# The Statistical Models for Estimating Heat Seal Parameters in the Bag-making Machine

Takashi Inomata and Toshinari Kamakura

*Abstract*—**Recently we need to manage statistical process controls to maintain product quality based on the sensor technology such as small thermometers and displacement sensors, which can give as vast information on production parameters.**

**In bag-making process, the strength of bags is very difficult to estimate without any distraction of products, and it sometimes needs a large sample size. In place of the destruction test we use the estimates by observations from important substitutional characteristics.**

**In this article, we shall focus on the heat-seal time parameter which is considered as one of the most important substitutional characteristics.**

**As we cannot directly observe heat-seal time, we measure several other variables, that may be used to estimate the function of these for the target parameters regarding heat-seal time. Therefore we measured the displacements of the seal-bars and that of the moving frame. Then we found that displacement data behaves as the skewed cyclic function own the time and we had to model the data taking into account this characteristics.**

**Therefore we proposed new statistical models based on the nested trigonometric function and discussed the procedure to estimate heat seal parameteres using our proposed models.**

*Index Terms*—**estimation of sealing parameters, link mechanism, mechanical engineering, nested trigonometric function.**

## I. Introduction

To perform high product quality we must well manage to control quality process. Information technology has developed in recent years in production environments and then we have to cope with the big data produced from IT technology. However, there are inspections with destruction of products and other test which are very difficult to carry out from the view point of cost. In such circumstances, we sometimes investigate the variables statistically related with target values like qualities and life times.

In this article, we focus on the process of producing thermoplastic bags for packaging foods and toiletries. The heat seal technology is used for the bag-making machine. The heat seal problems have been studied for a long time from the chemical and thermodynamical point of view([1],[2],[3],[4],[5]). Hishinuma [1] studied the temperatures of joint surface which make great effects on the strength of seal, and proposed the method of "MTMS". Komatsubara et al.[5] proposed the differential equation models for explaining the temperatures of joint parts of two thermoplastics.

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We study new problems on attaining stable and suitable temperatures for the joint surface of two thermoplastics in real bag-making machine[8]. It is very important for keeping products in good quality.

In this article, we discuss the models for estimating heat seal parameteres in the bag-making machine.

## II. Purpose of Research

Our purpose is to derive the distribution of seal time based on the displacements in the bag-making machine, whose major tasks are to press thermoplastics several films and joint together with heats, and cut them by specified sizes. The heat seal techniques are used for jointing materials of films. In this process, the materials are pressed and heated by the two seal-bars with a constant temperature. Then the heated areas of films with seal-bars with suitable temperature will be melted and jointed together.

The important parameters of the heat-sealing process are as follows:

Seal Temperature [°C] temperature of the bars
Seal Time [msec.] contact time of the films and the bars
Seal Load [kgf] load for pressing the films

The strength of adhesion depends on the temperature of the films and its temperature varies as the function of the seal temperature and the seal time under the condition certain constant pressure.

Fig.1 shows mechanical components of the bag-making machine. We describe the process of heat sealing in the bag making machine.

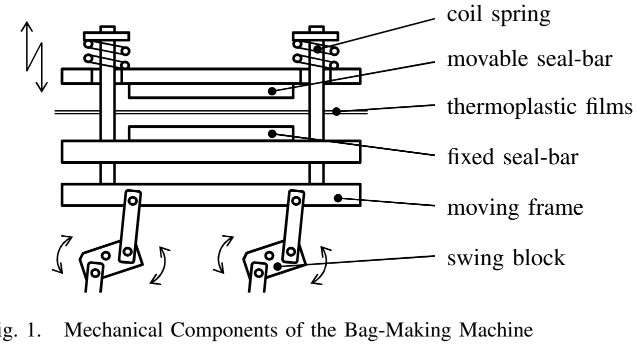


Fig. 1. Mechanical Components of the Bag-Making Machine

The several thermoplastic films used as materials for bags are placed between two seal-bars, whose temperatures must be controlled. The upper bar is movable, and the under bar is fixed. The movable seal-bar is connected to the moving frame by the spring. When the moving frame is operating, the movable seal-bar moves up and down. The movable seal-bar moves down and approaches the fixed seal-bar. During two seal-bars hold the films, heats are conveyed to films with pressing loads. The starting time of this operation is defined

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as $t_s$. Then, the movable seal-bar moves back to the home position. The time of this operations is corresponding to the end of the sealing time $t_e$. Then the heated film moves off, and a new one is supplied, and the next sealing operation is carried out.

Though we can observe the seal-bar temperature with thermocouples, we do not have any method of measuring seal time $ST$ and seal load $L$. We consider the load $L$ as follows:

$$L = W + Kl. \tag{1}$$

$W$ is weight of the movable seal-bar, $K$ is spring constant, and $l$ is amount of contraction of the coil spring. During machine operation, $W$ is constant, but $l$ is a random variable influenced by material thickness which may be changeable owing to sorts of materials and their spatial locations. For evaluating $L$ appropriately we need to observe the values of the $l$.

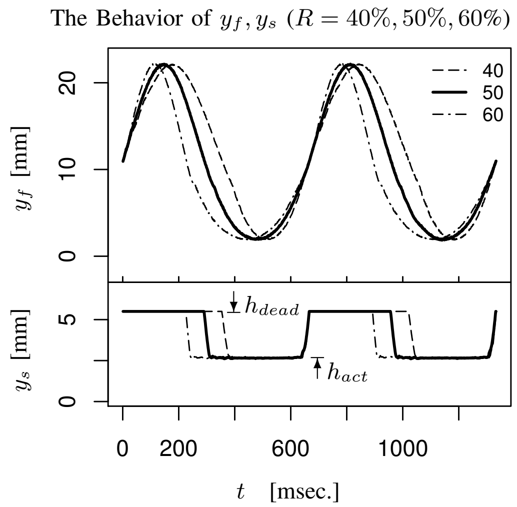



Fig. 2. The Observations of Moving Frame $y_f$ and That of Movable Seal-Bar $y_s$

Then we observe the distance between moving frame and fixed seal-bar, and the distance between movable seal-bar and fixed seal-bar (See Fig.2). They were observed under the following conditions; the bag-making machine works 90 times sealing per minute, the ratio of sealing time ($R$) in a cycle takes the three levels of 40%, 50%, and 60%. (This $R$ is explained in III-A.)

We found that the relative locations of moving frame $y_f$ behaves as distorted cosine wave, and that ones of movable seal-bar $y_s$ behaves as square wave.

The $h_{act}$ is the height of the bottom part of $y_s$ from origin, and the top part is corresponding to the value of the observation limit, over which we can not observe the locations.

Research purpose of this paper is to estimate the two process parameters $W$ and $ST$, by observed values $y_f$ and $y_s$ from the bag making machine.

## III. Modeling and Estimate of Process Parameters

We describe the modeling for the bag-making process and the estimation procedures of the model parameters and the process parameters.

The process parameters are the seal time and the seal load. If all that we have to estimate is just one parameter for the seal time, the model for the observations $y_s$'s may be built simply, but we need to estimate two parameters, the seal time and the seal load, jointly. For estimating the seal load, it is necessary to estimate the amount of contraction of the coil spring, and we create a model regarding the behavior of the movable frame causing the movement of movable seal-bar.

### A. Modeling

At first we examine the structure of the moving frame.

One of candidate models is a mechanical model using the design information. The working mechanism is shown in Fig.3. It consists of two four-bar mechanisms and a slider-crank mechanism. We describe the correspondences of the objects in Fig.1 and Fig.3. We can observe the following relationship: the swing block vs. side P3P4 about the fulcrum point C2, and the moving frame vs. the slider-crank mechanism M.

Now we are able to construct a theoretical model of this mechanism, but we cannot obtain the design information of each part. On account of such a reason, if we used a mechanical model, we would have to estimate a design parameters with $y_s$. However, this theoretical model requires many parameters included in trigonometric functions that do not have inverse functions explicitly (see Appendix); it's difficult to estimate these parameters.

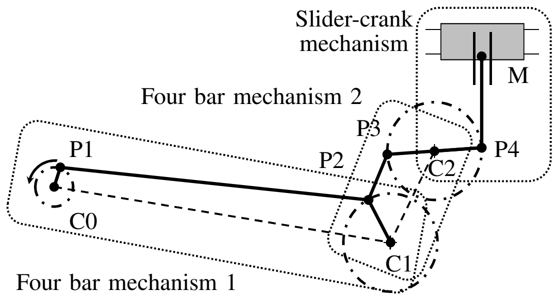



Fig. 3. The Working Mechanisms of the Sealing Part in the Bag-Making Machine.

That is why we need a simple model that can represent well the behavior of the moving frame.

In the bag-making machine, the position of each part is decided by phase angle $\theta$ of driving part C0. Fig.4 shows the data $y_u$; it is observed under the condition of Seal Ratio 50% (angular velocity of driving part is constant.). The $y_u$'s are standardized to -1 and 1 by maximum value and minimum value. The starting value of $y_u$ is 1 and the phase angle $\theta$ takes from 0 to $2\pi$. For comparison the cosine wave is drawn with $y_u$ curve, and the differences between $y_u$ and cosine wave are shown below.

A periodic difference of $y_u$ and $\cos\theta$ in Fig.4 suggests us to use a distorted cosine function for modeling. Now we propose new nested function model. The proposed nested trigonometric function will be shown below.

$$F_f(\theta \,|\, a, b, c) = a\cos(\theta + b\sin\theta) + c, \tag{2}$$

$$F_u(\theta \,|\, b) = \cos(\theta + b\sin\theta), \tag{3}$$

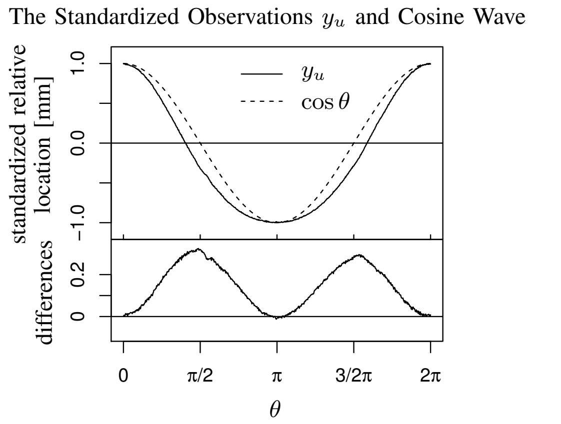



Fig. 4. The Standardized Observations of Moving Frame and Cosine Wave

where $a, b, c$ are parameters for adjusting shapes and locations. When $b$ takes value from -1 to 1, the function $F_u$ may seems to be similar to that of cosine, but the bottom part of this function greatly different with the cosine wave. This difference is of importance for estimating the two process parameters (the seal time and the seal load). Fig.5 shows the tail behaviors of the function of $F_u$ with different model parameters.

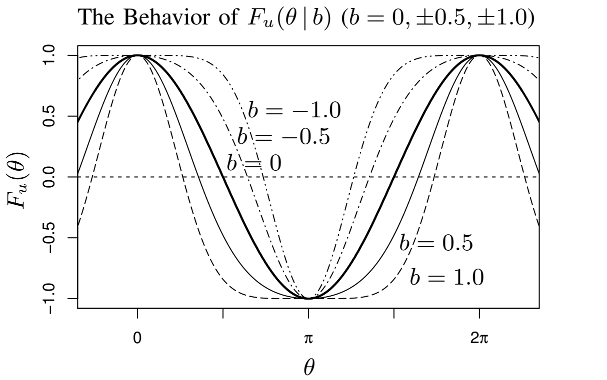



Fig. 5. The Behavior of the Function $F_u(\theta \mid b)$

The variable $y_s$ can be classified into three states from the mechanical view point: Coupling, Sealing, and Outlying. 'Coupling' is the state that movable seal-bar in the observation range of the sensor is moving; 'Sealing' is the state that seal operation is performed, and the height of movable seal-bar hold $h_{act}$. 'Outlying' is the state that the movable seal-bar is out of observation range of the sensor.

The above seal-bar model designated as $F_s$ can be expressed as the following equations.

$$
\begin{aligned}
&F_s(\theta \mid d, h_{act}, h_{dead}) \\
&\quad = \begin{cases} h_{dead} & \text{if } F_f(\theta \mid a, b, c) + d > h_{dead}, \\ h_{act} & \text{if } F_f(\theta \mid a, b, c) + d < h_{act}, \\ F_f(\theta \mid a, b, c) + d & \text{otherwise.} \end{cases}
\end{aligned} \tag{4}
$$

Here, we describe the relationship between $\theta$ and $t$.

The bag-making machine has two modes: one is the operation of sealing and the other is supplying the materials and these two modes are alternatively performed. The rotation angle of the driving part is divided into two equal parts, and each part is mapped to each mode. Each mode has each different angular velocity of the driving part.

For explanation of the phase angle, we have the following notation: we write angular velocities $\omega_f$ and $\omega_s$ for starting angle of supplying mode $\theta_f$ and that of sealing mode $\theta_s$, individually. For phase angle of driving part at $t = 0$ we write $\theta_0$. At the time $t$, The phase angle $\theta(t)$ is,

$$
\begin{aligned}
&\theta(t \mid \theta_s, \theta_f, \omega_s, \omega_f, \theta_0) \\
&= \begin{cases} \omega_f t + \theta_0 & \text{if } 0 < t \le \frac{\theta_s - \theta_0}{\omega_f} := t_1 \\ \omega_s (t - t_1) + \theta_s & \text{if } t_1 < t \le t_1 + \frac{\theta_f - \theta_s}{\omega_s} := t_2 \\ \omega_f (t - t_2) + \theta_f & \text{if } t_2 < t \le t_2 + \frac{2\pi - \theta_f}{\omega_f} := t_3 \\ \omega_f (t - t_3) & \text{otherwise } (t_3 < t \le t_3 + \frac{\theta_0}{\omega_f}). \end{cases}
\end{aligned} \tag{5}
$$

As each mode is assigned equally, we can express as

$$\theta_f = \theta_s + \pi. \tag{6}$$

Using sealing ratio $R$ and cycle time of driving part $T$, $\omega_f$ and $\omega_s$ can be expressed as follows:

$$\omega_f = \frac{\pi}{(1 - R)T}, \tag{7}$$

$$\omega_s = \frac{\pi}{RT}. \tag{8}$$

The parameters included in (5) are substituted for ($6 \sim 8$), and then (5) becomes

$$\theta(t \mid \theta_s, \theta_f, \omega_s, \omega_f, \theta_0) = \theta(t \mid \theta_0, \theta_s, R, T). \tag{9}$$

For example, the behavior of the function $\theta$ is shown in Fig. 6.

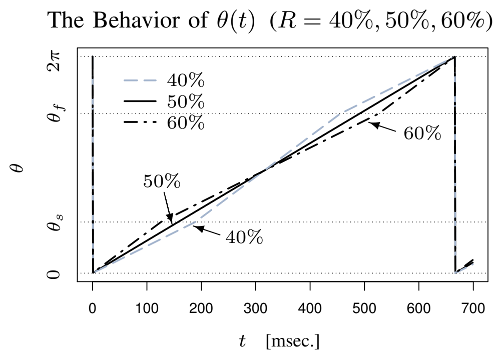



Fig. 6. The Behavior of Phase Angle $\theta(t)$

### B. Estimation of model parameters

We consider the observation data $\boldsymbol{y}_f = \{y_{f1}, \ldots, y_{fn}\}$, $\boldsymbol{y}_s = \{y_{s1}, \ldots, y_{sn}\}$, at the time $\boldsymbol{t} = \{t_1, \ldots, t_n\}$.

Because parameter $h_{dead}$ is corresponding to the height of top part of $y_s$ and is corresponding to maximum value of $y_s$, estimator of $h_{dead}$ included in (4) is,

$$\widehat{h_{dead}} = \max\{y_{s1}, \ldots, y_{sn}\}. \tag{10}$$

Run of the data corresponding to $h_{dead}$ appears in every cycle. We can use the top of the run as the landmark, and we can divide the dataset into smaller $m$ sets of cycle data by each landmark point. We write the data of $i$th cycle as $t_{[i]}, y_{f[i]}, y_{s[i]}$ $(i = 1, \ldots, m)$, then,

$$\begin{aligned} \boldsymbol{t} &= \{\boldsymbol{t}_{[1]}, \ldots, \boldsymbol{t}_{[i]} \ \ldots, \boldsymbol{t}_{[m]}\}, \\ \boldsymbol{y}_f &= \{\boldsymbol{y}_{f[1]}, \ldots, \boldsymbol{y}_{f[i]}, \ldots, \boldsymbol{y}_{f[m]}\}, \\ \boldsymbol{y}_s &= \{\boldsymbol{y}_{s[1]}, \ldots, \boldsymbol{y}_{s[i]}, \ldots, \boldsymbol{y}_{s[m]}\}. \end{aligned}$$

To estimate the parameters in each cycle, we take out $\boldsymbol{t}_{[i]}, \boldsymbol{y}_{f[i]}$ and $\boldsymbol{y}_{s[i]}$ for an $i$th cycle. Fitting a model to each data, we estimate parameters.

Here, the size of data $\boldsymbol{t}_{[i]}, \boldsymbol{y}_{f[i]}, \boldsymbol{y}_{s[i]}$ is $n_i$,

$$\begin{aligned} \boldsymbol{t}_{[i]} &= \{t_{[i]1}, \ldots, t_{[i]j}, \ldots, t_{[i]n_i}\}, \\ \boldsymbol{y}_{f[i]} &= \{y_{f[i]1}, \ldots, y_{f[i]j}, \ldots, y_{f[i]n_i}\}, \\ \boldsymbol{y}_{s[i]} &= \{y_{s[i]1}, \ldots, y_{s[i]j}, \ldots, y_{s[i]n_i}\}. \end{aligned}$$

We set

$$\beta_f = (\theta_0, \theta_s, R, T, a, b, c). \tag{11}$$

For every cycle data, we estimate $\beta_f$'s by non-linear least square methods.

$$S_{f[i]}(\beta_f) = \sum_{j=1}^{n_i} \left[F_f(\theta(t_{[i]j}) \,|\, \beta_f) - y_{f[i]j}\right]^2. \tag{12}$$

Estimator of parameter $\widehat{\beta_{f[i]}}$ is

$$\widehat{\beta_{f[i]}} = \arg\min_{\beta_f} S_{f[i]}(\beta_f). \tag{13}$$

To estimate the parameter $F_s$ we derive the following equation replacing the $\beta_f$ with the estimate obtained from (12):

$$\begin{aligned} &F_{s[i]}(\theta(t) \,|\, d, h_{act}, h_{dead}) \\ &= \begin{cases} h_{dead}, & \text{if } F_{f[i]}(\cdot) + d > h_{dead}, \\ h_{act}, & \text{if } F_{f[i]}(\cdot) + d < h_{act}, \\ F_{f[i]}(\cdot) + d, & \text{otherwise.} \end{cases} \end{aligned} \tag{14}$$

where

$$F_{f[i]}(\cdot) = F_{f[i]}(\theta(t) \,|\, \widehat{\beta_{f[i]}}), \tag{15}$$

$$\beta_s = (d, h_{act}, h_{dead}). \tag{16}$$

Residual sum of squares $S_{s[i]}$ is

$$S_{s[i]}(\beta_s) = \sum_{j=1}^{n_i} \left[F_s(\theta(t_{[i]j}) \,|\, \beta_s) - y_{s[i]j}\right]^2. \tag{17}$$

and estimator of parameter $\widehat{\beta_{s[i]}}$ is

$$\widehat{\beta_{s[i]}} = \arg\min_{\beta_s} S_{s[i]}(\beta_s). \tag{18}$$

### *C. Calculate of process parameters*

We describe the procedure to calculate of process parameters by using estimators of model parameters mentioned in the previous section.

We express the $i$th estimators of model parameters as

$$\widehat{\beta_{f[i]}} = (\widehat{\theta_{0[i]}}, \widehat{\theta_{s[i]}}, \widehat{R_{[i]}}, \widehat{T_{[i]}}, \widehat{a_{[i]}}, \widehat{b_{[i]}}, \widehat{c_{[i]}}), \tag{19}$$

$$\widehat{\beta_{s[i]}} = (\widehat{d_{[i]}}, \widehat{h_{act[i]}}, \widehat{h_{dead[i]}}.) \tag{20}$$

*1) Seal time:* Seal time $ST_{[i]}$ is calculated from the solution of equation as follows.

$$F_{s[i]}(\theta(t) \,|\, \widehat{\beta_{s[i]}}) - \widehat{h_{act[i]}} = 0. \tag{21}$$

Here the time $t$ is in the interval $[t_{[i]1}, t_{[i]n_i}]$. We can obtain two solutions $t_{s[i]}, t_{e[i]} (t_{s[i]} < t_{e[i]})$, by using a numerical method. Using these solutions, estimator of seal time $\widehat{ST_{[i]}}$ becomes

$$\widehat{ST_{[i]}} = t_{e[i]} - t_{s[i]}. \tag{22}$$

In addition, if we have an interest in the seal time of under the suitable seal load $L_s (L_s > W)$, we are able to refine (21) as follows:

$$F_{s[i]}(\theta(t) \,|\, \widehat{\beta_{s[i]}}) - \widehat{h_{act[i]}} - l_s = 0, \tag{23}$$

where $l_s$ is the amount of contraction of the coil spring corresponding to $L_s$:

$$l_s = \frac{L_s - W}{K}. \tag{24}$$

*2) Seal Load:* Seal load is related to (1). Amount of contraction of the coil spring change with time, but now, we focus on this maximum value. $l$ is

$$\widehat{l_{[i]}} = \widehat{h_{act[i]}} - (\widehat{d_{[i]}} - \widehat{c_{[i]}} - \widehat{a_{[i]}}). \tag{25}$$

## IV. The Results

In this section, by using three sets of data, shown Fig. 2, we estimate the process parameters, and discuss the results of calculation.

We can not obtain the measurements of seal time and amount of contraction of coil spring up to now. Therefore comparing estimated values with theoretical values, we can evaluate the validity of the results. To evaluate the advantage of the proposed model $F_f$ of nested trigonometric function, we compare with sine wave model $G_f$ and $G_s$.

### *A. Sine Wave Model $G_f$ and $G_s$*

The model $G_f$ for moving frame is

$$G_f(\theta(t) \,|\, a_r, c_r) = a_r \cos(\theta(t)) + c_r, \tag{26}$$

where $\theta$ is used (9). The model $G_s$ for the seal-bar is

$$\begin{aligned} &G_s(\theta(t) \,|\, d_r, h_{act\,r}, h_{dead\,r}) \\ &= \begin{cases} h_{dead\,r}, & \text{if } G_f(\cdot) + d_r > h_{dead\,r}, \\ h_{act\,r}, & \text{if } G_f(\cdot) + d_r < h_{act\,r}, \\ G_f(\cdot) + d_r, & \text{otherwise,} \end{cases} \end{aligned} \tag{27}$$

where

$$G_f(\cdot) = G_f(\theta(t) \,|\, a_r, c_r). \tag{28}$$

### *B. Results of fitting*

We check the result of applying the models of moving frame and movable seal-bar described in the previous section to the data.

Fig.7 shows the result of applying the model $F_f$ and $G_f$ to typical observation data $y_{f[i]}$ in seal ratio $R = 60\%$. The residual errors are shown in the lower. Fig.7 illuminates that the residual errors of the proposed model are much smaller than those of the $G_f$ model.

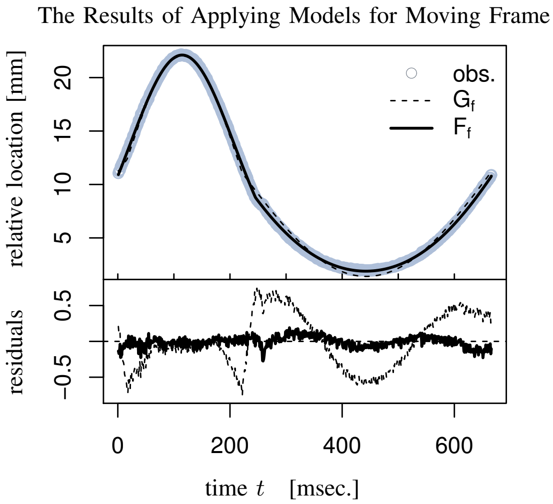



Fig. 7. The Results of Applying Models for Moving Frame

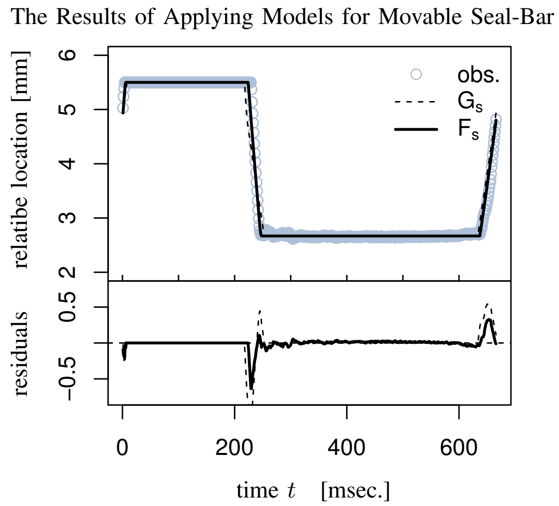



Fig. 8. The Results of Applying Models for Movable Seal-Bar

Fig.8 shows the result of applying the model $F_s$ and $G_s$ to the observations of movable seal-bar.

Most of the data are near the observation limit $h_{dead}$ or the height of the sealing movement $h_{act}$, and the lack of fits on $G_f$ in the moving frame data is small, and the proposed model $F_s$ is moderately superior to $G_s$ in the coupling part. The AIC( Akaike's Information Criteria) value of $G_s$ is $-483.96$, and $F_s$'s is $-1518.10$. Based on these findings, we consider that our proposed models are better.

In addition, Table I shows the process parameter estimated by the estimator of the model parameter. The design value of seal time is 400msec. As for the difference between design the value and each estimate, the difference of the proposed model $F_s$ is smaller. The estimate $\hat{l}_s$ using the sine wave model is larger than that by our proposed models. This comes from the difference of fitting level in each models for the moving frame.

### C. Estimated Process Parameters

We apply the models to every cycles in each observation data R = 40%, 50%, and 60%, and examine the behavior of process values calculated by estimated model parameters.

TABLE I
ESTIMATE VALUES OF PARAMETERS($R = 60\%, k = 10$)

| | $\widehat{ST}$ | $\hat{l}$ |
|---|---|---|
| Design Value | 400 | unknown |
| Proposed Model $F_s$ | 391 | 6.76 |
| Sine Model $G_s$ | 381 | 7.53 |

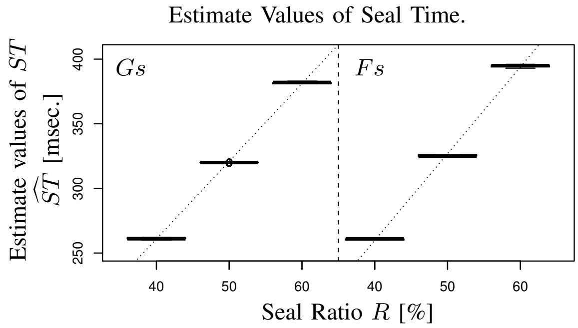



Fig. 9. The Box Plot of Estimate Value of Seal Time

The box plot in Fig. 9 shows calculated seal time every ratio $R$. Because variations in each level are small, each box is squashed. The relation of seal time and seal ratio are almost linear. Table II summarizes the several statistics of seal times estimates for each the design value. Comparing with $ST$ from each model, as the seal ratio increase, difference with each design value on sine curve model becomes larger than that on proposed model. The relationship between $R$ and $\widehat{ST}$, can be expressed in a simple regression model.

$$\begin{aligned} design &: ST = 6.67R, \\ G_s &: ST = 20.28 + 6.015R, \\ F_s &: ST = -8.35 + 6.706R, \end{aligned}$$

where the design model is described in the operation speed is 90 times per minutes. These regression model shows that the model of $F_s$ is similar to design model than that of $G_s$. According to these results, we conclude that the poposed model $F_s$ is much better than thta of $G_s$.

We examine the amount of contraction of coil spring in Fig.10.

Because there are no change the seal-bar and material film thickness, spring shrinkage calculated from each data should be constant values despite changing the seal ratio. The $G_s$ model has a positive trend for seal ratio $R$, but the proposed model gives us the desirable result that the effect of the seal ratio appears to be very small.

TABLE II
SUMMARY OF ESTIMATE VALUE OF SEAL TIME $\widehat{ST}$

| $R[\%]$ | 40 | | 50 | | 60 | |
|---|---|---|---|---|---|---|
| Model | $Gs$ | $Fs$ | $Gs$ | $Fs$ | $Gs$ | $Fs$ |
| min. | 261.0 | 261.0 | 319.0 | 321.0 | 380.0 | 383.0 |
| ave. | 261.2 | 261.6 | 319.5 | 325.2 | 380.6 | 389.7 |
| max. | 262.0 | 263.0 | 320.0 | 326.0 | 382.0 | 391.0 |
| sd. | 0.4 | 0.6 | 0.5 | 0.5 | 0.6 | 1.6 |
| Design Value | 266.7 | | 333.4 | | 400.0 | |

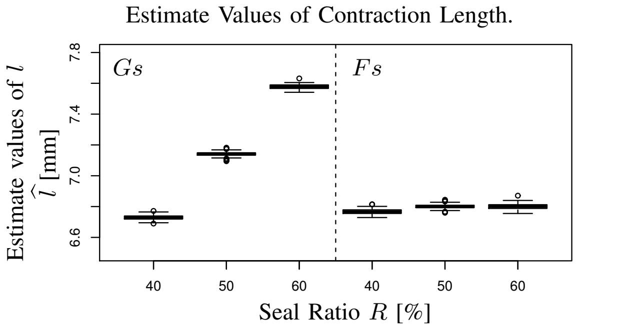



Fig. 10. The Box Plot of Estimate Values of Spring Contraction Length

## V. Conclusion and Future

The purpose of this article was to specify the state of the bag-making machine and to estimate their characteristic values. Then we proposed the statistical models for estimation.

Our proposed method is as follows: The model is constructed with several mathematical nested functions which correspond to each state of the bag-making machine. The model parameters were estimated by non-linear reast square error method. The state of object was specified and characteristic values were calculated. We applied this procedure to the observation data obtained from the heat seal process in the bag-making machine, and we could specify state of the seal process and calculated the characteristic values.

As for the observation data, we found they behaved as distorted sine wave. We were able to obtain good results by using nested trigonometric function for the model.

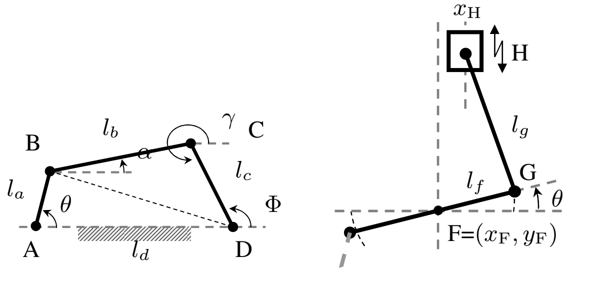
The proposed procedure was valid and useful to obtain characteristic values which has been obtained to be obtained in oparating bag-making machines.

We are expected to analyze the data set from the operating machine in real time because computing power has improved recently for managing production process.

## The Theoretical Model of The Moving Frame in The Bag-making Machine.

We consider the mechanisms which are used in the bag-making machine [6], [7].

In this appendix we have the following notation: we write $x_{\mathrm{A}}$ as coordinate x and $y_{\mathrm{A}}$ as coordinate y at point A, and $\angle\mathrm{AOB}$ as angle between line segment OA and OB, and $\overline{\mathrm{AB}}$ as length of line segment AB, $\angle_x\mathrm{AB}$ as angle between line segment AB and x-axis.




(a) Four-Bar mechanism (b) Slider-Crank mechanism

Fig. A.1. The Mechanism Frameworks

### A. Four-bar mecanism

In Fig.1(a), $\theta$ is the input angle of this mechanism. The following system of equations is satisfied.

$$\begin{cases} l_a\cos\theta + l_b\cos\alpha + l_c\cos\gamma = l_d \\ l_a\sin\theta + l_b\sin\alpha + l_c\sin\gamma = 0. \end{cases} \tag{A.1}$$

Here $\Phi$ is the output angle.

$$\Phi = \pi - (\angle\mathrm{ADB} + \angle\mathrm{BDC}). \tag{A.2}$$

According to the cosine theorem, the two triangles $\triangle\mathrm{ABD}$ and $\triangle\mathrm{BCD}$ are replaced as follows:

$$\Phi = \pi - \left(\tan^{-1}\frac{l_a\sin\theta}{l_d - l_a\cos\theta} + \cos^{-1}\frac{l_c^2 + L_a^2 - l_b^2}{2l_cL_a}\right), \tag{A.3}$$

where

$$L_a = \sqrt{l_a^2 - 2l_al_d\cos\theta + l_d^2}. \tag{A.4}$$

Paying attention to the angle of placement of the mechanism in xy coordinate, we reset $\theta$ as $\angle_x\mathrm{AB}$, and $\Phi$ as $\angle_x\mathrm{CD}$. Then (A.3) becomes

$$\begin{aligned} \Phi &= \pi + \theta_x \\ &\quad - \left(\tan^{-1}\frac{l_a\sin(\theta-\theta_x)}{l_d - l_a\cos(\theta-\theta_x)} + \cos^{-1}\frac{l_c^2 + L_a^2 - l_b^2}{2l_cL_a}\right), \\ &= F_\Phi(\theta \mid \beta_\Phi), \end{aligned} \tag{A.5}$$

where $\theta_x$ is $\angle_x\mathrm{AD}$ and

$$L = \sqrt{l_a^2 - 2l_al_d\cos(\theta-\theta_x) + l_d^2}, \tag{A.6}$$

$$\beta_\Phi = (l_a, l_b, l_c, l_d, \theta_x). \tag{A.7}$$

### B. Slider-crank mechanism

Fig.1(b) shows that this mechanism has the point G swinging around the point F, and the point H restricted x coordinate to $x_{\mathrm{H}}$. The point H reciprocates parallel to y axis.

$$\begin{aligned} y_{\mathrm{H}} &= y_{\mathrm{F}} + l_f\sin\theta + \sqrt{l_g^2 - \{l_f\cos\theta - (x_{\mathrm{H}} - x_{\mathrm{F}})\}^2}, \\ &= F_H(\theta \mid \beta_H), \end{aligned} \tag{A.8}$$

where

$$\beta_H = (l_f, l_g, x_{\mathrm{H}}, x_{\mathrm{F}}, y_{\mathrm{F}}). \tag{A.9}$$

### C. The Theoretical Model for Moving Frame

Fig.3 shows that these mechanisms are placed at a suitable angle on the xy coordinate in the bag-making machine.

The origin of the xy coordinate is placed on driving part C0, and the y axis is taken parallel to reciprocating motion of M, and $\theta$ is taken as $\angle_x\mathrm{C0P1}$ .

As regards the four-bar mechanism 1, the output angle $\Phi_1$ is as follows:

$$\Phi_1 = F_\Phi(\theta \mid \beta_{\Phi 1}), \tag{A.10}$$

where

$$\beta_{\Phi 1} = (\overline{C0P1}, \overline{P1P2}, \overline{P2C1}, \overline{C0C1}, \angle_x\mathrm{C0C1}). \tag{A.11}$$

As regards the four-bar mechanism 2, the output angle $\Phi_2$ is as follows:

$$\Phi_2 = F_\Phi(\Phi_1 \mid \beta_{\Phi 2}), \tag{A.12}$$

where

$$\beta_{\Phi 2} = (\overline{C1P2}, \overline{P2P3}, \overline{P3C2}, \overline{C1C2}, \angle_x \mathrm{C1C2}). \quad \text{(A.13)}$$

Then, $y_M$ is,

$$y_M = F_H(\Phi_2 - \pi \,|\, \beta_{H1}), \quad \text{(A.14)}$$

where

$$\beta_{H1} = (\overline{C2P4}, \overline{P4M}, x_{\mathrm{M}}, x_{\mathrm{C2}}, y_{\mathrm{C2}}). \quad \text{(A.15)}$$

As the result, theoretical model for moving frame is written by $F_H, F_\Phi$ as follows:

$$F_M = F_H(F_\Phi(F_\Phi(\theta)) - \pi \,|\, \beta_{\Phi 1}, \beta_{\Phi 2}, \beta_{H1}).$$

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