Reliability Assessment and Residual Life Prediction Method based on Wiener Process and Current Degradation Quantity

Huibing Hao, Chunping Li

Abstract—**In this article, the population reliability modeling and individual residual life prediction are discussed. Firstly, three kinds of different Wiener process models are used to characterize the degradation data, and the unknown parameters are estimated by using the Markov Chain Monte Carlo (MCMC) method. Secondly, under those degradation models, the individual residual life prediction method is also obtained on the basis of current degradation quantity. Finally, a fatigue cracks data example is given to illustrate the usefulness and validity of the proposed model and method. Numerical results show that the random effect model is well fitted with the actual degradation data, and this model has smallest prediction error. Meanwhile the prediction accuracy is acceptable, and this prediction method provides a foundation for maintenance decision.**

***Index Terms*—Wiener process, residual life, current degradation quantity, Markov chain Monte Carlo**

I. Introduction

DUE to the advances in material science and manufacture technology, the lifetime of modern product has been becoming longer and longer, usually it is difficult to obtain enough failure data during research and development period[1]. In this situation, degradation data can be used as an alternate resource for reliability analysis. In the last decades, degradation data have played a more important role in reliability assessment than ever before [2].

As we know that degradation (e.g. wear, erosion and fatigue) is a common phenomenon for electro-mechanical system and its components. Degradation can be mathematically described with a continuous process in terms of time. In Ref. [3], three kinds of method for degradation data analysis are proposed, i.e. linear regression method, degradation path method, and stochastic process method. One highlight of stochastic process model is that the lifetime can be defined as the first hitting time when the degradation process reaches a failure threshold. And stochastic process can flexibly describe the failure mechanism and characteristics of operating environment, it has been widely used to model the degradation path, including Gamma process [4], Wiener process [5,7,8,9,10], and Markov process [6], et al.

Among those stochastic processes, Wiener process is most widely used. For example, LEE et al [5] and TANG et al [8] used Wiener process model to describe the degradation of light emitting diode; SU and ZHANG [7] used it to deal with degradation data of laser device.

In reliability study, beyond evaluating products' reliability, how to obtain the residual lifetime of a product is also of great interest. In Ref. [9], REN et al used fixed effect Wiener process to estimate residual life of an aircraft engine, and the prediction accuracy is given.

However, the above studies consider only the fixed effect Wiener process. In this paper, the mixed effect and random effect Wiener process models are proposed, then the reliability assessment and individual residual life can be obtained. Considering Markov chain Monte Carlo (MCMC) method is convenient and efficient to sample from complex distribution, MCMC method is used to estimate the unknown parameters [15].

The rest of the paper is organized as follows. In Section 2, the different degradation models are introduced. Then, the residual life prediction methods are presented in Sections 3. In Section 4, the parameters estimation method based on the MCMC is presented. A numerical example with fatigue cracks data is given in Section 5. Finally, some conclusions are made in Section 6.

II. Population Degradation Model Based on Wiener Process

Due to the good mathematical properties and physical interpretations of Wiener process, it has been taken to describe the performance degradation of products. A well adopted form of Wiener process $\{X(t), t \geq 0\}$ can be expressed as M_1

$$X(t) = \mu t + \sigma B(t) \tag{1}$$

where μ and σ are drift and diffusion parameters, respectively; $B(t)$ is a standard Brownian motion which is used to describe time-correlated structure.

Let ξ be the threshold value of the product. It is assumed that the degradation path is described by the model M_1. Given the threshold value ξ, the product's lifetime T is defined as

$$T = \inf\{t \mid X(t) \geq \xi\} \tag{2}$$

and it is known that T follows inverse Gaussian distribution with probability distribution function (PDF)

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$$f_T(t\mid\mu,\sigma)=\frac{\xi}{\sqrt{2\pi\sigma^2t^3}}\exp\left(-\frac{(\xi-\mu t)^2}{2\sigma^2t}\right)$$
$$=\sqrt{\frac{\xi^2\upsilon}{2\pi t^3}}\exp\left(-\frac{\upsilon(\xi-\mu t)^2}{2t}\right) \tag{3}$$

Then, basis on the PDF of lifetime T, we can obtain the reliability at time t as follow

$$R(t)=\Pr(T>t)=\int_t^{+\infty}f_T(x)dx$$
$$=\Phi\left(-\frac{\mu t-\xi}{\sqrt{\sigma^2t}}\right)-\exp\left(\frac{2\mu\xi}{\sigma^2}\right)\Phi\left(-\frac{\mu t+\xi}{\sqrt{\sigma^2t}}\right) \tag{4}$$

where $\Phi(\cdot)$ is the cumulative distribution function (CDF) of standard normal distribution.

In most cases, each sample unit usually experiences different sources of variations during their operation. Thus, it is more appropriate to incorporate unit to unit variability in the degradation process, and the mixed effect model can describe the unit variability. A conventional mixed effect Wiener process model can be expressed as M_2

$$\begin{cases}Y(t)=\mu t+\sigma B(t)\\ \mu\sim N(\eta,\sigma_\eta^2)\end{cases} \tag{5}$$

Assume that the degradation path of product is described by the model M_2. Considering that the drift parameter μ is random variable, by using the total law of probability, the PDF of the lifetime T can be reconstructed in model M_2 as

$$f_T(t\mid\upsilon)=\int_{-\infty}^{+\infty}f_T(t\mid\mu,\sigma)\varphi\left(\frac{\mu-\eta}{\sigma_\eta}\right)d\mu$$
$$=\sqrt{\frac{\xi^2}{2\pi(\upsilon^{-1}+\sigma_\eta^2t)t^3}}\exp\left(-\frac{(\xi-\eta t)^2}{2(\upsilon^{-1}t+\sigma_\eta^2t^2)}\right) \tag{6}$$

where $\varphi(\cdot)$ is distribution function of the standard normal distribution.

Then, the reliability at time t can be expressed as

$$R(t)=\Phi\left(-\frac{\eta t-\xi}{\sqrt{\sigma_\eta^2t^2+\upsilon^{-1}t}}\right)-\exp\left(\frac{2\eta\upsilon^{-1}\xi+2\sigma_\eta^2\xi^2}{\upsilon^{-2}}\right)$$
$$\times\Phi\left(-\frac{2\sigma_\eta^2\xi t+\upsilon^{-1}(\eta t+\xi)}{\upsilon^{-1}\sqrt{\sigma_\eta^2t^2+\upsilon^{-1}t}}\right) \tag{7}$$

Up to now, some paper have considered mixed effect Wiener process model and their applications (see in [4], [10], [11] and [12]). But in those studies, only the drift parameter μ is considered to be random variable.

In this paper, a random effect Wiener process is used to characterize the degradation data, where μ and σ of this model are regarded as random variables. A random effect Wiener process model can be expressed as M_3

$$\begin{cases}Z(t)=\mu t+\sigma B(t)\\ \upsilon=\sigma^{-2}\sim G(\beta,\alpha)\\ \mu\mid\upsilon\sim N(\theta,\lambda/\upsilon)\end{cases} \tag{8}$$

where β, α, θ and λ are unknown parameters; $G(\cdot)$ and $N(\cdot,\cdot)$ are gamma distribution and normal distribution, respectively. It is noted that model M_3 can be used to describe both the variation from unit to unit and time correlated structure.

Similarly to the above, when μ and σ are random variables, by using the total law of probability, the PDF of lifetime T in model M_3 is given by

$$f_T(t)=\frac{\xi\alpha^\beta}{2\pi\sqrt{\lambda t^3}\Gamma(\beta)}\int_0^{+\infty}\left\{\upsilon^{\beta-1}\exp\left[-\left(\frac{(\xi-\theta t)^2}{2t(1+\lambda t)}+\alpha\right)\upsilon\right]\int_{-\infty}^{+\infty}\exp\left[-\frac{(\mu-\frac{\lambda\xi+\theta}{1+\lambda t})^2}{\frac{2\lambda}{(1+\lambda t)\upsilon}}\right]d\mu\right\}d\upsilon$$
$$=\frac{\Gamma(\beta+\frac{1}{2})\xi}{\sqrt{2\pi t^3[\alpha(\lambda t+1)]}\Gamma(\beta)}\left(1+\frac{(\xi-\theta t)^2}{2\alpha(\lambda t^2+t)}\right)^{-\beta-\frac{1}{2}} \tag{9}$$

and the reliability at time t can be expressed as

$$R(t)=1-\int_0^t\frac{\Gamma(\beta+\frac{1}{2})\xi}{\sqrt{2\pi x^3[\alpha(\lambda x+1)]}\Gamma(\beta)}\times\left(1+\frac{(\xi-\theta x)^2}{2\alpha(\lambda x^2+x)}\right)^{-\beta-\frac{1}{2}}dx \tag{10}$$

III. Individual Residual Life Prediction Based on Current Performance Degradation

As we known, Equations (4)、(7) and (10) provide a basis method for product reliability evaluation, and they can characterize the average survival rate of the population. Instead of the average population's characteristics, the residual life prediction of individual product has important practical applications, such as planning of maintenance activities, supply chain management, replenishment of inventory system, et al. In this section, the residual life prediction method based on current performance degradation is given by using the different Wiener process model.

Firstly, we focus on the residual life prediction method under the degradation model M_1. According to the independent increment property of the Wiener process, given μ and σ, we can get the following

$$X(t)\mid\mu,\sigma\sim N\left(\mu t,\sigma^2t\right) \tag{11}$$

and

$$\Delta X(t-s)\mid\mu,\sigma\sim N\left(\mu(t-s),\sigma^2(t-s)\right),\ \forall t>s\,. \tag{12}$$

where $\Delta X(t-s)=X(t)-X(s)$.

Supposing that a product has operated until time t_k without failure, and $X(t_k)$ is the corresponding degradation quantity at time t_k, the conditional reliability can be formulated as

$$R\left(t\mid X(t_k)=x\right)=\Pr\left(T>t\mid X(t_k)=x\right)$$
$$=\Pr\left(X(t)<\xi\mid X(t_k)=x\right)$$
$$=\frac{\Pr\left(X(t)<\xi,X(t_k)=x\right)}{\Pr\left(X(t_k)=x\right)}$$
$$=\frac{\Pr\left(\Delta X(t-t_k)<\xi-x,X(t_k)=x\right)}{\Pr\left(X(t_k)=x\right)}$$
$$=\frac{\Pr\left(\Delta X(t-t_k)<\xi-x\right)\Pr\left(X(t_k)=x\right)}{\Pr\left(X(t_k)=x\right)}$$

$$=\Pr\left(\Delta X(t-t_k)<\xi-x\right)=\Phi\left(\frac{\xi-x-\mu(t-t_k)}{\sigma\sqrt{t-t_k}}\right) \quad (13)$$

Given prediction reliability level p ($0<p<1$) and performance degradation $X(t_k)=x$ based on current time t_k, we can obtain the continuous operation time $L=t-t_k$ of the operated until as:

$$\frac{\xi-x-\mu L}{\sigma\sqrt{L}}=Z_{1-p} \quad (14)$$

where Z_{1-p} can be obtained through checking Normal distribution probability table.

Secondly, we focus on the residual life prediction method under the degradation model M_2. From the Equation (11), given μ and σ, we know that the PDF of $X(t)$ follows as

$$f(x)=\frac{1}{\sqrt{2\pi\sigma^2 t}}\exp\left(-\frac{(x-\mu t)^2}{2\sigma^2 t}\right) \quad (15)$$

Considering that the drift parameter μ is random variable, by using the total law of probability, the PDF of $Y(t)$ can be reconstructed in model M_2 as

$$g(x)=\int_{-\infty}^{+\infty} f(x)\varphi\left(\frac{\mu-\eta}{\sigma_\eta}\right)d\mu$$

$$=\frac{1}{\sqrt{2\pi(\sigma_\eta^2 t^2+\sigma^2 t)}}\exp\left(-\frac{(x-\eta t)^2}{2(\sigma_\eta^2 t^2+\sigma^2 t)}\right) \quad (16)$$

Then, we know that the PDF of $Y(t)$ follows as

$$Y(t)\mid\sigma\sim N\left(\eta t,\sigma_\eta^2 t^2+\sigma^2 t\right) \quad (17)$$

and

$$\Delta Y(t-s)\mid\sigma\sim N\left(\eta(t-s),\sigma_\eta^2(t-s)^2+\sigma^2(t-s)\right) \quad (18)$$

where $\Delta Y(t-s)=Y(t)-Y(s)$.

Similarly, suppose that a product has operated until time t_k without failure, and $Y(t_k)$ is the corresponding degradation quantity at time t_k, the conditional reliability can be formulated as

$$R\left(t\mid Y(t_k)=x\right)=\Pr\left(T>t\mid Y(t_k)=x\right)$$

$$=\Pr\left(\Delta Y(t-t_k)<\xi-x\right)$$

$$=\Phi\left(\frac{\xi-x-\eta(t-t_k)}{\sqrt{\sigma_\eta^2(t-t_k)^2+\sigma^2(t-t_k)}}\right) \quad (19)$$

Given prediction reliability level p ($0<p<1$) and performance degradation $Y(t_k)=x$ based on current time t_k, we can obtain the continuous operation time $L=t-t_k$ of the operated until as:

$$\frac{\xi-x-\eta L}{\sqrt{\sigma_\eta^2 L^2+\sigma^2 L}}=Z_{1-p} \quad (20)$$

where Z_{1-p} can be obtained through checking Normal distribution probability table.

Finally, we focus on the residual life prediction method under the degradation model M_3. If the drift parameter μ and the diffusion coefficient σ are random variables, by using the similarly method, the PDF of $Z(t)$ can be reconstructed in model M_3 as

$$h(x)=\frac{\alpha^\beta}{2\pi\Gamma(\beta)\sqrt{\lambda}}\int_{-\infty}^{+\infty}\int_0^{+\infty}\upsilon^\beta\times$$

$$\exp\left\{-\left(\frac{(x-\mu t)^2}{2}+\frac{(\mu-\theta)^2}{2\lambda}+\alpha\right)\upsilon\right\}d\upsilon d\mu$$

$$=\frac{\Gamma(\beta+\frac{1}{2})}{\sqrt{2\pi\alpha(\lambda t^2+t)]}\Gamma(\beta)}\left(1+\frac{(x-\theta t)^2}{2\alpha(\lambda t^2+t)}\right)^{-\beta-\frac{1}{2}} \quad (21)$$

Note that $\sqrt{\frac{\beta}{\alpha(\lambda t^2+t)}}\left(Z(t)-\theta t\right)$ has a T distribution with degrees of freedom 2β. That is to say

$$Z(t)\sim T_{2\beta}\left(\sqrt{\beta}(\theta t-\xi)\big/\sqrt{\alpha(\lambda t^2+t)}\right) \quad (22)$$

where $T_{2\beta}$ is the T distribution function with degrees of freedom 2β.

Similarly, supposing that a product has operated until time t_k without failure, and $Z(t_k)$ is the corresponding degradation quantity at time t_k, the conditional reliability can be formulated as

$$R\left(t\mid Z(t_k)=x\right)=\Pr\left(T>t\mid Z(t_k)=x\right)$$

$$=\Pr\left(\Delta Z(t-t_k)<\xi-x\right)$$

$$=T_{2\beta}\left(\frac{\sqrt{\beta}\left(\theta(t-t_k)-(\xi-x)\right)}{\sqrt{\alpha\left(\lambda(t-t_k)^2+(t-t_k)\right)}}\right) \quad (23)$$

Given prediction reliability level p ($0<p<1$) and performance degradation $Z(t_k)=x$ based on current time t_k, we can obtain the continuous operation time $L=t-t_k$ of the operated until as:

$$\frac{\sqrt{\beta}\left(\theta L-(\xi-x)\right)}{\sqrt{\alpha\left(\lambda L^2+L\right)}}=M_{1-p} \quad (24)$$

where M_{1-p} can be obtained through checking T distribution probability table. The residual life L can be obtained by resolving the above equation.

IV. Parameters Estimation via Bayesian MCMC Method

Bayesian inference is an efficient approach to evaluate the unknown parameters of a given model. When it is difficult to obtain the analytical posterior distribution, MCMC method can be used. It can generate samples from the posterior distribution, and these samples can also be used to estimate the desired features of the posterior distribution.

Suppose the degradation path of product is governed by M_3. To achieve parameters estimation, we assume that n units are tested, and $X_i(t_{ij})$ denotes the cumulative degradation values of product i at time t_{ij}, for $i=1,2,\cdots,n$; $j=0,1,2,\cdots,m$. Let

$$\Delta Z_i(t_{ij})=Z_i(t_{ij})-Z_i(t_{i(j-1)}),\ \Delta t_{ij}=t_{ij}-t_{i(j-1)},\ t_{i0}=0,\ Z_i(t_{i0})=0$$

From the Equation (22), the joint density can be obtained as

$$f(\Delta Z_i)=\frac{\Gamma(\beta+\frac{m}{2})}{(2\pi\alpha)^{m/2}|A|^{1/2}\Gamma(\beta)}\left(1+\frac{1}{2\alpha}(\Delta Z_i-\theta\Delta t_i)'A^{-1}(\Delta Z_i-\theta\Delta t_i)'\right)^{-\beta-\frac{m}{2}} \quad (25)$$

where

$\Delta Z_i = (\Delta Z_i(t_{i1}), \Delta Z_i(t_{i2}), \cdots, \Delta Z_i(t_{im}))$,

$\Delta t_i = (\Delta t_{i1}, \Delta t_{i2}, \cdots, \Delta t_{im})$, $[A_{pq}] = \begin{cases} \lambda \Delta t_{ip}^2 + \Delta t_{ip} & p = q \\ \lambda \Delta t_{ip} \Delta t_{iq} & p \neq q \end{cases}$

Due to the independence assumption of the degradation measurements of different product, the log-likelihood function can be expressed as

$$l(\alpha,\beta,\lambda,\theta \mid \Delta Z) = \sum_{i=1}^{n} \Big\{ \log\Gamma(\beta + \tfrac{m}{2}) - \log\Gamma(\beta) - \frac{m}{2}\log 2\pi\alpha - \frac{1}{2}\log|A| - \left(\beta + \frac{m}{2}\right)\log\left(1 + \frac{1}{2\alpha}(\Delta Z_i - \theta\Delta t_i)' A^{-1}(\Delta Z_i - \theta\Delta t_i)\right) \Big\} \quad (26)$$

From Equation (26), we know that the model not only has four parameters, but is also very complicated from a computational viewpoint. For this reason, the MCMC with the Gibbs sampling techniques is employed in this study to estimate model parameters. Let $\pi(\theta_j \mid \theta_{-j}, Z)$ denote the full conditional posterior distribution of θ_j , where $\theta_{-j} = (\theta_1, \cdots, \theta_{j-1}, \theta_{j+1}, \cdots, \theta_n)$ and Z is the observed data.

The algorithm of parameters estimation via the Gibbs sampling can be summarized as follow:

Step 1: Initialize $\rho^{(0)} = (\rho_1^{(0)}, \rho_2^{(0)}, \cdots, \rho_k^{(0)})$;

Step 2: Set $t = 1$;

Step 3: Generate $\rho_1^{(t)}$ from conditional distribution $\pi_1^*(\rho_1 \mid \rho_2, \rho_3, \cdots, \rho_k, Z)$;

Step 4: Generate $\rho_2^{(t)}$ from conditional distribution $\pi_2^*(\rho_2 \mid \rho_1, \rho_3, \cdots, \rho_k, Z)$;

Step 5: Generate $\rho_j^{(t)}$ from conditional distribution $\pi_j^*(\rho_j \mid \rho_1, \cdots, \rho_{j-1}, \rho_{j+1}, \cdots, \rho_k, Z)$;

Step 6: Generate $\rho_k^{(t)}$ from conditional distribution $\pi_k^*(\rho_k \mid \rho_1, \rho_2, \cdots, \rho_{k-1}, Z)$;

Step 7: Set $t = t+1$, and repeat Steps 3-6, $t = 1, 2, \cdots, N_1$;

Step 8: Estimate the desired features based on simulation samples of $\rho^{(1)}, \rho^{(2)}, \cdots, \rho^{(N_1)}$.

Using the Bayesian software package WinBUGS (see in Ref. [15]) and carrying out the Gibbs sampling, the estimator of the model parameters can be obtained.

V. Numerical Example

In this section, a numerical example about fatigue cracks data is given to demonstrate the validity of the proposed model and method. The fatigue crack dataset is presented in Ref. [1], and 21 degradation samples are collected. The observed measurement variable is the crack length over time, and all samples are measured every 0.01million cycles. The product is defined to be failed if the length of crack crosses 1.6 inch, and the testing stopped at 0.10 million cycles.

In the original degradation data, the degradation path of each sample over time is nonlinear function. Here, a proper transformation as *y=(x-0.9)/x* is adopted to make it approximately linear. The transformed crack length data are listed in Table I and part of samples are depicted in Fig.1. Based on this transformation method, the failure threshold value becomes 0.4375 inch.

TABLE I
The Crack Length Data (Inches)

	Crack length						
	0	0.01	0.02	…	0.08	0.09	0.10
1	0	0.052632	0.100000	…	0.391892	0.451220	
2	0	0.042553	0.081633	…	0.343066	0.387755	0.437500
3	0	0.042553	0.081633	…	0.333333	0.383562	0.430380
4	0	0.042553	0.081633	…	0.328358	0.370629	0.419355
5	0	0.042553	0.081633	…	0.328358	0.370629	0.419355
6	0	0.042553	0.081633	…	0.323308	0.361702	0.403974
7	0	0.042553	0.081633	…	0.318182	0.361702	0.407895
8	0	0.042553	0.081633	…	0.307692	0.352518	0.395973
9	0	0.021739	0.072165	…	0.296875	0.338235	0.375000
10	0	0.021739	0.062500	…	0.285714	0.328358	0.366197
11	0	0.032258	0.062500	…	0.274194	0.312977	0.352518
12	0	0.032258	0.072165	…	0.262295	0.302326	0.343066
13	0	0.021739	0.072165	…	0.250000	0.285714	0.312977
14	0	0.032258	0.062500	…	0.250000	0.285714	0.307692
15	0	0.021739	0.062500	…	0.256198	0.291339	0.323308
16	0	0.021739	0.052632	…	0.224138	0.262295	0.285714
17	0	0.032258	0.062500	…	0.224138	0.250000	0.274194
18	0	0.021739	0.042553	…	0.210526	0.243697	0.268293
19	0	0.021739	0.042553	…	0.196429	0.224138	0.250000
20	0	0.021739	0.042553	…	0.196429	0.224138	0.243697
21	0	0.021739	0.042553	…	0.189189	0.210526	0.237288

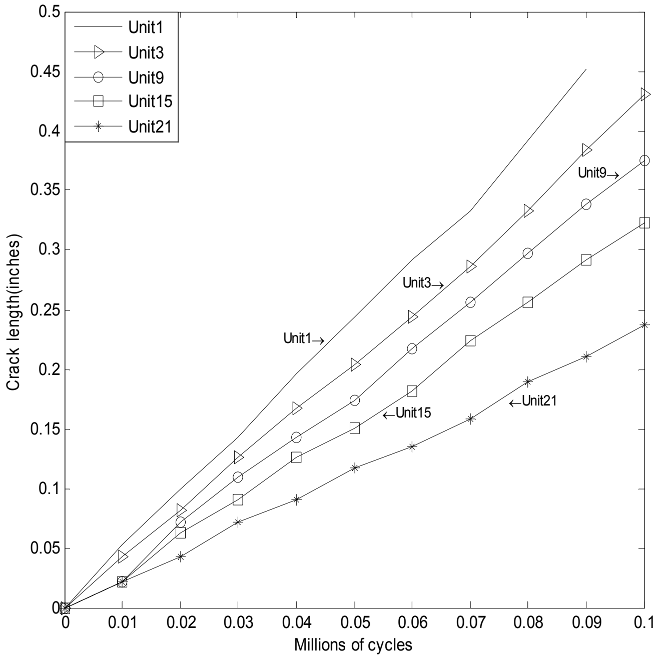

Fig. 1. The development of crack sizes over time.

A. Population reliability assessment

Based on the above data, we can estimate the reliability with Equations (4), (7) and (10) of different degradation models M_1, M_2 and M_3, respectively. In order to judge which model is more flexible, now we compare the results obtained with the above three models.

By using MCMC method, the estimation of the unknown parameters in those models can be obtained as follows:

$$\hat{\mu} = 3.377 \text{ , } \hat{\sigma} = 0.08746$$

and

$$\hat{\eta} = 3.377 \text{ , } \sigma_\eta = 0.649 \text{ , } \sigma = 0.062$$

and

$$\hat{\alpha} = 0.5293 \text{ , } \hat{\beta} = 143.1 \text{ , } \hat{\lambda} = 122.2 \text{ , } \hat{\theta} = 3.378$$

Correspondingly, we establish the reliability curves with three models respectively, as shown in Fig. 2. From Fig. 2, it can be found that reliability of the fatigue crack under model M_1 is not falling before the 0.10 million cycles. In fact, when the running time arrived at 0.10 million cycles, some units have failed, and the other units are also gradually close to fail, thus we can conclude that degradation assessment with M_1 is not so consistent with actual degradation data. On the contrary, the reliability curve under the model M_2 and model M_3 can well reflect the actual degradation situation of products.

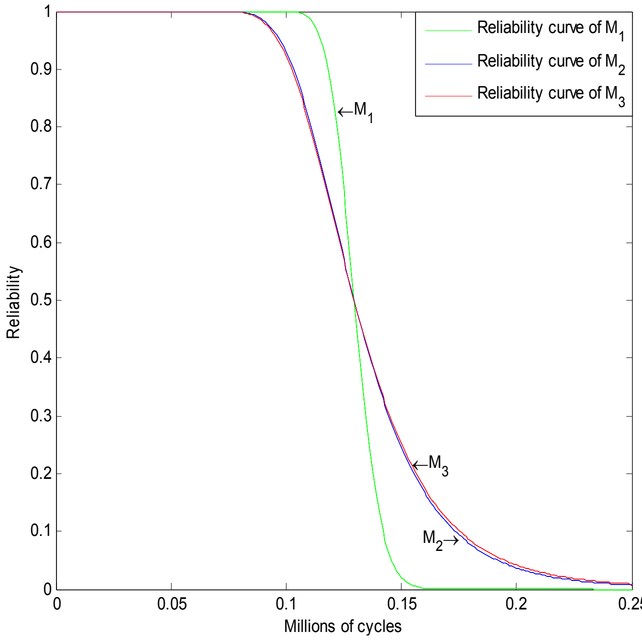

Fig.2 The reliability curve of three degradation models

B. Individual residual life prediction

Set the threshold ξ=0.4375, from Table 1, we can find that there are two units (i.e. unit 1 and unit 2) failed, and the corresponding failure times are 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 million cycles respectively. Given prediction reliability level p=0.95, we can obtain the residual life of product. In order to verify the superiority of the proposed model, we conduct some comparative studies by comparing the results obtained with model M_1, M_2 , M_3 and the true residual life, the corresponding result are shown as in Table □ and Table □.

From the Table 3 and Table 4, we can find that the mean error of unit 1 is 0.209, 0.074 and 0.069 in the different degradation models M_1, M_2 and M_3. The mean error of unit 2 is 0.1236, 0.0112 and 0.0102 in the different degradation models M_1, M_2 and M_3. Obviously, random effect model M_3 has smallest prediction error in the above three degradation models.

TABLE II
RESULT COMPARE OF UNIT 2

TV	0.10	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01
PV1	0.115	0.103	0.092	0.08	0.069	0.057	0.045	0.034	0.022	0.01
RE1	0.15	0.144	0.15	0.143	0.15	0.14	0.125	0.133	0.1	0
ME1	0.1236									
PV2	0.097	0.088	0.079	0.069	0.06	0.05	0.04	0.03	0.02	0.01
RE2	0.030	0.022	0.013	0.014	0	0	0	0.033	0	0
ME2	0.0112									
PV3	0.097	0.087	0.078	0.069	0.06	0.05	0.04	0.03	0.02	0.01
RE3	0.030	0.033	0.025	0.014	0	0	0	0	0	0
ME3	0.0102									

TV= True value of RL, ME1= mean error under M_1, RE1=relative error under M_1, PV1= predict value under M_1, ME2= mean error under M_2, RE2=relative error under M_2, PV3= predict value under M_3 ME3= mean error under M_3, RE3=relative error under M_3, PV3= predict value under M_3,RE=|PV-TV|/TV, ME is the arithmetic mean of RE.

TABLE □
RESULT COMPARE OF UNIT 1

TV1	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01
PV1	0.115	0.101	0.087	0.076	0.061	0.048	0.035	0.024	0.0094
RE1	0.278	0.256	0.243	0.267	0.218	0.2	0.167	0.2	0.06
ME1	0.209								
PV2	0.097	0.086	0.075	0.065	0.053	0.043	0.032	0.022	0.0094
RE2	0.078	0.075	0.071	0.083	0.060	0.075	0.067	0.1	0.06
ME2	0.074								
PV3	0.097	0.085	0.074	0.065	0.0528	0.0423	0.0317	0.022	0.0093
RE3	0.078	0.063	0.057	0.083	0.056	0.057	0.0567	0.1	0.07
ME3	0.069								

TV= True value of RL, ME1= mean error under M_1, RE1=relative error under M_1, PV1= predict value under M_1, ME2= mean error under M_2, RE2=relative error under M_2, PV3= predict value under M_3 ME3= mean error under M_3, RE3=relative error under M_3, PV3= predict value under M_3,RE=|PV-TV|/TV, ME is the arithmetic mean of RE.

VI. CONCLUSION

In this paper, three different Wiener process models are proposed to characterize the degradation path; the corresponding reliability assessment model and residual life prediction method are established. A case study of the fatigue crack data is given to validate the effectiveness of the proposed model and method. Main conclusions of this study are summarized below:

(1) Random effect Wiener process is well fitted to describe degradation.

(2) Since the likelihood function is so complicated, instead of directly maximizing the likelihood function, MCMC method can be used to estimate the unknown parameters.

(3) The residual life prediction method based on current performance degradation is obtained with the proposed model. Compared with fixed effect model M_1, the random effect model M_3 has smallest prediction error.

In this paper, we consider only the case when the performance degradation is governed by random effect Wiener process. In practice, it is also possible that the degradation paths of the product follow gamma process, Markov process, or other stochastic process.

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