Almost Periodic Solution for a Lotka-Volterra Recurrent Neural Networks with Harvesting Terms on Time Scales

Li Yang, Yonghong Yang, Yaqin Li and Tianwei Zhang

Abstract—**By using the theory of exponential dichotomy and Banach fixed point theorem, this paper is concerned with the problem of the existence and uniqueness of almost periodic solution in a harvesting Lotka-Volterra recurrent neural networks on time scales. To a certain extent, our work in this paper corrects the defect in [Y.G. Liu, B.B. Liu, S.H. Ling, The almost periodic solution of Lotka-Volterra recurrent neural networks with delays, Neurocomputing 74 (2011) 1062-1068]. Further, by constructing a suitable Lyapunov function, some simple sufficient conditions are obtained for the local asymptotical stability of the above model. Finally, an example is given to illustrate the feasibility and effectiveness of the main result.**

Index Terms—**Almost periodic solution; Lotka-Volterra; Neural networks; Banach fixed point theorem; Harvesting.**

I. INTRODUCTION

The Lotka-Volterra type neural networks, derived from conventional membrane dynamics of competing neurons, provide a mathematical basis for understanding neural selection mechanisms [1-5]. It was shown that the continuous-time recurrent neural networks can be embedded into Lotka-Volterra models by changing coordinates, which suggests that the existing techniques in the analysis of Lotka-Volterra systems can also be applied to recurrent neural networks [6]. In recent years, there are some papers concerning with the dynamic behaviours of Lotka-Volterra recurrent neural networks [6-8]. In [7], the convergence involving global exponential, or asymptotic, stability of the following Lotka-Volterra recurrent neural networks is discussed:

$$\begin{cases} \dot{x}_i(t) = x_i(t)\Big[r_i - \sum\limits_{j=1}^{n} a_{ij}x_j(t) \\ \qquad\qquad - \sum\limits_{j=1}^{n} b_{ij}x_j(t-\tau_{ij}(t))\Big], \quad t>0, \\ x_i(t) = \phi_i(t) > 0, \quad \forall t \in [-\tau, 0], \end{cases} \tag{1.1}$$

where $x_i(t)$ denotes the state of neuron ith at time t. Real numbers a_{ij} and b_{ij} represent the synaptic connection weights from neuron j to neuron i at time t and $t-\tau_{ij}(t)$, respectively, and r_i denotes the external input. The variable delays $\tau_{ij}(t)$ for $i,j = 1,2,\dots n$ are non-negative functions satisfying $\tau_{ij}(t) \in [0,\tau]$ for $t \geq 0$, where τ is a constant.

Manuscript received May 9, 2016; revised October 9, 2016.

Li Yang is with the School of Statistics and Mathematics, Yunnan University of Finance and Economics, Kunming 650221, China. (h-nynyl@163.com).

Yonghong Yang is with the School of Essential Education, Yunnan Minzu University, Kunming 650031, China. (mathyhyang@163.com).

Yaqin Li is with the Department of Mathematics, Kunming University, Yunnan 650214, China. (yqlikm@163.com).

Tianwei Zhang is with the City College, Kunming University of Science and Technology, Kunming 650051, China. (zhang@kmust.edu.cn).

Correspondence author: Tianwei Zhang. (zhang@kmust.edu.cn).

By using the theory of exponential dichotomy and contraction mapping principle, many scholars increasingly have their eye on the existence and uniqueness of almost periodic solutions of all kinds of neural networks (e.g., Shunting inhibitory cellular neural networks [9], Hopfield neural networks [10], Cohen-Grossberg neural networks [11-12], etc) in the recent ten years. Also, Liu et al. [8] focused on studying the existence and uniqueness of positive almost periodic solution of the networks extended from network (1.1):

$$\begin{cases} \dot{x}_i(t) = x_i(t)\Big[r_i(t) - \sum\limits_{j=1}^{n} a_{ij}(t)x_j(t) \\ \qquad\qquad - \sum\limits_{j=1}^{n} b_{ij}(t)x_j(t-\tau_{ij}(t))\Big], \quad t>0, \\ x_i(t) = \phi_i(t) > 0, \quad \forall t \in [-\tau, 0]. \end{cases} \tag{1.2}$$

By using the theory of exponential dichotomy and contraction mapping principle, the authors obtained some sufficient conditions for the existence and uniqueness of almost periodic solution of system (1.2). Unfortunately, the work in [8] is not perfect (see Remark 3.1 in Section 3).

It is well known that in celestial mechanics, almost periodic solutions to differential equations or difference equations are intimately related. In the same way, electronic circuits, ecological systems, neural networks, and so forth exhibit almost periodic behavior. A vast amount of researches have been directed toward studying these phenomena (see [13-16]). Also, the theory of calculus on time scales (see [17] and references cited therein) was initiated by Stefan Hilger in his Ph.D. thesis in 1988 [18] in order to unify continuous and discrete analysis, and it has a tremendous potential for applications and has recently received much attention since his foundational work. Therefore, it is meaningful to study that on time scales which can unify the continuous and discrete situations. However, there are no concepts of almost periodic time scales and almost periodic functions on time scales, so that it is impossible for us to study almost periodic solutions to differential equations on time scales. On the other hand, in many earlier studies, it has been shown that harvesting has a strong impact on dynamic evolution of a population, e.g., see [19-22]. So the study of the population dynamics with harvesting is becoming a very important subject in mathematical bio-economics. This paper is concerning with the almost periodic solution of the following delayed Lotka-Volterra recurrent neural networks with harvesting terms:

$$x_i^{\Delta}(t) = x_i(t)\Big[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t)$$

(Advance online publication: 26 November 2016)

$$-\sum_{j=1}^{n} b_{ij}(t)x_j(t-\tau_{ij}(t))\Bigg] - h_i(t), \ (1.3)$$

where $\mathbb{T}$ is a periodic time scale; $r_i(t) > 0$, $a_{ij}(t) > 0$, $b_{ij}(t) > 0$, $h_i(t) > 0$ and $\tau_{ij}(t) > 0$ are all almost periodic functions, τ_{ij} are transmission delays at time t and satisfy $t - \tau_{ij}(t) \in \mathbb{T}$, $h_i(t) > 0$ represent harvesting terms, $i, j = 1, 2, \ldots, n$. The meanings of the parameters are the same as the corresponding ones mentioned in system (1.1). From the point of view of biology, we focus our discussion on the existence and uniqueness of positive almost periodic solution of system (1.3) by using the theory of exponential dichotomy and Banach fixed point theorem. When h_i $(i = 1, 2, \ldots, n)$ is small enough and close to zero, then system (1.3) is approximately equivalent to system (1.2). Therefore, our work in this paper corrects the defect in article [8] to a certain extent.

For any bounded function $f \in C(\mathbb{T})$, $f^+ = \sup_{s\in\mathbb{T}} f(s)$, $f^- = \inf_{s\in\mathbb{T}} f(s)$. We list some assumptions which will be used in this paper.

(H_1) r_i, a_{ij}, b_{ij} and h_i are nonnegative almost periodic functions with $0 < h_i^- < r_i^+$, $i, j = 1, 2, \ldots, n$.

(H_2) There exist positive constants $\eta_i \in \left[\frac{r_i^+ h_i^+}{r_i^-}, \frac{(r_i^+)^2 h_i^+}{r_i^- h_i^-}\right)$ $(i = 1, 2, \ldots n)$ such that

$$\sup_{s\in\mathbb{R}}\Big\{ - r_i(s) + \sum_{j=1}^{n} 2a_{ij}(s) + \sum_{j=1}^{n} 2b_{ij}(s)\Big\} < -\eta_i < 0,$$

where $i = 1, 2, \ldots n$.

The organization of this paper is as follows. In Section 2, we give some basic definitions and necessary lemmas which will be used in later sections. In Section 3, by using Banach fixed point theorem, we obtain some sufficient conditions ensuring existence and uniqueness of positive almost periodic solution of system (1.3). Finally, an example is given to illustrate that the result of this paper is feasible.

II. PRELIMINARIES

Now, let us state the following definitions and lemmas, which will be useful in proving our main result.

Definition 1. [17] A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real set $\mathbb{R}$ with the topology and ordering inherited from $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ and the graininess $\mu, \nu : \mathbb{T} \to \mathbb{R}^+$ are defined, respectively, by

$$\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}, \quad \rho(t) := \sup\{s \in \mathbb{T} : s < t\},$$

$$\mu(t) := \sigma(t) - t, \quad \nu(t) := t - \rho(t).$$

The point $t \in \mathbb{T}$ is called left-dense, left-scattered, right-dense or right-scattered if $\rho(t) = t$, $\rho(t) < t$, $\sigma(t) = t$ or $\sigma(t) > t$, respectively. Points that are right-dense and left-dense at the same time are called dense. If $\mathbb{T}$ has a left-scattered maximum m_1, defined $\mathbb{T}^\kappa = \mathbb{T} - \{m_1\}$; otherwise, set $\mathbb{T}^\kappa = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m_2, defined $\mathbb{T}_\kappa = \mathbb{T} - \{m_2\}$; otherwise, set $\mathbb{T}_\kappa = \mathbb{T}$.

Definition 2. [17] A function $p : \mathbb{T} \to \mathbb{R}$ is said to be regressive provided $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}^k$, where $\mu(t) = \sigma(t) - t$ is the graininess function. The set of all regressive rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ is denoted by $\mathcal{R}$ while the set $\mathcal{R}^+$ is given by $\{f \in \mathcal{R} : 1+\mu(t)f(t) > 0\}$ for all $t \in \mathbb{T}$. Let $p \in \mathcal{R}$. The exponential function is defined by

$$e_p(t, s) = \exp\bigg(\int_s^t \xi_{\mu(\tau)}(p(\tau))\Delta\tau\bigg),$$

where $\xi_{h(z)}$ is the so-called cylinder transformation.

Lemma 1. [17] *Let $p, q \in \mathcal{R}$. Then*

(i) $e_0(t, s) \equiv 1$ *and* $e_p(t, t) \equiv 1$*;*
(ii) $\frac{1}{e_p(t,s)} = e_{\ominus p}(t, s)$*, where* $\ominus p(t) = -\frac{p(t)}{1+\mu(t)p(t)}$*;*
(iii) $e_p(t, s)e_p(s, r) = e_p(t, r)$*;*
(iv) $e_p^\Delta(\cdot, s) = pe_p(\cdot, s)$.

Definition 3. [17] For $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^k$, the delta derivative of f at t, denoted by $f^\Delta(t)$, is the number (provided it exists) with the property that given any $\epsilon > 0$, there is a neighborhood $U \subset \mathbb{T}$ of t such that

$$|f(\sigma(t)) - f(s) - f^\Delta(t)[\sigma(t) - t]| \le \epsilon|\sigma(t) - s|, \forall s \in U.$$

Lemma 2. [17] *Let f, g be Δ-differentiable functions on $\mathbb{T}$. Then*

(i) $(k_1 f + k_2 g)^\Delta = k_1 f^\Delta + k_2 g^\Delta$ *for any constants* k_1, k_2*;*
(ii) $(fg)^\Delta(t) = f^\Delta(t)g(t) + f(\sigma(t))g^\Delta(t) = f(t)g^\Delta(t) + f^\Delta(t)g(\sigma(t))$.

Lemma 3. [17] *Assume that $p(t) \ge 0$ for $t \ge 0$. Then $e_p(t, s) \ge 1$.*

Lemma 4. [17] *Suppose that $p \in \mathcal{R}^+$. Then*

(i) $e_p(t, s) > 0$ *for all* $t, s \in \mathbb{T}$*;*
(ii) *if* $p(t) \le q(t)$ *for all* $t \ge s$, $t, s \in \mathbb{T}$*, then* $e_p(t, s) \le e_q(t, s)$ *for all* $t \ge s$.

Lemma 5. [17] *Suppose that $p \in \mathcal{R}$ and $a, b, c \in \mathbb{T}$, then*

$$\big[e_p(c, \cdot)\big]^\Delta = -p\big[e_p(c, \cdot)\big]^\sigma,$$

$$\int_a^b p(t)e_p(c, \sigma(t))\Delta t = e_p(c, a) - e_p(c, b).$$

Definition 4. ([23]) A time scale $\mathbb{T}$ is called a periodic time scale if

$$\Pi := \{\tau \in \mathbb{R} : t + \tau \in \mathbb{T}, \forall t \in \mathbb{T}\} \neq \{0\}.$$

Definition 5. ([24]) Let $\mathbb{T}$ be a periodic time scale. A function $x : \mathbb{T} \to \mathbb{R}^n$ is called almost periodic on $\mathbb{T}$, if for any $\epsilon > 0$, the set

$$E(\epsilon, x) = \{\tau \in \Pi : |x(t + \tau) - x(t)| < \epsilon, \forall t \in \mathbb{T}\}$$

is relatively dense in $\mathbb{T}$; that is, there exists a constant $l = l(\epsilon) > 0$, for any interval with length $l(\epsilon)$, there exists a number $\tau = \tau(\epsilon)$ in this interval such that

$$\|x(t + \tau) - x(t)\| < \epsilon, \quad \forall t \in \mathbb{T}.$$

The set $E(\epsilon, x)$ is called the ϵ-translation set of x, τ is called the ϵ-translation number of x, and $l(\epsilon)$ is called the inclusion of $E(\epsilon, x)$.

Lemma 6. ([24]) *If $x \in C(\mathbb{T}, \mathbb{R}^n)$ is an almost periodic function, then x is bounded on $\mathbb{T}$.*

Lemma 7. ([24]) *If $x, y \in C(\mathbb{T}, \mathbb{R}^n)$ are almost periodic functions, then $x+y$, xy are also almost periodic*

Definition 6. ([25]) Let $y \in C(\mathbb{T}, \mathbb{R}^n)$ and $P(t)$ be a $n \times n$ continuous matrix defined on $\mathbb{T}$. The linear system

$$y^{\Delta}(t) = P(t)y(t), \quad t \in \mathbb{T}$$

is said to be an exponential dichotomy on $\mathbb{T}$ if there exist constants $k, \lambda > 0$, projection S and the fundamental matrix $Y(t)$ satisfying

$$\|Y(t)SY^{-1}(s)\| \le ke_{\ominus\lambda}(t,s), \quad \forall t \ge s,$$
$$\|Y(t)(I-S)Y^{-1}(s)\| \le ke_{\ominus\lambda}(s,t), \quad \forall t \le s, \ t, s \in \mathbb{T}.$$

Lemma 8. ([24]) *If the linear system $y^{\Delta}(t) = P(t)y(t)$ has an exponential dichotomy, then almost periodic system*

$$y^{\Delta}(t) = P(t)y(t) + g(t), \quad t \in \mathbb{T}$$

has a unique almost periodic solution $y(t)$ which can be expressed as follows:

$$y(t) = \int_{-\infty}^{t} Y(t)SY^{-1}(\sigma(s))g(s)\,\Delta s$$
$$-\int_{t}^{\infty} Y(t)(I-S)Y^{-1}(\sigma(s))g(s)\,\Delta s.$$

Lemma 9. ([25]) *If $P(t) = (a_{ij}(t))_{n\times n}$ is a uniformly bounded rd-continuous matrix-valued function on $\mathbb{T}$, and there is a $\delta > 0$ such that*

$$|a_{ii}(t)| - \sum_{j\ne i}|a_{ij}(t)| - \frac{1}{2}\mu(t)\Big[\sum_{j\ne i}|a_{ij}(t)|\Big]^2$$
$$-\delta^2\mu(t) \ge 2\delta, \quad t \in \mathbb{T}, \ i = 1, 2, \dots, n,$$

then $y^{\Delta}(t) = P(t)y(t)$ admits an exponential dichotomy on $\mathbb{T}$.

Lemma 10. (**Banach fixed point theorem** [26]) *Assume that $(\mathbb{B}, \rho)$ is a complete metric space, $T : (\mathbb{B}, \rho) \to (\mathbb{B}, \rho)$ is a contraction mapping, i.e., there exists $\lambda \in (0, 1)$, such that*

$$\rho(Tx, Ty) \le \lambda\rho(x, y), \ \forall x, \, y \in \mathbb{B}.$$

Then T has a unique fixed point in $\mathbb{B}$.

III. Main result

In this section, we study the existence and uniqueness of almost periodic solution of system (1.3) by using Banach fixed point theorem.

Let

$$k_i := \frac{h_i^-}{r_i^+}, \quad l_i := \frac{r_i^+ h_i^+}{r_i^- \eta_i}, \quad i = 1, 2, \dots n.$$

By (H_2), it is easy to see that $k_i < l_i \le 1$, $i = 1, 2, \dots n$. Set

$$\mathbb{B} = \Big\{x = (x_1, x_2, \dots, x_n)^T \in AP(\mathbb{T}, \mathbb{R}^n) :$$
$$k_i \le x_i(t) \le l_i, \forall t \in \mathbb{T}, i = 1, 2, \dots n\Big\}$$

with the distance $\rho(x, y)$ from x to y is defined by

$$\rho(x, y) = \max_{1\le i\le n}\{\sup_{t\in\mathbb{R}}|x_i(t) - y_i(t)|\},$$

where $x(t) = (x_1(t), x_2(t), \dots, x_n)^T$, $y(t) = (y_1(t), y_2(t), \dots, y_n)^T \in \mathbb{B}$. Obviously, $(\mathbb{B}, \rho)$ is a complete metric space.

Theorem 1. *Assume that (H_1)-(H_2) hold, then system (1.3) has a unique almost periodic solution in $\mathbb{B}$.*

Proof: For $\forall\, \varphi = (\varphi_1, \varphi_2, \dots, \varphi_n)^T \in \mathbb{B}$, we consider the almost periodic solution of nonlinear almost periodic differential

$$x_i^{\Delta}(t) = r_i(t)x_i(t) - \varphi_i(t)\Big[\sum_{j=1}^{n} a_{1j}(t)\varphi_j(t)$$
$$+\sum_{j=1}^{n} b_{ij}(t)\varphi_j(t - \tau_{ij}(t))\Big] - h_i(t), \qquad (3.1)$$

where $i = 1, 2, \dots n$. Thus, by Lemma 8, we obtain that the system (3.1) has exactly one almost periodic solution:

$$x^{\varphi}(t) = (x_1^{\varphi}(t), x_2^{\varphi}(t), \dots, x_n^{\varphi}(t))^T,$$

where

$$x_i^{\varphi}(t) = \int_t^{+\infty} e_{r_i}(t, \sigma(s))\Big\{\varphi_i(s)\Big[\sum_{j=1}^{n} a_{ij}(s)\varphi_j(s)$$
$$+\sum_{j=1}^{n} b_{ij}(s)\varphi_j(s - \tau_{ij}(s))\Big] + h_i(s)\Big\}\Delta s,$$

in which $i = 1, 2, \dots n$.

Now, we give a mapping T defined on $\mathbb{B}$ by setting

$$T(\varphi) = (T_1(\varphi), T_2(\varphi), \dots, T_n(\varphi))^T$$
$$= (x_1^{\varphi}, x_2^{\varphi}, \dots, x_n^{\varphi})^T, \quad \forall\varphi \in \mathbb{B}.$$

First, we prove that the mapping T is a self-mapping from $\mathbb{B}$ to $\mathbb{B}$. In fact, $\forall\varphi \in \mathbb{B}$, in view of definition of T, we have

$$T_i(\varphi)(t) \ \ge \ \int_t^{+\infty} e_{r_i}(t, \sigma(s))h_i(s)\,\Delta s$$
$$\ge \frac{h_i^-}{r_i^+} = k_i, \quad \forall t \in \mathbb{T}, \ i = 1, 2, \dots n. (3.2)$$

On the other hand, it follows that

$$T_i(\varphi)(t) = \int_t^{+\infty} e_{r_i}(t, \sigma(s))\Big\{\varphi_i(s)\Big[\sum_{j=1}^{n} a_{ij}(s)\varphi_j(s)$$
$$+\sum_{j=1}^{n} b_{ij}(s)\varphi_j(s - \tau_{ij}(s))\Big] + h_i(s)\Big\}\,\Delta s$$
$$\le \int_t^{+\infty} e_{r_i}(t, \sigma(s))\Big\{l_i\Big[\sum_{j=1}^{n} a_{ij}(s)l_j$$
$$+\sum_{j=1}^{n} b_{ij}(s)l_j\Big] + h_i(s)\Big\}\,\Delta s$$
$$\le \int_t^{+\infty} e_{r_i}(t, \sigma(s))\Big\{l_i\Big[\sum_{j=1}^{n} a_{ij}(s)$$
$$+\sum_{j=1}^{n} b_{ij}(s)\Big] + h_i(s)\Big\}\,\Delta s$$
$$\le \int_t^{+\infty} e_{r_i}(t, \sigma(s))\Big\{l_i\Big[\sum_{j=1}^{n} 2a_{ij}(s)$$

$$
\begin{aligned}
&\quad +\sum_{j=1}^{n} 2b_{ij}(s)\Big] + h_i(s)\Big\}\,\Delta s \\
&\le \int_t^{+\infty} e_{r_i}(t,\sigma(s))\Big\{l_i\big[r_i(s)-\eta_i\big] + h_i(s)\Big\}\,\Delta s \\
&\le \int_t^{+\infty} \Big[r_i(s)e_{r_i}(t,\sigma(s)) - \eta_i e_{r_i^+}(t,\sigma(s))\Big] l_i\,\Delta s \\
&\quad + \frac{h_i^+}{r_i^-} \\
&= l_i \int_t^{+\infty} \Big[r_i(s)e_{r_i}(t,\sigma(s))\Big]\,\Delta s \\
&\quad -l_i \int_t^{+\infty} \eta_i e_{r_i^+}(t,\sigma(s))\,\Delta s + \frac{h_i^+}{r_i^-} \\
&= -l_i \int_t^{+\infty} \Big[e_{r_i}(t,\sigma(s))\Big]^{\Delta}\,\Delta s \\
&\quad -l_i \int_t^{+\infty} \eta_i e_{r_i^+}(t,\sigma(s))\,\Delta s + \frac{h_i^+}{r_i^-} \\
&= l_i - l_i \int_t^{+\infty} \eta_i e_{r_i^+}(t,\sigma(s))\,\Delta s + \frac{h_i^+}{r_i^-} \\
&\le (1-\frac{\eta_i}{r_i^+})l_i + \frac{h_i^+}{r_i^-} \\
&= l_i, \quad \forall t\in\mathbb{T},\ i=1,2,\ldots n. \qquad (3.3)
\end{aligned}
$$

By (3.2) and (3.3), we get that T is a self-mapping from $\mathbb{B}$ to $\mathbb{B}$.

Next, we show that $T:\mathbb{B}\to\mathbb{B}$ is a contraction mapping. In fact, for $\forall\,\varphi,\psi\in\mathbb{B}$, we have

$$
\begin{aligned}
&T_i(\varphi) - T_i(\psi) \\
&= \int_t^{+\infty} e_{r_i}(t,\sigma(s))\Big\{\varphi_i(s)\Big[\sum_{j=1}^{n} a_{ij}(s)\varphi_j(s) \\
&\quad + \sum_{j=1}^{n} b_{ij}(s)\varphi_j(s-\tau_{ij}(s))\Big] + h_i(s)\Big\}\,\Delta s \\
&\quad - \int_t^{+\infty} e_{r_i}(t,\sigma(s))\Big\{\psi_i(s)\Big[\sum_{j=1}^{n} a_{ij}(s)\psi_j(s) \\
&\quad + \sum_{j=1}^{n} b_{ij}(s)\psi_j(s-\tau_{ij}(s))\Big] + h_i(s)\Big\}\,\Delta s.
\end{aligned}
$$

Under definition of $\rho(x,y)$, we have

$$
\begin{aligned}
&|T_i(\varphi) - T_i(\psi)|_0 \\
&:= \sup_{t\in\mathbb{T}} |T_i(\varphi)(t) - T_i(\psi)(t)| \\
&= \sup_{t\in\mathbb{T}} \Big|\int_t^{+\infty} e_{r_i}(t,\sigma(s))\Big\{\varphi_i(s)\Big[\sum_{j=1}^{n} a_{ij}(s)\varphi_j(s) \\
&\quad + \sum_{j=1}^{n} b_{ij}(s)\varphi_j(s-\tau_{ij}(s))\Big] + h_i(s)\Big\}\,\Delta s \\
&\quad - \int_t^{+\infty} e_{r_i}(t,\sigma(s))\Big\{\psi_i(s)\Big[\sum_{j=1}^{n} a_{ij}(s)\psi_j(s) \\
&\quad + \sum_{j=1}^{n} b_{ij}(s)\psi_j(s-\tau_{ij}(s))\Big] + h_i(s)\Big\}\,\Delta s\Big| \\
&\le \sup_{t\in\mathbb{T}} \int_t^{+\infty} e_{r_i}(t,\sigma(s)) \\
&\quad \times\Big\{\sum_{j=1}^{n} a_{ij}(s)\Big|\varphi_i(s)\varphi_j(s) - \psi_i(s)\psi_j(s)\Big| \\
&\quad + \sum_{j=1}^{n} b_{ij}(s)\Big|\varphi_i(s)\varphi_j(s-\tau_{ij}(s)) \\
&\quad -\psi_i(s)\psi_j(s-\tau_{ij}(s))\Big|\Big\}\,\Delta s \\
&= \sup_{t\in\mathbb{T}} \int_t^{+\infty} e_{r_i}(t,\sigma(s)) \\
&\quad \times\Big\{\sum_{j=1}^{n} a_{ij}(s)\Big|\Big(\varphi_i(s)-\psi_i(s)\Big)\varphi_j(s) \\
&\quad +\psi_i(s)\Big(\varphi_j(s)-\psi_j(s)\Big)\Big| \\
&\quad + \sum_{j=1}^{n} b_{ij}(s)\Big|\Big(\varphi_i(s)-\psi_i(s)\Big)\varphi_j(s-\tau_{ij}(s)) \\
&\quad +\psi_i(s)\Big(\psi_j(s-\tau_{ij}(s)) - \varphi_j(s-\tau_{ij}(s))\Big)\Big|\Big\}\,\Delta s \\
&\le \sup_{t\in\mathbb{T}} \int_t^{+\infty} e_{r_i}(t,\sigma(s)) \times \Big\{\sum_{j=1}^{n} a_{ij}(s)\Big|\varphi_j(s)+\psi_i(s)\Big| \\
&\quad + \sum_{j=1}^{n} b_{ij}(s)\Big|\varphi_j(s-\tau_{ij}(s)) + \psi_i(s)\Big|\Big\}\,\Delta s\cdot\rho(\varphi,\psi) \\
&\le \sup_{t\in\mathbb{T}} \int_t^{+\infty} e_{r_i}(t,\sigma(s)) \times \Big\{\sum_{j=1}^{n} a_{ij}(s) \\
&\quad + \sum_{j=1}^{n} b_{ij}(s)\Big\}\big(l_j + l_i\big)\,\Delta s\cdot\rho(\varphi,\psi) \\
&\le \sup_{t\in\mathbb{T}} \int_t^{+\infty} e_{r_i}(t,\sigma(s))\Big\{\sum_{j=1}^{n} 2a_{ij}(s) \\
&\quad + \sum_{j=1}^{n} 2b_{ij}(s)\Big\}\,\Delta s\cdot\rho(\varphi,\psi) \\
&\le \sup_{t\in\mathbb{T}} \int_t^{+\infty} e_{r_i}(t,\sigma(s))[r_i(s)-\eta_i]\,\Delta s\cdot\rho(\varphi,\psi) \\
&\le \sup_{t\in\mathbb{T}} \Big(\int_t^{+\infty} r_i(s)e_{r_i}(t,\sigma(s))\,\Delta s \\
&\quad - \int_t^{+\infty} e_{r_i^+}(t,\sigma(s))\eta_i\,\Delta s\Big)\cdot\rho(\varphi,\psi) \\
&\le \big(1-\frac{\eta_i}{r_i^+}\big)\rho(\varphi,\psi), \quad i=1,2,\ldots n. \qquad (3.4)
\end{aligned}
$$

It follows from (3.4) that

$$
\rho(T(\varphi),T(\psi)) \le \max_{1\le i\le n}\Big\{1-\frac{\eta_i}{r_i^+}\Big\}\rho(\varphi,\psi) = \lambda\rho(\varphi,\psi),
$$

where $\lambda = \max_{1\le i\le n}\big\{1-\frac{\eta_i}{r_i^+}\big\}\in[0,1)$, which implies that the mapping $T:\mathbb{B}\to\mathbb{B}$ is a contraction mapping. Therefore, by Lemma 10, the mapping T possesses a unique fixed point

$$
x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T \in \mathbb{B}, \quad Tx^* = x^*.
$$

So system (1.3) has a unique almost periodic solution. This completes the proof. ■

Remark 1. In article [8], by using the contraction mapping principle, Liu et al. obtained that system (1.2) has a unique positive almost periodic solution in Ω, where

$$\Omega = \Big\{ x = (x_1, x_2, \dots, x_n)^T \in AP(\mathbb{R}, \mathbb{R}^n) :$$

$$x_i(t) \geq 0^+, \sum_{i=1}^{n} \rho_i x_i(t) \leq 1, \forall t \in \mathbb{R}, i = 1, 2, \dots n \Big\},$$

in which $\rho_i\,(i = 1, 2, \dots, n)$ are positive constants and 0^+ is defined as follows:

Definition 7. (See Definition 1 in [8]) Define 0^+ as a positive number, which is infinitely close to, yet not equal to, 0, and satisfying

$$0^+ = \alpha 0^+, \quad \forall\, |\alpha| \in (0, +\infty).$$

However, think carefully and we find that the number 0^+ defined by Definition 3.1 (i.e., Definition 1 of [8]) does not exist. Therefore, Ω defined in [8] is invalid and their work is not perfect.

Remark 2. When $\mathbb{T} = \mathbb{R}$ and $h_i\,(i = 1, 2, \dots, n)$ is small enough and close to zero, then system (1.3) is approximately equivalent to system (1.2). Therefore, our work in this paper corrects the defect in article [8] to a certain extent.

IV. Local asymptotical stability

In this section, we will construct some suitable Lyapunov functions to study the local asymptotical stability of system (1.3).

Theorem 2. *Assume that*

(H_3) $\Theta = r^- - A - B > 0$, *where*

$$r^- := \min_{1\leq i\leq n} r_i^-,$$

$$A := \max_{1\leq i\leq n} \sum_{j=1}^{n} (a_{ij}^+ + b_{ij}^+) l_j,$$

$$B := \max_{1\leq j\leq n} \sum_{i=1}^{n} (a_{ij}^+ + b_{ij}^+) l_i.$$

Then system (1.3) *is locally asymptotically stable.*

Proof: Assume that $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in \mathbb{B}$ and $x^*(t) = (x_1^*(t), x_2^*(t), \dots, x_n^*(t))^T \in \mathbb{B}$ are any two solutions of system (1.3). In view of system (1.3), for $t \in \mathbb{T}^+$, $i = 1, 2, \dots, n$, we have

$$\begin{aligned}
&(x_i(t) - x_i^*(t))^\Delta \\
&= r_i(t)\big[x_i(t) - x_i^*(t)\big] \\
&\quad - \sum_{j=1}^{n} [a_{ij}(t) + b_{ij}(t)] x_j(t) \big[x_i(t) - x_i^*(t)\big] \\
&\quad - \sum_{j=1}^{n} [a_{ij}(t) + b_{ij}(t)] x_i^*(t) \big[x_j(t) - x_j^*(t)\big].
\end{aligned}$$

Let

$$V(t) = \sum_{i=1}^{n} |x_i(t) - x_i^*(t)|.$$

Hence we can obtain from (H_7)-(H_9) that

$$\begin{aligned}
D^+ V^\Delta(t) &= D^+ \sum_{i=1}^{n} |x_i(t) - x_i^*(t)|^\Delta \\
&\geq \sum_{i=1}^{n} r_i^- |x_i(t) - x_i^*(t)| \\
&\quad - \sum_{i=1}^{n}\sum_{j=1}^{n} [a_{ij}^+ + b_{ij}^+] l_j |x_i(t) - x_i^*(t)| \\
&\quad - \sum_{i=1}^{n}\sum_{j=1}^{n} [a_{ij}^+ + b_{ij}^+] l_i |x_j(t) - x_j^*(t)| \\
&\geq (r^- - A - B) V(t) = \Theta V(t).
\end{aligned}$$

Integrating the last inequality from T_0 to t leads to

$$V(T_0) + \Theta \sum_{i=1}^{n} \int_{T_0}^{t} |x_i(s) - x_i^*(s)|\, \Delta s \leq V(t) < +\infty,$$

that is,

$$\sum_{i=1}^{n} \int_{T_0}^{+\infty} |x_i(s) - x_i^*(s)|\, \Delta s < +\infty,$$

which implies that

$$\sum_{i=1}^{n} \lim_{s\to+\infty} |x_i(s) - x_i^*(s)| = 0.$$

Thus, system (1.3) is locally asymptotically stable. This completes the proof. ∎

Theorem 3. *Assume that* (H_1)-(H_3) *hold. Then the unique almost periodic solution of system* (1.3) *is locally asymptotically stable.*

V. An example

Example 1. Consider the following harvesting Lotka-Volterra recurrent neural networks on time scales:

$$\begin{aligned}
x_i^\Delta(t) &= x_i(t)\Big[1 - \sum_{j=1}^{2} a_{ij}(t) x_j(t) \\
&\quad - \sum_{j=1}^{2} b_{ij}(t) x_j(t-1)\Big] - 0.1,
\end{aligned} \tag{5.1}$$

where $b_{ij}(s) = 0.1 \sin^2(\sqrt{3}s)$ $(i, j = 1, 2)$ and

$$\begin{pmatrix} a_{11}(s) & a_{12}(s) \\ a_{21}(s) & a_{22}(s) \end{pmatrix} = 0.1 \begin{pmatrix} \sin^2(\sqrt{2}s) & \cos^2(\sqrt{3}s) \\ \cos^2(\sqrt{5}s) & \cos^2(\sqrt{7}s) \end{pmatrix}.$$

Then system (5.1) has a unique positive almost periodic solution, which is locally asymptotically stable.

Proof: Corresponding to system (1.3), $a_{ij}^+ = 0.1$, $b_i^+ = 0.1$, $r_i^- = 1$ and $h_i^+ = h_i^- = 0.1$, $i, j = 1, 2$. Taking $\eta_1 = \eta_2 = 0.2$. By a easy calculation, we obtain

$$\begin{aligned}
&\sup_{s\in\mathbb{R}} \Big\{ -r_i(s) + \sum_{j=1}^{2} 2a_{ij}(s) + \sum_{j=1}^{n} 2b_{ij}(s) \Big\} \\
&< -1 + 4 \times 0.1 + 4 \times 0.1 \\
&= -0.2 = -\eta_i < 0, \\
&r^- = 1, \quad A = B = 0.2, \quad \Theta = 0.6,
\end{aligned}$$

where $i = 1, 2$, which implies that (H_2)-(H_3) in Theorems 1 and 3 hold. It is easy to verify that (H_1) in Theorem 1 is satisfied and the result follows from Theorems 1 and 3. This completes the proof. ∎

VI. Conclusion

In this paper, some sufficient conditions are established for the existence, uniqueness and local asymptotical stability of almost periodic solution for a harvesting Lotka-Volterra recurrent neural networks on time scales. The main result obtained in this paper are completely new even in case of the time scale $\mathbb{T} = \mathbb{R}$ or $\mathbb{Z}$ and complementary to the previously known results. Besides, the method used in this paper may be used to study many other ecological models such as predator-prey models [27-28], facultative mutualism models [29-30], and so on.

VII. Acknowledgements

This work was supported by Tian Yuan Fund of NSFC (No.11526180), Yunnan University of Finance and Economics Scientific Research Found Project (No.YC2015D09), Yunnan Province Education Department Scientific Research Fund Project (No.2015Y275), Scientific Research Fund of Yunnan Provincial Education Department (2014Y388) and the Research Program of Kunming University in China (XJL15010).

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