Service and Selling Effort Decisions in a Two-Echelon Decentralized Supply Chain

Shengju Sang

Abstract—This paper investigates the service and selling effort decisions of a product in a two-echelon supply chain with one manufacturer and one retailer. Assuming that the market demand is dependent on the retail price, service level provided by the manufacturer and selling effort chosen by the retailer, three different kinds of game structures are Manufacturer-Stackelberg, considered, i.e., **Retailer-**Stackelberg and Vertical-Nash, and their optimal solutions are also derived. Finally, the results of the proposed models are analyzed via a numerical example. It is shown that the manufacturer and the retailer make the largest profits in the Vertical-Nash and Retailer-Stackelberg games, respectively, and the customer obtains the highest service level in the Retailer-Stackelberg game.

Index Terms—three-stage supply chain, spanning revenue sharing contract, fuzzy demand, trapezoidal fuzzy number

I. INTRODUCTION

RENCENTLY, as the living standard of people is improving, people become more and more sensitive to service level they could enjoy rather than a single price factor. Hence, how the manufacturer choosing his optimal service level and selling price for the product has become a hot topic faced by manager. Recently, more and more scholars and market administrators have begun take price and service into consideration to solve the supply chain management problems.

In a traditional supply chain, Iyer [1] studied the channel coordination mechanism when the retailers competed in price and service. Tsay and Agrawal [2] analyzed the price and service choices of two non-cooperating and cooperating retailers, and found that the supply chain members could achieve coordination only under very limiting conditions. Xiao and Yang [3] formulated a price and service competition model of two supply chains with one risk neutral supplier and one risk aversion retailer under demand uncertainty. Xiao and Yang [4] also developed a price and service competition model between a retailer and a manufacturer with a risk sharing rule under demand uncertainty. Lu et al. [5] proposed a price and service

competition model with two manufacturers and a common retailer in a liner demand function, where the customers were sensitive to both selling price and service level of the manufacturers. Wu [6] focused on a price and service decisions model with two manufacturers and a retailer, where the manufacturers produced the new and remanufactured products. Han et al. [7] studied a price and service competition problem with one manufacturer and two retailers where the manufacturer acted as the Stackelberg leader and two retailers were the follower, and made their optimal retail prices independently. In a dual-channel supply chain, Yan and Pei [8] analyzed the pricing and retail service decisions with a liner demand function. Dan et al. [9] also studied the optimal prices and retail services decisions in a centralized and a decentralized dual channel supply chain. In addition, Wang and Zhao [10] studied the price and service decisions in a dual supply chain where the manufacturer offered direct channel service and retail service. Xu et al. [11] examined the selling price and service strategies in a dual-channel market when the manufacturer and the retailer had fairness concerns. Some researches also studied the price and service decisions in a fuzzy environment, where the demand was a fuzzy liner function of selling price and service level. For instance, Zhao et al. [12] analyzed prices and services competition problems with two competing manufacturers and one retailer in fuzzy environments. Zhao and Wang [13] studied the pricing and retail service decisions between one manufacturer and two retailers with fuzzy demands. Samadi et al. [14] proposed a fuzzy inventory model with shortages, where the demand was a function of price, service expenditure and marketing expenditure. Koide and Sandoh [15] took the reference price effects of consumers into account and discussed a clearance pricing optimization in two periods. Sang [16] studied a revenue sharing contract with fuzzy demand in a three-echelon supply chain. Yang et al. [17] developed a fuzzy three-echelon inventory model with defective products and rework under credit period.

Most of the existing literatures have discussed the market demand depended on the retail price and the manufacturer's efforts. However, in real life, the retailer also has the opportunity to affect the final demand by choosing the selling efforts, such as providing self space, advertising and other demand enhancing activities. In this paper, we will present the two-echelon supply chain models with a manufacturer and a retailer, in which the market demand depends not only on the service, but also on the selling effort. We mainly discuss the conditions where the manufacturer and the retailer pursue three different power structures: pursuing the Manufacturer-Stackelberg game, playing the Retailer-Stackelberg game and acting in the Vertical-Nash game.

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Shengju Sang is with the Department of Economics and Management, Heze University, Heze, 274015, China (phone: +86 15853063720; e-mail: sangshengju@163.com).

The remainder of this paper is organized as follows. In Section II, the problem and notations related to this paper are described. Sections III, IV and V respectively develop three non-cooperative games, followed by a numerical example in Section VI. The conclusions are given in the last Section.

II. PROBLEM DESCRIPTIONS

Consider a two-echelon supply chain consisting of one manufacturer and one retailer. The manufacturer sells his product to the retailer, and then the retailer retails it to the customer. We assume the manufacturer produces only one product and the retailer sells only single product.

We model the demand faced by the manufacturer and retailer as a function of retail price p, service level s and selling effort level e, which is given by

$$q = \alpha - \beta p + \gamma s + \lambda e \tag{1}$$

where the parameter α represents the market potential, the parameter β represents the sensitivity of demand to price changes, the parameter γ represents the demand expansion effectiveness coefficient of the service level by the manufacturer, and the parameter λ represents the demand expansion effectiveness coefficient of the selling effort by the retailer.

Further, let *w* denote the wholesale price per unit charged to the retailer by the manufacturer, *c* the manufacturer's cost of producing its product and *m* the retailer's profit margin on the product. As the retail price can be treated as the total of the profit margin and wholesale price, we consider retail price as p = m + w. Then the demand for the product can be rewritten as

$$q = \alpha - \beta (w + m) + \gamma s + \lambda e \tag{2}$$

It is assumed that the marginal cost of the manufacturer is not affected by the service level. Further, the cost of achieving service level requires fixed investment, which is a quadratic function of service level *s*. It is given by $\frac{1}{2}\theta s^2$, where the parameter θ is the investment coefficient. We also assume that the retailer's cost at selling effort level *e* is $\frac{1}{2}\eta e^2$, where the parameter η is the effort cost coefficient.

According to the problem descriptions, the profit functions of the manufacturer and the retailer can be derived as follows

$$\Pi_{M} = (w-c) \left[\alpha - \beta (w+m) + \gamma s + \lambda e \right] - \frac{1}{2} \theta s^{2}$$
(3)

$$\Pi_{R} = m \left[\alpha - \beta \left(w + m \right) + \gamma s + \lambda e \right] - \frac{1}{2} \eta e^{2}$$
(4)

To insure that the manufacturer and the retailer will gain positive profits, we impose the following conditions on the parameters:

$$\begin{aligned} &2\beta\eta - \lambda^2 > 0 \ , \ 2\beta\theta - \gamma^2 > 0 \ , \ 4\beta\theta\eta - 2\lambda^2\theta - \gamma^2\lambda > 0 \ , \\ &4\beta\theta\eta - 2\gamma^2\eta - \lambda^2\theta > 0 \ \text{ and } \ 3\beta\theta\eta - \lambda^2\theta - \gamma^2\eta > 0 \ . \end{aligned}$$

III. MANUFACTURER-STACKELBERG GAME

The MS (Manufacturer-Stackelberg) game scenario arises in the market where the size of the retailer is smaller

compared to the manufacturer. In this case, the manufacturer is the leader, and the retailer is the follower. That is, firstly, the manufacturer sets the wholesale price w and the service level s using the retailer's reaction function. Then, the retailer sets the sale margin m and the selling effort level e so as to maximize his expected profit.

Theorem 1. The optimal solutions in the MS game are

$$n^* = \frac{\eta \theta (\alpha - \beta c)}{4\beta \theta \eta - 2\lambda^2 \theta - \gamma^2 \lambda}$$
(5)

$$e^* = \frac{\lambda \theta (\alpha - \beta c)}{4\beta \theta \eta - 2\lambda^2 \theta - \gamma^2 \lambda}$$
(6)

$$w^{*} = \frac{\theta \left(2\beta\eta - \lambda^{2}\right) \left(\alpha - \beta c\right)}{\beta \left(4\beta\theta\eta - 2\lambda^{2}\theta - \gamma^{2}\lambda\right)} + c \tag{7}$$

$$s^* = \frac{\eta \gamma \left(\alpha - \beta c\right)}{4\beta \theta \eta - 2\lambda^2 \theta - \gamma^2 \lambda} \tag{8}$$

Proof. We first derive the optimal decisions of the retailer. Assuming an interior solution, we get the first-order derivatives of Π_R from Equation (4) with respect to *m* and *e* as follows

$$\frac{\partial \Pi_R}{\partial m} = -2\beta m + \lambda e + \alpha - \beta w + \gamma s \tag{9}$$

$$\frac{\partial \Pi_R}{\partial e} = -\eta e + \lambda m \tag{10}$$

Therefore, the Hessian matrix of Π_{R} is

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 \Pi_R}{\partial m^2} & \frac{\partial^2 \Pi_R}{\partial m \partial e} \\ \frac{\partial^2 \Pi_R}{\partial e \partial m} & \frac{\partial^2 \Pi_R}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -2\beta & \lambda \\ \lambda & -\eta \end{bmatrix}$$
(11)

Note that the Hessian matrix of Π_R is negative definite, since $\beta > 0$, $\eta > 0$ and $2\beta\eta - \lambda^2 > 0$. Thus Π_R is strictly jointly concave in *m* and *e*. Hence, let the first-order conditions be zero, we get the optimal response functions of the retailer as

$$m^{*}(w,s) = \frac{\eta}{2\beta\eta - \lambda^{2}} (\alpha - \beta w + \gamma s)$$
(12)

$$e^{*}(w,s) = \frac{\lambda}{2\beta\eta - \lambda^{2}} (\alpha - \beta w + \gamma s)$$
(13)

Substituting $m^*(w, s)$ in Equation (12) and $e^*(w, s)$ in Equation (13) into Equation (3), we get the manufacturer's profit as

$$\Pi_{M} = \frac{\beta\eta}{2\beta\eta - \lambda^{2}} (w - c) (\alpha - \beta w + \gamma s) - \frac{1}{2}\theta s^{2} \qquad (14)$$

The first-order derivatives of Equation (14) with respect to w and s can be derived as follows

$$\frac{\partial \Pi_{M}}{\partial w} = \frac{\beta \eta}{2\beta \eta - \lambda^{2}} \left(-2\beta w + \gamma s + \alpha + \beta c \right)$$
(15)

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$$\frac{\partial \Pi_M}{\partial s} = -\theta s + \frac{\beta \eta \gamma}{2\beta \eta - \lambda^2} (w - c)$$
(16)

Therefore, the Hessian matrix of Π_M is

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 \Pi_M}{\partial w^2} & \frac{\partial^2 \Pi_M}{\partial w \partial s} \\ \frac{\partial^2 \Pi_M}{\partial s \partial w} & \frac{\partial^2 \Pi_M}{\partial s^2} \end{bmatrix} = \begin{bmatrix} -\frac{2\beta^2 \eta}{2\beta\eta - \lambda^2} & \frac{\beta\eta\gamma}{2\beta\eta - \lambda^2} \\ \frac{\beta\eta\gamma}{2\beta\eta - \lambda^2} & -\theta \end{bmatrix}$$
(17)

Note that the Hessian matrix of Π_M is negative definite, since $\beta > 0$, $\eta > 0$, $\theta > 0$, $2\beta\eta - \lambda^2 > 0$ and $4\beta\theta\eta - 2\lambda^2\theta$ $-\gamma^2\lambda > 0$. Thus Π_M is strictly jointly concave in *w* and *s*.

Let the first-order conditions be zero, we get w^* and s^* as follows

$$w^{*} = \frac{\theta (2\beta\eta - \lambda^{2})(\alpha - \beta c)}{\beta (4\beta\theta\eta - 2\lambda^{2}\theta - \gamma^{2}\lambda)} + c$$
(18)

$$s^* = \frac{\eta \gamma (\alpha - \beta c)}{4\beta \theta \eta - 2\lambda^2 \theta - \gamma^2 \lambda}$$
(19)

Substituting w^* and s^* into Equations (12) and (13), we can easily obtain m^* and e^* as follows

$$m^* = \frac{\eta \theta (\alpha - \beta c)}{4\beta \theta \eta - 2\lambda^2 \theta - \gamma^2 \lambda}$$
(20)

$$e^* = \frac{\lambda \theta (\alpha - \beta c)}{4\beta \theta \eta - 2\lambda^2 \theta - \gamma^2 \lambda}$$
(21)

The proof of Theorem 1 is completed.

By combining Equations (5), (6), (7) and (8) with Equations (3) and (4), we derive the optimal profits of the manufacturer and the retailer in the MS case as follows

$$\Pi_{M}^{*} = \frac{\theta \eta \left(\alpha - \beta c\right)^{2}}{2\left(4\beta \theta \eta - 2\lambda^{2} \theta - \gamma^{2} \lambda\right)}$$
(22)

$$\Pi_{R}^{*} = \frac{\theta^{2} \eta \left(2\beta\eta - \lambda^{2}\right) \left(\alpha - \beta c\right)^{2}}{2 \left(4\beta\theta\eta - 2\lambda^{2}\theta - \gamma^{2}\lambda\right)^{2}}$$
(23)

Remark 1. If we do not take the selling effort into consideration, then the results of the Theorem 1 reduce to

$$m^{*} = \frac{\theta(\alpha - \beta c)}{4\beta\theta - \gamma^{2}}$$
$$w^{*} = \frac{2\theta(\alpha - \beta c)}{4\beta\theta - \gamma^{2}} + c$$
$$s^{*} = \frac{\gamma(\alpha - \beta c)}{4\beta\theta - \gamma^{2}}$$
$$\Pi_{M}^{*} = \frac{\theta(\alpha - \beta c)^{2}}{2(4\beta\theta - \gamma^{2})}$$

$$\Pi_{R}^{*} = \frac{\beta\theta^{2} (\alpha - \beta c)^{2}}{\left(4\beta\theta - \gamma^{2}\right)^{2}}$$

IV. RETAILER-STACKELBERG GAME

The RS (Retailer-Stackelberg) game scenario arises in the market where the size of the manufacturer is smaller compared to the retailer. This case implies the retailer becomes the leader and the manufacturer is the follower. The retailer first sets sale margin m and selling effort level e using the reaction functions of the manufacturer. Then the manufacturer observes the decisions made by the retailer and makes his response to those decisions by setting wholesale price w and service level s.

Theorem 2. The optimal solutions in the RS game are

$$m^{**} = \frac{\eta \left(2\beta\theta - \gamma^2\right) (\alpha - \beta c)}{\beta \left(4\beta\theta\eta - 2\gamma^2\eta - \lambda^2\theta\right)}$$
(24)

$$e^{**} = \frac{\theta \lambda \left(\alpha - \beta c\right)}{4\beta \theta \eta - 2\gamma^2 \eta - \lambda^2 \theta}$$
(25)

$$w^{**} = \frac{\theta \eta \left(\alpha - \beta c\right)}{4\beta \theta \eta - 2\gamma^2 \eta - \lambda^2 \theta} + c \tag{26}$$

$$S^{**} = \frac{\gamma \eta \left(\alpha - \beta c\right)}{4\beta \theta \eta - 2\gamma^2 \eta - \lambda^2 \theta}$$
(27)

Proof. We first derive the optimal decisions of the manufacturer. Assuming an interior solution, we get the first-order derivatives of Π_M from Equation (3) with respect to *w* and *s* as follows

$$\frac{\partial \Pi_{M}}{\partial w} = -2\beta w + \gamma s + \alpha - \beta m + \lambda e + \beta c \qquad (28)$$

$$\frac{\partial \Pi_M}{\partial s} = -\theta s + \gamma \left(w - c \right) \tag{29}$$

Therefore, the Hessian matrix of Π_M is

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 \Pi_M}{\partial w^2} & \frac{\partial^2 \Pi_M}{\partial w \partial s} \\ \frac{\partial^2 \Pi_M}{\partial s \partial w} & \frac{\partial^2 \Pi_M}{\partial s^2} \end{bmatrix} = \begin{bmatrix} -2\beta & \gamma \\ \gamma & -\theta \end{bmatrix}$$
(30)

Note that the Hessian matrix of Π_M is negative definite, since $\beta > 0$, $\theta > 0$ and $2\beta\theta - \gamma^2 > 0$. Thus Π_M is strictly jointly concave in *w* and *s*. Hence, let the first-order conditions be zero, we get the optimal response functions of the manufacturer as

$$w^{**}(m,e) = \frac{\theta}{2\beta\theta - \gamma^2} (\alpha - \beta m + \lambda e - \beta c) + c \qquad (31)$$

$$s^{**}(m,e) = \frac{\gamma}{2\beta\theta - \gamma^2} (\alpha - \beta m + \lambda e - \beta c)$$
(32)

Substituting $w^{**}(m, e)$ and $s^{**}(m, e)$ into Equation (4), we get the retailer's profit as

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$$\Pi_{R} = \frac{\beta\theta}{2\beta\theta - \gamma^{2}} m \left(\alpha - \beta m + \lambda e - \beta c\right) - \frac{1}{2}\eta e^{2}$$
(33)

We get the first-order derivatives of Π_R from Equation (22) with respect to *m* and *e* as follows

$$\frac{\partial \Pi_{R}}{\partial m} = \frac{\beta \theta}{2\beta \theta - \gamma^{2}} \left(-2\beta m + \lambda e + \alpha - \beta c \right)$$
(34)

$$\frac{\partial \Pi_R}{\partial e} = -\eta e + \frac{\beta \theta \lambda}{2\beta \theta - \gamma^2} m \tag{35}$$

Therefore, the Hessian matrix of Π_R is

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 \Pi_R}{\partial m^2} & \frac{\partial^2 \Pi_R}{\partial m \partial e} \\ \frac{\partial^2 \Pi_R}{\partial e \partial m} & \frac{\partial^2 \Pi_R}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -\frac{2\beta^2\theta}{2\beta\theta - \gamma^2} & \frac{\beta\theta\lambda}{2\beta\theta - \gamma^2} \\ \frac{\beta\theta\lambda}{2\beta\theta - \gamma^2} & -\eta \end{bmatrix}$$
(36)

Note that the Hessian matrix of Π_R is negative definite, since $\theta > 0$, $\eta > 0$, $2\beta\theta - \gamma^2 > 0$ and $4\beta\theta\eta - 2\gamma^2\eta - \lambda^2\theta > 0$. Thus Π_R is strictly jointly concave in *m* and *e*. Hence, let the first-order conditions be zero, we get the optimal solutions m^{**} and e^{**} of the retailer as follows

$$m^{**} = \frac{\eta \left(2\beta\theta - \gamma^2\right) (\alpha - \beta c)}{\beta \left(4\beta\theta\eta - 2\gamma^2\eta - \lambda^2\theta\right)}$$
(37)

$$e^{**} = \frac{\theta \lambda (\alpha - \beta c)}{4\beta \theta \eta - 2\gamma^2 \eta - \lambda^2 \theta}$$
(38)

Substituting m^{**} and e^{**} in Equation (37) and (38) into Equations (31) and (32), we obtain w^{**} and s^{**} as follows

$$w^{**} = \frac{\theta \eta \left(\alpha - \beta c\right)}{4\beta \theta \eta - 2\gamma^2 \eta - \lambda^2 \theta} + c \tag{39}$$

$$s^{**} = \frac{\gamma \eta \left(\alpha - \beta c\right)}{4\beta \theta \eta - 2\gamma^2 \eta - \lambda^2 \theta} \tag{40}$$

The proof of Theorem 2 is completed.

By combining Equations (24), (25), (26) and (27) with Equations (3) and (4), we derive the optimal profits for the manufacturer and retailer in the RS case as follows

$$\Pi_{M}^{**} = \frac{\theta \eta^{2} \left(\alpha - \beta c\right)^{2}}{2 \left(4\beta \theta \eta - 2\gamma^{2} \eta - \lambda^{2} \theta\right)^{2}}$$
(41)

$$\Pi_{R}^{**} = \frac{\theta \eta (\alpha - \beta c)^{2}}{2(4\beta \theta \eta - 2\gamma^{2}\eta - \lambda^{2}\theta)}$$
(42)

Remark 2. If we do not take the selling effort into consideration, then the results of the Theorem 2 reduce to

$$m^{**} = \frac{\alpha - \beta c}{2\beta}$$
$$w^{**} = \frac{\theta(\alpha - \beta c)}{2(2\beta\theta - \gamma^2)} + c$$

$$s^{**} = \frac{\gamma(\alpha - \beta c)}{2(2\beta\theta - \gamma^2)}$$
$$\Pi_M^{**} = \frac{\theta(\alpha - \beta c)^2}{8(2\beta\theta - \lambda^2)}$$
$$\Pi_R^{**} = \frac{\theta(\alpha - \beta c)^2}{4(2\beta\theta - \lambda^2)}$$

V. VERTICAL-NASH GAME

The VN (Vertical-Nash) game scenario arises in the market where the manufacturer and retailer have equal market power. In this case the manufacturer determines the wholesale price w and the service level s, and the retailer makes the profit margin m and the selling effort level e simultaneously and independently.

Theorem 3. The optimal solutions in the VN game are

$$m^{***} = \frac{\eta \theta (\alpha - \beta c)}{3\beta \theta \eta - \lambda^2 \theta - \gamma^2 \eta}$$
(43)

$$e^{***} = \frac{\lambda\theta(\alpha - \beta c)}{3\beta\theta\eta - \lambda^2\theta - \gamma^2\eta}$$
(44)

$$w^{***} = \frac{\eta \theta (\alpha - \beta c)}{3\beta \theta \eta - \lambda^2 \theta - \gamma^2 \eta} + c$$
(45)

$$s^{***} = \frac{\eta \gamma (\alpha - \beta c)}{3\beta \theta \eta - \lambda^2 \theta - \gamma^2 \eta}$$
(46)

Proof. We get the first-order derivatives of Π_M from Equation (3) with respect to *w* and *s*, and Π_R from Equation (4) with respect to *m* and *e* as follows

$$\frac{\partial \Pi_M}{\partial w} = -2\beta w + \gamma s + \alpha - \beta m + \lambda e + \beta c \tag{47}$$

$$\frac{\partial \Pi_M}{\partial s} = -\theta s + \gamma \left(w - c \right) \tag{48}$$

$$\frac{\partial \Pi_R}{\partial m} = -2\beta m + \lambda e + \alpha - \beta w + \gamma s \tag{49}$$

$$\frac{\partial \Pi_R}{\partial e} = -\eta e + \lambda m \tag{50}$$

Therefore, the Hessian matrix of Π_M and Π_R are

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 \Pi_M}{\partial w^2} & \frac{\partial^2 \Pi_M}{\partial w \partial s} \\ \frac{\partial^2 \Pi_M}{\partial s \partial w} & \frac{\partial^2 \Pi_M}{\partial s^2} \end{bmatrix} = \begin{bmatrix} -2\beta & \gamma \\ \gamma & -\theta \end{bmatrix}$$
$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 \Pi_R}{\partial m^2} & \frac{\partial^2 \Pi_R}{\partial m \partial e} \\ \frac{\partial^2 \Pi_R}{\partial e \partial m} & \frac{\partial^2 \Pi_R}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -2\beta & \lambda \\ \lambda & -\eta \end{bmatrix}$$

Note that the Hessian matrix of Π_M and Π_R are negative definite, since $\beta > 0$, $\theta > 0$, $\eta > 0$, $2\beta\theta - \gamma^2 > 0$ and $2\beta\eta - \lambda^2 > 0$. Thus Π_M is strictly jointly concave in w

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and S, and Π_R is strictly jointly concave in m and e.

Let the first-order condition be zero, we get the optimal solutions m^{***} , e^{***} , w^{***} and s^{***} of the retailer and the manufacturer as follows

$$m^{***} = \frac{\eta \theta (\alpha - \beta c)}{3\beta \theta \eta - \lambda^2 \theta - \gamma^2 \eta}$$
(51)

$$e^{***} = \frac{\lambda\theta(\alpha - \beta c)}{3\beta\theta\eta - \lambda^2\theta - \gamma^2\eta}$$
(52)

$$w^{***} = \frac{\eta \theta (\alpha - \beta c)}{3\beta \theta \eta - \lambda^2 \theta - \gamma^2 \eta} + c$$
(53)

$$s^{***} = \frac{\eta \gamma (\alpha - \beta c)}{3\beta \theta \eta - \lambda^2 \theta - \gamma^2 \eta}$$
(54)

The proof of Theorem 3 is completed.

By combining Equations (43), (44), (45) and (46) with Equations (3) and (4), we derive the optimal profits for the manufacturer and the retailer in the VN case as follows

$$\Pi_{M}^{***} = \frac{\eta^{2} \theta \left(2\beta\theta - \gamma^{2}\right) \left(\alpha - \beta c\right)^{2}}{2 \left(3\beta\theta\eta - \lambda^{2}\theta - \gamma^{2}\eta\right)^{2}}$$
(55)

$$\Pi_{R}^{***} = \frac{\eta \theta^{2} \left(2\beta\eta - \lambda^{2}\right) \left(\alpha - \beta c\right)^{2}}{2 \left(3\beta\theta\eta - \lambda^{2}\theta - \gamma^{2}\eta\right)^{2}}$$
(56)

Remark 3. If we do not take the selling effort into consideration, then the results of the Theorem 3 reduce to

$$m^{***} = \frac{\theta(\alpha - \beta c)}{3\beta\theta - \gamma^2}$$
$$w^{***} = \frac{\theta(\alpha - \beta c)}{3\beta\theta - \gamma^2} + c$$
$$s^{***} = \frac{\gamma(\alpha - \beta c)}{3\beta\theta - \gamma^2}$$
$$\Pi_M^{***} = \frac{\theta(\alpha - \beta c)^2 (2\beta\theta - \gamma^2)}{2(3\beta\theta - \gamma^2)^2}$$
$$\Pi_R^{***} = \frac{\beta\theta^2 (\alpha - \beta c)^2}{(3\beta\theta - \gamma^2)^2}$$

VI. NUMERICAL EXAMPLE

In this section, we tend to further elucidate the above proposed three different non-cooperative games. The following parameters are used for illustration:

 $\alpha = 100$, $\beta = 5.0$, $\gamma = 5.0$, $\lambda = 4.0$, $\theta = 4.0$ and c = 6.0.

Based on the analysis showed in the Sections III, IV and V, we present the results of the optimal prices, service level, selling effort level and profits of the supply chain members in the MS, RS and VN games in Tables I and II.

TABLE I THE OPTIMAL PRICES FOR THREE DIFFERENT GAMES WITH DIFFERENT η

			1		
	η	т	е	w	S
MS	4.50	9.55	8.48	18.30	11.93
	5.00	8.14	6.51	17.07	10.17
	5.50	7.26	5.28	16.30	9.08
	6.00	6.67	4.44	15.78	8.33
	6.50	6.23	3.84	15.40	7.80
RS	4.50	13.31	15.77	23.75	22.18
	5.00	12.21	13.02	22.28	20.35
	5.50	11.44	11.09	21.25	19.06
	6.00	10.86	9.66	20.48	18.10
	6.50	10.42	8.55	19.89	17.34
VN	4.50	13.48	11.98	19.48	16.84
	5.00	12.61	10.09	18.61	15.77
	5.50	11.98	8.72	17.98	14.98
	6.00	11.51	7.67	17.51	14.38
	6.50	11.13	6.85	17.13	13.91

TABLE II THE OPTIMAL PROFITS FOR THREE DIFFERENT GAMES WITH DIFFERENT η

		1		
	η	Π_M	Π_R	Π_{sc}
MS	4.50	334.09	293.60	627.69
	5.00	284.88	225.26	510.14
	5.50	254.24	187.09	441.33
	6.00	233.33	162.96	396.29
	6.50	218.15	146.43	364.58
RS	4.50	39.37	621.13	660.50
	5.00	33.13	569.77	602.90
	5.50	29.06	533.66	562.72
	6.00	26.22	509.90	533.12
	6.50	24.13	486.26	510.39
VN	4.50	340.50	585.16	925.66
	5.00	298.27	540.87	839.14
	5.50	269.30	509.22	778.52
	6.00	248.26	485.49	733.75
	6.50	232.33	467.05	699.38

Based on the results showed in Table1, we find:

(1) The profit margin of the retailer m is the highest in the VN case followed by the RS and then the MS cases. The optimal selling effort level e is the highest in the RS game this is because under this case the retailer is a pricing leader. The wholesale price w and service level s are the highest in the RS case followed by the VN and then the MS cases.

(2) The retailer makes the largest profit in the RS case, and the smallest in the MS case, and the manufacturer makes the largest profit in the VN case, and the smallest in the RS case. The profit of the manufacturer is larger than that of the retailer in the MS case, and the profit of the retailer is larger than that of the manufacturer in the RS case. It indicates that the actor who is the leader in the supply chain holds advantage in obtaining higher profit. The profit of the supply chain system is the largest in the VN case when no actor is a pricing leader.

(3) As the service investment coefficient η increases, the profit margin of the retailer, selling effort level, wholesale price, service level, and profits of the manufacturer and the retailer will all decrease.

VII. CONCLUSIONS

This paper proposes a two-echelon supply chain

management, where the manufacturer and retailer pursue three different kinds of scenarios: Manufacturer-Stackelberg, Retailer-Stackelberg and Vertical-Nash games. The models in our case contain three strategic variables: price, service level and selling effort, which is truly representative of the electronic industry. The limitation of this paper is that we only consider one manufacturer and one retailer. There are some possible extensions to improve our supply chain models, for example, the decisions model with multiple competitive manufacturers and retailers, the coordination mechanism of the supply chain under the decentralized decision case can be studied in the future.

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Shengju Sang Received his B.M. Degree from Qingdao University, Qingdao, China, in 2005 and M.M. Degree from Qingdao University in 2008, received his Ph.D. Degree in School of Management and Economics, Beijing Institute of Technology, Beijing, China, in 2011. He is currently working as an associate professor at department of Economics and Management, Heze

University, Heze, China. His papers appeared in some journals such as Fuzzy Optimization and Decision Making, Journal of Intelligent & Fuzzy Systems, Mathematical Problems in Engineering and other journals. His current research interests are in the areas of supply chain management, fuzzy decisions and its applications.