Correlation coefficient

In order to determinate the degree to which external markers and functional targets movements are associated we use Pearson correlation coefficient. Correlation coefficient between two variables $x = x_1, x_2, \ldots, x_n$ and $y = y_1, y_2, \ldots, y_n$ is calculated by:

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x}) \sum_{i=1}^{n}(y_i - \bar{y})}{n\sigma_x\sigma_y}, \quad (8)$$

where $\bar{x}, \bar{y}$ are averages of x and y, σ_x, σ_y are standard deviations of x and y, n is the sample size.

Coefficient values ranges from -1 to 1. The closer the absolute value of r_{xy} gets to 1, the stronger linear relationship between the variables is, see table I.

E. Linear regression

Linear regression assumes that two variables are systematically linked by a linear relationship:

$$y = \beta_0 + \beta_1 x + \varepsilon, \quad (9)$$

where x is the input variable, y is the response (predicted) variable, (β_0, β_1) are model parameters, and ε is a random error. Ordinary least squares method [13] is a typical approach for estimating the unknown model parameters (β_0, β_1), given a set of observations (x, y).

The extension to multiple predictor variables is known as multiple linear regression. Formally, the model for multiple linear regression can be written as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p + \varepsilon, \quad (10)$$

where p is the number of independent variables and other notations remain unchanged.

One of the key assumptions behind the linear regression models is that the errors ε are independent from each other, i.e. $\mathrm{E}(\varepsilon_t \varepsilon_s) = 0$, when $t \neq s$ and when $t = s$, $\mathrm{E}(\varepsilon_t)^2 = \sigma^2$, where $\mathrm{E}(x)$ denotes the mean of x, and σ^2 denotes the variance of ε_t. If $\mathrm{E}(\varepsilon_t \varepsilon_s) \neq 0$, then the assumption is violated,

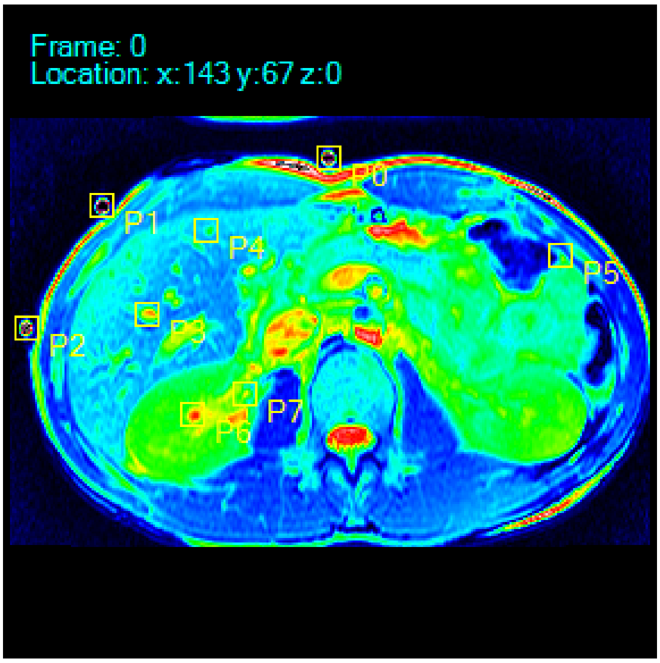

Fig. 1. Points of interest in DICOM image

and the regression model has a problem of autocorrelation. Mathematically first–order autocorrelation means that the model errors satisfy a recursive relationship [14]:

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t, \quad (11)$$

where $\{u_t, t = 1, 2, \ldots n\}$ is a sequence of independent random variables, which are normally distributed with zero mean and a constant variance, and ρ is the autoregressive coefficient ($|\rho| < 1$). When $\rho = 0$, errors ε_t are uncorrelated. The most typical case is the first–order autoregressive error. For determining whether the errors are following the first–order autoregressive process, we use the Durbin–Watson test [15].

If autocorrelation is found, we modify the model by including the estimated first–order autoregressive coefficient of the error term:

$$y_i = \beta_0 + \beta_1 x_i + \ldots + \beta_p x_p + e_i, \quad (12)$$

$$e_i = \rho e_{i-1} + \varepsilon_i, \quad (13)$$

where e_i are regression model residuals, ε random error and ρ is autoregressive coefficient which can be computed using residuals of initial model (equation (9)):

$$\hat{\rho} = \frac{\sum_{i=2}^{n} e_i e_{i-1}}{\sum_{i=2}^{n} e_{i-1}^2}. \quad (14)$$

III. DATA COLLECTION

Respiratory motion data was collected with MRT Achieva XR (Philips Medical Systems) (with a 16–channel SENSE XR Torso coil). 8 sets of 2D signals (see fig. 2) were collected, using 3 surrogate markers. Two persons data was collected from using three external markers placed at the different positions . Records were produced in DICOM[1].

[1]Digital Imaging and Communications in Medicine (DICOM) is *de facto* standard for handling, storing, printing, and transmitting information in medical imaging.

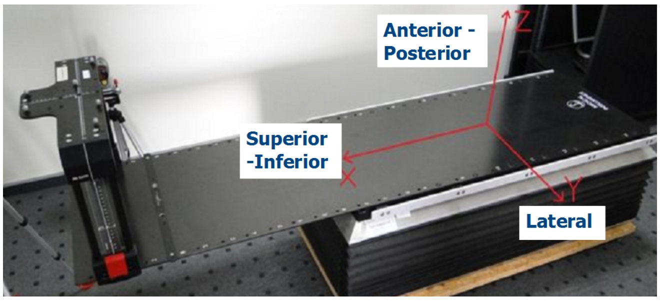

Fig. 2. Motion directions

TABLE II
SIGNALS SUMMARY: MIN AND MAX DISPLACEMENT (MIN, MAX), STANDARD DEVIATION (SD) AND DOMINANT PERIOD (T)

Direction	Min	Max	SD	T
P0 x	25.9	30	0.97	11.91
P0 y	74.8	75.5	0.18	11.91
P1 x	23.7	24.7	0.24	11.91
P1 y	48.4	48.6	0.05	11.91
P2 x	23.8	24.8	0.22	11.91
P2 y	20.9	21.4	0.11	11.91
P3 x	45.7	50.7	1.21	11.63
P3 y	56.8	61.8	1.42	*
P4 x	35.1	37.8	0.62	11.91
P4 y	34.3	40.4	1.53	11.91
P5 x	29.2	29.8	0.12	11.91
P5 y	27.3	31.5	1.16	11.91
P6 x	41.8	46.8	0.88	11.91
P6 y	29.3	34	1.5	11.91
P7 x	44.5	46	0.33	11.91
P7 y	36.5	43.3	1.7	11.91
P8 x	50.6	53.8	0.74	11.91
P8 y	29.9	38	2.02	11.91
P9 x	42.5	46.2	0.64	*
P9 y	24.9	30.5	1.45	11.91

Time series from the records were extracted using in–house tools, where several (6—10) points–of–interest (POI) were tracked instead of tumors. The duration of the records varied from 300 to 500 frames, i.e. 150—400 sec. Overall, 87 signal–pairs were obtained. However, some signals (6) were deemed useless, because either target or marker did not move. All signals were defined by two components: one part of the signals had lateral and anterior-posterior directions (superior inferior direction was ignored), and another part had anterior–posterior and superior inferior directions (lateral direction was ignored), see fig. 1 for an example of POIs.

In this paper part of data, consisting of three external markers (P0 — P2) and seven functional targets (P3 — P9) was used, see summary in table II.

Analysis of collected signals shows that maximum range of the internal POI's (functional targets) is higher than external markers, i.e. functional targets move more. Moreover, external markers move more in x (anterior – posterior) direction and targets move more in y (superior – inferior) direction. In this case lateral direction was not observed. When anterior – posterior and lateral directions were observed (superior – inferior direction was ignored) targets and markers move more in anterior – posterior direction.

To identify the periodic tendencies in the observed time series we use periodograms. Based on this analysis results (see table II), it can be concluded that each signal usually has one period which is more or less constant for all signals of each series. Exceptional cases (marked by the symbol *) are observed for signals with failed detection: in these cases period can not be identified because periodograms have a lot of dominant spikes.

TABLE III
CORRELATION MATRIX

	P0 x	P0 y	P1 x	P1 y	P2 x	P2 y
P3 x	0.05	-0.06	0.07	-0.07	0.06	0.03
P3 y	-0.33	0.37	-0.33	0.26	-0.3	-0.24
P4 x	0.98	-0.91	0.97	-0.82	0.93	0.89
P4 y	-0.99	0.93	-0.98	0.85	-0.94	-0.89
P5 x	0.77	-0.64	0.79	-0.65	0.9	0.85
P5 y	-0.97	0.88	-0.95	0.82	-0.96	-0.91
P6 y	-0.72	0.72	-0.74	0.64	-0.73	-0.66
P7 x	0.92	-0.91	0.92	-0.79	0.84	0.79
P7 y	-0.99	0.93	-0.97	0.84	-0.93	-0.88
P8 x	0.97	-0.91	0.96	-0.82	0.91	0.86
P8 y	-0.97	0.91	-0.96	0.83	-0.92	-0.88
P9 x	-0.09	0.04	-0.05	0.03	-0.01	-0.06
P9 y	-0.79	0.78	-0.8	0.7	-0.79	-0.73

Data was transformed for the analysis, i.e. each time series was normalized such that the minimum value is zero, and the maximum is equal to $\max(P_i) - \min(P_i)$, as follows

$$x'_{ij} = x_{ij} - \min(x_{i1}, x_{i2}, \ldots, x_{in}), \quad (15)$$
$$y'_{ij} = y_{ij} - \min(y_{i1}, y_{i2}, \ldots, y_{in}), \quad (16)$$

where $P_i = \{p_{i1}, p_{i2}, \ldots, p_{in}\}$ is a time series consisting of n observations and $p_{ij} = \{x_{ij}, y_{ij}\}$.

IV. EXPERIMENTAL ANALYSIS

A. Correlation

To evaluate the strength of the (linear) relationship between markers and targets we use Pearson correlation coefficient. Calculated measures are provided in table III. To simplify the analysis of correlation results we use heatmap, where white colour indicates weak correlation between signals and black – strong correlation. Correlation values in this graph are represented in terms of absolute size (see fig. 4).

Correlation analysis shows, that most of the signals are linked by strong or medium relationships. Overall data correlation varies from 0.001 to 0.991 with average equal to 0.492. In this particular case the strongest correlation was observed between target P0 and marker P4. The lowest degree of correlation were obtained between markers P3, P9 and all external markers due to the failed detection of P3 and P9. Despite the fact that correlation between target P6 and all external markers is quite strong this target detection has failed as well [2].

Moreover, correlation analysis shows that the strength of the (linear) relationship between targets and markers also depends on the external marker position. In almost all cases, the strongest correlation was observed between targets and markers from position P0 (abdomen area).

B. Prediction of Functional Target Position

Multiple linear regression is another approach for predicting position of the functional target from an external marker. In this case it is assumed that the position of each

[2] Markers with failed detection were eliminated from the further analysis.

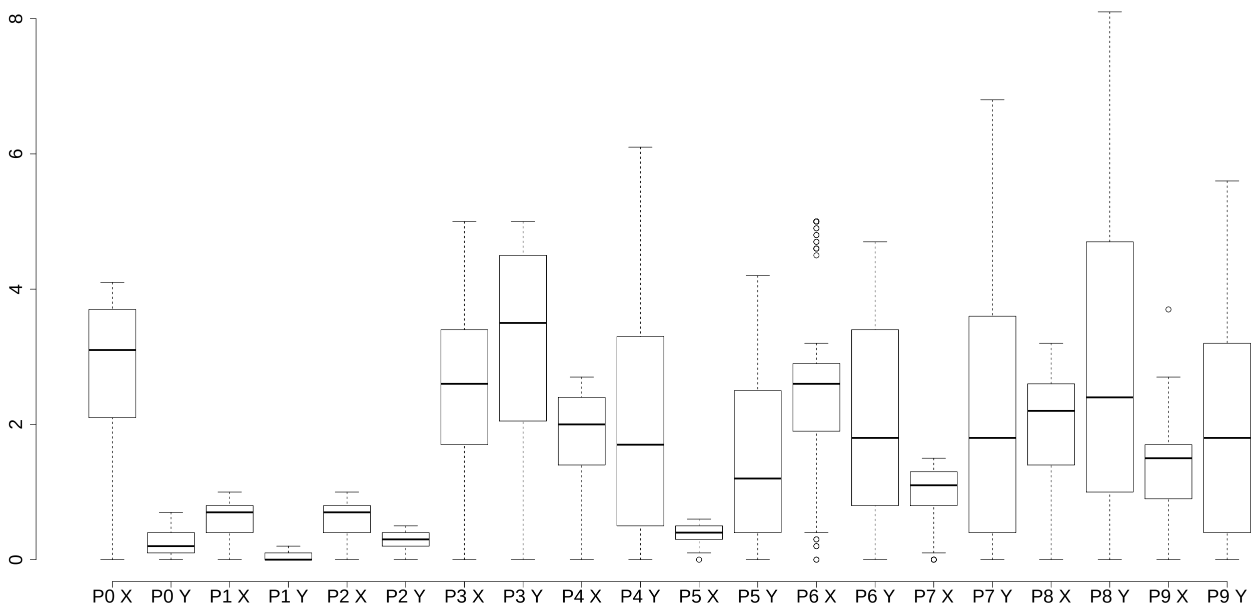

Fig. 3. Distribution of collected signals (in mm)

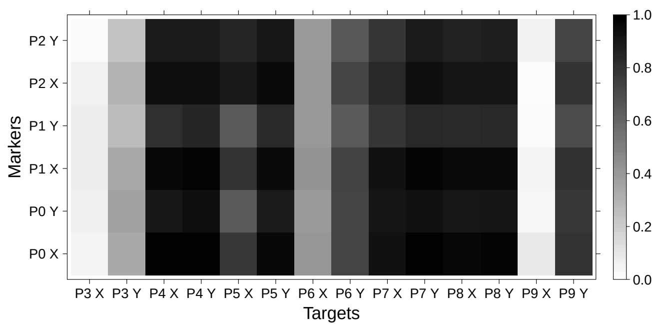

Fig. 4. Signals correlation heatmap

coordinate of the internal point depends on all coordinates of the external marker:

$$x_i^{(t)} = \hat{\beta}_0^x + \hat{\beta}_1^x x_i^{(m)} + \hat{\beta}_2^x y_i^{(m)} \quad (17)$$

$$y_i^{(t)} = \hat{\beta}_0^y + \hat{\beta}_1^y x_i^{(m)} + \hat{\beta}_2^y y_i^{(m)}. \quad (18)$$

At this stage dataset was split into training and test sets as 50:50. All possible pairs between external markers and internal points–of–interest were analyzed. Based on the correlation analysis results linear model was chosen.

C. Simple linear regression

Using simple linear regression each coordinate of internal signal is predicted separately based on the corresponding coordinate of the external marker:

$$x_i^{(t)} = \hat{\beta}_0^x + \hat{\beta}_1^x x_i^{(m)}, \quad (19)$$

$$y_i^{(t)} = \hat{\beta}_0^y + \hat{\beta}_1^y y_i^{(m)}. \quad (20)$$

Table IV for some results. Using MAE and RMSE measures we evaluated performance of the predictors on the two-dimensional trajectories (2D) and separate x and y predictions. The table also shows p–values of Durbin–Watson test and adjusted coefficient of determination values (R^2_{adj}) that indicate how well a regression line fits the data.

The table shows that better results are obtained by predicting x coordinate: average value of MAE for coordinate x is 0.14 mm and for y –0.75 mm; average value of RMSE for x is 0.31 mm and for y –0.93 mm. For both directions regression models are characterized by relatively high values of R^2_{adj}. However, almost all the standard linear regression models suffer from the problem of *autocorrelation*, since in the most of cases p–values of the Durbin–Watson test are lower than 0.05. This may lead to the conclusion that values of R^2_{adj} may be overrated. In order to solve this problem we apply a modified version of the regression model that takes into account the first–order autoregressive errors. The results are reported in the Table V.

Comparing the results obtained by the linear regression model with the first–order autoregressive errors we can see that all p–values of Durbin–Watson test are greater than 0.05, which suggests that the problem of autocorrelation is solved. Moreover, the proposed models have larger values for the adjusted coefficient of determination (R^2_{adj}), i.e. models with first–order autoregressive errors fit the data better.

In table VIII we present performance of all the models. We can see that the average values are slightly higher due to the first-order dependencies. More substantial changes are observed in maximum values of the performance measures. Since the main goal is to find a model with the best forecasting performance it can be concluded that simple linear regression model is more suitable for the functional target tracking in comparison with the simple linear model with AR(1) errors.

TABLE IV
PREDICTION ERROR USING SIMPLE LINEAR REGRESSION

Model	MAE, mm			RMSE, mm			p–value		R^2_{adj}	
	x	y	$2D$	x	y	$2D$	x	y	x	y
P4∼P0	0.1	0.53	0.55	0.14	0.67	0.69	0	0	0.96	0.89
P4∼P1	0.15	0.76	0.79	0.18	0.92	0.94	0	0	0.93	0.74
P4∼P2	0.2	0.85	0.89	0.26	1.01	1.04	0	0	0.87	0.81
P5∼P0	0.07	0.49	0.51	0.09	0.63	0.64	0	0	0.53	0.8
P5∼P1	0.07	0.6	0.61	0.09	0.73	0.73	0	0	0.55	0.7
P5∼P2	0.05	0.61	0.61	0.06	0.73	0.73	0	0.03	0.72	0.86
P7∼P0	0.1	0.6	0.62	0.13	0.76	0.77	0	0	0.82	0.88
P7∼P1	0.11	0.85	0.87	0.14	1.04	1.05	0	0	0.83	0.72
P7∼P2	0.15	0.93	0.95	0.19	1.1	1.12	0	0	0.66	0.8
P8∼P0	0.15	0.81	0.85	0.19	1.01	1.03	0.13	0	0.93	0.85
P8∼P1	0.22	0.99	1.04	0.26	1.24	1.26	0.03	0	0.91	0.68
P8∼P2	0.29	1.03	1.1	0.38	1.26	1.32	0	0	0.85	0.77

TABLE V
PREDICTION ERROR USING SIMPLE LINEAR REGRESSION WITH AR(1) ERRORS

Model	MAE, mm			RMSE, mm			p–value		R^2_{adj}	
	x	y	$2D$	x	y	$2D$	x	y	x	y
P4∼P0	0.10	0.53	0.56	0.14	0.68	0.7	0.82	0.96	0.97	0.93
P4∼P1	0.15	0.75	0.79	0.18	0.91	0.93	0.35	0.50	0.94	0.75
P4∼P2	0.18	0.81	0.84	0.24	0.94	0.97	0.36	0.90	0.91	0.83
P5∼P0	0.07	0.52	0.53	0.09	0.67	0.68	0.09	0.59	0.68	0.9
P5∼P1	0.07	0.60	0.61	0.09	0.72	0.73	0.1	0.85	0.68	0.74
P5∼P2	0.04	0.59	0.6	0.06	0.71	0.71	0.72	0.91	0.74	0.86
P7∼P0	0.11	0.60	0.62	0.14	0.76	0.77	0.3	0.93	0.86	0.92
P7∼P1	0.10	0.85	0.87	0.13	1.03	1.04	0.77	0.36	0.85	0.73
P7∼P2	0.16	0.88	0.91	0.19	1.03	1.05	0.34	0.82	0.77	0.82
P8∼P0	0.15	0.82	0.85	0.19	1.01	1.03	0.13	0.74	0.93	0.88
P8∼P1	0.22	0.99	1.04	0.26	1.23	1.26	0.91	0.37	0.91	0.7
P8∼P2	0.27	0.97	1.04	0.35	1.19	1.24	0.47	0.62	0.86	0.8

D. Multiple linear regression

Multiple linear regression results are provided in Table VI. The results are similar to the simple linear regression case: x-coordinate predictions are more accurate than y-coordinate, regression models are characterized by relatively high values of R^2_{adj}, but suffer from the problem of autocorrelation, i.e. p–values of the Durbin–Watson test are lower than 0.05. However, analyzing the testing accuracies over all the models (Table VIII) we can see a noticeable decrease in average values of MAE and RMSE. Improvement in accuracy was caused by more precise predictions of coordinate y. In this case the use of multiple linear regression does not have a significant impact on the x-coordinate predictions. Relation between prediction accuracy and the range of signal motion was observed in all models: in cases where x is lateral direction and y is anterior – posterior, coordinate y is the one with a greater range of movement and has more accurate predictions. When lateral direction was ignored, prediction of anterior–posterior direction was more successful than superior–inferior and the range of markers motion is wider for anterior–posterior direction. This may lead to the conclusion, that more accurate results are obtained using coordinate with a greater range of movement. In the future we are planning to test the hypothesis, that quite high accuracy can be obtained using only one coordinate with the largest amplitude.

In order to solve the above-mentioned problem of the autocorrelation an inclusion of the estimated first–order autoregressive coefficient of the error term was investigated. The results (see Table VII) show that this modification solved the problem of autocorrelation. Moreover, models with the first–order autoregressive errors have larger values for the adjusted coefficient of determination (R^2_{adj}). However, analyzing the prediction error over all the models (Table VIII) we can see that more accurate results are obtained using multiple linear regression model. This leads to the conclusion that models with the first–order autoregressive errors suffer from the problem of overfitting. In summary, it can be concluded that multiple linear regression model can be identified as the most suitable method for functional target motion prediction in context of the analyzed methods.

E. Relation between predictor performance and marker position

Previous results showed that multiple linear regression provides the most accurate predictions. More detailed analysis of these results (Table VI) suggest that prediction performance depends on the position of an external marker POI. It is easy to see that more accurate predictions are obtained using external markers placed in position P0 –area of the abdomen. Also we can see that the minimum value of MAE is observed for relation P5∼P0 (see fig. 7, fig 8), while RMSE - for relation P4∼P0 (see fig. 5, fig 6).

V. CONCLUSION

A comprehensive analysis of selected signals shows that functional targets move more than external markers. Signals movement range depends on their directions as well: markers move more in anterior–posterior direction; targets –in superior inferior direction, if this direction is ignored, then in anterior–posterior. Experiments show that multiple linear regression model is the most suitable method for functional target motion prediction from the analyzed methods. Furthermore, better result are obtained using external markers with

TABLE VI
PREDICTION ERROR USING MULTIPLE LINEAR REGRESSION

Model	MAE, mm			RMSE, mm			p–value		R^2_{adj}	
	x	y	$2D$	x	y	$2D$	x	y	x	y
P4∼P0	0.09	0.2	0.24	0.12	0.25	**0.27**	0	0	0.97	0.98
P4∼P1	0.15	0.32	0.37	0.18	0.39	0.43	0	0	0.93	0.95
P4∼P2	0.22	0.55	0.61	0.3	0.76	0.82	0	0	0.9	0.9
P5∼P0	0.06	0.21	**0.23**	0.08	0.27	0.28	0	0	0.58	0.96
P5∼P1	0.07	0.34	0.35	0.09	0.4	0.41	0	0	0.56	0.91
P5∼P2	0.04	0.4	0.4	0.06	0.54	0.54	0	0	0.72	0.95
P7∼P0	0.11	0.22	0.26	0.15	0.27	0.31	0	0.07	0.85	0.97
P7∼P1	0.11	0.36	0.39	0.14	0.44	0.46	0	0	0.83	0.93
P7∼P2	0.15	0.6	0.63	0.19	0.82	0.85	0	0	0.67	0.89
P8∼P0	0.15	0.42	0.47	0.19	0.49	0.53	0.28	0	0.94	0.94
P8∼P1	0.21	0.52	0.59	0.25	0.63	0.67	0.05	0	0.91	0.89
P8∼P2	0.32	0.73	0.83	0.43	0.97	1.07	0	0	0.87	0.87

TABLE VII
PREDICTION ERROR USING MULTIPLE LINEAR REGRESSION

Model	MAE, mm			RMSE, mm			p–value		R^2_{adj}	
	x	y	$2D$	x	y	$2D$	x	y	x	y
P4∼P0	0.09	0.21	0.24	0.12	0.25	0.28	0.78	0.6	0.97	0.98
P4∼P1	0.16	0.33	0.38	0.19	0.4	0.45	0.42	0.18	0.94	0.96
P4∼P2	0.2	0.47	0.53	0.27	0.63	0.69	0.44	0.69	0.93	0.94
P5∼P0	0.06	0.23	0.25	0.08	0.29	0.3	0.41	0.77	0.69	0.97
P5∼P1	0.07	0.39	0.41	0.09	0.46	0.47	0.2	0.1	0.67	0.95
P5∼P2	0.04	0.36	0.37	0.06	0.5	0.5	0.76	0.55	0.74	0.96
P7∼P0	0.11	0.22	0.26	0.15	0.27	0.31	0.7	0.07	0.87	0.97
P7∼P1	0.11	0.37	0.4	0.14	0.45	0.47	0.68	0.34	0.85	0.95
P7∼P2	0.16	0.53	0.57	0.19	0.7	0.73	0.45	0.6	0.78	0.93
P8∼P0	0.15	0.43	0.48	0.19	0.5	0.53	0.28	0.27	0.94	0.94
P8∼P1	0.21	0.51	0.58	0.25	0.62	0.67	0.05	0.57	0.91	0.92
P8∼P2	0.3	0.66	0.76	0.4	0.88	0.97	0.58	0.49	0.88	0.91

TABLE VIII
PREDICTION ERROR OVER ALL MODELS

	Simple lin. regr.		Simple lin. regr. with AR(1)		Multiple lin. regr.		Multiple lin. regr. with AR(1) errors	
	MAE,mm	RMSE, mm	MAE,mm	RMSE, mm	MAE,mm	RMSE, mm	MAE,mm	RMSE, mm
Average	1.069	1.245	1.074	1.252	0.81	0.958	0.812	0.962
Min	0.25	0.29	0.25	0.29	0.13	0.15	0.14	0.15
Max	3.44	4.01	3.76	4.28	3.35	4.05	3.47	4.17

a greater range of movement and that is usually abdomen area.

Our plans include testing the hypothesis, that quite high accuracy can be obtained using only one coordinate with the largest amplitude. We plan to analyze respiratory motion prediction and design cases of an overall system radiation therapy system with respiratory motion compensation.

ACKNOWLEDGMENT

The authors would like to thank G. Čaplinskas for extracting trajectories from MRI images.

REFERENCES

[1] G. D. R. and C. J. Y, "Adaptive radiation for lung cancer," *Journal of Oncology*, vol. 2011, 2010.

[2] B. S. A. and N. C. M., "Recent advances in radiotherapy," *BMC Medicine*, vol. 8, no. 1, 2010.

[3] D. Ruan, J. A. Fessler, and J. M. Balter, "Real-time prediction of respiratory motion based on local regression methods," *Physics in Medicine and Biology*, vol. 52, no. 23, pp. 7137–7152, 2007. [Online]. Available: http://stacks.iop.org/0031-9155/52/i=23/a=024

[4] H. Wu, G. C. Sharp, B. Salzberg, D. Kaeli, H. Shirato, and S. B. Jiang, "A finite state model for respiratory motion analysis in image guided radiation therapy," *Physics in Medicine and Biology*, vol. 49, no. 23, p. 5357, 2004. [Online]. Available: http://stacks.iop.org/0031-9155/49/i=23/a=012

[5] M. G. S. Keall P.J and et. al., "The management of respiratory motion in radiation oncology report of aapm task group 76," *Medical Physics*, vol. 33, no. 10, pp. 3874–3900, 2006.

[6] M. M. J., "Tracking moving organs in real time," *Seminars in Radiation Oncology*, vol. 14, no. 1, pp. 91–100, 2004.

[7] F. Ernst and A. Schweikard, "A survey of algorithms for respiratory motion prediction in robotic radiosurgery," in *GI Jahrestagung'09*, 2009, pp. 1035–1043.

[8] N. S. S. H. Ionascu D., Jiang S. B. and B. R. I., "Internal-external correlation investigations of respiratory induced motion of lung tumors," *Medical Physics*, vol. 34, no. 10, 2007.

[9] S. Y. K. K. T. T. N. A. Tsunashima Y., Sakae T. and A. Y., "Correlation between the respiratory waveform measured using a respiratory sensor and 3d tumor motion in gated radiotherapy," *International Journal of Radiation Oncology Biology Physics*, vol. 60, no. 3, pp. 951–958, 2004.

[10] v. I. Krilavičius T., Užupytė R. and S. H., "Correlation of external markers and functional targets for respiration compensation in radiotherapy," in *Proc. of Medical Physics in the Baltic States, Kaunas, Lithuania, Oct, 2013*, pp. 42–45.

[11] K. T. v. I. Užupytė R., Man KL. and S. H., "Indirect tracking of functional target for respiration compensation in radiotherapy," in *Lecture Notes in Engineering and Computer Science: Proceedings of The International MultiConference of Engineers and Computer Scientists 2015, IMECS 2015*, 18-20 March, 2015, Hong Kong, pp. 733–736.

[12] P. Bloomfield, *Fourier analysis of time series: an introduction*, 2nd ed. John Wiley & Sons.

[13] J. F. T. Hastie, R. Tibshirani, *The elements of statistical learning : data mining, inference, and prediction*, 2nd ed., ser. Springer series in statistics. New York: Springer, 2009.

[14] H. Barreto and F. Howland, *Introductory Econometrics: Using Monte Carlo Simulation with Microsoft Excel*. Cambridge, 2006.

[15] D. Conniffe, "Tightening durbin-watson bounds," *The Economic and Social Review*, vol. 28, pp. 351–356, 1997.

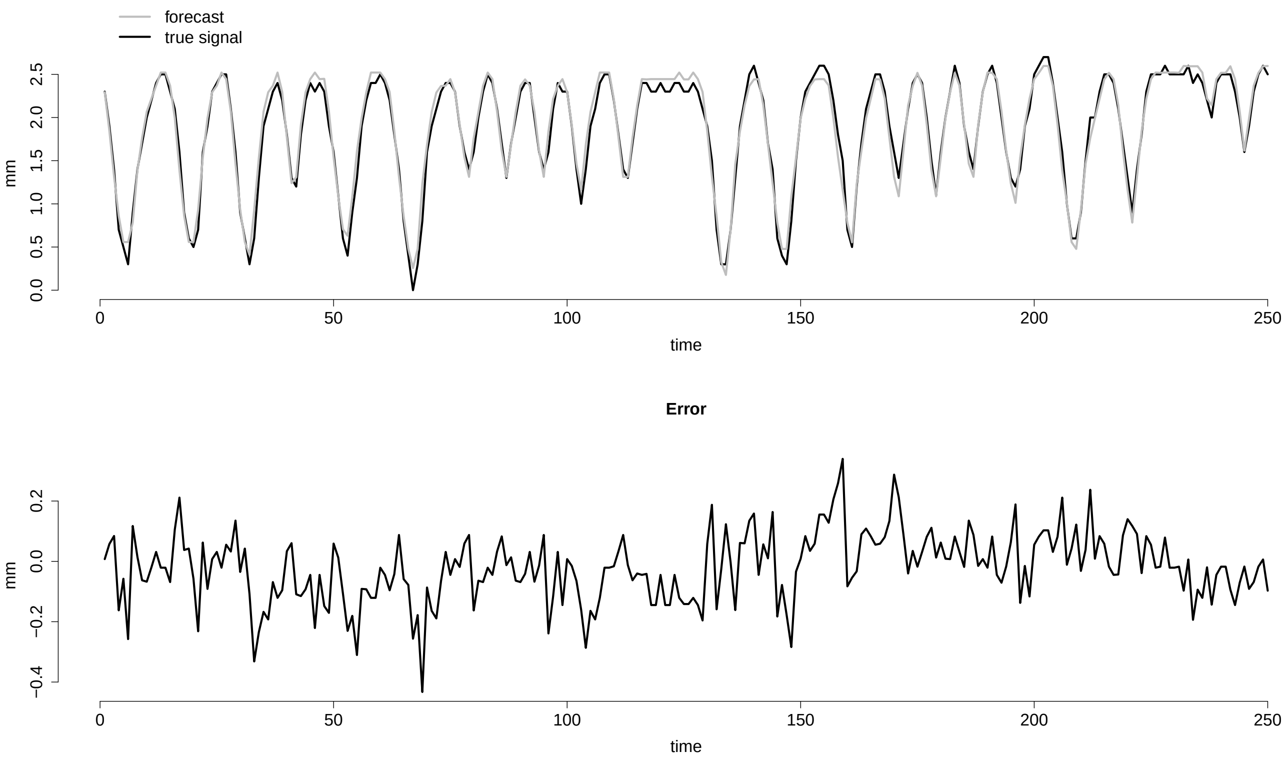

Fig. 5. Forecast and error term of x from relation P4∼P0 (Multiple Linear Regression)

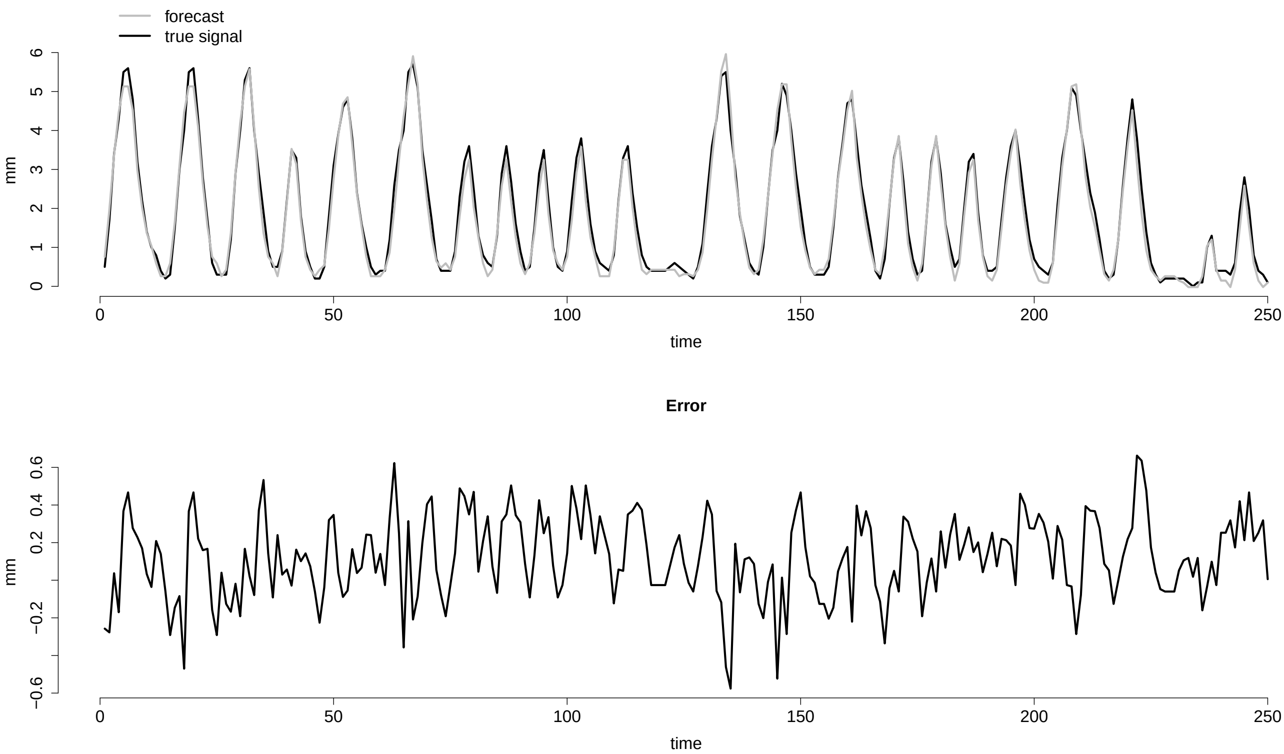

Fig. 6. Forecast and error term of y from relation P4∼P0 (Multiple Linear Regression)

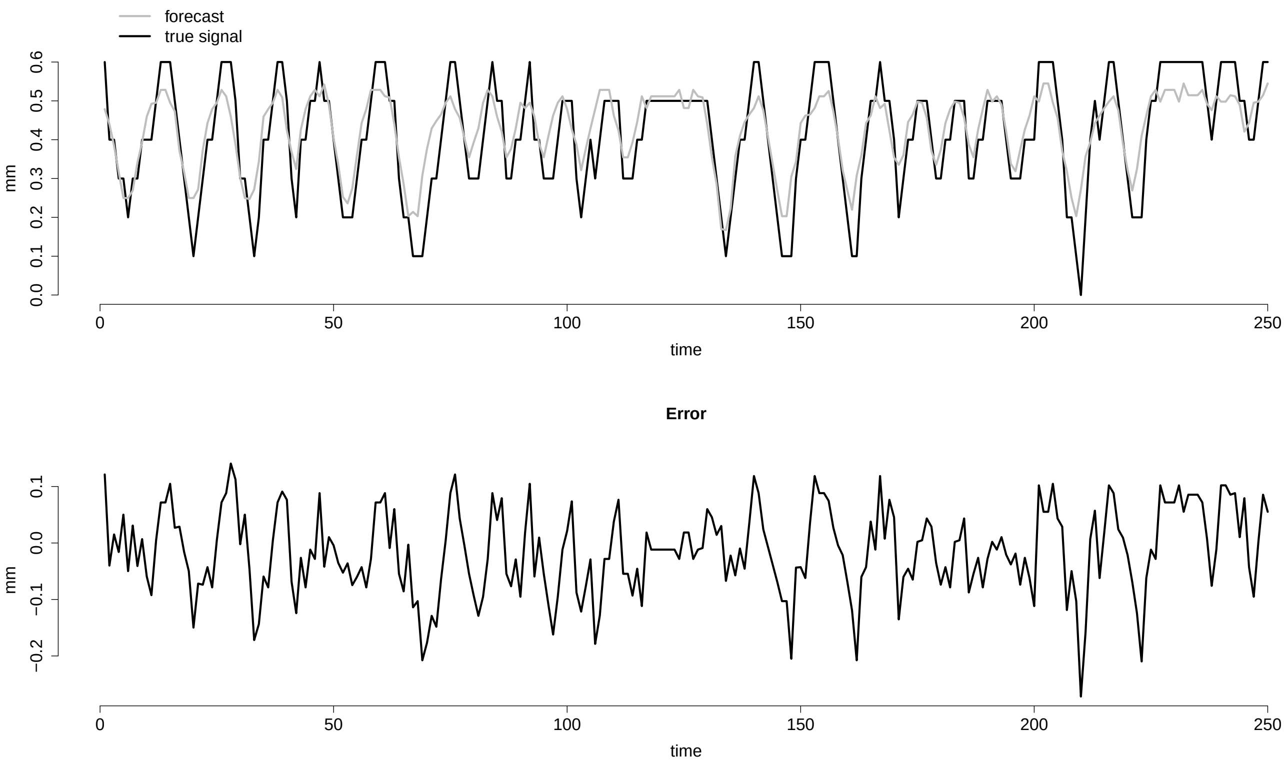

Fig. 7. Forecast and error term of x from relation P5∼P0 (Multiple Linear Regression)

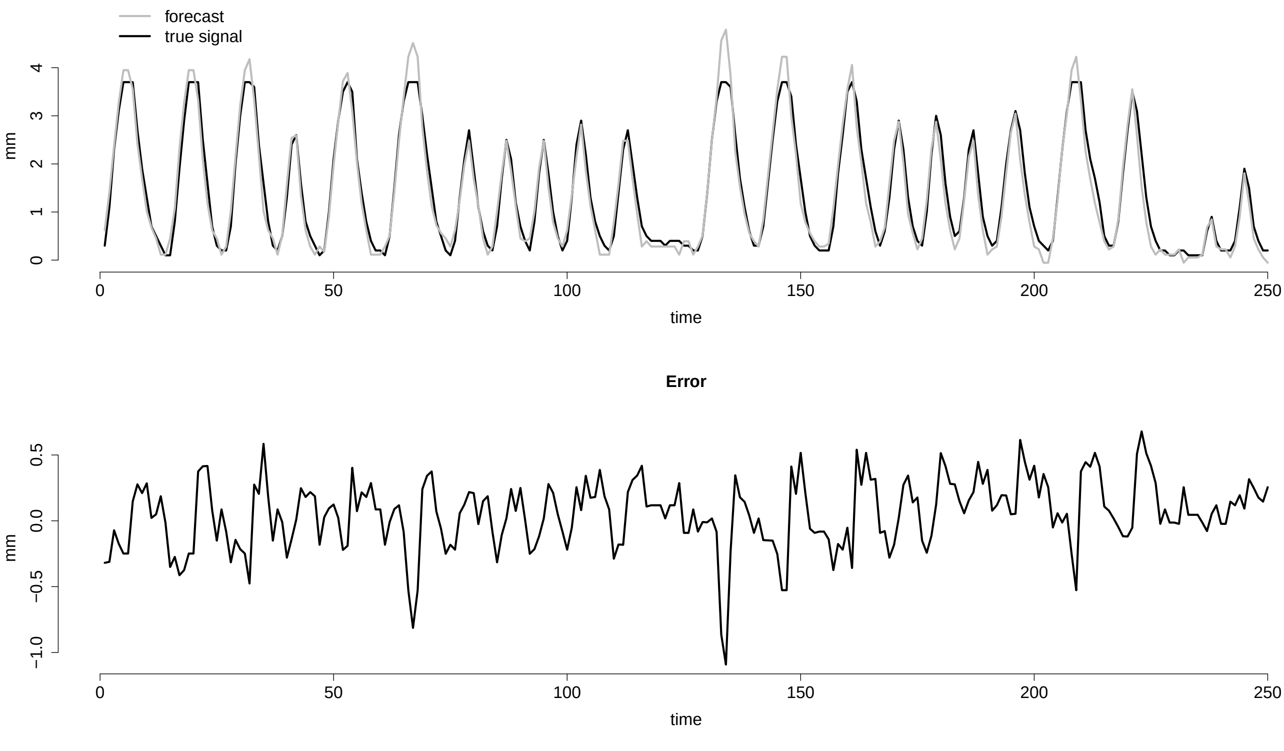

Fig. 8. Forecast and error term of y from relation P5∼P0 (Multiple Linear Regression)