Stackelberg Game in a Supply Chain Led by Retailers in a Fuzzy Environment

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Abstract—**In this paper, a two-echelon supply chain that includes one manufacturer and one retailer is considered. In this supply chain, the retailer plays the dominant role. Considering the various factors of uncertainty in a real economy, the market demand function, manufacturing costs, and retail operating costs are considered as the fuzzy variables. The Stackelberg game is adopted to solve the problem between the retailer and the manufacturer. The expected value model and the chance-constrained model are introduced to solve for the optimal decision. The optimal wholesale price and marginal profit per unit that are at equilibrium in each model are provided to determine the maximum profit for the retailer and the manufacturer. Finally, a numerical example illustrates the effectiveness of the supply chain game model.**

Key words—**Supply chain models, Stackelberg game, Fuzzy environment, Optimal strategy**

I. INTRODUCTION

WITH the globalization of science and technology and the accompanying productivity improvements, the variety and quality of goods today have improved significantly. As a result, consumers now expect retailers to provide a wide-range and adequate supply of goods. Such a change in market demand has caused a gradual increase in the number of large-scale retailers whose retail powers have improved and with that their positions in the industrial supply chains. Recently, a supply chain model dominated by retailers has attracted considerable attention, with many researchers focusing on this.

Reference [1] conducted an in-depth analysis of price competition under a retailer duopoly, concluding that the equilibrium prices in a Stackelberg game model (leader/follower model) are higher than those under the Nash game model (bargaining model). Further, they found that product differentiation benefits the manufacturer and not the retailer, but shop differentiation benefits the retailer and not the manufacturer. Reference [2] analyzed the pricing and ordering strategy of the two-stage supply chain led by a retailer in the situation of demand uncertainty, proving the existence and uniqueness of an optimal strategy. Reference [3] considered one supply chain that consisted of two manufacturers and one retailer, and another that consisted of one manufacturer and two retailers; they then analyzed the different outputs under wholesale price contracts and revenue sharing contracts in order to determine the advantage of a revenue sharing contract.

Reference [4] summed up the pricing decision process of a one supplier, multiple retailer supply chain, considering different degrees of product substitution and retailer supply. Reference [5] studied the coordination of the supply chain with quantity discount contracts. Reference [6] studied the price coordination problem in a three-echelon supply chain and considered three types of channel structures. Reference [7] analyzed a discount-pricing problem over two periods and revealed that the profit function over the two periods is concave if the target consumers are loss neutral.

Reference [8] considered the pricing decisions for two substitutable products in a supply chain with one common retailer and two competitive manufacturers, analyzing the effects of the two different manufacturing competitive strategies and the different channel members' power structures on the optimal pricing decisions. In order to explore how the manufacturer and the retailer make their decisions about wholesale price, retail price, and collection rate under symmetric and asymmetric information conditions, reference [9] discussed the optimal decision problem in a closed-loop supply chain with symmetric and asymmetric information structures using game theory. They established four game models that examined the strategies of each firm and explored the role of the manufacturer and the retailer in the different game scenarios under symmetric and asymmetric information structures. Reference [10] chose a two-level supply chain led by a retailer, consisting of two retailers and one manufacturer in a retailer market, and made a comparative analysis of the equilibrium results of Cournot and Stackelberg competition between the retailers; they found that the differences between the retailers were beneficial to the manufacturer.

The data and parameters used in a Stackelberg model must be determinate [11]. However, in a real decision-making process, fluctuations in market demand and manufacturing costs often lead to uncertainty. Essentially, uncertainty greatly limits the application range of the determined value model. Therefore, to enhance the ability of the classical game theory model to explain realistic problems, it is necessary to extend the deterministic model to non-deterministic cases.

The proposition of fuzzy sets and fuzzy logic [12] provide an effective method of measuring and understanding elements with unclear boundaries. For example, reference [13] converted a fuzzy number into a certain value and under the condition that demand and supply are fuzzy linear

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functions, analyzed the consumer surplus and producer surplus at market equilibrium. Reference [14] studied the Cournot model in a fuzzy environment, obtained the optimal production of manufacturers in a fuzzy environment by using a triangular fuzzy number, and analyzed the impact of fuzzy parameters of the inverse demand function and the cost function on manufacturer profits. Reference [15] considered supply chain models with two competitive manufacturers acting as leaders, and a retailer acting as a follower in a fuzzy decision environment; the two manufacturers were assumed to pursue Cournot competitive behavior, and the expected value optimum policy and chance-constrained programming models were derived; reference [15] concluded that the supply chain member confidence in the level of profits affects the final optimal solution.

Based on the above literature, this paper discusses the equilibrium solving method and the corresponding equilibrium results of the Stackelberg game model with fuzzy demand and fuzzy costs. References [16-20] studied supply chain game problems and the supply chain coordination mechanism under a fuzzy demand environment. The fuzzy objects are mainly the consumer's demand function and the manufacturer's cost. In order to find the optimal price strategies for the manufacturer and the retailer to realize maximum profits, this paper uses the supply chain expected value model and the chance-constrained mechanism model [21-24]. Expanding on the existing literature, this paper not only considers manufacturing costs, but also retail operating costs, thus further extending the scope of the fuzzy variables, bringing this research scenario closer to economic reality.

II. Preliminaries

The fuzzy set theory has developed very quickly since its inception. The corresponding fuzzy techniques encompass almost all areas of economic research. Fuzzy theory uses $Pos\{A\}$ to describe the probability of event occurrence of A. In order to guarantee the rationality of $Pos\{A\}$ in practice, it needs to display certain mathematical properties [13].

Provided that Θ is a nonempty set, $P(\Theta)$ is the power set of Θ; then,

Axiom 1. $P\{\Theta\}=1$;

Axiom 2. $P\{\Phi\}=1$;

Axiom 3. For any set $\{A_i\}$ in $P(\Theta)$, $Pos\{U_i A_i\}=\sup_i Pos\{A_i\}$;

If the above three axioms are met, it is referred to as a possibility measure and the triple ($\Theta, P(\Theta), Pos$), as a possibility space.

The following definitions and properties in the analysis serve as the premise and foundation of this research:

Definition 1. [14] Provided that fuzzy variable ξ is a function from a possibility space ($\Theta, P(\Theta), Pos$) to a real line R, then ξ can be said to be a fuzzy variable defined on a possibility space ($\Theta, P(\Theta), Pos$).

Definition 2. [15] Fuzzy variable ξ is a non-negative (or positive) variable, if and only if $Pos\{\xi<0\}=0$ (or $Pos\{\xi\le 0\}=0$).

Proposition 1. [15] Provided that ξ_i is a mutually independent fuzzy variable, function $f_i: R\to R, i=1,2,...m$, then $f_1(\xi_1)$, $f_2(\xi_2)$, …, $f_m(\xi_m)$ are also mutually independent fuzzy variables.

Definition 3. [15] Provided that ξ is a fuzzy variable defined in the possibility space ($\Theta, P(\Theta), Pos$) and $\alpha\in(0,1]$, then

$\xi_\alpha^L=\inf\{r\mid Pos\{\xi\le r\}\ge\alpha\}$ and $\xi_\alpha^U=\sup\{r\mid Pos\{\xi\ge r\}\ge\alpha\}$ are, respectively, referred to as the α-pessimistic and α-optimistic values of fuzzy variable ξ.

Here, r is the value that fuzzy variable ξ achieves with possibility α. The α-pessimistic value ξ_α^L is the infimum value that ξ achieves with possibility α, and α-optimistic value ξ_α^U is the supremum value that ξ achieves with possibility α.

Example 1. The triangular fuzzy variable $\xi=(a,b,c)$ has its α-pessimistic value and α-optimistic value as $\xi_\alpha^L=a+(b-a)\alpha$ and $\xi_\alpha^U=c-(c-b)\alpha$.

Proposition 2. [16], [17] If there are two mutually independent fuzzy variables, which are expressed as ξ and η, then we can conclude the following:

(1) For any $\alpha\in(0,1]$, $(\xi+\eta)_\alpha^L=\xi_\alpha^L+\eta_\alpha^L$

(2) For any $\alpha\in(0,1]$, $(\xi+\eta)_\alpha^U=\xi_\alpha^U+\eta_\alpha^U$

(3) For any $\alpha\in(0,1]$, $(\xi\cdot\eta)_\alpha^L=\xi_\alpha^L\cdot\eta_\alpha^L$

(4) For any $\alpha\in(0,1]$, $(\xi\cdot\eta)_\alpha^U=\xi_\alpha^U\cdot\eta_\alpha^U$.

Definition 4. [16] Let ξ be a fuzzy variable, and r_0 be a real number defined from $-\infty$ to ∞. The expected value of ξ is defined by

$$E[\xi]=\int_0^{+\infty}C_r\{\xi\ge r_0\}dr_0-\int_{-\infty}^0 C_r\{\xi\le r_0\}dr_0.$$

Provided that at least one of the two integrals is finite, especially, if ξ is a non-negative fuzzy variable, then

$$E[\xi]=\int_0^{+\infty}C_r\{\xi\ge r_0\}dr_0.$$

Example 2. The triangular fuzzy variable $\xi=(a,b,c)$ has an expected value of

$$E[\xi]=\frac{a+2b+c}{4}.$$

Proposition 3. [16] Provided that ξ is a fuzzy variable with limited expectations, then

$$E[\xi]=\frac{1}{2}\int_0^1(\xi_\alpha^L+\xi_\alpha^U)d\alpha.$$

Proposition 4. [16] Provided that ξ and η are mutually independent fuzzy variables with limited expectations, then for any number a and b, the formula is as follows:

$E[a\xi + b\eta] = aE[\xi] + bE[\eta]$.

III. MODEL DESCRIPTIONS

This paper examines a two-stage supply chain consisting of a supply chain with a manufacturer and a retailer. The manufacturer sells wholesale goods to the retailer, and then the retailer sells the ordered goods to the customer. In order to maximize their profits, the manufacturer and the retailer formulate the optimal wholesale price and the marginal profit per unit. When the retailer is dominant in the supply chain, that company becomes the core enterprise in the supply chain. In the Stackelberg model, the dominant player makes the decisions first; the follower then makes decisions according to the dominant player's decisions. Therefore, in the two-stage supply chain Stackelberg game led by a retailer, the retailer first decides the marginal profit per product unit and after observing the retailer's unit profit, the manufacturer decides on the wholesale price. Thus, the retailer and the manufacturer maximize their profits. In this model, the retailer's operating costs will be considered to move the model closer to reality. In order to construct a two-stage supply chain model in a fuzzy environment, the following basic symbols will be used.

Notation

w The wholesale price per product unit;

c_m The manufacturing cost per product unit;

c_r The retailer's operating cost per product unit;

v_m The manufacturer's single product profit

$v_m = w - c_m$

m The retailer's wholesale purchase price and customer sale price differential, referred to as marginal profit per unit;

Π_M The profits of the manufacturer, and the function of w and m;

Π_R The profits of the retailer, and the function of w and m.

Consider that the customer demand function is a linear decreasing function on the wholesale prices and the marginal profit per unit is denoted as $D = a - b(w+m)$. Here, a and b are two mutually independent non-negative fuzzy variables. Parameter a represents the maximum market capacity and parameter b represents the demand to price change rate; the customer demand D is also a fuzzy variable. As the demand in reality is positive, $Pos\{a - b(w+m) \le 0\} = 0$.

The profit function of the manufacturer and the retailer can be expressed, respectively, as:

$$\Pi_M(w,m) = (w - c_m)D \tag{1}$$

$$\Pi_R(w,m) = (m - c_r)D. \tag{2}$$

IV. FUZZY TWO-ECHELON SUPPLY CHAIN MODELS

We analyzed the situation where a retailer has a dominant role. Here, the retailer becomes the key enterprise in the supply chain, and the manufacturer becomes the follower. Suppose that the information between the retailer and manufacturer is symmetric, according to the Stackelberg game model the retailer will make decisions first. Further, since this paper considers the operational costs of the retailer, the decision variable of the retailer is the profit of the unit product. Hereafter, the manufacturer will formulate the product's wholesale price according to the observed profit of the unit product; In addition, both the retailer as well as the manufacturer can realize their biggest profit. Based on the previously stated basic assumptions, we can build the expected value model of the supply chain with the retailer in the dominant role.

$$\begin{cases} \max\limits_{m} E[\Pi_R(m,w)] = \max\limits_{m} E\{(m - c_r)(a - b(w+m))\} \\ s.t. \\ \quad m - c_r > 0 \\ \quad w^* \text{ is the optimal solution of the model in the lower level} \\ \quad \begin{cases} \max\limits_{w} E[(w - c_m)(a - b(w+m))] \\ s.t. \\ \quad Pos\{a - b(w+m) \le 0\} = 0 \end{cases} \end{cases} \tag{3}$$

Suppose $E[\Pi_M(w)]$ is the expected profit of the manufacturer, with regard to the above-mentioned two-echelon planning model, the following is a tenable conclusion:

Theorem 1. Suppose the unit product profit m is constant; then, if

$$Pos\{a - b\frac{E[a] + E[bc_m] + E[b]m}{2E[b]} \le 0\} = 0$$

and

$$Pos\{c_m \ge \frac{E[a] + E[bc_m] - E[b]m}{2E[b]}\} = 0,$$

the best reaction function of the manufacturer to the unit profit margin is

$$w^* = \frac{E[a] + E[bc] - E[b]m}{2E[b]}. \tag{4}$$

Proposition 5. The best reaction function of the manufacturer w^* decreases strictly with m.

Proof.

$$\begin{aligned} & E[\Pi_M(w)] \\ & = \frac{1}{2}\int_0^1 \{[(w - c_m)(a - b(w+m))]_\alpha^L + [(w - c_m)(a - b(w+m))]_\alpha^U\} d\alpha \\ & = \frac{1}{2}\int_0^1 \{[(w - c_m)_\alpha^L (a - b(w+m))_\alpha^L] + [(w - c_m)_\alpha^U (a - b(w+m))_\alpha^U]\} d\alpha \\ & = \frac{1}{2}\int_0^1 [(w - c_{m\alpha}^U)(a_\alpha^L - b_\alpha^U(w+m)) + (w - c_{m\alpha}^L)(a_\alpha^U - b_\alpha^L(w+m))] d\alpha \\ & = -E[b]w^2 + (E[a] + E[bc_m] - mE[b])w + mE[bc_m] \\ & - \frac{1}{2}\int_0^1 (a_\alpha^L c_{m\alpha}^U + a_\alpha^U c_{m\alpha}^L) d\alpha \end{aligned} \tag{5}$$

With regard to the above equations, the first-order and second-order derivatives of w are:

$$\frac{dE[\Pi_{\mathrm{M}}(w)]}{dw}=(E[a]+E[bc_m]-mE[b])-2E[b]w,$$

$$\frac{d^2E[\Pi_{\mathrm{M}}(w)]}{dw^2}=-2E[b]<0.$$

Therefore $E[\Pi_{\mathrm{M}}(w)]$ is a concave function that can obtain the max value in the following equation:

$$w^*(m)=-\frac{1}{2}m+\frac{E[a]+E[bc_m]}{2E[b]} \quad . \tag{6}$$

Apparently, w^* is a strict decreasing function related to m.

Suppose $E[\Pi_{\mathrm{R}}(m,w^*(m))]$ is the expected profit of the retailer, with regard to the above-mentioned two-echelon planning model, the following is a tenable conclusion:

Theorem 2.

If $Pos\{c_m\geq\frac{E[a]}{2E[b]}\}=0$

and

$Pos\{a-b(E[a]+\frac{E[bc]}{2E[b]}\leq 0\}=0$,

then the optimal unit marginal profit and the optimal wholesale price, respectively, are

$$m^*=\frac{E[a]-E[bc_m]}{2E[b]},$$

$$w^*=\frac{E[a]+3E[bc_m]}{4E[b]}.$$

Proposition 6. In $(m^*,w^*(m^*))$, the manufacturer and retailer achieve their max expected profit, respectively, as:

$$E[\Pi_{\mathrm{M}}(m^*,w^*(m^*))]$$
$$=\frac{(E[a]^*)^2+8E[a]E[bc_m]-(E[bc_m])^2)}{8E[b]},$$
$$-\frac{1}{2}\int_0^1(a_\alpha^{\mathrm{L}}c_{m\alpha}^{U}+a_\alpha^{\mathrm{U}}c_{m\alpha}^{L})d\alpha$$

$$E[\Pi_{\mathrm{R}}(m^*,w^*(m^*))]$$
$$=\frac{(E[a]-E[bc_m])^2}{8E[b]}+\frac{E[a]+E[bc_m]}{2E[b]}E[bc_r].$$
$$-\frac{1}{2}\int_0^1(a_\alpha^{\mathrm{L}}c_{r\alpha}^{U}+a_\alpha^{\mathrm{U}}c_{r\alpha}^{L})d\alpha$$

Proof. The process is the same as Proposition 5. By substituting w^* in the above equations, we obtain

$$E[\Pi_{\mathrm{R}}(m,w^*(m))]$$
$$=\text{-E[b]m}^2+(E[a]+E[bc_r]-wE[b])m+wE[bc_r]$$
$$-\frac{1}{2}\int_0^1(a_\alpha^{\mathrm{L}}c_{r\alpha}^{\mathrm{U}}+a_\alpha^{\mathrm{U}}c_{r\alpha}^{\mathrm{L}})d\alpha \tag{7}$$
$$=-\frac{\mathrm{E[b]}}{2}\mathrm{m}^2+\frac{\mathrm{E[a]}-E[bc_m]}{2}m+\frac{\mathrm{E[a]}+E[bc_m]}{2\mathrm{E[b]}}E[bc_r]$$
$$-\frac{1}{2}\int_0^1(a_\alpha^{\mathrm{L}}c_{r\alpha}^{\mathrm{U}}+a_\alpha^{\mathrm{U}}c_{r\alpha}^{\mathrm{L}})d\alpha$$

Calculate the first- and second-order derivatives of the above equations with respect to m, respectively, as:

$$\frac{dE[\Pi_{\mathrm{R}}(m,w^*(m))]}{dm}=-mE[b]+\frac{E[a]-E[bc_m]}{2}$$

$$\frac{dE[\Pi_{\mathrm{R}}(m,w^*(m))]}{dm}=-E[b]<0.$$

Therefore, $E[\Pi_{\mathrm{R}}(m,w^*(m))$ is a concave function, which realizes its max value in

$$m^*=\frac{E[a]-E[bc_m]}{2E[b]}. \tag{8}$$

The max profit of the retailer is

$$E[\Pi_{\mathrm{R}}(m^*,w^*(m^*))]$$
$$=\frac{(E[a]-E[bc_m])^2}{8E[b]}+\frac{E[a]+E[bc_m]}{2E[b]}E[bc_r].$$
$$-\frac{1}{2}\int_0^1(a_\alpha^{\mathrm{L}}c_{r\alpha}^{U}+a_\alpha^{\mathrm{U}}c_{r\alpha}^{L})d\alpha$$

The max profit of the manufacturer is

$$E[\Pi_{\mathrm{M}}(m^*,w^*(m^*))]$$
$$=\frac{(E[a]^*)^2+8E[a]E[bc_m]-(E[bc_m])^2)}{8E[b]}$$
$$-\frac{1}{2}\int_0^1(a_\alpha^{\mathrm{L}}c_{m\alpha}^{U}+a_\alpha^{\mathrm{U}}c_{m\alpha}^{L})d\alpha$$

Strategy $(m_*,w^*(m^*))$ is the Stackelberg–Nash equilibrium solution for the supply chain expected value model.

In addition, we can also build the $\max i\max$ chance-constrained model and $\min i\max$ chance-constrained model.

First, we construct the $\max i\max$ chance-constrained model as follows:

$$\begin{cases}\max\limits_{m}\Pi_{\mathrm{R}}\\ s.t.\\ \quad Pos\{m(a-b(w^*(m)+m)\geq\Pi_{\mathrm{R}}\}\geq\alpha\\ \quad m-c_r>0\\ \quad w^*\text{ is the optimal solution for the lower-level plan}\\ \quad\begin{cases}\max\limits_{w}\Pi_{\mathrm{M}}\\ s.t.\\ \quad Pos\{(w-c_M)(a-b(w+m))\geq\Pi_{\mathrm{M}}\}\geq\alpha\\ \quad Pos\{a-b(w+m)\leq 0\}=0\\ \quad Pos\{w-c_m\leq 0\}=0.\end{cases}\end{cases} \tag{9}$$

wherein α is the predefined confidence level for the manufacturer and the retailer, for all provided available (m,w) strategies, $\max\limits_{m}\Pi_{\mathrm{R}}$ and $\max\limits_{w}\Pi_{\mathrm{M}}$ are the α-optimistic values of profit for the manufacturer and retailer, respectively, and therefore, the model represented in (9) is equivalent to the model below:

$$\begin{cases} \max\limits_{m}(m(a-b(w^*(m)+m)))_\alpha^{\mathrm{U}} \\ s.t. \\ \quad m-c_r>0 \\ \quad w^* \text{ is the optimal solution for the lower-level plan} \\ \quad \begin{cases} \max\limits_{w}((w-c_m)(a-b(w+m)))_\alpha^{\mathrm{U}} \\ s.t. \\ \quad Pos\{a-b(w+m)\le 0\}=0 \\ \quad Pos\{w-c_m\le 0\}=0. \end{cases} \end{cases} \tag{10}$$

Wherein $(\Pi_{\mathrm{R}}(m,w^*(m)))_\alpha^{\mathrm{U}}$ and $(\Pi_{\mathrm{M}}(w))_\alpha^{\mathrm{U}}$ are the α -optimistic values of profit for the manufacturer and retailer, respectively.

Proposition 7.

If $Pos\{c_m \ge \frac{a_\alpha^{\mathrm{U}}+b_\alpha^L c_{r\alpha}{}^L+b_\alpha^L c_{m\alpha}{}^L}{4b_\alpha^{\mathrm{L}}}\}=0$

and

$Pos\{a-b\frac{3a_\alpha^{\mathrm{U}}+2b_\alpha^{\mathrm{L}}c_{R\alpha}{}^{\mathrm{L}}}{4b_\alpha^{\mathrm{L}}}\le 0\}=0$,

the model represented in (11) has the one and only α -optimistic value, the Stackelberg–Nash equilibrium solution $(\frac{a_\alpha^{\mathrm{U}}+b_\alpha^L c_{r\alpha}{}^L-b_\alpha^L c_{m\alpha}{}^L}{2b_\alpha^{\mathrm{L}}},\frac{a_\alpha^{\mathrm{U}}+b_\alpha^L c_{r\alpha}{}^L+b_\alpha^L c_{m\alpha}{}^L}{4b_\alpha^{\mathrm{L}}})$.

Proof. The optimistic value of the manufacturer's profit is

$$\begin{aligned} \max_{w}(\Pi_{\mathrm{M}}(w))_\alpha^{\mathrm{U}} &= ((w-c_m)(a-b(m+w)))_\alpha^{\mathrm{U}} \\ &=(w-c_{m\alpha}{}^{\mathrm{L}})(a_\alpha^{\mathrm{U}}-b_\alpha^{\mathrm{L}}(m+w)) \\ &=-b_\alpha^{\mathrm{L}}w^2+(a_\alpha^{\mathrm{U}}+b_\alpha^{\mathrm{L}}c_{m\alpha}{}^{\mathrm{L}}-b_\alpha^{\mathrm{L}}m)w+b_\alpha^{\mathrm{L}}c_{m\alpha}{}^{\mathrm{L}}m-a_\alpha^{\mathrm{U}}c_{m\alpha}{}^{\mathrm{L}} \end{aligned}$$

Calculate the first- and second-order derivatives of the above equations with regard to w

$$\frac{d\max\limits_{w}(\Pi_{\mathrm{M}}(w))_\alpha^{\mathrm{U}}}{dw}=-2b_\alpha^{\mathrm{L}}w+a_\alpha^{\mathrm{U}}+b_\alpha^{\mathrm{L}}c_{m\alpha}{}^{\mathrm{L}}-b_\alpha^{\mathrm{L}}m \tag{11}$$

$$\frac{d^2\max\limits_{w}(\Pi_{\mathrm{M}}(w))_\alpha^{\mathrm{U}}}{dw^2}=-2b_\alpha^{\mathrm{L}}<0.$$

This, $\max\limits_{w}(\Pi_{\mathrm{M}}(w))_\alpha^{\mathrm{U}}$ is a concave function, and realizes its max value in

$$w^*(m)=\frac{a_\alpha^{\mathrm{U}}+b_\alpha^{\mathrm{L}}c_{m\alpha}{}^{\mathrm{L}}-b_\alpha^{\mathrm{L}}m}{2b_\alpha^{\mathrm{L}}}. \tag{12}$$

w^* is a strict decreasing function with regard to m.

To derive the optimistic value of the retailer's profit, substituting w^* in (13) yields

$$\begin{aligned} &\max_{m}(\Pi_{\mathrm{R}}(m,w^*(m)))_\alpha^{\mathrm{U}} \\ &=((m-c_r)(a-b(m+w)))_\alpha^{\mathrm{U}} \\ &=(w-c_{r\alpha}{}^{\mathrm{L}})(a_\alpha^{\mathrm{U}}-b_\alpha^{\mathrm{L}}(m+w)) \\ &=-\frac{b_\alpha^{\mathrm{L}}}{2}m^2+\frac{a_\alpha^{\mathrm{U}}+b_\alpha^{\mathrm{L}}c_{r\alpha}{}^{\mathrm{L}}-b_\alpha^{\mathrm{L}}c_{m\alpha}{}^{\mathrm{L}}}{2}m \\ &+\frac{b_\alpha^{\mathrm{L}}c_{m\alpha}{}^{\mathrm{L}}c_{r\alpha}{}^{\mathrm{L}}-a_\alpha^{\mathrm{U}}c_{r\alpha}{}^{\mathrm{L}}}{2} \end{aligned} \tag{13}$$

Calculate the first- and second-order derivatives of the above equations with regard to m

$$\frac{d\max\limits_{m}(\Pi_{\mathrm{R}}(m,w^*(m)))_\alpha^{\mathrm{U}}}{dm}=-b_\alpha^{\mathrm{L}}m+\frac{a_\alpha^{\mathrm{U}}+b_\alpha^{\mathrm{L}}c_{r\alpha}{}^{\mathrm{L}}-b_\alpha^{\mathrm{L}}c_{m\alpha}{}^{\mathrm{L}}}{2},$$

$$\frac{d^2\max\limits_{m}(\Pi_{\mathrm{M}}(w))_\alpha^{\mathrm{U}}}{dm^2}=-b_\alpha^{\mathrm{L}}<0.$$

Thus, $\max\limits_{m}(\Pi_{\mathrm{R}}(m,w^*(m)))_\alpha^{\mathrm{U}}$ is a concave function that realizes its max value in

$$m^*=\frac{a_\alpha^{\mathrm{U}}+b_\alpha^{\mathrm{L}}c_{r\alpha}{}^{\mathrm{L}}-b_\alpha^{\mathrm{L}}c_{m\alpha}{}^{\mathrm{L}}}{2b_\alpha^{\mathrm{L}}}. \tag{14}$$

Apparently, w^* is a strict decreasing function with regard to m. Substituting m^* in w^* can result in

$$w^*=\frac{a_\alpha^U+b_\alpha^L c_{r\alpha}{}^L+b_\alpha^L c_{m\alpha}{}^L}{4b_\alpha^L}. \tag{15}$$

Therefore, (m^*,w^*) is the only equilibrium solution of α -optimistic values for the manufacturer and retailer.

On the other hand, we can build a $\min i\max$ chance-constrained programming model for the two-echelon supply chain.

$$\begin{cases} \max\limits_{m}\min\limits_{\Pi_{\mathrm{R}}}\Pi_{\mathrm{R}} \\ s.t. \\ \quad Pos\{m(a-b(w^*(m)+m)\le \Pi_{\mathrm{R}}\}\ge\alpha \\ \quad m-c_r>0 \\ \quad w^* \text{ is the optimal solution for the lower-level plan} \\ \quad \begin{cases} \max\limits_{m}\min\limits_{\Pi_{\mathrm{M}}}\Pi_{\mathrm{M}} \\ s.t. \\ \quad Pos\{(w-c_M)(a-b(w+m))\le\Pi_{\mathrm{M}}\}\ge\alpha \\ \quad Pos\{a-b(w+m)\le 0\}=0 \\ \quad Pos\{w-c_m\le 0\}=0. \end{cases} \end{cases} \tag{16}$$

Wherein, α is the predefined confidence level for the manufacturer and retailer, for all provided available (m,w) strategies, $\min\limits_{m}\Pi_{\mathrm{R}}$ 和 $\min\limits_{w}\Pi_{\mathrm{M}}$ are the α -pessimistic values for the manufacturer and retailer, respectively. Therefore the model represented in (16) is equivalent to the following model:

$$\begin{cases} \max\limits_{m}(m(a-b(w^*(m)+m)))_\alpha^{\mathrm{L}} \\ s.t. \\ \quad m-c_r>0 \\ \quad w^* \text{ is the optimal solution for the lower-level plan} \\ \quad \begin{cases} \max\limits_{w}((w-c_M)(a-b(w+m)))_\alpha^{\mathrm{L}} \\ s.t. \\ \quad Pos\{a-b(w+m)\le 0\}=0 \\ \quad Pos\{w-c_m\le 0\}=0. \end{cases} \end{cases} \tag{17}$$

Wherein, $(\Pi_{\mathrm{R}}(m,w^*(m)))_\alpha^{\mathrm{L}}$, $(\Pi_{\mathrm{M}}(w))_\alpha^{\mathrm{L}}$ are the α -pessimistic values for the manufacturer and retailer, respectively.

Regarding the model represented in (16) and (17), there are tenable conclusions below:

Proposition 8.

If $Pos\{c_m \geq \frac{a_\alpha^{\mathrm{L}} + b_\alpha^U c_{r\alpha}^{U} + b_\alpha^U c_{m\alpha}^{U}}{4b_\alpha^{\mathrm{U}}}\} = 0$

and

$$Pos\{a - b\frac{3a_\alpha^{\mathrm{L}} + 2b_\alpha^{\mathrm{U}} c_{R\alpha}^{\mathrm{U}}}{4b_\alpha^{\mathrm{U}}} \leq 0\} = 0,$$

the model represented in (17) has the one and only α-optimistic value, Stackelberg–Nash equilibrium solution

$$(\frac{a_\alpha^{\mathrm{L}} + b_\alpha^U c_{r\alpha}^{U} - b_\alpha^U c_{m\alpha}^{U}}{2b_\alpha^{\mathrm{U}}}, \frac{a_\alpha^{\mathrm{L}} + b_\alpha^U c_{r\alpha}^{U} + b_\alpha^U c_{m\alpha}^{U}}{4b_\alpha^{\mathrm{U}}}).$$

Proof. This is similar to the proof of Proposition 7.

With respect to the analysis above, a conclusion for the game equilibrium in the two-echelon supply chain is shown in Table I.

Table I
Summary of the fuzzy two-echelon supply chain model dominated by the retailer

Ranking criterion	Optimal unit product profit m^*	Optimal wholesale price w^*
Expectation criterion	$\frac{E[a] - E[bc_m]}{2E[b]}$	$\frac{E[a] + 3E[bc_m]}{4E[b]}$
α-optimistic value criterion	$\frac{a_\alpha^{\mathrm{U}} + b_\alpha^L c_{r\alpha}^{L} - b_\alpha^L c_{m\alpha}^{L}}{2b_\alpha^{\mathrm{L}}}$	$\frac{a_\alpha^{\mathrm{U}} + b_\alpha^L c_{r\alpha}^{L} + b_\alpha^L c_{m\alpha}^{L}}{4b_\alpha^{\mathrm{L}}}$
α-pessimistic value criterion	$\frac{a_\alpha^{\mathrm{L}} + b_\alpha^U c_{r\alpha}^{U} - b_\alpha^U c_{m\alpha}^{U}}{2b_\alpha^{\mathrm{U}}}$	$\frac{a_\alpha^{\mathrm{L}} + b_\alpha^U c_{r\alpha}^{U} + b_\alpha^U c_{m\alpha}^{U}}{4b_\alpha^{\mathrm{U}}}$
	Max profit of retailer	Max profit of manufacturer
Expectation criterion	$\frac{(E[a] - E[bc_m])^2}{8E[b]} + \frac{E[a] + E[bc_m]}{2E[b]} E[bc_r] - \frac{1}{2}\int_0^1 (a_\alpha^{\mathrm{L}} c_{r\alpha}^{U} + a_\alpha^{\mathrm{U}} c_{r\alpha}^{L}) d\alpha$	$\frac{(E[a]^*)^2 + 8E[a]E[bc_m] - (E[bc_m])^2)}{8E[b]} - \frac{1}{2}\int_0^1 (a_\alpha^{\mathrm{L}} c_{m\alpha}^{U} + a_\alpha^{\mathrm{U}} c_{m\alpha}^{L}) d\alpha$
α-optimistic value	$\frac{(a_\alpha^U - b_\alpha^L c_{R\alpha}^{L} + b_\alpha^L c_{m\alpha}^{L})^2}{8b_\alpha^L}$	$\frac{(a_\alpha^U - b_\alpha^L c_{r\alpha}^{L} + b_\alpha^L c_{m\alpha}^{L})^2}{16b_\alpha^L} + \frac{b_\alpha^L c_{r\alpha}^{L} c_{m\alpha}^{L}}{2} + \frac{3b_\alpha^L (c_{r\alpha}^{L})^2}{16} - \frac{b_\alpha^L (c_{m\alpha}^{L})^2}{4}$
α-pessimistic value	$\frac{(a_\alpha^L - b_\alpha^U c_{r\alpha}^{U} + b_\alpha^U c_{m\alpha}^{U})^2}{8b_\alpha^U}$	$\frac{(a_\alpha^L - b_\alpha^U c_{r\alpha}^{U} + b_\alpha^U c_{m\alpha}^{U})^2}{16b_\alpha^U} + \frac{b_\alpha^U c_{r\alpha}^{U} c_{m\alpha}^{U}}{2} + \frac{3b_\alpha^U (c_{r\alpha}^{U})^2}{16} - \frac{b_\alpha^U (c_{m\alpha}^{U})^2}{4}$

V. Numerical Experiment

The above discussion solves for the pricing strategies of various manufacturers in a two-echelon supply chain where the retailer plays the dominant role. A numerical example is given here to illustrate the effectiveness of this game model. For example, manufacturing costs, operational costs, market capacity, and demand change rate are normally evaluated by the management decision makers and experts. During evaluation, terms such as "low costs," "big market capacity," and "sensitive demand changing rate" are frequently used to describe the approximate evaluation values. The estimators depend on experience to determine the relationship between fuzzy language variables and the triangle fuzzy value, shown in Table II.

Table II
Relationship between linguistic expression and triangular fuzzy variable

	Language variable	Triangle fuzzy value
Manufacturing costs	low (approx. 3)	(2,3,4)
	medium (approx. 5)	(4,5,6)
	high (approx. 7)	(6,7,8)
Operational costs	lower (approx. 2)	(1,2,3)
	medium (approx. 4)	(3,4,5)
	high (approx. 6)	(5,6,7)
Market capacity	Very big (approx. 5000)	(4900,5000,5100)
	Rather small (approx. 3000)	(2900,3000,3100)
Demand changing rate	Very sensitive (approx. 500)	(450,500,550)
	Sensitive (approx. 300)	(280,300,320)

Suppose the current condition is as follows: the evaluated product market capacity is very large (approximately 5000), the demand changing rate is very sensitive (approximately 500), the manufacturing costs are average (approximately 5), the operational cost of the retailer is rather low (approximately 2), according to the expected value model and fuzzy variable equation, the conclusions can be obtained in Tables III and IV .

Table III
The optimal strategy of the Stackelberg game of a supply chain when the retailer plays a dominant role

Ranking criterion	Optimal unit product profit m^*	Optimal wholesale price w^*
Expected value criterion	2.517	6.275
	Max profit of retailers	Max profit of manufacturers
	11696.669	5012.219

Table IV
Analysis on optimal pricing strategy and the sensitivity of α variable

α value	Optimistic value criterion		Pessimistic value criterion	
	m^*	w^*	m^*	w^*
$\alpha = 1$	3.500	3.200	3.500	4.250
$\alpha = 0.95$	3.530	3.238	3.470	4.260
$\alpha = 0.9$	3.561	3.276	3.441	4.270
$\alpha = 0.85$	3.591	3.315	3.411	4.281
$\alpha = 0.8$	3.622	3.354	3.382	4.291

In Table II, we can observe that in the Stackelberg game of a two-echelon supply chain, the dominant retailer can obtain larger profits. The retailer can force the manufacturer to decrease its wholesale price so that a larger quantity of products can be purchased, and the retailer's profits increase.

In Tables III and IV, we can see that the Stackelberg game optimal strategies and the maximum profits change with the predefined confidence levels of manufacturers and retailers. Under the optimistic value criterion, as the confidence level decreases, the optimal wholesale prices and optimal unit margin profits gradually increase. However, when the max profits of retailers gradually increase, the max profits of manufacturers gradually decrease. Under the pessimistic value criterion, as the confidence levels decrease, only the optimal wholesale prices gradually increase. However, when optimal unit margin profits and retailer max profits gradually increase, the max profits of manufacturers gradually increase.

VI. CONCLUSION

This paper considers market demand, manufacturing costs, and operational costs to be fuzzy variables, and establishes the chance-constrained programming model—and the related models of optimistic and pessimistic values—as the expected value model of a two-echelon supply chain with a dominant retailer. Using game theory, an analysis was conducted on the optimal pricing strategies and maximum profits for both the manufacturer and the retailer in the various models when the retailer is dominant. In the Stackelberg game equilibrium, the dominant role yields more profit for retailers, and the optimal pricing strategies are related to the confidence levels predefined by manufacturers and retailers. Thus, the conclusions for the optimal strategies relate to the certainty of the environment.

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