The Fluid Mechanics of Mitral Regurgitation in the Human Heart: Towards a Better Understanding of Echocardiograms

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***Abstract*—This research focuses on using analytical fluid mechanics to determine whether or not a mitral regurgitation (MR) patient's Echocardiogram reading can be trusted, to assess the severity of his Mitral Regurgitation (MR). MR occurs when the mitral valve inlet, separating the left atrium and the left ventricle does not fully close during ventricular systole, allowing regurgitant blood to flow backwards from the left ventricle into the left atrium. For clinicians, knowing the severity of this back-flow is important in preventing eventual heart failure. The Coanda Effect phenomenon in a particular patient is believed to cause discrepancies in echocardiogram results, leading clinicians to sometimes incorrectly diagnose a patient's MR severity. We propose to generate a tool for reading undebatable solid geometries in an echocardiogram and using these to judge the prevalence of wall effect to distort a fluid jet being read. Using control volume analysis for a regurgitant jet in the mitral valve, an analytical solution for the expected flow profile, based on wall effects around the regurgitant jet is derived. Using a turbulent 3D ANSYS model, detailed velocity and momentum mappings of the blood flowing backwards from the mitral valve into the left atrium are obtained. We believe these are first steps towards understanding wall effects in MR and determining the level of trustworthiness of any echocardiogram reading of MR severity.**

***Index Terms*—Mitral Regurgitation, Echocardiogram, Coanda Effect, Wall-effects.**

I. Introduction

Regurgitant Mitral Valve is a medical condition which, if untreated, leads to eventual heart failure. A regurgitant Mitral Valve allows back flow of blood into the left atrium and in severe cases, into the pulmonary vein at the inlet of the left atrium. The mitral valve is located between the Left Atrium and the Left Ventricle. It allows blood to flow from the Left Atrium to the left Ventricle and ideally prevents back flow from the left ventricle into the left Atrium when contraction of the ventricle (ventricular systole) occurs. During ventricular systole, the mitral valve is supposed to be fully closed. Mitral regurgitation happens when the mitral valve is not fully closed during a ventricular systole. Individuals with very severe cases of mitral regurgitation actually present pressure rises due to back flow to the pulmonary veins, and immediately undergo mitral valve repair or replacement surgery.

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The problem is with less severe cases where a pressure rise is not seen in the pulmonary vein due to less back flow. In such a case, only visual evidence of the regurgitant jet is present, likely detected via Echocardiogram. Many times, depending on a patient's anatomy, clinicians are unable to know if they can trust solely Echocardiogram evidence in determining MR severity.

There are various methods clinicians use to determine the severity of mitral regurgitation [5], but the most common method is via 2D Echocardiogram realtime images. A less common method, cardiac magnetic resonance imaging method (cardiac MRI method), provides detailed 3D spatial visual and geometric data towards the diagnosis of mitral regurgitation severity. Recent studies have shown significant discordance between more detailed/accurate cardiac MRI readings of MR severity and less accurate 2D Echocardiogram readings of MR severity, where cardiac MRI readings (harder to procure) show more detailed, accurate results [8]. Due to the widespread availability and use of the echocardiogram method, the present work seeks to enhance the clinician's certainty in assessing echocardiogram readings to determine MR severity.

II. Coanda Effect

The Coanda Effect is the adherence of a fluid jet to a nearby curved surface resulting in a change in the fluid path [4].

In this paper, we study the Coanda effect in the regurgitant Mitral Valve. The angle of coaptation of the Mitral Valve ("cone angle") is believed to influence the Coanda Effect ("divergence angle") in a Fluid Jet (see [3]). Foster concludes that Coanda Effect only shows up if the Mitral Valve coaptation angle is small enough.

The presence of the Coanda Effect can distort the appearance of a regurgitant fluid jet in a 2D echocardiogram reading. Thus, a clinician may wrongly assume a particular patient has non-severe mitral regurgitation based on the distorted appearance of their severe mitral regurgitant jet via the 2D echocardiogram reading. In fortunate cases, further investigation, perhaps via the images generated from cardiac magnetic resonance imaging (MRI) or additional echocardiogram perspectives, the patient's MR can be re-diagnosed as severe, and remedial steps taken. The converse of this scenario can also be true, where a mild patient, because of jet distortion, can appear to have severe MR and even be inadvertently sent to unnecessary, expensive surgery.

A fluid-mechanics-based *Coanda Number* can be used to define the extent to which Coanda effect is present in an MR patient's left atrium. This *"Coanda Number"* can be defined as the ratio between the momentum of the fluid flow near the valve leaflets (wall flow) and the rest of the flow far from the valve leaflets in the middle of the regurgitant volume (central jet flow). In such a regime, a high *"Coanda Number"* indicates a strong presence of wall effects on a regurgitant jet and therefore the unsuitability of a standard echocardiogram reading of MR severity. We believe the *"Coanda Number"* is a function of valve coaptation geometry, which can be clearly read from echocardiograms. This can be proven using MRI velocity-encoded maps of MR patients' regurgitant jets.

Since the velocity data being used to build the present enhancement of echocardiograms is MRI-derived, an important part of the present work is to build a strong connection between MRI-read coaptation angles of the 4 chamber view and echocardiogram-read coaptation angles of the Mitral valve in the 4 chamber view, and this investigation will be discussed first.

III. Data Analysis of Coaptation Angles

Valve coaptation angle has been seen play a key role in the presence of Coanda Effect in valves [3]. We observe 104 echocardiogram records (taken from www.echopedia.org) of the ventricular systoles of 51 separate patients and measure the mitral valve coaptation angles (see Figure 1), producing a histogram of the distribution of coaptation angles' scaled statistical z-values (see Figure 2).

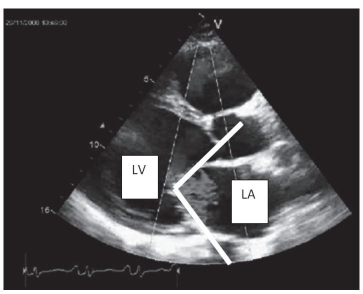

Fig. 1. Coaptation angles are approximated from echocardiogram records [2].

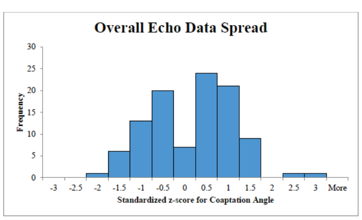

Fig. 2. The distribution of scaled coaptation angles generated from echocardiogram records available at www.echopedia.org.

Due to the fact that velocity encoded cardiac MRI data is a good reference point for understanding the accuracy of an echocardiogram reading, we also observe 133 cardiac MRI records [7] of the ventricular systoles of 45 separate patients and measure the mitral valve coaptation angles (see Figure 3), producing a histogram of the distribution of coaptation angles' scaled statistical z-values (see Figure 4).

The strong similarities between the distributions of the mitral valve coaptation angles derived from two separate types

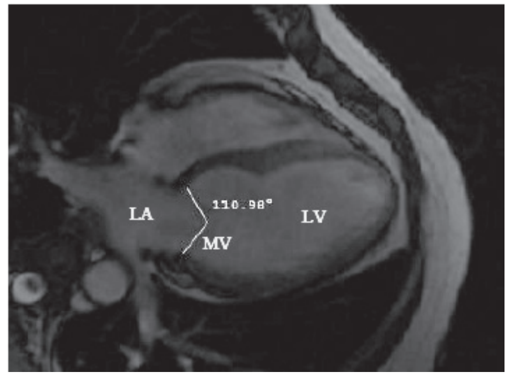

Fig. 3. Coaptation angles were approximated from MRI records taken from [7]. Here we see the 4-chamber view of the heart showing the left atrium (LA), the mitral valve (MV) and the left ventricle (LV). The echocardiogram-derived angle of coaptation of the mitral valve is measured as indicated by the solid white lines.

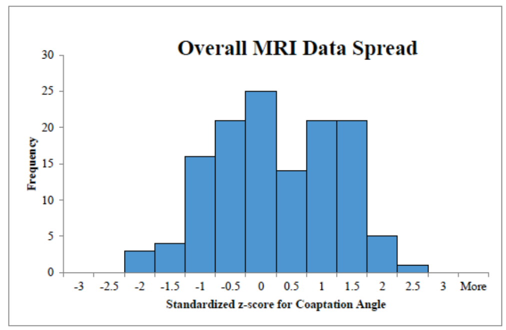

Fig. 4. The distribution of scaled coaptation angles generated from MRI records taken from [7]. Here we see the 4-chamber view of the heart showing the left atrium (LA), the mitral valve (MV) and the left ventricle (LV). The MRI-derived angle of coaptation of the mitral valve is measured as indicated by the solid white lines.

of records of the mitral valve coaptation angle establishes the statistically significant relationship between echocardiogram readings and MRI readings. It is an indicator that the use of concurrent MRI and Echocardiogram readings can be rigourously used towards understanding the fluid mechanics of mitral regurgitation. In velocity-encoded MRI records, we can move towards better understanding whether or not to trust an MR patient's echocardiogram results.

IV. Analytical Model

The extent of the influence of Coanda effect in a fluid jet can be quantified by comparing the momentum in the viscous portion influenced by the surrounding curved surface (wall flow) to the momentum in the turbulent portion nearer to the center of the jet. An analytical model of mitral regurgitation is developed towards calculating the expected ratio between momentum in the wall flow and momentum in the center of the jet. A control volume model of the left atrium is generated (see Figure 5). Mass conversation (see Equation 1) and Momentum conservation (see Equation 2) are enforced in this control volume.

$$\iiint \frac{\partial \rho}{\partial t} + \iint \rho v_i n_i dS = 0 \qquad (1)$$

$$\iint \rho \overline{v}_i (\overline{v_i}.n) dS = \sum \overline{F} \qquad (2)$$

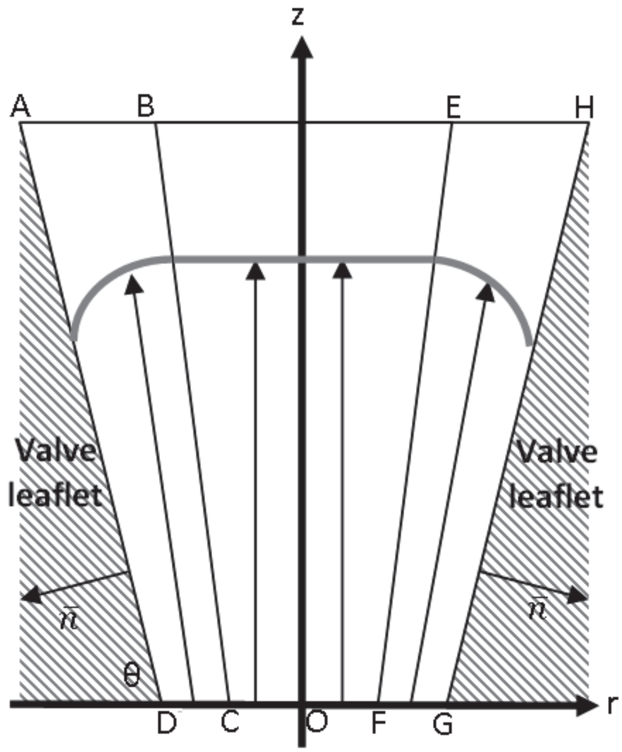

Fig. 5. Control volume in which Mitral Regurgitation is modeled in wall flow regions showing viscous effects and therefore evidence of Coanda Effect, and a central flow region, where turbulence pervades and Coanda Effect is absent.

Based on control volume analysis of the left atrium, with the regurgitant flow split into two separate sections (wall flow where viscous effects dominate fluid flow and center flow where turbulence dominates fluid flow), the velocity profiles for the two regions are calculated. Using these velocities, momentum is calculated in each fluid region. The following derivation shows an approximate theoretical ratio between the wall flow momentum and the center flow momentum. This is termed the "Coanda Number" in the present work. It is assumed that a large Coanda Number corresponds with less trustworthy echocardiogram MR severity readings, while a smaller Coanda Number corresponds with more trustworthy echocardiogram MR severity readings. In turn, it is postulated that large mitral valve coaptation angles are associated with a smaller Coanda Number while small mitral valve coaptation angles are associated with larger Coanda Number.

Consider the axi-symmetrical control-volume model of fluid in the regurgitant mitral valve jet. The flow in near-wall regions ABCD and EFGH in Figure 5 dominated by the presence of the mitral valve leaflets is considered a viscous flow, and a parabolic flow profile [6, pp.302] is enforced as shown. The flow further from the wall, at the center of the jet, as represented in region BCFE in Figure 5, is modeled as turbulent, therefore having a flat flow profile [6, pp.368]. Given these characteristics of the flow regions, and the fact that the gradients of the control volume are $m = \frac{r_E - r_F}{z_E} = \frac{r_B - r_C}{z_B}$. The limit $a = m\,(z - z_B) + r_B$ and $b = m\,(z - z_A) + r_A$. We represent the velocities as follows:

$$u(r) = \begin{cases} u_{\max}\left(1 - \frac{r^2}{R^2}\right), & \text{if } a < |r| < b. \\ u_{\max}, & \text{if } |r| < m\,(z - z_E) + r_E. \end{cases} \tag{3}$$

At the flow outlet near the valve leaflets, i.e. surface AB in Figure 5, for a small time duration, Δt, the momentum flux can be roughly approximated to be,

$$P_{\text{wall}} = \left(\int_{r_A}^{r_B} \int_0^{2\pi} \rho \overline{u(r)} (\overline{u(r)} \cdot \overline{n}) r \; d\phi \; \text{dr} \right) \Delta t. \tag{4}$$

After integrating this expression,

$$P_{\text{wall}} = \left(\pi u_{\max}^2 \rho sin^2\theta \left(r_B^2 - r_A^2 \right) \right) \cdot \Delta t. \tag{5}$$

At the flow outlet near the center of the regurgitant jet, i.e. surface BE in Figure 5, for a small time duration, Δt, the momentum flux can similarly be roughly approximated to be,

$$P_{\text{center}} = \left(\int_0^{r_B} \int_0^{2\pi} \rho u_{\max}^2 r \; d\phi \; \text{dr} \right) \Delta t. \tag{6}$$

After integrating this expression,

$$P_{\text{center}} = \pi \rho u_{\max}^2 r_B^2 \Delta t. \tag{7}$$

From these two expressions, we can deduce an approximate "Coanda Number", Co, as follows:

$$\text{Co} = \frac{P_{\text{wall}}}{P_{\text{center}}} = \frac{sin^2\theta \left(r_B^2 - r_A^2 \right)}{r_B^2} \tag{8}$$

The angle θ illustrated in Figure 5 is directly related to coaptation angle, **C**, as such:

$$\theta = \frac{\pi - C}{2}. \tag{9}$$

Thus we find a simplified approximation to a relationship between coaptation angle and Coanda Effect, as quantified by the Coanda Number.

V. 3D SIMULATION OF MITRAL REGURGITATION

Similar to the analytical model described in Section IV, a turbulent two-equation model is modeled in ANSYS. A realistic 3D geometrical model of the left atrium is built in AutoCAD using geometrical relationships found in [1].

The mass conservation equation is enforced within the boundaries of the 3D simulated left atrium as follows:

$$\frac{\partial(p)}{\partial(t)} + \bigtriangledown \cdot (\rho(\overrightarrow{v})) = s_m \tag{10}$$

where s_m represents the sum of body forces within the model.

The momentum conservation is represented in Reynolds-average Navier-Stokes (RANS) equations:

$$\rho(\frac{\partial(u)}{\partial(t)} + u \cdot (\bigtriangledown)u) = - \bigtriangledown (p) + \\ \bigtriangledown \cdot (\mu(\bigtriangledown)u + (\bigtriangledown(u)^T) - \frac{2}{3}\mu(\bigtriangledown \cdot (u))I) + F,$$

where the term $\rho(\frac{\partial(u)}{\partial(t)} + u \cdot (\bigtriangledown)u)$ represents the intertial forces acting on the moving fluid, the term $-\bigtriangledown(p)$ represents the changes in static pressure, **F** represents the external forces acting on the fluid, the term $\bigtriangledown \cdot (\mu(\bigtriangledown)u + (\bigtriangledown(u)^T) - \frac{2}{3}\mu(\bigtriangledown \cdot$

$(u))I)$ depicts the viscous effects and the term $\bigtriangledown(\cdot)u^T$ is used to simulate turbulent effects within the 3D geometry shown in Figures 6 and 7.

Velocity and pressure values are obtained at 0.4 seconds of the total simulation time span, which is the time during which ventricular systole occurs and this is the time in which MR is believed to occur during the cardiac cycle [5]. The variation of velocity from the free stream to the left atrial walls is found for different mitral valve coaptation angles.

The momentum ratios between flow near the walls and flow near the center of the regurgitant jet can be calculated via integration according to what is done in Equations 4 and 6.

Velocity results for different mitral valve coaptation angles 70 ^{o} and 150 ^{o} are shown in Figures 7, and 6, respectively. Using the same methodology and turbulent equations as described above, graphs were plotted to observe the variation of momentum ratio, based not only on proximal distance from the central regurgitate jet and wall, but also on coaptation angle variation.

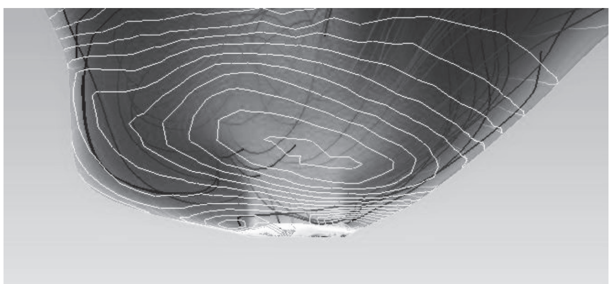

Fig. 6. A coaptation Angle of 150^o corresponds with barely noticeable wall effects or Coanda effect.

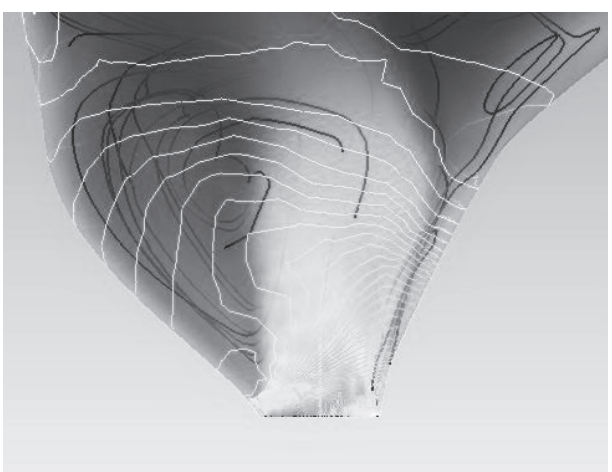

Fig. 7. A coaptation Angle of 70^o corresponds with more pronounced wall effects or Coanda Effect.

Momentum ratios can very readily be generated from this 3D velocity field computation, and can then be used to computationally arrive at the Coanda Number from calculations at all the discretized grid-points at specific cross sections along the Z-axis moving away from the inlet in the simulation. Such ratios can be compared to momentum derived from velocity-encoded cardiac MRI patient data, from MR patients, to verify and refine the present computational work.

VI. Discussions and Future work

We establish a link between the coaptation angle of the 4-chamber view of mitral valve walls originating from MRI versus those originating from echocardiograms. We then analytically and computationally show that coaptation angle influences the presence of Coanda Effect in MR. With further probes on this phenomenon, using velocity encoded MRI data, we can understand the presence of wall effects or Coanda Effect based simply on Mitral Valve coaptation angle.

Eventually the present findings will be applied to the diagnosis of less severe cases of MR, where only visual Echocardiogram evidence of their regurgitant jet is present. In that case, depending on a patient's anatomy, i.e. their mitral valve coaptation angles, clinicians will know if they can trust solely Echocardiogram evidence in determining the patient's MR severity.

References

[1] Delgado, Victoria, et al. "Assessment of mitral valve anatomy and geometry with multislice computed tomography." *JACC: Cardiovascular Imaging*, vol. 2, no.5, pp. 556-565, 2009.

[2] EchoPedia Online, "Diseases of the Mitral Valve". *EchoPedia.org*, accessed June 2015.

[3] G. P. Foster, E. M. Isselbacher, D. Gilon, A. E. Weyman, and R. A. Levine, "Proximal jet divergence by the coanda effect: Potential for overestimation of aortic insufficiency severity." *Journal of the American College of Cardiology*, vol. 27, no.2, pp. 285, 1996.

[4] C. Ginghina, "The Coanda effect in cardiology." *Journal of Cardiovascular Medicine*, vol. 8, no. 6, pp. 411-413, 2007.

[5] P. A. Grayburn, "How to measure severity of mitral regurgitation." *Heart*, vol. 94, no.3, pp. 376-383, 2008.

[6] Kundu, P. K., and L. M. Cohen. "Fluid Mechanics" *Elsevier*, 2008.

[7] P. Radau, Y. Lu, K. Connelly, G. Paul, A. J. Dick, G. A. Wright, "Evaluation Framework for Algorithms Segmenting Short Axis Cardiac MRI."*The MIDAS Journal Cardiac MR Left Ventricle Segmentation Challenge*,2009.

[8] Uretsky, S., Gillam, L., Lang, R., et al., "Discordance between echocardiography and MRI in the assessment of mitral regurgitation severity: a prospective multicenter trial." *Journal of the American College of Cardiology*, vol.65, no.11, 2015.