Pricing and Service Decisions of Supply Chain in an Uncertain Environment

Shengju Sang

Abstract—**In a two-echelon supply chain system with one manufacturer and one retailer, the pricing and service decisions of three non-cooperative games under an uncertain environment are researched. The market demand is assumed as a function of the retail price and service level, and the market base, manufacturing cost and selling cost are all considered as uncertain variables. Three different kinds of scenarios including two Stackelberg games and one Vertical-Nash game are pursued by the manufacturer and the retailer, and their optimal solutions are also given. Finally, the results of the proposed models are analyzed via a numerical example. It is shown that the manufacturer makes the largest expected profit in the Manufacturer-Stackelberg game, and the smallest in the Retailer-Stackelberg game. The retailer makes the smallest expected profit in the Manufacturer-Stackelberg game.**

Index Terms—**supply chain, pricing and service, game theory, uncertain variable**

I. Introduction

In current competitive and uncertain environment, manufacturers and retailers often play a price war to attract customers. Besides price, service is also an important factor that determines the buying decisions of customers. For instance, in electronic industry, maintenance service has important impact on the consumer's decision to buy a product. So, how the price and service effect the decisions of the manufacturer and the retailer becomes a hot issue among practitioners and scholars.

There are several studies that deal with the price and service decisions in supply chain. Iyer [1] studied a supply chain with one manufacturer and two competitive retailers who competed in price and service. Tsay and Agrawal [2] discussed the choices of price and service decisions with two non-cooperating and cooperating retailers, and found that the supply chain members could achieve coordination only under very limiting conditions. Xiao and Yang [3] analyzed a price and service competition model of two supply chains and examined the role of risk and information in the channel decisions. Xiao and Yang [4] also developed a price and service competition model between a retailer and a manufacturer with a risk sharing rule under demand uncertainty. Lu et al. [5] proposed a price and service competition model with two manufacturers and a common retailer, in which three different game scenarios were examined. Wu [6] focused on a price and service decisions model with two manufacturers and a retailer, where the manufacturers produced the new and remanufactured products. Han et al. [7] studied a price and service competition problem with one manufacturer and two retailers where the manufacturer acted as the Stackelberg leader and two retailers were the follower, and made their optimal retail prices independently. Yan and Pei [8] analyzed pricing and retail service decisions in a dual-channel supply chain. Dan et al. [9] also studied the optimal prices and retail services decisions in a centralized and a decentralized dual channel supply chain. In addition, Wang and Zhao [10] studied the price and service decisions in a dual supply chain where the manufacturer offered direct channel service and retail service. Sang [11] studied the service and selling effort decisions **a** two-echelon supply chain in which the manufacturer and the retailer pursue three different power structures.

All studies mentioned above discussed price and service decisions in a crisp environment, such as a linear market demand and known production cost. However, in real world, especially for some new electronic products, the relevant precise dates are difficult to obtain due to lack of historical data. In this situation, the market base and production cost can usually be predicted by some experts. Thus, fuzzy set theory proposed by Zadeh [12], is used to deal with the price decisions of supply chain by some scholars. For instance, Wei and Zhao [13] studied price decisions in a closed-loop supply chain, in which the demand and costs were fuzziness. Similar issue was studied by Wei and Zhao [14] in a reverse supply chain. In addition, Zhao et al. [15] studied price decisions with two competitive retailers in fuzzy environments. Zhao et al. [16] also considered a pricing competition problem with two manufacturers in fuzzy environments. Some researches also studied price and service decisions in a fuzzy environment, where the demand was a fuzzy liner function of selling price and service level. For instance, Zhao et al. [17] analyzed prices and services competition problems with two competing manufacturers and one retailer in fuzzy environments. Zhao and Wang [18] studied the pricing and retail service decisions between one manufacturer and two retailers with fuzzy demands. Sang [19] proposed one expected value and two chance-constrained programming models with two competitive manufacturers and a common retailer in a fuzzy demand environment. Hong [20] studied a Stackelberg game led by the retailer in a two-echelon fuzzy supply chain. Sang [21]

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analyzed a revenue sharing contract in a three-echelon supply chain where the demand was considered as a trapezoidal fuzzy number. Yano et al. [22] studied multi-objective fuzzy random linear programming problems based on coefficients of variation.

When we use fuzzy set theory to solve expert's prediction, there may be some problems. For example, the market base predicted by expert may be "about 2000". If we measure "about 2000" by fuzzy set theory, we may obtain the market base is "about 2000" with belief degree 1 by possibility measure, and is "not 2000" with belief degree 1 as well. That is to say, "about 2000" and "not 2000" have the same belief degree in possibility measure. It seems that nobody can accept this conclusion. Hence, those imprecise quantities like "about 2000" cannot be quantified by possibility measure, and then they are not fuzzy concepts. To deal these problems, Liu [23] proposed an uncertainty theory. Later, Ding [24] applied the uncertainty theory to solve the supply chain problem.

To the best of our knowledge, there is no study that deals with the pricing and service decisions of supply chain with uncertainty theory in an uncertain environment. Therefore, in this paper, we discuss the pricing and service decisions with a manufacturer and a retailer, in which the market base, manufacturing cost and selling cost are all uncertain variables. We mainly discuss the conditions where the manufacturer and the retailer pursue three different power structures: pursuing the Manufacturer-Stackelberg game, playing Retailer-Stackelberg game and acting in Vertical-Nash game.

The rest of paper is organized as follows. In section II, we introduce the uncertain theory related to the paper. In Section III, we briefly describe the problem and the notations in our models. In Section IV, we develop three non-cooperative games between the manufacturer and the retailer in an uncertain environment. In Section V, a numerical example is provided to illustrate the results of the proposed models. The last section summarizes the work done in this paper and further research areas.

II. Preliminaries

Definition 1. [23] Let L be a σ-algebra on a nonempty set Γ and M be a set function from L to $[0,1]$. Then M is called an uncertain measure if it satisfies the following three axioms:

Axiom 1. $M(\Gamma)=1$.

Axiom 2. $M(\Lambda)+M(\Lambda^c)=1$.

Axiom 3. For every countable sequence of events $\{\Lambda_i\}$, $i=1,2,\ldots$, we have

$$M\left(\bigcup_{i=1}^{\infty}\Lambda_i\right)\le\sum_{i=1}^{\infty}M(\Lambda_i)$$

Definition 2. [23] Let ξ be an uncertain variable, and its uncertainty distribution Φ is defined by

$$\Phi(x)=M(\xi\le x)$$

for any real number x.

Definition 3. [18]An uncertain variable $\xi=L(a,b)$ is called linear if it has a linear uncertainty distribution

$$\Phi(x)=\begin{cases}0, & if\ x\le a\\ (x-a)/(b-a), & if\ a\le x\le b\\ 1, & if\ x\ge b\end{cases}$$

where a and b are real numbers with $a<b$.

Lemma 1. [25] Let ξ be an uncertain variable with uncertainty distribution Φ. If the expected value of ξ exists, then

$$E[\xi]=\int_0^1\Phi^{-1}(\alpha)\mathrm{d}\alpha$$

where Φ^{-1} is the inverse function of Φ.

Lemma 2. Let $\xi=L(a,b)$ be a linear uncertain variable, then

$$E[\xi]=\int_0^1\Phi^{-1}(\alpha)\mathrm{d}\alpha=\int_0^1\Phi^{-1}(1-\alpha)\mathrm{d}\alpha$$

Proof: $E[\xi]=\int_0^1\Phi^{-1}(\alpha)\mathrm{d}\alpha=\int_0^1\big(a+(b-a)\alpha\big)\mathrm{d}\alpha=\frac{a+b}{2}$

$$\int_0^1\Phi^{-1}(1-\alpha)\mathrm{d}\alpha=\int_0^1\big(a+(b-a)(1-\alpha)\big)\mathrm{d}\alpha=\frac{a+b}{2}$$

The proof of Lemma 2 is completed.

Lemma 3. [25] Let $\xi_1,\xi_2,\ldots,\xi_n$ be independent uncertain variables with uncertainty distributions $\Phi_1,\Phi_2,\ldots,\Phi_n$, respectively. A function $f(x_1,x_2,\ldots,x_n)$ is strictly increasing with respected to $x_1,x_2,\ldots,x_m$ and strictly decreasing with respected to $x_{m+1},x_{m+2},\ldots,x_n$. Then the expected value of $\xi=f(\xi_1,\xi_2,\ldots,\xi_n)$ is

$$E[\xi]=\int_0^1 f\left(\Phi_1^{-1}(\alpha),\ldots,\Phi_m^{-1}(\alpha),\Phi_{m+1}^{-1}(1-\alpha),\ldots,\Phi_n^{-1}(1-\alpha)\right)\mathrm{d}\alpha$$

III. Problem Descriptions

This paper considers a two-echelon supply chain consisting of one manufacturer and one retailer. The manufacturer sells his product to the retailer, and then the retailer retails it to the customer. The uncertain demand faced by the manufacturer and the retailer is assumed as a linear function of the retail price p and the service level s, which is given by

$$q=\tilde{d}-\beta p+\gamma s$$

where the parameter $\tilde{d}$ is a positive linear uncertain variable and represents the market potential, the parameter β represents the sensitivity of demand to price changes and the parameter γ represents the demand expansion effectiveness coefficient of the service level offered by the manufacturer.

Further, let $\tilde{c}_m$ and $\tilde{c}_r$ denote the manufacturer's cost of producing its product and retailer's cost of selling its product, respectively, which are uncertain variables, w the wholesale price per unit charged to the retailer by the manufacturer, and m the retailer's profit margin on the product. As the retail price p can be treated as the total of the profit margin m and the wholesale price w, we consider retail price as $p=m+w$. Then the demand for the product can be rewritten as

$$\tilde{q}=\tilde{d}-\beta(m+w)+\gamma s$$

It is assumed that the marginal cost of the manufacturer is not affected by the service level. Further, the cost of achieving service level requires fixed investment, which is a quadratic function of service level s. It is given by $\frac{1}{2}\lambda s^2$, where the parameter λ is the investment coefficient.

The profits of the manufacturer and the retailer can be expressed as follows

$$\tilde{\Pi}_M=(w-\tilde{c}_m)\left(\tilde{d}-\beta(m+w)+\gamma s\right)-\tfrac{1}{2}\lambda s^2 \quad (1)$$

$$\tilde{\Pi}_R=(m-\tilde{c}_r)\left(\tilde{d}-\beta(m+w)+\gamma s\right) \quad (2)$$

Theorem 1. The expected profits of the manufacturer and the retailer can be transformed as follows

$$\begin{aligned}E\left[\tilde{\Pi}_M\right]=&-\beta w^2-\tfrac{1}{2}\lambda s^2+\gamma ws+\left(E\left[\tilde{d}\right]-\beta m+\beta E[\tilde{c}_m]\right)w\\&-\gamma E[\tilde{c}_m]s+\beta m-\int_0^1\Phi_{\tilde{d}}^{-1}(\alpha)\Phi_{\tilde{c}_m}^{-1}(1-\alpha)\,\mathrm{d}\alpha \quad (3)\end{aligned}$$

$$\begin{aligned}E\left[\tilde{\Pi}_R\right]=&-\beta m^2+\left(E\left[\tilde{d}\right]-\beta m+\gamma s+\beta E[\tilde{c}_r]\right)m+\beta E[\tilde{c}_r]w\\&-\gamma E[\tilde{c}_r]s-\gamma ws-\int_0^1\Phi_{\tilde{d}}^{-1}(\alpha)\Phi_{\tilde{c}_r}^{-1}(1-\alpha)\,\mathrm{d}\alpha \quad (4)\end{aligned}$$

Proof: Let $\tilde{c}_m$, $\tilde{c}_r$ and $\tilde{d}$ be positive uncertain variables with uncertainty distributions $\Phi_{\tilde{c}_m}$, $\Phi_{\tilde{c}_r}$ and $\Phi_{\tilde{d}}$. From (1), we can find that $E\left[\tilde{\Pi}_M\right]$ is monotone decreasing with $\tilde{c}_m$, and monotone increasing with $\tilde{d}$. Then referring to Lemma 1, 2 and 3, we have

$$\begin{aligned}E\left[\tilde{\Pi}_M\right]=&\int_0^1\Big[\left(w-\Phi_{\tilde{c}_m}(1-\alpha)\right)\left(\Phi_{\tilde{d}}(\alpha)-\beta(m+w)+\gamma s\right)\\&-\tfrac{1}{2}\lambda s^2\Big]\mathrm{d}\alpha\\=&\,wE\left[\tilde{d}\right]-\beta w(w+m)+\gamma ws-\int_0^1\Phi_{\tilde{d}}^{-1}(\alpha)\Phi_{\tilde{c}_m}^{-1}(1-\alpha)\,\mathrm{d}\alpha\\&+\beta(w+m)E[\tilde{c}_m]-\gamma sE[\tilde{c}_m]-\tfrac{1}{2}\lambda s^2\\=&-\beta w^2-\tfrac{1}{2}\lambda s^2+\gamma ws+\left(E\left[\tilde{d}\right]-\beta m+\beta E[\tilde{c}_m]\right)w-\gamma E[\tilde{c}_m]s\\&+\beta m-\int_0^1\Phi_{\tilde{d}}^{-1}(\alpha)\Phi_{\tilde{c}_m}^{-1}(1-\alpha)\,\mathrm{d}\alpha\end{aligned}$$

In the same way, we can derive $E\left[\tilde{\Pi}_R\right]$ showed as in (4). The proof of Theorem 1 is completed.

IV. Models Analysis

In this section, we examine the supply chain actors how to set their optimal solutions when they pursue different power structures in an uncertain environment. We mainly discuss the conditions where the manufacturer and the retailer pursue three non-cooperative games: the manufacturer dominates the channel, the retailer dominates the channel, and the manufacturer and the retailer have an equal bargaining power.

A. Manufacturer-Stackelberg game

The MS (Manufacturer-Stackelberg) game scenario arises in the market where the size of the retailer is smaller compared to the manufacturer. In this case, the manufacturer is the leader, and the retailer is the follower. That is, firstly, the manufacturer sets the wholesale price w and the service level s using the retailer's reaction function. Then, the retailer sets the profit margin m so as to maximize his expected profit. Thus, the MS game model can be given as follows

$$\begin{cases}\max\limits_{w,s} E\left[\tilde{\Pi}_M\right]=E\left[(w-\tilde{c}_m)\left(\tilde{d}-\beta(m^*+w)+\gamma s\right)-\tfrac{1}{2}\lambda s^2\right]\\ \text{s.t.}\\ M\{w-\tilde{c}_m\le 0\}=0\\ \text{where } m^* \text{ solves the following problem}\\ \max\limits_{m} E\left[\tilde{\Pi}_R\right]=E\left[(m-\tilde{c}_r)\left(\tilde{d}-\beta(m+w)+\gamma s\right)\right]\\ \text{s.t.}\\ M\{m-\tilde{c}_r\le 0\}=0\\ M\left\{\tilde{d}-\beta(m+w)+\gamma s\right\}=0\end{cases}$$

Theorem 2. If $M\left\{\tilde{d}-\beta\left(m^*(w,s)+w\right)+\gamma s\right\}=0$ and $M\left\{m^*(w,s)-\tilde{c}_r\le 0\right\}=0$, then the optimal response function of the retailer in the MS game model is

$$m^*(w,s)=\frac{1}{2\beta}\left(E\left[\tilde{d}\right]+\beta E[\tilde{c}_r]-\beta w+\gamma s\right) \quad (5)$$

Proof: From (4), the first-order and second-order derivatives of $E\left[\tilde{\Pi}_R\right]$ with respect to m can be obtained as

$$\frac{\mathrm{d}E\left[\tilde{\Pi}_R\right]}{\mathrm{d}m}=-2\beta m+E\left[\tilde{d}\right]+\beta E[\tilde{c}_r]-\beta w+\gamma s$$

$$\frac{\mathrm{d}^2E\left[\tilde{\Pi}_R\right]}{\mathrm{d}m^2}=-2\beta$$

Note that the second-order derivative of $E\left[\tilde{\Pi}_R\right]$ is negative definite, since $\beta>0$. Consequently, $E\left[\tilde{\Pi}_R\right]$ is strictly concave in m. Hence, let the first-order condition be zero, we can get the optimal response function of the retailer $m^*(w,s)$ as showed in (5).

The proof of Theorem 2 is completed.

Theorem 3. If $M\left\{w^*-\tilde{c}_m\le 0\right\}=0$, $M\left\{m^*-\tilde{c}_r\le 0\right\}=0$, $M\left\{\tilde{d}-\beta(m^*+w^*)+\gamma s^*\le 0\right\}=0$ and $4\beta\lambda-\gamma^2>0$, then the optimal solutions of the manufacturer and retailer in the MS game are

$$w^*=\frac{2\lambda\left[E\left[\tilde{d}\right]-\beta\left(E[\tilde{c}_m]+E[\tilde{c}_r]\right)\right]}{4\beta\lambda-\gamma^2}+E[\tilde{c}_m] \quad (6)$$

$$s^* = \frac{\gamma\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{4\beta\lambda-\gamma^2} \quad (7)$$

$$m^* = \frac{\lambda\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{4\beta\lambda-\gamma^2}+E\left[\tilde{c}_r\right] \quad (8)$$

$$p^* = \frac{3\lambda\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{4\beta\lambda-\gamma^2}+E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right] \quad (9)$$

Proof: Substituting $m^*(w,s)$ in (5) into (3), we can get the expected profit of the manufacturer as

$$\begin{aligned} E\left[\tilde{\Pi}_M\right] &= -\tfrac{1}{2}\beta w^2-\tfrac{1}{2}\lambda s^2+\tfrac{1}{2}\gamma ws \\ &+\tfrac{1}{2}\left(E\left[\tilde{d}\right]+\beta E\left[\tilde{c}_m\right]-\beta\left[\tilde{c}_r\right]\right)w-\tfrac{1}{2}\gamma E\left[\tilde{c}_m\right]s \\ &+\tfrac{1}{2}E\left[\tilde{c}_m\right]\left(E\left[\tilde{d}\right]+\beta\left[\tilde{c}_r\right]\right)-\int_0^1\Phi_{\tilde{d}}^{-1}(\alpha)\Phi_{\tilde{c}_m}^{-1}(1-\alpha)\,\mathrm{d}\alpha \quad (10)\end{aligned}$$

From (10), we can get the first-order derivatives of $E\left[\tilde{\Pi}_M\right]$ with respect to w and s as follows

$$\frac{\partial E\left[\tilde{\Pi}_M\right]}{\partial w} = -\beta w+\tfrac{1}{2}\gamma s+\tfrac{1}{2}\left(E\left[\tilde{d}\right]+\beta E\left[\tilde{c}_m\right]-\beta\left[\tilde{c}_r\right]\right)$$

$$\frac{\partial E\left[\tilde{\Pi}_M\right]}{\partial s} = -\lambda s+\tfrac{1}{2}\gamma w-\tfrac{1}{2}\gamma E\left[\tilde{c}_m\right]$$

Thus, the Hessian matrix of $E\left[\tilde{\Pi}_M\right]$ is

$$\mathrm{H} = \begin{bmatrix} -\beta & \tfrac{1}{2}\gamma \\ \tfrac{1}{2}\gamma & -\lambda \end{bmatrix}$$

Note that the Hessian matrix of $E\left[\tilde{\Pi}_M\right]$ is negative definite, since $\beta>0$, $\lambda>0$ and $4\beta\lambda-\gamma^2>0$. Thus, $E\left[\tilde{\Pi}_M\right]$ is strictly jointly concave in w and s.

Let the first-order conditions be zero, we get w^* and s^* as shown in (6) and (7). Substituting w^* and s^* into (5), we can easily obtain m^* showed in (8).

The optimal retail price p^* in the MS game is

$$\begin{aligned} p^* &= w^*+m^* \\ &= \frac{3\lambda\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{4\beta\lambda-\gamma^2}+E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\end{aligned}$$

The proof of Theorem 3 is completed.

B. Retailer-Stackelberg game

The RS (Retailer-Stackelberg) game scenario arises in the market where the size of the manufacturer is smaller compared to the retailer. This case implies the retailer becomes the leader and the manufacturer is the follower. The retailer first sets the profit margin using the reaction functions of the manufacturer. Then the manufacturer observes the decision made by the retailer and makes his response to this decision by setting wholesale price w and service level s. Thus, the RS game model can be given as follows

$$\begin{cases} \max\limits_{m} E\left[\tilde{\Pi}_R\right] = E\left[\left(m-\tilde{c}_r\right)\left(\tilde{d}-\beta\left(m+w^{**}\right)+\gamma s^{**}\right)\right] \\ \text{s.t.} \\ M\left\{m-\tilde{c}_r\le 0\right\}=0 \\ \text{where } w^{**} \text{ and } s^{**} \text{ solves the following problem} \\ \max\limits_{w,s} E\left[\tilde{\Pi}_M\right] = E\left[\left(w-\tilde{c}_m\right)\left(\tilde{d}-\beta\left(m+w\right)+\gamma s\right)-\tfrac{1}{2}\lambda s^2\right] \\ \text{s.t.} \\ M\left\{w-\tilde{c}_m\le 0\right\}=0 \\ M\left\{\tilde{d}-\beta\left(m+w\right)+\gamma s\le 0\right\}=0 \end{cases}$$

Theorem 4. If $M\left\{\tilde{d}-\beta\left(m+w^{**}(m)\right)+\gamma s^{**}(m)\le 0\right\}=0$, $M\left\{w^{**}(m)-\tilde{c}_m\le 0\right\}=0$, and $2\beta\lambda-\gamma^2>0$, then the optimal response functions of the manufacturer in the RS game are

$$w^{**}(m) = \frac{\lambda E\left[\tilde{d}\right]+\left(\beta\lambda-\gamma^2\right)E\left[\tilde{c}_m\right]-\beta\lambda m}{2\beta\lambda-\gamma^2} \quad (11)$$

$$s^{**}(m) = \frac{\gamma\left(E\left[\tilde{d}\right]-\beta E\left[\tilde{c}_m\right]-\beta m\right)}{2\beta\lambda-\gamma^2} \quad (12)$$

Proof: From (3), we can get the first-order derivatives of $E\left[\tilde{\Pi}_M\right]$ with respect to w and s as follows

$$\frac{\partial E\left[\tilde{\Pi}_M\right]}{\partial w} = -2\beta w+\gamma s+E\left[\tilde{d}\right]+\beta E\left[\tilde{c}_m\right]-\beta m$$

$$\frac{\partial E\left[\tilde{\Pi}_M\right]}{\partial s} = -\lambda s+\tfrac{1}{2}\gamma w-\gamma E\left[\tilde{c}_m\right]$$

Thus, the Hessian matrix of $E\left[\tilde{\Pi}_M\right]$ is

$$\mathrm{H} = \begin{bmatrix} -2\beta & \gamma \\ \gamma & -\lambda \end{bmatrix}$$

Note that the Hessian matrix of $E\left[\tilde{\Pi}_M\right]$ is negative definite, since $\beta>0$, $\lambda>0$ and $2\beta\lambda-\gamma^2>0$. Thus, $E\left[\tilde{\Pi}_M\right]$ is strictly jointly concave in w and s.

Let the first-order conditions be zero, we get $w^{**}(m)$ and $s^{**}(m)$ as shown in (11) and (12).

The proof of Theorem 4 is completed.

Theorem 5. If $M\left\{w^{**}-\tilde{c}_m\le 0\right\}=0$, $M\left\{m^{**}-\tilde{c}_r\le 0\right\}=0$, $M\left\{\tilde{d}-\beta\left(m^{**}+w^{**}\right)+\gamma s^{**}\le 0\right\}=0$ and $2\beta\lambda-\gamma^2>0$, then the optimal solutions of the manufacturer and retailer in the RS game are

$$w^{**} = \frac{\lambda\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{2\beta\lambda-\gamma^2}+E\left[\tilde{c}_m\right] \quad (13)$$

$$s^{**} = \frac{\gamma\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{2\beta\lambda-\gamma^2} \quad (14)$$

$$m^{**} = \frac{E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)}{2\beta}+E\left[\tilde{c}_r\right] \quad (15)$$

$$p^{**}=\frac{\left(4\beta\lambda-\gamma^2\right)\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{2\beta\lambda-\gamma^2}+E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right] \quad (16)$$

Proof: Substituting $w^*(m)$ and $s^*(m)$ in (11) and (12) into (4), we can get the expected profit of the retailer as

$$\begin{aligned}E\left[\tilde{\Pi}_R\right]=&-\frac{\beta^2\lambda}{2\beta\lambda-\gamma^2}m^2+\frac{\beta\lambda\left(E\left[\tilde{d}\right]-\beta E\left[\tilde{c}_m\right]+\beta E\left[\tilde{c}_r\right]\right)}{2\beta\lambda-\gamma^2}m\\&+\frac{E\left[\tilde{c}_r\right]\left[\left(\beta\lambda-\gamma^2\right)E\left[\tilde{d}\right]+\beta^2\lambda E\left[\tilde{c}_m\right]\right]}{2\beta\lambda-\gamma^2}\\&-\int_0^1\Phi_{\tilde{d}}^{-1}(\alpha)\Phi_{\tilde{c}_r}^{-1}(1-\alpha)\,\mathrm{d}\alpha\end{aligned} \quad (17)$$

From (17), the first-order and second-order derivatives of $E\left[\tilde{\Pi}_R\right]$ with respect to m can be obtained as

$$\frac{\mathrm{d}E\left[\tilde{\Pi}_R\right]}{\mathrm{d}m}=-\frac{2\beta^2\lambda}{2\beta\lambda-\gamma^2}m+\frac{\beta\lambda\left(E\left[\tilde{d}\right]-\beta E\left[\tilde{c}_m\right]+\beta E\left[\tilde{c}_r\right]\right)}{2\beta\lambda-\gamma^2}$$

$$\frac{\mathrm{d}^2E\left[\tilde{\Pi}_R\right]}{\mathrm{d}m^2}=-\frac{2\beta^2\lambda}{2\beta\lambda-\gamma^2}$$

Note that the second-order derivative of $E\left[\tilde{\Pi}_R\right]$ is negative definite, since $\beta>0$, $\lambda>0$ and $2\beta\lambda-\gamma^2>0$. Consequently, $E\left[\tilde{\Pi}_R\right]$ is strictly concave in m.

Hence, let the first-order condition be zero, we can get m^{**} as showed in (15).

Substituting m^{**} into (11) and (12), we can easily obtain w^{**} and s^{**} shown in (13) and (14).

The optimal retail price p^{**} in the RS game is

$$\begin{aligned}p^{**}&=w^{**}+m^{**}\\&=\frac{\left(4\beta\lambda-\gamma^2\right)\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{2\beta\lambda-\gamma^2}+E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\end{aligned}$$

The proof of Theorem 5 is completed.

C. Vertical-Nash game

The VN (Vertical-Nash) game scenario arises in the market where the manufacturer and retailer have equal market power. In this case the manufacturer determines the wholesale price w and service level s, and the retailer makes the profit margin m simultaneously and independently. Thus, the RS game model can be given as follows

$$\begin{cases}\max\limits_{w,s}E\left[\tilde{\Pi}_M\right]=E\left[\left(w-\tilde{c}_m\right)\left(\tilde{d}-\beta(m+w)+\gamma s\right)-\frac{1}{2}\lambda s^2\right]\\\max\limits_{m}E\left[\tilde{\Pi}_R\right]=E\left[\left(m-\tilde{c}_r\right)\left(\tilde{d}-\beta(m+w)+\gamma s\right)\right]\\\text{s.t.}\\M\left\{w-\tilde{c}_m\le 0\right\}=0\\M\left\{m-\tilde{c}_r\le 0\right\}=0\\M\left\{\tilde{d}-\beta(m+w)+\gamma s\le 0\right\}=0\end{cases}$$

Theorem 6. If $M\left\{w^{***}-\tilde{c}_m\le 0\right\}=0$, $M\left\{m^{***}-\tilde{c}_r\le 0\right\}=0$, $M\left\{\tilde{d}-\beta\left(w^{***}+m^{***}\right)+\gamma s^{***}\le 0\right\}=0$ and $2\beta\lambda-\gamma^2>0$, then the optimal solutions of the manufacturer and retailer in the VN game are

$$w^{***}=\frac{\lambda\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{3\beta\lambda-\gamma^2}+E\left[\tilde{c}_m\right] \quad (18)$$

$$s^{***}=\frac{\gamma\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{3\beta\lambda-\gamma^2} \quad (19)$$

$$m^{***}=\frac{\lambda\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{3\beta\lambda-\gamma^2}+E\left[\tilde{c}_r\right] \quad (20)$$

$$p^{***}=\frac{2\lambda\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{3\beta\lambda-\gamma^2}+E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right] \quad (21)$$

Proof: The first-order derivatives of $E\left[\tilde{\Pi}_M\right]$ with respect to w and s can be obtained from (3) as follows

$$\frac{\partial E\left[\tilde{\Pi}_M\right]}{\partial w}=-2\beta w+\gamma s+E\left[\tilde{d}\right]+\beta E\left[\tilde{c}_m\right]-\beta m$$

$$\frac{\partial E\left[\tilde{\Pi}_M\right]}{\partial s}=-\lambda s+\tfrac{1}{2}\gamma w-\gamma E\left[\tilde{c}_m\right]$$

The Hessian matrix of $E\left[\tilde{\Pi}_M\right]$ is

$$\mathrm{H}=\begin{bmatrix}-2\beta & \gamma\\ \gamma & -\lambda\end{bmatrix}$$

Note that the Hessian matrix of $E\left[\tilde{\Pi}_M\right]$ is negative definite, since $\beta>0$, $\lambda>0$ and $2\beta\lambda-\gamma^2>0$. Thus, $E\left[\tilde{\Pi}_M\right]$ is strictly jointly concave in w and s.

The first-order and second-order derivatives of $E\left[\tilde{\Pi}_R\right]$ with respect to m can be obtained from (4) as

$$\frac{\mathrm{d}E\left[\tilde{\Pi}_R\right]}{\mathrm{d}m}=-2\beta m+E\left[\tilde{d}\right]+\beta E\left[\tilde{c}_r\right]-\beta w+\gamma s$$

$$\frac{\mathrm{d}^2E\left[\tilde{\Pi}_R\right]}{\mathrm{d}m^2}=-2\beta$$

Note that the second-order derivative of $E\left[\tilde{\Pi}_R\right]$ is negative definite, since $\beta>0$. Consequently, $E\left[\tilde{\Pi}_R\right]$ is strictly concave in m.

Hence, let the first-order conditions be zero simultaneously, we can get w^{***}, s^{***} and m^{***} as showed in (19), (20) and (21).

The optimal retail price p^{***} in the VN game is

$$\begin{aligned}p^{***}&=w^{***}+m^{***}\\&=\frac{2\lambda\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{3\beta\lambda-\gamma^2}+E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\end{aligned}$$

The proof of Theorem 6 is completed.

D. Analysis of the optimal solutions under three non-cooperative games

In this subsection, we compare the optimal solutions namely, the wholesale prices, service levels, profit margins and retail prices. For convenience of analysis, superscripts *MS*, *RS*, and *VN* are used to denote the MS game, RS game and VN game, respectively. To insure that the optimal solutions are all positive, we impose the condition on the parameters: $2\beta\lambda-\gamma^2>0$, that is $\lambda>\frac{\gamma^2}{2\beta}$.

Theorem 7. The wholesale prices are in the order

$$w^{RS}>w^{MS}>w^{VN}$$

Proof: It is easy to verify that

$$w^{RS}-w^{MS}=\frac{\lambda\gamma^2\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{\left(4\beta\lambda-\gamma^2\right)\left(2\beta\lambda-\gamma^2\right)}>0$$

$$w^{MS}-w^{VN}=\frac{\left(2\beta\lambda-\gamma^2\right)\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{\left(4\beta\lambda-\gamma^2\right)\left(3\beta\lambda-\gamma^2\right)}>0$$

The proof of Theorem 7 is completed.

Theorem 8. The service levels are in the order

$$s^{RS}>s^{VN}>s^{MS}$$

Proof: It is easy to verify that

$$s^{RS}-s^{VN}=\frac{\beta\lambda\gamma\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{\left(2\beta\lambda-\gamma^2\right)\left(3\beta\lambda-\gamma^2\right)}>0$$

$$s^{VN}-s^{MS}=\frac{\beta\lambda\gamma\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{\left(3\beta\lambda-\gamma^2\right)\left(4\beta\lambda-\gamma^2\right)}>0$$

The proof of Theorem 8 is completed.

Theorem 9. The profit margins satisfy the following

If $\lambda>\frac{\gamma^2}{\beta}$, then $m^{RS}>m^{VN}>m^{MS}$;

If $\frac{\gamma^2}{2\beta}<\lambda<\frac{\gamma^2}{\beta}$, then $m^{VN}>m^{RS}>m^{MS}$;

If $\lambda=\frac{\gamma^2}{\beta}$, then $m^{RS}=m^{VN}>m^{MS}$.

Proof: It is easy to verify that

$$m^{RS}-m^{MS}=\frac{\left(2\beta\lambda-\gamma^2\right)\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{2\beta\left(4\beta\lambda-\gamma^2\right)}>0$$

$$m^{VN}-m^{MS}=\frac{\beta\lambda^2\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{\left(3\beta\lambda-\gamma^2\right)\left(4\beta\lambda-\gamma^2\right)}>0$$

$$m^{RS}-m^{VN}=\frac{\left(\beta\lambda-\gamma^2\right)\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{2\beta\left(4\beta\lambda-\gamma^2\right)}$$

When $\beta\lambda>\gamma^2$, we obtain $m^{RS}>m^{VN}$. When $\frac{1}{2}\gamma^2<\beta\lambda<\gamma^2$, we obtain $m^{RS}<m^{VN}$. When $\beta\lambda=\gamma^2$, we obtain $m^{RS}=m^{VN}$.

The proof of Theorem 9 is completed.

Theorem 10. The retail prices satisfy the following

If $\lambda>\frac{\gamma^2}{\beta}$, then $p^{RS}>p^{MS}>p^{VN}$;

If $\frac{\gamma^2}{2\beta}<\lambda<\frac{\gamma^2}{\beta}$, then $p^{RS}>p^{VN}>p^{MS}$;

If $\lambda=\frac{\gamma^2}{\beta}$, then $p^{RS}>p^{MS}=p^{VN}$.

Proof: It is easy to verify that

$$p^{RS}-p^{MS}=\frac{\left[2\beta\lambda\left(2\beta\lambda-\gamma^2\right)+\gamma^4\right]\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{2\beta\left(2\beta\lambda-\gamma^2\right)\left(4\beta\lambda-\gamma^2\right)}>0$$

$$p^{RS}-p^{VN}=\frac{\left[4\left(\beta\lambda-\frac{3}{8}\gamma^2\right)^2+\frac{7}{16}\gamma^4\right]\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{2\beta\left(2\beta\lambda-\gamma^2\right)\left(4\beta\lambda-\gamma^2\right)}>0$$

$$p^{MS}-p^{VN}=\frac{\left(\beta\lambda-\gamma^2\right)\left[E\left[\tilde{d}\right]-\beta\left(E\left[\tilde{c}_m\right]+E\left[\tilde{c}_r\right]\right)\right]}{\left(3\beta\lambda-\gamma^2\right)\left(4\beta\lambda-\gamma^2\right)}$$

When $\beta\lambda>\gamma^2$, we have $p^{MS}>p^{VN}$. When $\frac{1}{2}\gamma^2<\beta\lambda<\gamma^2$, we obtain $p^{MS}<p^{VN}$. When $\beta\lambda=\gamma^2$, we obtain $p^{MS}=p^{VN}$.

The proof of Theorem 10 is completed.

V. Numerical Example

In this section, we tend to further elucidate the above proposed three different non-cooperative games. Due to lack of the historical date, the costs of the manufacturer and retailer, and market base are predicted by the experiences of experts showed in Table I. We further assume $\beta=90$ and $\gamma=60$.

TABLE I
THE UNCERTAIN VARIABLES

Parameter	Linguistic description	Distribution
Market base $\tilde{D}$	About 2000	$L=(150,250)$
Manufacturing cost $\tilde{c}_m$	Between 7 and 9	$L=(7,9)$
Selling cost $\tilde{c}_r$	Between 1 and 3	$L=(1,3)$

From Table I, we have

$$E\left[\tilde{D}\right]=\frac{1500+2500}{2}=2000,$$

$$E\left[\tilde{c}_m\right]=\frac{7+9}{2}=8,$$

$$E\left[\tilde{c}_r\right]=\frac{1+3}{2}=2$$

$$\int_0^1\Phi_{\tilde{d}}^{-1}(\alpha)\Phi_{\tilde{c}_m}^{-1}(1-\alpha)\,\mathrm{d}\alpha$$

$$=\int_0^1(1500+1000\alpha)\left(7+2(1-\alpha)\right)\mathrm{d}\alpha=\frac{47500}{3},$$

$$\int_0^1\Phi_{\tilde{d}}^{-1}(\alpha)\Phi_{\tilde{c}_r}^{-1}(1-\alpha)\,\mathrm{d}\alpha$$

$$= \int_0^1 (1500 + 1000\alpha)(1 + 2(1-\alpha))\,\mathrm{d}\alpha = \frac{11500}{3}.$$

Based on the analysis showed in the Section IV, we present the optimal solutions in the MS, RS and VN games in Table II.

TABLE II
THE OPTIMAL SOLUTIONS WITH DIFFERENT λ

	λ	w	s	m	p	$E\left[\tilde{\Pi}_M\right]$	$E\left[\tilde{\Pi}_R\right]$
MS	30.00	17.17	9.17	6.58	23.75	2020.83	1036.88
	35.00	16.56	7.33	6.28	22.83	1852.78	1416.02
	40.00	16.15	6.11	6.07	22.22	1740.74	1647.00
	45.00	15.86	5.24	5.93	21.79	1660.71	1801.31
	50.00	15.64	4.58	5.82	21.46	1600.69	1911.21
RS	30.00	26.33	36.67	8.11	34.44	166.67	6888.89
	35.00	22.26	24.44	8.11	30.37	166.67	4648.15
	40.00	20.22	18.33	8.11	28.33	166.67	3527.78
	45.00	19.00	14.67	8.11	27.11	166.67	2855.56
	50.00	18.19	12.22	8.11	26.30	166.67	2407.41
VN	30.00	15.33	14.67	9.33	24.67	1780.00	5006.67
	35.00	14.58	11.28	8.58	23.16	1170.61	4064.76
	40.00	14.11	9.17	8.11	22.22	1180.55	3527.78
	45.00	13.79	7.72	7.79	21.58	1175.90	3183.29
	50.00	13.56	6.67	7.56	21.11	1166.67	2944.44

Based on the results showed in Table II, we find:

(1) The optimal solutions w and s for the manufacturer decrease in the three game cases, as the parameter λ increases. The wholesale price w under the RS case is the highest, followed by MS and then VN cases. The optimal service level s is the highest in the RS case when the retailer has more bargaining power. The MS case provides the lowest service level this is because under this case the full costs of service are afforded by the manufacturer.

(2) When the parameter λ increases, the retail price p for the retailer decreases in the three game cases, the profit margin m decreases in the MS and VN cases, while does not vary in the RS case. When $\lambda < 40$, the profit margin m in the case of VN is the highest, followed by RS and then MS cases, and the retail price p in the case of RS is the highest, followed by VN and then MS cases. When $\lambda = 40$, the profit margin m in the case of MS is the lowest, and the profit margin m in the RS case is equal to that in the VN case. The retail price p in the case of RS is the highest, and the retail price p in the MS case is equal to that in the VN case. When $\lambda > 40$, the retailer makes the largest profit margin m in the RS case, and the smallest in the MS case. The retail price p under the RS case is the highest, followed by MS and then VN cases.

(3) The manufacturer's expected profit decreases in the MS and VN cases, and does not vary in the RS case, as the parameter λ increases. The retailer's expected profit increases in the MS case, and decreases in the RS and VN cases. The manufacturer makes the largest expected profit in the MS case, and the smallest in the RS case. The retailer makes the smallest expected profit in the MS case. When $\lambda < 40$, the expected profit of the retailer in the RS case is higher than that in the VN case. When $\lambda = 40$, the retailer makes the same expected profit in the RS and VN cases. When $\lambda < 40$, the expected profit of the retailer in the RS case is smaller than that in the VN case.

VI. CONCLUSION

This paper proposes a two-echelon supply chain management in an uncertain environment, where the manufacturer and retailer pursue three different kinds of scenarios: Manufacturer-Stackelberg, Retailer-Stackelberg and Vertical-Nash games. The models in our case contain two strategic variables, price and service level, three uncertain variables, market base, manufacturing cost and selling cost, which is truly representative of the electronic industry. The main contribution of the paper is that we consider the pricing and service decisions of supply chain in an uncertain environment. One limitation of this paper is that we only consider one manufacturer and one retailer. Therefore, one possible extension work is to study the pricing and service decisions with multiple competing manufacturers or retailers in an uncertain environment. The other limitation is that the uncertain variables are only considered as linear uncertain variables, the other types of the uncertain variables can be considered in the future.

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