Fish Consumption Impact on Coronary Heart Disease Mortality in Morocco: A Mathematical Model with Optimal Control

Mohamed Lamlili E.N.*, Abdesslam Boutayeb, Mohamed Derouich, Wiam Boutayeb and Abderrahmane Moussi

Abstract—**Extending the results of a previous paper on the relationship between fish consumption and coronary heart disease, an optimal control approach is proposed in the present paper.**
The Pontryagin's minimum principle is used to characterize the optimal control, to minimize the population of susceptible individuals and also to reduce the mortality rate of coronary heart disease.
A numerical simulation is carried out to show the impact of the proposed optimal control. Indeed, the model shows that the prevalence and mortality of coronary heart disease can be significantly reduced in a period of 10 years.

Index Terms—**Fish consumption, CHD, Harvesting, Modelling, Optimal control, Simulation.**

I. Introduction

ONE of the biggest challenges currently facing humanity is chronic diseases which are sweeping the entire globe, with an increasing trend in developing countries [1]. According to the World Health Statistics 2014, the top three causes of years of life lost due to premature death are coronary heart disease (CHD), lower respiratory infections and stroke [2]. The joint WHO/FAO Expert report on diet, nutrition and the prevention of chronic diseases indicated that, fish oils are rich in eicosapentaenoic acid (EPA) and docosahexaenoic acid (DHA) which are the main sources of polyunsaturated fatty acids (PUFAs) of the family omega-3 [3]. Consequently, fish consumption is recommended as a protective action against coronary heart disease and ischaemic stroke. Indeed, most of the epidemiological evidence related to omega-3 PUFAs is derived from studies of fish consumption in populations or interventions involving fish diets in clinical trials [4], [5], [6], [7], [8], [9], [10]. Fish oils have been used in the Gruppo Italiano per lo Studio della Sopravvivenza nell'Infarto Miocardico (GISSI) trial involving survivors of myocardial infarction [4]. After 3.5 years of follow-up, the group that received fish oil had a 20% reduction in total mortality, a 30% reduction in cardiovascular death and a 45% decrease in sudden death. Hu et al. found that, among women, higher consumption of fish and omega-3 fatty acids was associated with a lower risk of CHD, particularly CHD deaths [5]. Kris-Etherton et al. have studied the effects on cardiovascular diseases (CVD) and concluded that Omega3 (n3) acid supplements which are highly contained in fatty fish can reduce cardiac events [6]. Virtanen et al. pointed out that consuming fish can reduce risk of major chronic diseases (cardiovascular disease, cancer...) [7]. A meta-analysis of 13 cohort studies examined the association between fish intake and CHD mortality. Significant inverse associations were reported between fish consumption and fatal CHD [8]. Another meta-analysis of observational studies on fish intake and CHD indicated that fish consumption was associated with significantly lower risk of fatal and total CHD [9]. In 2010 an updated meta-analysis of seven cohort studies concluded that fish consumption has a significant protective effect on fatal CHD [10].

Morocco has an important stock of fish living along the 3500 kms of Mediterranean and Atlantic coasts, with sardine as the most abundant species [11]. However, the Moroccan consumption of fish is relatively low (8kg/person/year).

Most of the mathematical models devoted to the dynamics of fish populations deal with the bionomic equilibrium which is achieved when the total revenue obtained by selling the harvested biomass equals the total cost utilized in harvesting it [12], [13], [14], [15], [16]. Consequently, maximizing a benefit function is very often indicated as the optimal harvesting policy under sustainability of fish populations. Mathematical models with optimal control were used for optimal prevention and vaccination strategies for communicable diseases like HIV/AIDS, Chikungunya disease and more general SIR epidemic models [17], [18], [19]. Other updated references dealing with optimal control and broadly with mathematical modeling and simulation can be found in [20], [21], [22], [23], [24], [25].

In this paper, we propose a mathematical model using optimal control for non communicable diseases, going beyond the mere economic benefit by minimizing the rate of people at risk of CHD based on the level of fish consumption.

II. The mathematical model

We consider the mathematical model developed by Lamlili et al. [26], studying the dynamics of a population at risk of CHD and that of fish population living along the Moroccan coasts, presented as follows:

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$$
\begin{cases}
\dfrac{dY}{dt} = \Lambda - (\mu+\alpha)Y + \beta\overline{Y} \\[2ex]
\dfrac{d\overline{Y}}{dt} = -(\mu+\delta R_R+\beta)\overline{Y} + \alpha Y \\[2ex]
\dfrac{dX}{dt} = r\left(1-\dfrac{X}{K}\right)X - qEX \\[2ex]
X(0)>0,\ Y(0)>0,\ \overline{Y}(0)>0\ .
\end{cases}
\tag{1}
$$

Where:
$Y=Y(t)$: the number of persons without CHD risk,
$\overline{Y}=\overline{Y}(t)$: the number of persons with CHD risk,
$X=X(t)$: the biomass of the fish population in Moroccan coasts,
r: the biotic potential,
K: the carrying capacity,
E: the total harvesting effort,
q: the catchability coefficient,
Λ: the recruitment of persons without CHD risk,
μ: the natural mortality rate,
δ: the mortality rate due to CHD,
α: the probability to have CHD,
β: the rate of patients with CHD who are cured,
R_R: is the relative risk depending on fish consumption.

Note that the relative risk is a logistic function which describes (in order to compare) the likelihood of developing CHD in a group of individuals eating fish compared to individuals who have a little or no consumption ($R_R=1$). We recall that R_R is given by the following function:

$$R_R = \frac{a_1}{a_2+e^{a_3\gamma X}}\ ,$$

with

$$\gamma = \frac{qE}{P_t} aT_f\ ,$$

where a_1, a_2, a_3 are negative coefficients, a is the rate of fish consumed from the total harvested and T_f is a coefficient that transforms the consumption of fish per capita and per year to a frequency per month.

Most cardiovascular diseases can be prevented by addressing metabolic risk factors like obesity, high blood pressure, diabetes and raised lipids, and behavioural risk factors such as tobacco use, alcohol, physical inactivity and unhealthy diet [27]. Consequently, an optimal control strategy can be obtained by acting on these risk factors. In particular, a reduction of fish price is required by the Moroccan authorities and a sensitisation is needed to convince Moroccan people to consume more fish, using healthy eating habits.

Mathematically, introducing a control $u=u(t)$ in the model above (1) leads to the following controlled model:

$$
\begin{cases}
\dfrac{dY}{dt} = \Lambda - (\mu+\alpha(1-u))Y + \beta\overline{Y} \\[2ex]
\dfrac{d\overline{Y}}{dt} = -(\mu+\delta R_R+\beta)\overline{Y} + \alpha(1-u)Y \\[2ex]
\dfrac{dX}{dt} = r\left(1-\dfrac{X}{K}\right)X - qEX \\[2ex]
X(0)>0,\ Y(0)>0,\ \overline{Y}(0)>0\ .
\end{cases}
\tag{2}
$$

With,

$$N(t) = Y(t) + \overline{Y}(t) \text{ for all } t.$$

III. The optimal control

In order to reduce the rate of people at risk of CHD, we use the optimal control theory to analyse the behaviour of the controlled model (2) on the basis of minimizing the objective functional $\mathcal{J}(u)$ given by:

$$\mathcal{J}(u) = \int_0^T \left(\overline{Y} + Au^2(t)\right) dt\ . \tag{3}$$

Where A is a positive weight that balances the size of the terms. U is the control set defined by:

$$U = \{u/u \text{ is measurable},\ 0\le u(t)\le 1,\ t\in[0,T]\}\ .$$

Our goal is to characterize an optimal control $u^*\in U$ satisfying:

$$\mathcal{J}(u^*) = \min_{u\in U}\mathcal{J}(u)\ .$$

A. Existence and Positivity of Solutions

Proposition 3.1: The following set
$\Omega = \{(Y,\ \overline{Y},\ X)\ \in\mathbb{R}^3/0\le Y,\ \overline{Y}\le\frac{\Lambda}{\mu} \text{ and } 0\le X\le K\}$
is positively invariant under system (2).

Proof:

We have:

$$\frac{dY(t)}{dt} = \Lambda - (\mu+\alpha(1-u(t)))\,Y(t) + \beta\overline{Y}(t)\ ,$$
$$\Rightarrow \frac{d(Y(t)\ e^{(\mu+\alpha)t})}{dt} = e^{(\mu+\alpha)t}\left[\Lambda + \alpha u(t)Y(t) + \beta\overline{Y}(t)\right] \tag{4}$$

Assume that there exists $t^*>0$ such that $Y(t^*)=0$ while $Y(t)$, $\overline{Y}(t)$ and $X(t)$ are all positive for $t\in[0,t^*[$.

By integrating the equation (4) from 0 to t^*, we obtain:

$$Y(t^*) = e^{-(\mu+\alpha)t^*}\left[Y(0) + \int_0^{t^*} e^{(\mu+\alpha)t}\left[\Lambda + \alpha u(t)Y(t) + \beta\overline{Y}(t)\right] dt\right].$$

$\Rightarrow$ $Y(t^*)>0$, which is in contradiction with the assumption that $Y(t^*)=0$.
Thus,

$$Y(t)>0 \text{ for all } t\in[0,T]\ .$$

Let

$$\frac{d\overline{Y}}{dt} = -\left(\mu + \delta R_R(t) + \beta\right)\overline{Y}(t) + \left(\alpha(1-u(t))Y(t)\right),$$

with

$$\alpha\left(1-u(t)\right)Y(t) \geq 0 \;,$$

and

$$0 < R_R(t) \leq 1 \;.$$

$$\Rightarrow \quad \frac{d\overline{Y}(t)}{dt} \geq -\left(\mu + \delta + \beta\right)\overline{Y}(t) \;,$$

by using the Gronwall's inequality, we obtain

$$\overline{Y}(t) \geq \overline{Y}(0)e^{-(\mu+\delta+\beta)dt} > 0 \;.$$

Let us return to the last equation of the system (2) above

$$\frac{dX(t)}{dt} = r\left(1 - \frac{X(t)}{K}\right)X(t) - qEX(t) \;,$$

$$\frac{d(X(t)e^{qEt})}{dt} = \frac{dX(t)}{dt}e^{qEt} + qEX(t)e^{qEt} \;.$$

Assume that there exists $t^* > 0$ such that $X(t^*) = 0$ and $X(t) > 0$ for $t \in [0, t^*[$.
By integrating the equation (5) from 0 to t^*, we obtain:

$$X(t^*) = e^{-qEt^*}\left[X(0) + \int_0^{t^*} e^{qEt}\left[r\left(1 - \frac{X(t)}{K}\right)X(t)\right]\right].$$

This implies $X(t^*) > 0$, which is in contradiction with $X(t^*) = 0$.
Therefore,

$$X(t) > 0 \text{ for all } t \in [0, T] \;.$$

The fact that $X \leq K$ is relative to the ecological assumption that the biomass X can't be greater than the carrying capacity K.

Finally,

$$\begin{aligned}
\frac{dN(t)}{dt} &= \Lambda - \mu Y(t) - \mu\overline{Y}(t) - \delta R_R(t)\overline{Y}(t) \\
&= \Lambda - \mu N(t) - \delta R_R(t)\overline{Y}(t) \\
&\leq \Lambda - \mu N(t) \\
\Rightarrow N(t) &\leq \left(N(0) - \frac{\Lambda}{\mu}\right)e^{-\mu t} + \frac{\Lambda}{\mu} \\
&\leq \frac{\Lambda}{\mu} \;.
\end{aligned}$$

Proposition 3.2: The controlled system (2) that satisfies a given initial condition $(Y(0), \overline{Y}(0), X(0)) \in \Omega$ has an unique solution.

Proof:
The proof of the Proposition 3.2 is based on the following Lemma.

Lemma 3.3 ([28], Lemma 3.3):
If $f(t,x)$ and $\left[\frac{\partial f}{\partial x}\right](t,x)$ are continuous on $[a,b] \times \mathbb{R}^n$, then f is globally Lipschitz in x on $[a,b] \times \mathbb{R}^n$ if and only if $\left[\frac{\partial f}{\partial x}\right](t,x)$ is uniformly bounded on $[a,b] \times \mathbb{R}^n$.
Let
$Y = Y(t)$, $\overline{Y} = \overline{Y}(t)$, $X = X(t)$, $R_R = R_R(t)$,

$$\phi = \begin{pmatrix} Y \\ \overline{Y} \\ X \end{pmatrix} \text{ and } f(\phi) = \begin{pmatrix} \frac{dY}{dt} \\ \frac{d\overline{Y}}{dt} \\ \frac{dX}{dt} \end{pmatrix}.$$

From the system (2) we obtain:

$$\frac{\partial f}{\partial \phi} = M \;.$$

Where M takes the following form:

$$\begin{pmatrix} -\mu - \alpha(1-u) & \beta & 0 \\ \alpha(1-u) & -\beta - \mu - \dfrac{\delta\, a_1}{a_2 + \mathrm{e}^{a_3\gamma X}} & \dfrac{\delta\, a_1 a_3 \gamma\, \mathrm{e}^{a_3\gamma X}\overline{Y}}{(a_2 + \mathrm{e}^{a_3\gamma X})^2} \\ 0 & 0 & r - 2r\frac{X}{K} - qE \end{pmatrix}$$

Hence,

$$\begin{aligned}
\left|\left|\frac{\partial f}{\partial \phi}\right|\right|_1 = \quad & \max\Bigg(|\mu + \alpha(1-u)| + |\alpha(1-u)|, \\
& |\beta| + \left|\beta + \mu + \frac{\delta\, a_1}{a_2 + \mathrm{e}^{a_3\gamma X}}\right|, \\
& \left|\frac{\delta\, a_1 a_3\gamma\, \mathrm{e}^{a_3\gamma X}\overline{Y}}{(a_2 + \mathrm{e}^{a_3\gamma X})^2}\right| + \left|r - \tfrac{2rX}{K} - qE\right|\Bigg) .
\end{aligned}$$

Since $0 \leq 1 - u \leq 1$, it follows that:

$$|\mu + \alpha(1-u)| + |\alpha(1-u)| \leq \mu + 2\alpha \;.$$

From the definition $R_R = \dfrac{a_1}{a_2 + e^{a_3\gamma X}} \leq 1$, we find

$$|\beta| + \left|\beta + \mu + \frac{\delta\, a_1}{a_2 + \mathrm{e}^{a_3\gamma X}}\right| \leq 2\beta + \mu + \delta,$$

and finally,

$$\left|\frac{\delta\, a_1 a_3\gamma\, \mathrm{e}^{a_3\gamma X}\overline{Y}}{(a_2 + \mathrm{e}^{a_3\gamma X})^2}\right| + \left|r - \frac{2rX}{K} - qE\right| \leq \frac{a_3\delta\Lambda}{a_1\mu}\gamma + 3r + qE.$$

Therefore,

$$\left|\left|\frac{\partial f}{\partial \phi}\right|\right|_1 \leq \max\left(\mu + 2\alpha, 2\beta + \mu + \delta, \frac{a_3\delta\Lambda}{a_1\mu}\gamma + 3r + qE\right).$$

Consequently, from Lemma (3.3) f is globally Lipschitz. From the definition of the control $u(t)$ and the restriction on $Y(t) > 0, \overline{Y}(t) > 0$ and $X(t) > 0$, one can conclude that an unique solution of the system (2) exists.

B. Existence of an optimal control

Proposition 3.4: There exists an optimal control u^* and solutions Y^*, $\overline{Y}^*$ and X^* of the corresponding state system (2), such that

$$\mathcal{J}(u^*) = \min_{u \in U} \mathcal{J}(u) \;.$$

subject to the control system (2) with initial conditions.

Proof:

The existence of the optimal control can be obtained by checking the following steps (Theorem III.4.1 from [28]):

- From Propositions 3.1 and 3.2 and given that coefficients of the system (2) are bounded, the set of controls and corresponding state variables is nonempty.
- $\mathcal{J}(u) = \displaystyle\int_0^T \left(\overline{Y}(t) + Au^2(t)\right)dt$ is convex in u.
- The control space:

$$U = \{u/u \text{ are measurable, } 0 \le u(t) \le 1, t \in [0, T]\}$$

is convex and closed by definition.

- All the right hand sides of equations of system (2) are continuous, bounded above by a sum of bounded control and state variables, and can be written as a linear function of u with coefficients depending on time and state.
- The integrand in the objective functional, $\overline{Y}(t)+Au^2(t)$, is clearly convex on U.
- There exists constants $\alpha_1, \alpha_2 > 0$, and $\alpha > 1$ such that $\overline{Y}(t) + Au^2(t)$ satisfies: $\overline{Y}(t) + Au^2(t) \ge \alpha_1 + \alpha_2|u|^\alpha$. The state variables being bounded, let $\alpha_1 = \frac{1}{2}\overline{Y}(t), \alpha_2 = A$ and $\alpha = 2$ then it follows that:

$$\overline{Y}(t) + Au^2(t) \ge \alpha_1 + \alpha_2|u|^2 \ .$$

Theorem III.4.1 from [28] assures the existence of an optimal control.

C. Characterization of the optimal control

The necessary conditions for the optimal control arise from the Pontryagin's minimum principle [29]

Proposition 3.5: Given an optimal control u^* and solutions Y^*, $\overline{Y}^*$ and X^* of the corresponding state system (2), there exist adjoint variables λ_1, λ_2 and λ_3 satisfying

$$\Longrightarrow \begin{cases} \frac{d\lambda_1}{dt} &= (\lambda_1 - \lambda_2)(1-u^*)\alpha + \mu\lambda_1 \\ \frac{d\lambda_2}{dt} &= -1 + \lambda_2(\mu + \beta + \delta R_R^*) - \lambda_1\beta \\ \frac{d\lambda_3}{dt} &= \lambda_3\left(-r + \frac{2rX^*}{K} + qE\right) - \frac{\lambda_2\delta\, a_1 a_3 \gamma\, e^{a_3\gamma X^*}\overline{Y}^*}{(a_2 + e^{a_3\gamma X^*})^2} \end{cases}$$

With transversality conditions:

$$\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = 0 \ .$$

Moreover, the optimal control is given by

$$u^* = \min(1, \max(0, \frac{\alpha Y^*(\lambda_2 - \lambda_1)}{2A})) \ .$$

Proof:

The Hamiltonian H for the control problem is given by:

$$H(\vec{\lambda}, \vec{f}, u, t) = \overline{Y} + Au^2 + \ {}^t\vec{\lambda}.\vec{f},$$

with $\vec{\lambda} = \begin{pmatrix}\lambda_1 \\ \lambda_2 \\ \lambda_3\end{pmatrix}$, $\vec{f} = \begin{pmatrix} f_1(Y, \overline{Y}, X, u, t) \\ f_2(Y, \overline{Y}, X, u, t) \\ f_3(Y, \overline{Y}, X, u, t)\end{pmatrix}$,

where,

$$f_1(Y, \overline{Y}, X, u, t) = \Lambda - (\mu + \alpha(1-u))Y + \beta\overline{Y} \ ,$$

$$f_2(Y, \overline{Y}, X, u, t) = -(\mu + \delta R_R + \beta)\overline{Y} + \alpha(1-u)Y \ ,$$

$$f_3(Y, \overline{Y}, X, u, t) = r\left(1 - \frac{X}{K}\right)X - qEX \ .$$

Let

$$\vec{f^*} = \begin{pmatrix} f_1(Y^*, \overline{Y}^*, X^*, u^*, t) \\ f_2(Y^*, \overline{Y}^*, X^*, u^*, t) \\ f_3(Y^*, \overline{Y}^*, X^*, u^*, t)\end{pmatrix} \ ,$$

then the optimal control is determined by assuming that $\frac{dH}{du}(\vec{\lambda}, \vec{f^*}, u^*, t) = 0$, which is the optimality condition. Thus

$$u^* = \frac{\alpha Y^*(\lambda_2 - \lambda_1)}{2A}$$

By standard control arguments involving the bounds on the controls, we conclude

$$u^* = \begin{cases} = \frac{\alpha Y^*(\lambda_2 - \lambda_1)}{2A} & , \ \texttt{if} \ 0 < \frac{\alpha Y^*(\lambda_2 - \lambda_1)}{2A} < 1 \\ = 1 & , \ \texttt{if} \ \frac{\alpha Y^*(\lambda_2 - \lambda_1)}{2A} \ge 1 \\ = 0 & , \ \texttt{if} \ \frac{\alpha Y^*(\lambda_2 - \lambda_1)}{2A} \le 0 \end{cases}$$

In compact notation,

$$u^* = \min(1, \max(0, \frac{\alpha Y^*(\lambda_2 - \lambda_1)}{2A})) \ .$$

The adjoint variables λ_1, λ_2 and λ_3 are obtained by the following system:

$$\begin{aligned} \frac{d\lambda_1}{dt} &= -\frac{dH}{dY} \\ &= (\lambda_1 - \lambda_2)\,\alpha\,(1-u^*) + \lambda_1\mu \\ \frac{d\lambda_2}{dt} &= -\frac{dH}{d\overline{Y}} \\ &= -1 + \lambda_2\,(\mu + \beta + \delta\,R_R^*) - \lambda_1\beta \\ \frac{d\lambda_3}{dt} &= -\frac{dH}{dX} \\ &= \lambda_3\left(\frac{2rX^*}{K} + qE - r\right) - \frac{\lambda_2\delta\,a_1 a_3\gamma\,e^{a_3\gamma\,X^*}\overline{Y}^*}{(a_2 + e^{a_3\gamma\,X^*})^2} \ . \end{aligned}$$

IV. Numerical simulation

The resolution of the optimality system (2) is based on the Gauss Seidel-like implicit finite-difference method developed in [30] and denoted $GSS1$ method.

The time interval $[t_0, T]$ is discretized with a step h (time step size) such that $t_i = t_0 + ih$ for $i = 0, 1, \ldots, n$ and $t_n = T$.

At each point t_i, let

$$Y_i = Y(t_i),\ \overline{Y}_i = \overline{Y}(t_i),\ X_i = X(t_i) \ ,$$

$$R_{R_i} = R_R(t_i),\ \lambda_1^i = \lambda_1(t_i),\ \lambda_2^i = \lambda_2(t_i) \ ,$$

$$\lambda_3^i = \lambda_3(t_i) \text{ and } u_i = u(t_i) \ .$$

For the approximation of the following derivatives we use the first-order forward-difference.

$$\left.\begin{aligned} \frac{dY_{i+1}}{dt} &\approx \frac{Y_{i+1} - Y_i}{h} \\ \frac{d\overline{Y}_{i+1}}{dt} &\approx \frac{\overline{Y}_{i+1} - \overline{Y}_i}{h} \\ \frac{dX_{i+1}}{dt} &\approx \frac{X_{i+1} - X_i}{h} \end{aligned}\right\} \quad \text{for} \quad i = 0, \cdots, n-1.$$

Similarly, we approximate the time derivative of the adjoint variables by their first-order backward-difference:

$$\left.\begin{aligned} \frac{d\lambda_1^{n-i}}{dt} &\approx \frac{\lambda_1^{n-i} - \lambda_1^{n-i-1}}{h} \\ \frac{d\lambda_2^{n-i}}{dt} &\approx \frac{\lambda_2^{n-i} - \lambda_2^{n-i-1}}{h} \\ \frac{d\lambda_3^{n-i}}{dt} &\approx \frac{\lambda_3^{n-i} - \lambda_3^{n-i-1}}{h} \end{aligned}\right\} \quad \text{for} \quad i = 0, \cdots, n-1.$$

Hence the problem is given by the following numerical scheme for $i = 0, \cdots, n-1$.

$$
\left\{
\begin{array}{ll}
\dfrac{Y_{i+1}-Y_i}{h} & = \Lambda - \left(\mu + \alpha\,(1-u_i)\right) Y_{i+1} + \beta\,\overline{Y}_i \\[2ex]
\dfrac{\overline{Y}_{i+1}-\overline{Y}_i}{h} & = -\left(\mu\,\delta\,R_{R_i} + \beta\right)\overline{Y}_{i+1} \\
& \quad +\alpha\,(1-u_i)\,Y_{i+1} \\[2ex]
\dfrac{X_{i+1}-X_i}{h} & = r\left(1-\dfrac{X_i}{K}\right)X_{i+1} - qEX_{i+1} \\[2ex]
\dfrac{\lambda_1^{n-i}-\lambda_1^{n-i-1}}{h} & = \left({\lambda_1}^{n-i-1} - {\lambda_2}^{n-i}\right)\alpha\,(1-u_i) \\
& \quad +{\lambda_1}^{n-i-1}\mu \\[2ex]
\dfrac{\lambda_2^{n-i}-\lambda_2^{n-i-1}}{h} & = -1 + \lambda_2^{n-i-1}\left(\mu+\beta+\delta\,R_{Ri+1}\right) \\
& \quad -\lambda_1^{n-i-1}\beta \\[2ex]
\dfrac{\lambda_3^{n-i}-\lambda_3^{n-i-1}}{h} & = \lambda_3^{n-i-1}\left(\dfrac{2rX_{i+1}}{K} + qE - r\right) \\[2ex]
& \quad -\dfrac{\lambda_2^{n-i-1}\delta\,a_1a_3\gamma\,e^{a_3\gamma\,X_{i+1}}\overline{Y}_{i+1}}{\left(a_2+e^{a_3\gamma\,X_{i+1}}\right)^2}
\end{array}
\right.
$$

So we find

$$
\left\{
\begin{array}{l}
Y_{i+1} = \dfrac{\left(\Lambda+\beta\,\overline{Y}_i\right)h + Y_i}{1+h\mu+h\alpha-h\alpha\,u_i} \\[2ex]
\overline{Y}_{i+1} = \dfrac{\alpha\,(1-u_i)\,Y_{i+1}h + \overline{Y}_i}{1+h\mu\,\delta\,R_{R_i}+h\beta} \\[2ex]
X_{i+1} = \dfrac{X_i}{\left(1-r\left(\frac{K-X_i}{K}\right)h + qEh\right)} \\[2ex]
\lambda_1^{n-i-1} = \dfrac{\lambda_1^{n-i}+\alpha\,h\lambda_2^{n-i}-\alpha\,h\lambda_2^{n-i}u_i}{1+\alpha\,h-\alpha\,hu_i+\mu\,h} \\[2ex]
\lambda_2^{n-i-1} = \dfrac{\left(1+\lambda_1^{n-i-1}\beta\right)h+\lambda_2^{n-i}}{1+\mu\,h+h\beta+h\delta\,R_{Ri+1}} \\[2ex]
\lambda_3^{n-i-1} = \dfrac{\left(\dfrac{\lambda_2^{n-i-1}\delta\,a_1a_3\gamma\,e^{a_3\gamma\,X_{i+1}}\overline{Y}_{i+1}hK}{\left(a_2+e^{a_3\gamma\,X_{i+1}}\right)^2}+K\lambda_3^{n-i}\right)}{\left(K+2\,hrX_{i+1}+hqEK-hrK\right)} \\[2ex]
u_{i+1} = \dfrac{\alpha Y_{i+1}(\lambda_2^{n-i-1}-\lambda_1^{n-i-1})}{2A}\ .
\end{array}
\right.
$$

Simulations are performed by taking the following parameters [26]:

$$
\begin{gathered}
\Lambda = 500000,\ \mu = 0.014,\ \alpha = 0.06,\ \beta = 0.005,\ \delta = 0.006\ , \\
r = 1,\ K = 1.1\times 3.75\times 10^9,\ q = 0.04,\ E = 10\ , \\
P_t = 32\times 10^6,\ Y_0 = P_t\times 0.75\ , \\
\overline{Y}_0 = P_t\times 0.25,\ X_0 = 225\times 10^7\ .
\end{gathered}
$$

Since control and state functions are on different scales, the weight constant value is chosen as follows:

$$A = 3500000\ .$$

We consider different scenarios depending on the parameter a (the proportion of fish consumed from the total harvested) as shown in Fig 1 and Fig 2.

a=0

No fish is consumed at all: we find that the total mortality due to CHD in ten years is 7.9×10^5 (without control) and 5.9×10^5 (with control).

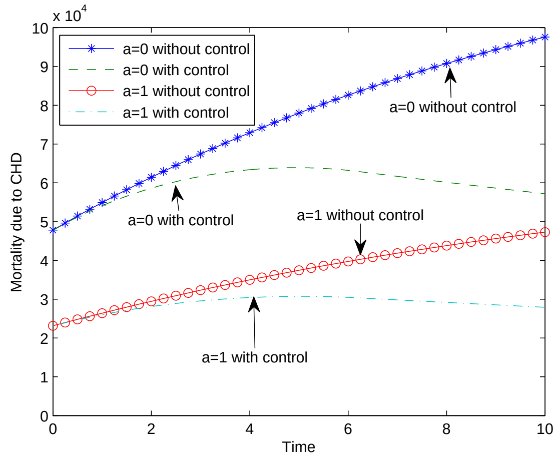

Fig. 1. Mortality due to CHD with $a = 0$ and $a = 1$ (with and without control)

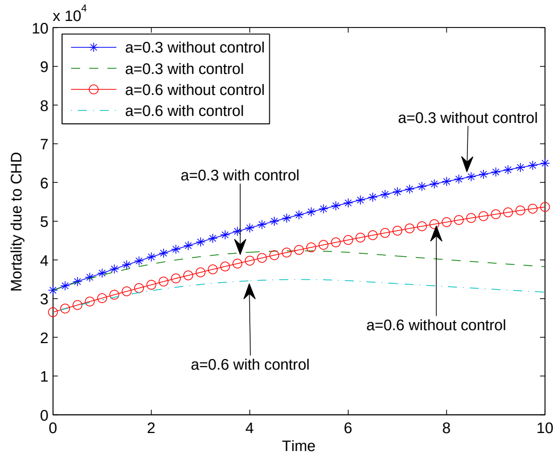

Fig. 2. Mortality due to CHD with $a = 0.3$ and $a = 0.6$ (with and without control)

a=0.30

30% of the total harvested fish is consumed: we find that the total mortality due to CHD in ten years is 5.2×10^5 (without control) and 3.9×10^5 (with control).

a=0.60

60% of the total harvested fish is consumed: we find that the total mortality due to CHD in ten years is 4.3×10^5 (without control) and 3.2×10^5 (with control).

a=1

The total harvested fish is completely consumed: we find that the total mortality due to CHD in ten years is 3.7×10^5 (without control) and 2.8×10^5 (with control).
These simulations prove that when the consumption of harvested fish goes from 30% to 60%, CHD mortality rate is reduced by 38% in ten years with control.

The report of the National Observatory of Human Development [31] states that household food expenditure accounts to 34% of income. 18.7% of its expenses are dedicated to poultry meat and red meat consumption, while

TABLE I
ESTIMATED PRICE OF EACH FOOD PRODUCT ACCORDING TO THE MINISTRY OF AGRICULTURE [32]:

Food products	Estimated price in dollars
Red meat	7 Dollars/kg
poultry	1.8 Dollars /kg
Sardine	From 1.2 to 1.5 Dollars /kg
White fish	From 5 to 12 Dollars/kg

only 3.5% of food expenditures are allocated to fish.

In light of the data of TABLE I and to encourage the Moroccan population to double their fish consumption to 7% of food expenditures (proposed strategy) without affecting their food consumption budget, it is necessary and sufficient to affect the share of food expenditures dedicated to red meat and poultry expenditure to consume more fish. This is clearly possible (in view of price table above) by substituting a portion of the share allocated to red meats and white fish and the substitution of a part of share allocated to poultry and sardines.

V. CONCLUSION

In this paper, extending the model proposed in [26], an optimal control approach was used to deal with the relationship between fish consumption and CHD mortality as a part of a whole strategy.
Existence and positivity of solutions as well as existence of an optimal control were proved. Given an optimal control and the corresponding solutions, adjoint variables were characterized. Finally a simulation was carried out showing that the use of an optimal control reduces the mortality rate by 25% at all levels of fish consumption.
It was already seen in the previous work that consuming fish rich in omega3 can reduce the mortality due to CHD, when the parameter a changes from 0.3 (actual strategy) to 0.6 (proposed strategy) by 17%. This work shows that a further 25% reduction can be obtained by using optimal control.

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