A Novel Joint Pilot Design Scheme for Sparse Channel Estimation in OFDM Systems

Tengfei Ren and Yanping Li

Abstract—**Pilot placement allocation is a key factor to improve the channel estimation performance in orthogonal frequency division multiplexing (OFDM) systems. However, conventional pilot optimization scheme is not attractive due to its high computational complexity under exhaustive search. To address this problem, a novel joint pilot design scheme utilizing the stochastic optimization scheme is proposed in this paper. It is focused on pilot placement optimization to improve the performance of sparse channel estimation. The proposed scheme combines the coherence minimization criterion and effective feedback of channel state information (CSI) at the receiver to jointly optimize the pilot placement to obtain the near-optimal pilot pattern, it is a tradeoff between accurate channel estimation and computational complexity without affecting the systems bandwidth usage. Simulation results show that the proposed scheme achieves better bit error rate (BER) performance when compared with the known stochastic optimization algorithm and random method.**

Index Terms—**OFDM, sparse channel estimation, compressed sensing, pilot design, feedback**

I. Introduction

As a promising modulation technique for high speed communication systems, OFDM technique has been widely adopted by various broadband wireless communication. However, high peak-to-average power ratio (PAPR) [1] and channel estimation accuracy [2] is the major task for OFDM systems. Considering the importance of channel estimation to the system performance, this paper investigates the improvement of channel estimation accuracy. Generally, the channel estimation method can be categorized into the following three classes: 1) pilot-aided channel estimation, e.g., the least square [2], minimum mean square error (MMSE) [3]; 2) semiblind channel estimation [4]; 3) blind channel estimation [5]. Since the pilot-aided channel estimation does not need the underlying multipath channel, many investigations employed the channel estimation based on pilot-aided.

In high-speed broadband communication, the wireless channel delay spread could be very large, and the number of non-zero paths are relatively small, thus the wireless channel can be modeled as a sparse channel [4]. The inherent sparsity of the wireless channels can be exploited by using the compressed sensing (CS) techniques [6]. Not only a high accuracy can be acquired based on CS channel estimation, but also the system transmission efficiency can be improved when compared with the conventional channel estimation method, therefore the CS techniques have been employed in the sparse channel estimation.

Recently, sparse channel estimation methods can be summarized as two main branches, one branch is the greedy algorithm, e.g., orthogonal matching pursuit (OMP). Another branch is the optimization of pilot pattern. Since the calculation of an exhaustive search could be huge and large memory would be required, it is impractical to select the optimal pilot pattern. Hence, the restricted isometry property (RIP) criterion is proposed to justify the pilot pattern selection [8], [9]. However, there is no standard polynomial time criterion to check whether a matrix satisfies RIP, the intractability of checking for the RIP of matrix motivated researchers to propose other methods to optimize pilot, such as the channel estimation based on cross-entropy optimization method [7], [10], [11] and the channel estimation based on the coherence criterion [12]-[17]. In [7], a cross-entropy optimization method to search for optimal pilot positions in sparse channel estimation scenarios is introduced. In [10], a pilot design method that utilizes convex optimization together with the cross entropy (CE) optimization is proposed to minimize the channel estimate mean squared error (MSE) of frequency selective channel. A novel scheme that utilizes constrained cross-entropy optimization to obtain an optimized pilot pattern is proposed in [11].

Unlike the work in [7], [10] and [11], a number of approaches based on coherence criterion are proposed. In [12], a new method based on lower coherence of the submatrix of the unitary discrete fourier transform (DFT) matrix associated with the pilot subcarriers is proposed to optimize pilot pattern. A modified discrete stochastic approximation method has been used to optimize the pilot placement in OFDM systems is proposed in [13]. Moreover, a fast pilot optimization algorithm based on minimizing the cross correlation of the measurement matrix is demonstrated in [14]. The [15] presented a deterministic procedure that jointly optimized for pattern and power of pilots for OFDM sparse channel estimation. The [16] proposed three pilot design schemes based on the mutual incoherence property (MIP) for sparse channel estimation in OFDM systems to obtain a near-optimal pilot pattern. However, the pilot pattern with lower coherence does not guarantee a better channel

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estimation performance compared to the pilot pattern with larger coherence [17].

In this paper, a novel joint pilot design scheme by using the CS techniques to obtain the near-optimal pilot pattern is proposed. By combining the coherence minimization criterion and the effective feedback of channel state information (CSI) at the receiver, the proposed scheme jointly optimizes the pilot placement to obtain the near-optimal pilot. Meanwhile, the proposed scheme achieves a better channel estimation performance and the accepted computational complexity.

This paper is organized as follows. Section II introduces the sparse channel model for OFDM systems based on CS theory. Section III presents an optimization algorithm for pilot pattern design. Simulation and performance evaluation are given in Section IV. Conclusions are drawn in Section V.

The notations used in this paper are defined as follows. A bold-face uppercase letter denotes a matrix, and a bold-face lowercase letter denotes a vector. $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, $\mathrm{diag}\{\cdot\}$, $|x|$, $\arg\{\cdot\}$, $0^{M\times N}$, $\delta(\cdot)$ and $\|\cdot\|_2$ denote transpose, conjugate transpose (Hermitian), the diagonal matrix, the absolute value of x, the argument function, the $M\times N$ zero matrix, the Kronecker delta function and the ℓ_2-norm, respectively.

II. System Model

An OFDM system with N subcarriers is considered. Among which N_p subcarriers are selected as pilots, other subcarriers are used for data transmission. A non-equally-spaced comb-type pilot pattern is employed, and denote the pilot position as $P=\{p_1,p_2,\cdots,p_{N_p}\}$, $\left(1\le p_1<p_2<...<p_{N_p}\le N\right)$. The corresponding transmitted pilot symbols and the received pilot symbols are denoted as X_P and Y_P.

The channel impulse response (CIR) is given by

$$h(\tau,t)=\sum_{l=1}^{L}h_l(t)\delta(\tau-\tau_l) \tag{1}$$

The multipath channel can be modeled as a finite impulse response (FIR) filter with impulse response $h=\left[h(1),h(2),\cdots,h(L)\right]^{\mathrm{T}}$, where L is the number of paths, $h_l(t)$ is the complex-valued channel impulse response, and τ_l is the delay spread for the lth path.

The received pilot symbols in OFDM can be expressed as

$$Y_P=diag\{X_P\}F_{P\times L}h+\mathrm{W}_P \tag{2}$$

where $diag\{X_P\}=diag\left\{X_{p_1},X_{p_2},\cdots,X_{p_{N_p}}\right\}$, $h=\left[h(1),h(2),\cdots,h(L)\right]^{\mathrm{T}}$, $\mathrm{W}_P=\left[W_{p_1},W_{p_2},\cdots,W_{p_{N_p}}\right]^{\mathrm{T}}$ is white complex Gaussian noise (AWGN) vector, and $\mathrm{Y}_P=\left[Y_{p_1},Y_{p_2},\cdots,Y_{p_{N_p}}\right]^{\mathrm{T}}$, and $F_{P\times L}$ is selected from a standard $N\times N$ DFT sub-matrix, given by

$$F_{P\times L}=\begin{bmatrix}1 & \omega^{p_1\cdot 1} & \cdots & \omega^{p_1\cdot(L-1)}\\ 1 & \omega^{p_2\cdot 1} & \cdots & \omega^{p_2\cdot(L-1)}\\ \vdots & \vdots & \ddots & \vdots\\ 1 & \omega^{p_{N_p}\cdot 1} & \cdots & \omega^{p_{N_p}\cdot(L-1)}\end{bmatrix}_{N_p\times L},\quad \omega=e^{-j2\pi/N}$$

We assume

$$A=diag\{X_P\}F_{P\times L} \tag{3}$$

Therefore, the equation (2) can be rewritten as

$$\mathrm{Y}_P=Ah+\mathrm{W}_P \tag{4}$$

When matrix A has more rows than columns, i.e., $N_p>L$, the channel in (4) can be solved by least squares (LS) estimation. However, when matrix A has more columns than rows, i.e., $N_p<L$, the problem becomes underdetermined. In this case, the LS estimation could not provide an accurate solution [7]. Fortunately, it was found that CS theory can be used to solve this problem because the wireless channel possess the feature of sparsity, i.e., h is a sparse vector. However, the RIP criterion is poor to test whether a matrix satisfies RIP in polynomial time [15]. Therefore, a novel joint pilot design scheme is proposed and presented in the Section III. The OFDM system model is illustrated in Fig.1.

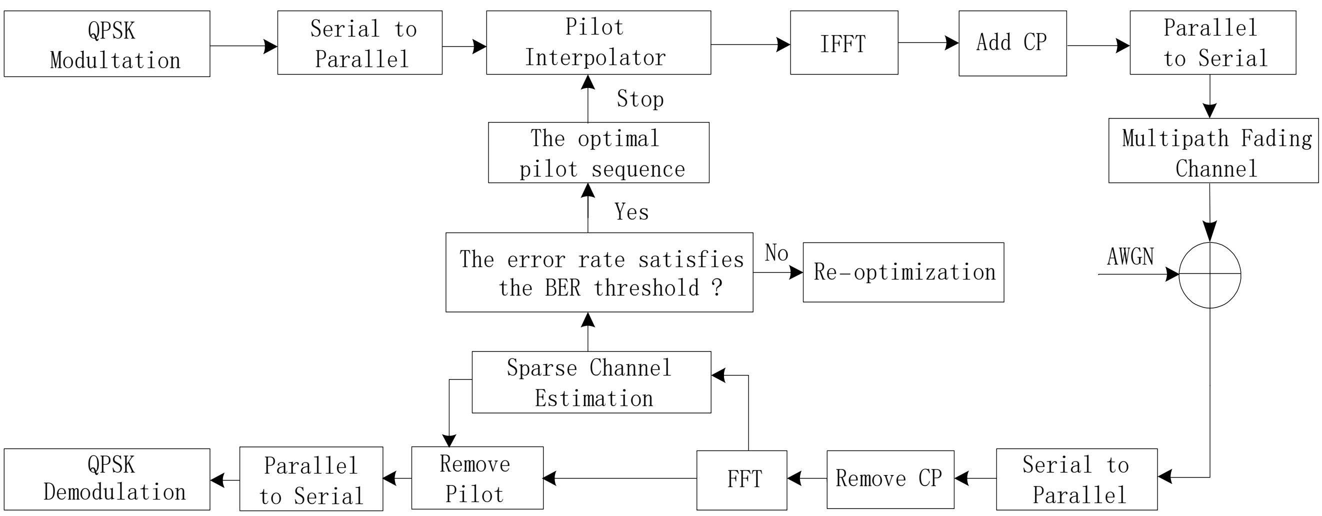

Fig. 1. OFDM system model

III. PROPOSED JOINT PILOT DESIGN SCHEME

In this section, a novel joint pilot design scheme to obtain the near-optimal pilot is given as follows.

A. Definition

We define the coherence of the matrix A is the maximum absolute of inner product between two different columns of A:

$$\mu(A)=\max_{m\neq n}\left|A^{\mathrm{H}}(m)A(n)\right| \tag{5}$$

where $A(m)$ and $A(n)$ are different columns of A, respectively.

Assume that the initial pilot pattern is $P=\{p_1,p_2,\cdots,p_{N_p}\}$, where $1\le p_1<p_2<...<p_{N_p}\le N$, then (5) can be written as

$$\mu(A)=\max_{1\le j\le L}\left|\sum_{i=1}^{N_p}\left|x(p_i)\right|^2 w^{p_i(j-1)}\right| \tag{6}$$

where $\left|x(p_i)\right|^2$ is the power of the i th pilot symbol. For simplicity, we assume $\left|x(p_i)\right|^2=E$ and $\sum_{i=1}^{N_p}w^{p_i(j-1)}=g_{i,j}$, then (6) can be rewritten as

$$\mu(A)=E\cdot\max_{1\le j\le L}\left|g_{i,j}\right| \tag{7}$$

The objective of pilot design scheme is the minimization of $\mu(A)$, which can be transformed to minimize $g_{i,j}$ by using the equation (7) to optimize the pilot pattern. Therefore, the optimal pilot sequence can be represented as

$$P_{opt}=\arg\min_{p}\mu(A) \tag{8}$$

B. Proposed Scheme

As mentioned above, the key idea of the proposed scheme is to search the near-optimal pilot pattern through utilizing the stochastic optimization method instead of exhaustive search. Since we have known that the pilot sequence with smaller $\mu(A)$ does not necessarily generate a better channel estimation performance compared to that of pilot sequence with larger $\mu(A)$, here the low coherence criterion and the effective feedback of the channel state information at the receiver is combined to jointly optimize the pilot placement to obtain the near-optimal pilot pattern. The detailed procedure of the proposed joint pilot design algorithm, namely Algorithm 1, was introduced in Table I.

TABLE I
OUR PROPOSED ALGORITHM 1

Input: randomly generate $P\subset \mathrm{N}$, BER threshold η_{th}

Output: the optimized pilot pattern P_{opt}

Initialization: $\Omega=0^{N_p\times N_p}$, $\Xi=0^{M\times N_p}$

for $l=1,2,\cdots,M$

  if $P=P_{opt}$

    Return P_{opt}

  else

    Perform the next step

  end if

  for $m=1,2,\cdots,N_p$

    Denote the set of fixing elements except for the mth position element of P_m and its candidate set as $K_m=P_m\setminus P_m(m)$ and $I_m=N\setminus K_m$, respectively;

    Remove one element from I_m and place on $P_m(m)$;

    Select one pilot pattern with smallest $\mu(A)$ from all placements by using the equation (7), and save the result to Ω;

  end for (m)

    Select one pilot pattern corresponding to the smallest $\mu(A)$ from the Ω as the next optimization pilot P_n;

  for $n=1,2,\cdots,N_p$

    Denote the set of fixing elements except for the nth position element of P_n and its candidate set as $K_n=P_n\setminus P_n(n)$ and $I_n=N\setminus K_n$, respectively;

    Remove one element from the I_n and place on the nth position of P_n;

    Select the pilot corresponding to the smallest $\mu(A)$ from all placements as the next pilot P_n';

    Sequentially optimize other location of P_n' until all the position elements updated;

  end for (n)

    Select the pilot corresponding to the global minimum as the final result P_{opt};

end for (l)

Select the P_{opt} from Ξ, and apply the P_{opt} to compute the BER at the receiver, and justify whether the generated BER satisfies η_{th} until meet the BER condition, output the optimized pilot P_{opt}.

The set of subcarrier N is set as $1,...,N$, the iteration number of outer loop and inner loop is set as $l=1,2,\cdots,M$ and

$m=1,2,\cdots,N_p$, respectively. We start with extracting N_p elements from the N to generate a random pilot sequence P, i.e., $p_1, p_2, \cdots, p_{N_p}$. Assume that Ω is a zero vector of $N_p \times N_p$ length to save each optimized result from the inner loop. If the random pilot sequence has the same result as optimization of the last inner loop, the inner loop process will not perform. Only when the value is different, the inner loop begins. Once the optimization of the inner loop is completed, the operation for the outer loop starts.

The detailed procedure of the inner loop can be described as follows. For each position of P_m, i.e., $m=1,2,\cdots,N_p$, fix N_p-1 positions except for the mth position. Assume the set constituted by the fixed elements is K_m, the candidate set I_m is $N \setminus K_m$. At each iteration, one element will be selected from the candidate set I_m and placed on the mth position of P_m, then choose the pilot sequence with a smaller $\mu(A)$ from all placements by using the equation (7) and save in Ω. After that, search one placement that have the smallest $\mu(A)$ from the Ω as the next optimization initial value P_n. For each position of P_n, i.e., $n=1,2,\cdots,N_p$, similarly, fix on N_p-1 positions except for the nth position, assume the set K_n consist of that have been fixed elements, and denote the candidate set I_n as $N \setminus K_n$. We remove one element from the I_n and place it on the nth position of P_n, search the pilot sequence with lower $\mu(A)$ from all placements as the next pilot P_n', then perform the optimization for other location of P_n' sequentially until all position elements have been updated, which can guarantee the optimization direction towards the global optimum value. Finally, the pilot sequence with minimum $\mu(A)$ according to the low coherence criterion will be selected as the optimization result of the inner loop.

In the outer loop, M iterations were performed, each iteration result will save in Ξ, from which select the pilot sequence that have the smallest $\mu(A)$ as the optimal pilot sequence P_{opt}. Meanwhile, the bit error rate (BER) threshold criterion have been introduced to justify whether the BER generated at the receiver meet the given BER threshold condition η_{th}. Once the parametric condition meets η_{th}, the pilot sequence will be selected as the near-optimal pilot pattern and feed back to the quaternary phase-shift keying (QPSK) modulation data, otherwise another optimization process will be performed until the result satisfies the given η_{th}.

In the next section, the effectiveness of our design scheme on BER will be tested via Monte-Carlo simulations.

IV. Simulation And Performance Evaluation

A simulation was carried out in this section to evaluate the BER performance of the proposed Algorithm 1 and other methods. In this paper, the OFDM systems adopted the QPSK modulation and a sparse multipath channel h with $L=60$ taps, where the position of 4 non-zero taps are generated randomly and the attenuation of each path is independent and identically distributed (i.i.d.) complex Gaussian distributed with zero mean and unit variance. Furthermore, data symbol and pilot symbol set as the same power.

In our simulation, the proposed Algorithm1 with stochastic sequential search (SSS) and stochastic parallel search (SPS) scheme which are proposed in [16] and the random method were compared. The above mentioned methods were based on the convex channel estimation, the BER threshold η_{th} was set to $5\times10^{-4} \le \eta_{\text{th}} \le 1\times10^{-2}$ and the simulation was performed 200 iterations. Furthermore, four methods were validated under different N and N_p. The pilot placements generated from four methods corresponding to different N and N_p were presented in Table II, Table III, Table IV and Table V, respectively. Since the BER performance is the ultimate measurement of the OFDM systems, the BER performance of four methods were mainly investigated and the BER performance of four methods with different values of N and N_p were given in the Fig. 2, Fig. 3, Fig. 4 and Fig. 5, respectively.

Since the optimization of pilot sequence is generated offline before the transmission of OFDM signals, we only need to choose the pilot sequence corresponding to the best performance from these generated pilot sequences, thus the computational complexity of Algorithm 1 is still acceptable.

TABLE II

Comparison of Pilot Placement $(N,\ N_p)=(128,8)$

Method	Numbers of pilots (N_p=8)							
Algorithm 1	3	25	45	47	77	81	91	105
SSS	5	25	45	47	77	81	87	105
SPS	31	42	46	47	64	87	113	116
Random	88	96	101	103	107	110	111	112

TABLE III

Comparison of Pilot Placement $(N,\ N_p)=(128,16)$

Method	Numbers of pilots (N_p=16)								
Algorithm 1	12	21	25	33	62	69	75	79	81
	88	89	103	105	107	113	128		
SSS	1	3	24	33	59	62	69	75	79
	88	89	103	105	107	113	128		
SPS	4	5	22	31	41	54	62	79	81
	97	109	112	121	123	125	127		
Random	4	9	12	15	16	19	23	26	32
	34	35	37	41	43	49	50		

TABLE IV

Comparison of Pilot Placement $(N,\ N_p)=(256,16)$

Method	Numbers of pilots (N_p=16)								
Algorithm 1	16	69	77	105	110	140	153	165	181
	184	192	196	204	230	251	254		
SSS	5	25	38	45	47	69	72	77	87
	105	140	153	184	192	196	204		
SPS	11	51	60	91	92	99	119	123	127
	134	152	170	174	187	190	244		
Random	7	16	20	23	33	34	46	59	63
	72	93	94	101	106	115	126		

TABLE V
COMPARISON OF PILOT PLACEMENT $(N,\ N_p)=(256,\ 32)$

Method	Numbers of pilots (N_p =32)								
Algorithm 1	24	41	69	78	82	85	90	96	102
	110	125	127	136	148	152	157	160	163
	170	190	201	209	215	218	226	228	229
	244	248	249	251	252				
SSS	6	24	41	48	50	81	85	90	95
	96	102	110	125	127	148	157	160	163
	168	170	190	194	201	208	210	218	228
	229	245	248	250	251				
SPS	12	28	29	40	48	53	66	73	76
	88	92	96	106	110	113	115	118	122
	126	138	152	156	165	178	181	199	209
	212	216	228	233	236				
Random	3	4	7	11	13	15	17	33	36
	40	43	45	51	56	57	69	72	77
	80	84	86	91	92	97	232	233	237
	238	240	242	253	254				

The pilot placement generated from four methods are given in table II when $(N,\ N_p)=(128,\ 8)$, and the BER performance corresponding to pilot placements obtained from above methods are presented in Fig. 2. From Fig. 2, we observe that the optimized pilot placements can significantly improve the BER performance of OFDM systems compared to random method, which means that the proposed Algorithm 1, SSS and SPS are superior to random method. Fig. 2 also shows Algorithm 1 achieves better performance than SSS and SPS, among which SSS performs slightly better than SPS. The Algorithm 1 obtains about 1~3 dB gain in signal-to-noise ratios (SNR) at the BER=5×10^{-3} compared to SSS and SPS respectively. Besides, the BER value of Algorithm 1 is reduced by nearly 0.001 and 0.002 at SNR=30 dB when compared with SSS and SPS respectively.

Table III presents the pilot placement for above methods when $(N,\ N_p)=(128,\ 16)$, Fig. 3 shows the BER performance of different methods when $(N,\ N_p)=(128,\ 16)$. As shown in Fig. 3, the Algorithm 1 achieves better performance than other methods, and the BER value of the Algorithm 1 is BER=6×10^{-4} at SNR=30 dB.

Table IV and Table V respectively show comparison of pilot placements by employing four methods when $(N,\ N_p)=(256,\ 16)$ and $(N,\ N_p)=(256,\ 32)$. Fig. 4 and Fig. 5 respectively plot the BER performance of different pilot placements under four channel estimation methods when $(N,\ N_p)=(256,\ 16)$ and $(N,\ N_p)=(256,\ 32)$. In Fig. 4, the Algorithm 1 have similar BER performance with SSS and SPS when SNR is lower than 15dB, the advantage of Algorithm 1 can be reflected clearly in the following 15dB. In Fig. 5, we see that the considerable BER performance compared to Fig. 4 can be obtained by using more pilots, this is because more pilots can help the receiver more accurately estimate the channel, however, it will reduces the spectral efficiency.

From Fig. 2 and Fig. 3, we observe that the BER of the Algorithm 1 decreases from 8×10^{-3} to 4×10^{-3} at SNR=20 dB, which demonstrate the considerable BER performance can be obtained by using more pilots, this is because the more pilots can help the receiver more accurately estimate the channel, however , it will reduces the spectral efficiency. As shown in Fig. 3 and Fig. 4, the BER performance of the above methods obviously degrades with N increases when N_p keep unchanged.

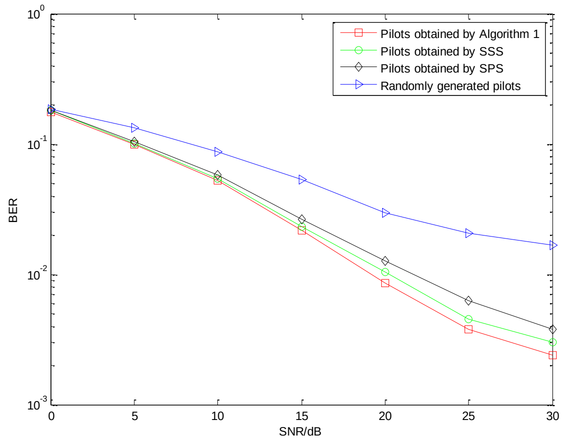

Fig. 2. BER Versus SNR of Different Methods for $(N,\ N_p)=(128,\ 8)$

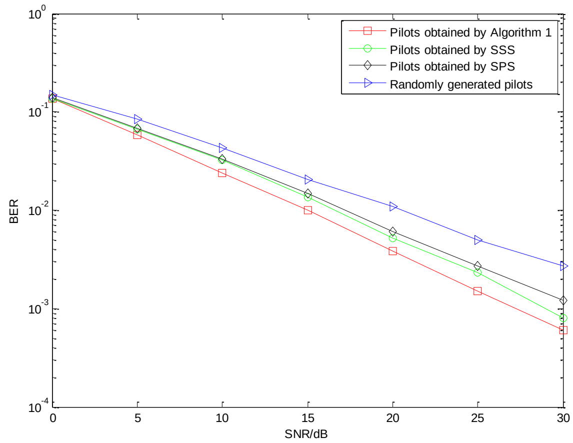

Fig. 3. BER Versus SNR of Different Methods for $(N,\ N_p)=(128,\ 16)$

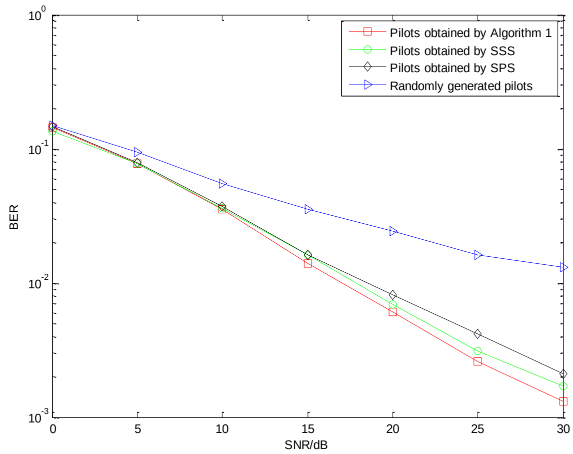

Fig. 4. BER Versus SNR of Different Methods for $(N,\ N_p)=(256,\ 16)$

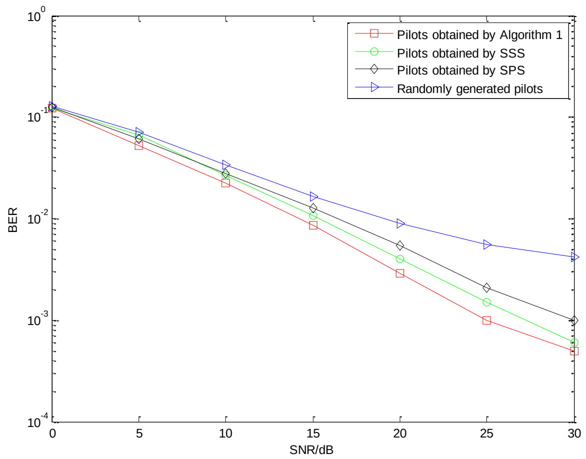

Fig. 5. BER Versus SNR of Different Methods for (N, N_p)=(256, 32)

It is seen from the above simulation results that the Algorithm1, SSS and SPS have better performance than random method, this is due to the Algorithm 1, SSS and SPS fully exploits the inherent sparsity of channel. Furthermore, the simulation results demonstrated the superior BER performance of the Algorithm1 compared to other methods.

V. CONCLUSIONS

In this paper, we proposed a novel joint pilot design scheme to allocate pilot placement in OFDM systems. By utilizing the low coherence minimization criterion in the procedure of pilot optimization, a better channel estimation performance can be obtained. Meanwhile, the proposed scheme combined the rule of effective feedback of optimal pilot placement at the receiver to ensure the better channel estimation result. Simulation results have validated the BER performance improvement of our proposed scheme compared to SSS, SPS and random method.

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