Generalized Mathematic Model of Electromechanical Energy Transducers with Non-contact Bearings

Flur Ismagilov, Irek Khayrullin, Vacheslav Vavilov, Aynur Yakupov

Abstract— **Development of aerospace and engineering industries, robotics and stand-alone generation forces the world electric machine industry to solve a new task – to create high-speed electromechanical energy transducers with high-coercivity permanent magnets with the capacity of 10 – 700 kWt, rotor speed of up to 200 000 rpm and ultrahigh-speed EMETs with the capacity of up to 1 kWt and rotor speed of up to 1 000 000 rpm. In this paper, the authors develop a mathematical model of high-speed electromechanical energy transducers with with non-contact bearing supports. And on the basis of the proposed mathematical model investigations the authors have developed a peculiar rotor position control algorithm in hybrid magnetic bearings.**

Keywords—**Air gap, High-coercitive permanent magnets (HCPM), High-frequency electromechanical energy transducer (EMET).**

I. INTRODUCTION

Development of aerospace and engineering industries, robotics and stand-alone generation forces the world electric machine industry to solve a new task – to create high-speed electromechanical energy transducers with high-coercivity permanent magnets (EMETs with HCPM) with the capacity of 10 – 700 kWt, rotor speed of up to 200 000 rpm and ultrahigh-speed EMETs with the capacity of up to 1 kWt and rotor speed of up to 1 000 000 rpm [1–3].

Application of these EMETs with HCPM is one of the prospective trends. They allow expanding functional opportunities of the facilities they are located in, as well as minimizing weight-size parameters of these facilities and increasing their production capacity. Thus research, development and production of such EMETs are being conducted by different manufacturing companies and corporations [3–10].

So, Calnetics company designs high-speed EMETs with HCPM with the capacity of from 3 to 700 kWt for aviation, ship-building, oil and gas industries. To minimize friction losses caused by EMET rotor high speed Calnetics company uses magnetic bearings in its EMETs with HCPM [4].

Capstone Turbine Corporation produces in lots several standard sizes of microturbine units where high-speed EMETs with HCPM (speed of from 60000 to 96000rpm, capacity of from 35 to 1200 kWt) are used. Aerodynamic bearing supports are used by Capstone company for minimizing friction losses in EMETs with HCPM [5].

NASA (Glenn Research Center, Cleveland, Ohio) is working on developing a high-speed EMET with HCPM for small space vehicles. EMET capacity is to be 7 kWt at the rotor speed of 50000 rpm. An electromagnetic bearing and a magnetic bearing with permanent magnets are used for decreasing friction losses [6].

Such researches are being carried out by ABB, ETH Zurich, Cryostar Group, E+A Elektromaschinen und Antriebe AG and other companies [5–10].

Having analyzed industrially produced EMETs with HCPM [5-10] it was found out that in order to increase their efficiency and minimize losses most of such EMETs are produced with non-contact bearing supports (NCBS), aerodynamic or magnetic bearing supports or with elastic bearing supports (e.g. hydrodynamic bearings).

NCBS usage leads to significant complication of the structural design of EMETs with HCPM, as well as complication of engineering process as it is necessary to take into account not only electromechanical, electromagnetic and thermal processes in EMETs but also interactions between them. Therefore, for providing process accuracy at designing a high-speed EMET with NCBS it is required to create new mathematical tools for their designing. These tools would allow considering interactions between NCBS and EMETs with HCPM during the operation process.

II. FORMULATION OF THE RESEARCH PROBLEM

In paper [12] a mathematical model of the EMETs with HCPM is represented, also its software implementation is designed in Matlab Simulink. This model includes only electromagnetic processes in high-speed EMETs. A multidisciplinary analysis of thermal and mechanical processes in high-speed EMETs is carried out with the help of the software package ANSYS in [13]. In paper [14] the task of multidisciplinary research of high-speed EMETs is solved in the software package ANSYS as

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well. Paper [15] represents a mathematical model for air-gap-conductor EMETs but only electromagnetic processes in high-speed EMETs are taken into account. A multidisciplinary task of high-speed EMET thermal condition estimation is solved in [16]. Paper [17] shows joint researches of thermal and electromagnetic processes in EMETs. A mathematical model of magnetic bearings is designed in [18], however, their influence on high-speed EMETs is not considered. The same results for gas and hybrid magnetic bearings have been obtained in papers [19–21]. So, in scientific literature there are a lot of mathematical models and multidisciplinary researches of high-speed EMETs with HCPMs and NCBS conducted separately. However, there are no muldisciplinary researches of a high-speed EMET – non-contact bearing support system taking into account their interactions. Although this task is rather vital both for basic sciences (as generalized objective function for optimization of high-speed EMETs with HCPM can be formulated based upon it) and for applied problems (solution of this task will allow increasing accuracy of EMET with HCPM designing significantly; a new control algorithm for rotor position of high-speed EMETs with HCPM can be formulated according to the solution results of the task mentioned above).

Therefore, this paper has an aim to create a generalized analytical model of high-speed EMETs with HCPM with elastic bearing supports considering interactions between the processes in EMETs and NCBS. As it is seen from the review this task is new and vital, and important to modern power-plant industry. At the same time analytical solution of the task will have a general and global character, in spite of solving this task by computer-based simulation methods (Femm, Ansys).

III. A GENERALIZED MATHEMATICAL MODEL OF A HIGH-SPEED EMET MOUNTED ON ELASTIC BEARING SUPPORTS.

At developing a generalized mathematical model which describes high-speed EMETs with NCBS a rigid nondeformable rotor on elastic bearing supports is taken. Active magnetic bearings, gas bearings and hybrid magnetic bearings can be used as elastic bearing supports. A rotor with weight mp is mounted on two supports (Fig.1). Static unbalance of the rotary mass is characterized by eccentricity e, dynamic one can be characterized by angular parameter β.

A non-salient pole EMET with HCPM is taken as an EMET while developing the mathematical model, Fig.1. For simplification of mathematical transformations the following allowances are taken:

– environmental and air-gap permeability is equal to vacuum permeability;

– non-magnetic gap value is much less than the radius of its curvature;

– a stator winding has a form of a thin conductive layer diametrically-spaced along the stator magnetic circuit boring k_L;

– induced currents density on thin copper layer thickness is constant;

– thermal conductivity of EMET active elements is constant in all the coordinate axes (EMET active elements are isotropic);

– axial component of the magnetic field strength in rotor butt end surfaces is 0, i.e. the EMET of infinite length is considered;

– temperature in the air-gap is considered to be acquainted.

While solving the task mentioned above simultaneous solution of several equation systems is being sought:

– of the equation system describing electromagnetic processes in a high-speed EMET (Maxwell`s equations for slow-moving media):

$$\mathrm{rot}\vec{H} = j + j_{CT} + \frac{\partial D}{\partial t}, \tag{1}$$

$$\mathrm{rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{2}$$

$$\vec{j} = \sigma\left[\vec{E} + \left(\vec{V}\times\vec{B}\right)\right], \tag{3}$$

$$\mathrm{div}\vec{B} = 0, \tag{4}$$

$$\mathrm{div}\vec{j} = 0, \tag{5}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0}, \tag{6}$$

$$\mathrm{div}\vec{D} = \rho_e \tag{7}$$

where $\vec{B}$ is the vector of resulting magnetic field magnetic induction; $\vec{E}$, $\vec{H}$ are the vectors of electric and magnetic fields; $\vec{V}$ is the vector of rotor motion rate; σ is the stator winding electric conductivity; $\vec{j}$ is the vector of induced currents density; $\vec{j}_{ст}$ is the vector of extraneous currents density;

– of the mathematical model describing rotor mechanical motion [21]:

$$m_1\ddot{x}_1 - m_{12}\ddot{x}_2 + h_0(\dot{y}_1 - \dot{y}_2) = Q_{1m} + Q_1 + Q_{1\nu}(t), \tag{8}$$

$$m_1\ddot{y}_1 - m_{12}\ddot{y}_2 - h_0(\dot{x}_1 - \dot{x}_2) = Q_{2m} + Q_2 + Q_{2\nu}(t), \tag{9}$$

$$-m_{12}\ddot{x}_1 + m_2\ddot{x}_2 - h_0(\dot{y}_1 - \dot{y}_2) = Q_{3m} + Q_3 + Q_{3\nu}(t), \tag{10}$$

$$-m_{12}\ddot{y}_1 - m_2\ddot{y}_2 + h_0(\dot{x}_1 - \dot{x}_2) = Q_{4m} + Q_4 + Q_{4\nu}(t), \tag{11}$$

where $Q_{1m} - Q_{4m}$ are support reaction forces; $Q_1 - Q_4$ are external forces determining the rotor motion character;

$$Q_{1\nu}(t) = \frac{m_\nu e\omega^2}{l_v}(z_2\cos\omega t) - \frac{(J_1 - J_3)e\omega^2}{l_v}(\sin\omega t);$$

$$Q_{2\nu}(t) = \frac{m_\nu e\omega^2}{l_v}(z_2\sin\omega t) - \frac{(J_1 - J_3)e\omega^2}{l_v}(-\cos\omega t);$$

$$Q_{3\nu}(t) = \frac{m_\nu e\omega^2}{l_\text{v}}(-z_1\cos\omega t) - \frac{(J_1 - J_3)e\omega^2}{l_\text{v}}(-\sin\omega t);$$

$$Q_{4\nu}(t) = \frac{m_\nu e\omega^2}{l_\text{v}}(-z_1\sin\omega t) - \frac{(J_1 - J_3)e\omega^2}{l_\text{v}}(\cos\omega t);$$

e is the rotor eccentricity; m_v the mass of a shift with the rotor; l_v is the rotor shift length; ω is the rotor speed; J_1 is the equatorial moment of shift inertia; J_3 is the axial moment of shift inertia; $m_1 = \frac{J_1 + m_\text{v} z_2^2}{l_\text{v}^2}$, $m_{12} = \frac{J_1 + m_\text{v} z_1 z_2}{l_\text{v}^2}$, $m_2 = \frac{J_1 + m_\text{v} z_1^2}{l_\text{v}^2}$ are the rotor mass changings; $h_0 = \frac{J_3\omega}{l_\text{v}^2}$ is the gyroscopic coefficient; x_1, x_2, y_1, y_2 are the shift radial throws in the elastic bearing support; z_1, z_2 are the axial coordinates of the first and second NCBS; $\ddot{x}_1, \ddot{x}_2, \ddot{y}_1, \ddot{y}_2$ are second differential coefficients upon motions in time.

To give a joint analysis of physical processes in the EMET with HCPM the developed mathematical model must include relationship of magnetic field parameters in the EMET air-gap and the temperature.

Considering that the EMET air-gap flux density is proportional to the HCPM residual induction, in this case the EMET with HCPM magnetic field – temperature relationship can be described as follows:

$$B(\Theta) = B_0\left(1 - \frac{k_{Br}(\Theta_\text{HCPM} - 23)}{100}\right)$$

where $B(\Theta)$ is the effective values of the HCPM residual induction and the coercivity force, respectively; Θ_HCPM is the HCPM temperature; k_Br is the HCPM residual induction temperature coefficient.

This equation can be given as follows:

$$\partial B = B_0 - \frac{B_0 k_{Br}\partial\Theta_\text{HCPM}}{100},$$

Having divided both parts into ∂t we get:

$$\frac{\partial B}{\partial t} = \frac{B_0}{\partial t} - \frac{B_0 k_{Br}}{100}\frac{\partial\Theta_\text{HCPM}}{\partial t},$$

Considering Maxwell`s equation set the equation can be written as follows:

$$-\frac{\partial B}{\partial t} = \text{rot}\vec{E}.$$

While analyzing electromagnetic processes it is considered that the resulting magnetic field is determined by the sum of two magnetic fields (rotor fields and stator field induced in the winding):

$$\vec{H} = \vec{H}_1 + \vec{H}_2, \quad (12)$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2. \quad (13)$$

Then Maxwell`s equation set can be rewritten as follows:

$$\text{rot}\vec{H} = j + j_\text{CT} + \frac{\partial D}{\partial t}, \quad (14)$$

$$\text{rot}\vec{E} = -\frac{\partial B_1}{\partial t} - \frac{\partial B_2}{\partial t} - \left(\frac{B_0}{\partial t} - \frac{B_0 k_{Br}}{100}\frac{\partial\Theta_\text{HCPM}}{\partial t}\right), \quad (15)$$

$$\vec{j} = \sigma\left[\vec{E} + \left(\vec{V}\times\vec{B}\right)\right], \quad (16)$$

$$\text{div}\vec{B} = 0, \quad (17)$$

$$\text{div}\vec{j} = 0, \quad (18)$$

$$\vec{H} = \mu_0\vec{B}, \quad (19)$$

$$\text{div}\vec{D} = \rho_\text{e}, \quad (20)$$

From the equation (15) it is seen that if $\frac{\partial B_1}{\partial t} = 0, \frac{\partial B_2}{\partial t} = 0$ (the rotor is motionless), but $\frac{\partial\Theta_\text{HCPM}}{\partial t} \neq 0$ (i.e. the EMET temperature is time variant), so the EMET with HCPM can generate electric power the capacity of which is determined by the HCPM temperature flow rate and its features. It is obvious that rate-of-change of the HCPM temperature is much less than the rotation speed of the high-speed EMET. Moreover, the proved feature of the EMET with HCPM can be used at developing different types of sensors and microthermoelectric generators.

This conclusion is one of the features of the EMETs with HCPM which is not common for other types of electric machines.

Solving the equation set (1)–(11) regarding the second field we get the following:

$$\begin{aligned}&\Delta\vec{H}_2 - \mu_0\sigma\left(\frac{\text{d}\vec{H}_2}{\text{d}t} - \text{rot}\left(\vec{V}\times\vec{H}_2\right)\right) = \\ &= -\mu_0\sigma\left(\frac{\text{d}\vec{H}_1}{\text{d}t} - \text{rot}\left(\vec{V}\times\vec{H}_1\right)\right)\end{aligned}. \quad (21)$$

While solving the Eq. (21) it is considered that the EMET rotor has a single-degree of freedom and the rotor speed does not depend on the coordinates. When taking an EMET on elastic bearing supports its rotor has five degrees of freedom. It means that this allowance is not appropriate to solution for the EMETs on elastic bearing supports

$$\text{rot}\left(\vec{V}\times\vec{H}_2\right)$$

is given as follows:

$$\begin{aligned}&\text{rot}\left(\vec{V}\times\vec{H}_2\right) = \left(\vec{H}_2\cdot\nabla\right)\vec{V} - \left(\vec{V}\cdot\nabla\right)\vec{H}_2 + \\ &+\vec{V}\text{div}\vec{H}_2 - \vec{H}_2\text{div}\vec{V}\end{aligned}, \quad (22)$$

$$\operatorname{rot}\left(\vec{V}\times\vec{H}_1\right)=\left(\vec{H}_1\cdot\nabla\right)\vec{V}-\left(\vec{V}\cdot\nabla\right)\vec{H}_1+ \\ +\vec{V}\operatorname{div}\vec{H}_1-\vec{H}_1\operatorname{div}\vec{V}, \quad (23)$$

where ∇ is Hamiltonian operator.

As $\operatorname{div}\vec{H}_2=0,\ \operatorname{div}\vec{H}_1=0$, so:

$$\operatorname{rot}\left(\vec{V}\times\vec{H}_2\right)=\left(\vec{H}_2\cdot\nabla\right)\vec{V}- \\ -\left(\vec{V}\cdot\nabla\right)\vec{H}_2-\vec{H}_2\operatorname{div}\vec{V}, \quad (24)$$

$$\operatorname{rot}\left(\vec{V}\times\vec{H}_1\right)=\left(\vec{H}_2\cdot\nabla\right)\vec{V}- \\ -\left(\vec{V}\cdot\nabla\right)\vec{H}_2-\vec{H}_2\operatorname{div}\vec{V}. \quad (25)$$

$$\Delta\vec{H}_2-\mu_0\sigma\left(\frac{\mathrm{d}\vec{H}_2}{\mathrm{d}t}-\left(\left(\vec{H}_2\cdot\nabla\right)\vec{V}- \\ \left(\vec{V}\cdot\nabla\right)\vec{H}_2-\vec{H}_2\operatorname{div}\vec{V}\right)\right)= \\ =-\mu_0\sigma\left(\frac{\mathrm{d}\vec{H}_1}{\mathrm{d}t}- \\ -\left(\left(\vec{H}_1\cdot\nabla\right)\vec{V}-\left(\vec{V}\cdot\nabla\right)\vec{H}_1-\vec{H}_1\operatorname{div}\vec{V}\right)\right). \quad (26)$$

From the vector analyses [22-24] it is known that:

$$\left(\vec{H}_1\cdot\nabla\right)\vec{V}=H_{x1}\frac{\partial\vec{V}}{\partial x}+H_{y1}\frac{\partial\vec{V}}{\partial y}+H_{z1}\frac{\partial\vec{V}}{\partial z}, \quad (27)$$

$$\left(\vec{V}\cdot\nabla\right)\vec{H}=V_x\frac{\partial\vec{H}}{\partial x}+V_y\frac{\partial\vec{H}}{\partial y}+V_z\frac{\partial\vec{H}}{\partial z}. \quad (28)$$

At designing the EMETs on elastic supports it is more reasonable to operate with variables in Cartesian coordinates. This is due to the fact that almost all the NCBS control systems are designed using Cartesian coordinates [21].

In this case taking into account Eq. (27), (28) can be written as follows:

The set of the given formulas is a generalized mathematical description of the high-speed EMET with HCPM on NCBS.

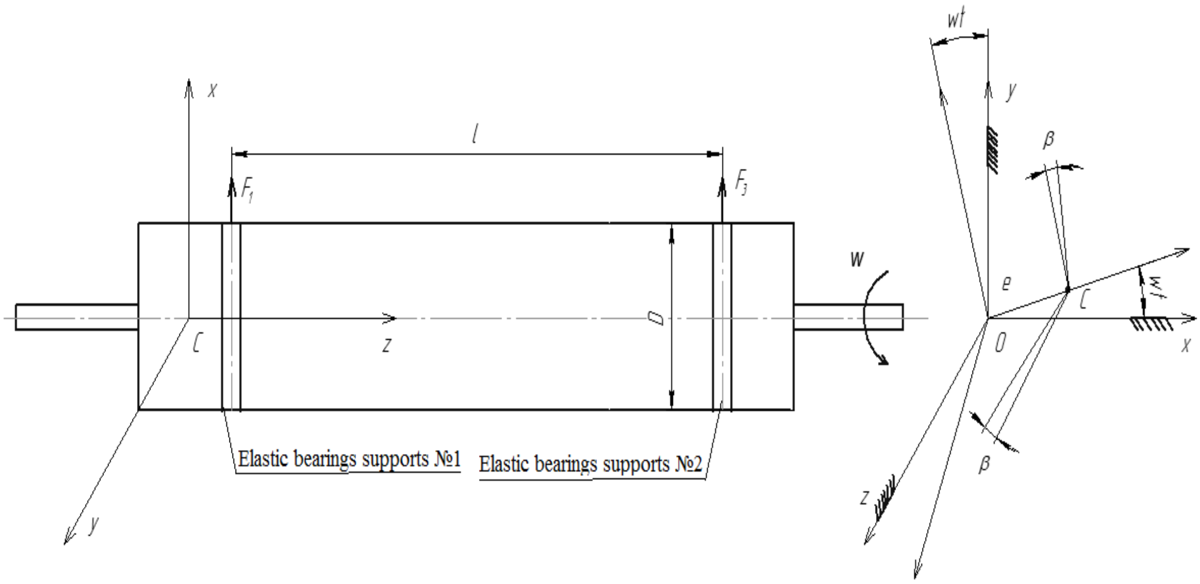

Fig. 1. Structural design of the EMET rotor on the elastic bearing supports

$$\Delta\vec{H}_2 - \mu_0\sigma\left(\frac{d\vec{H}_2}{dt} - \left(-\vec{H}_2 \mathrm{div}V + \left(H_{x2}\frac{\partial\vec{V}}{\partial x} + H_{y2}\frac{\partial\vec{V}}{\partial y} + H_{z2}\frac{\partial\vec{V}}{\partial z}\right) - \left(V_x\frac{\partial\vec{H}_2}{\partial x} + V_y\frac{\partial\vec{H}_2}{\partial y} + V_z\frac{\partial\vec{H}_2}{\partial z}\right)\right)\right) = \\ = -\mu_0\sigma\left(\frac{d\vec{H}_1}{dt} - \left(-\vec{H}_1 \mathrm{div}\vec{V} + \left(H_{x1}\frac{\partial\vec{V}}{\partial x} + H_{y1}\frac{\partial\vec{V}}{\partial y} + H_{z1}\frac{\partial\vec{V}}{\partial z}\right) - \left(V_x\frac{\partial\vec{H}_2}{\partial x} + V_y\frac{\partial\vec{H}_2}{\partial y} + V_z\frac{\partial\vec{H}_2}{\partial z}\right)\right)\right) \tag{29}$$

It is assumed that the rotor is rigidly fixed along the z-axis:

$$\Delta\vec{H}_2 - \mu_0\sigma\left(\frac{d\vec{H}_2}{dt} - \left(-\vec{H}_2 \mathrm{div}V + \left(H_{x2}\frac{\partial\vec{V}}{\partial x} + H_{y2}\frac{\partial\vec{V}}{\partial y}\right) - \left(V_x\frac{\partial\vec{H}_2}{\partial x} + V_y\frac{\partial\vec{H}_2}{\partial y}\right)\right)\right) \\ = -\mu_0\sigma\left(\frac{d\vec{H}_1}{dt} - \left(-\vec{H}_1 \mathrm{div}\vec{V} + \left(H_{x1}\frac{\partial\vec{V}}{\partial x} + H_{y1}\frac{\partial\vec{V}}{\partial y} + H_{z1}\frac{\partial\vec{V}}{\partial z}\right) - \left(V_x\frac{\partial\vec{H}_2}{\partial x} + V_y\frac{\partial\vec{H}_2}{\partial y} + V_z\frac{\partial\vec{H}_2}{\partial z}\right)\right)\right) . \tag{30}$$

IV. Computational Investigations of the Developed Mathematical Model and Its Practical Application.

As labor intensity is high while investigating the action of mechanical processes in the elastic bearing supports on parameters of the EMET with HCPM by analytical methods, so investigations of the proposed mathematical model were carried out in Matlab software package using the numerical method. Herewith a high-speed EMET with HCPM was put at a generator mode (with capacity of 120 kWt, rotor speed 35000 rpm, outlet linear stress of 200V, rotor outer diameter of 65 mm, effective length of 135 mm, stator bore diameter of 74 mm, stator outer diameter of 150 mm). To solve the task put above the developed mathematical model was realized in the form of a simulated computer model, Fig.2.

The developed simulated model consists of several blocks. Fig.3 shows a block describing the parameters of the EMET with HCPM along d, q coordinates.

While modeling the action of mechanical processes in the elastic bearing supports on electromagnetic processes in the EMET the following parameters were considered: discontinuous balance change of the forces influencing the EMET rotor, rotor displacement caused by the change mentioned above with further development of the NCBS control system for rotor homing action.

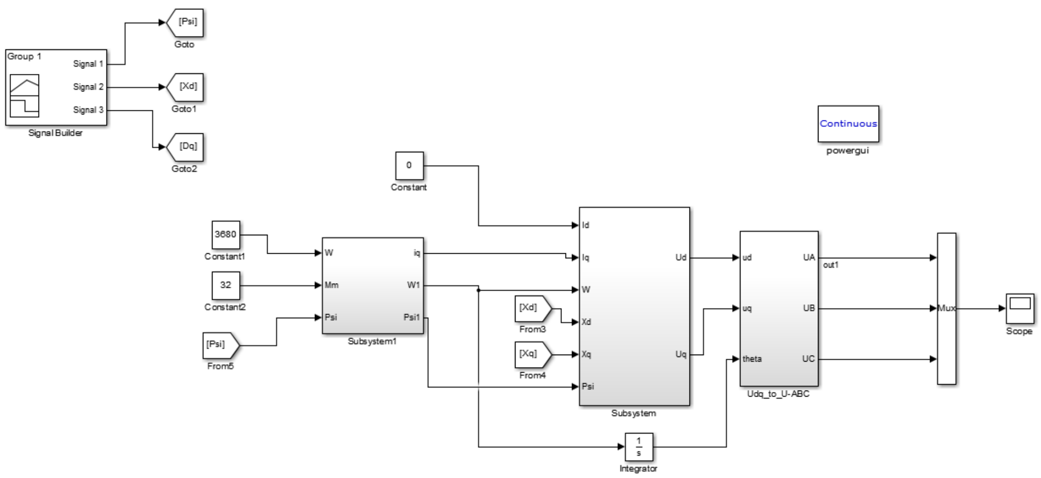

Fig. 2. Structural design of the EMET rotor on the elastic bearing supports Simulated model of the high-speed EMET with HCPM which realizes the developed generalized mathematical model

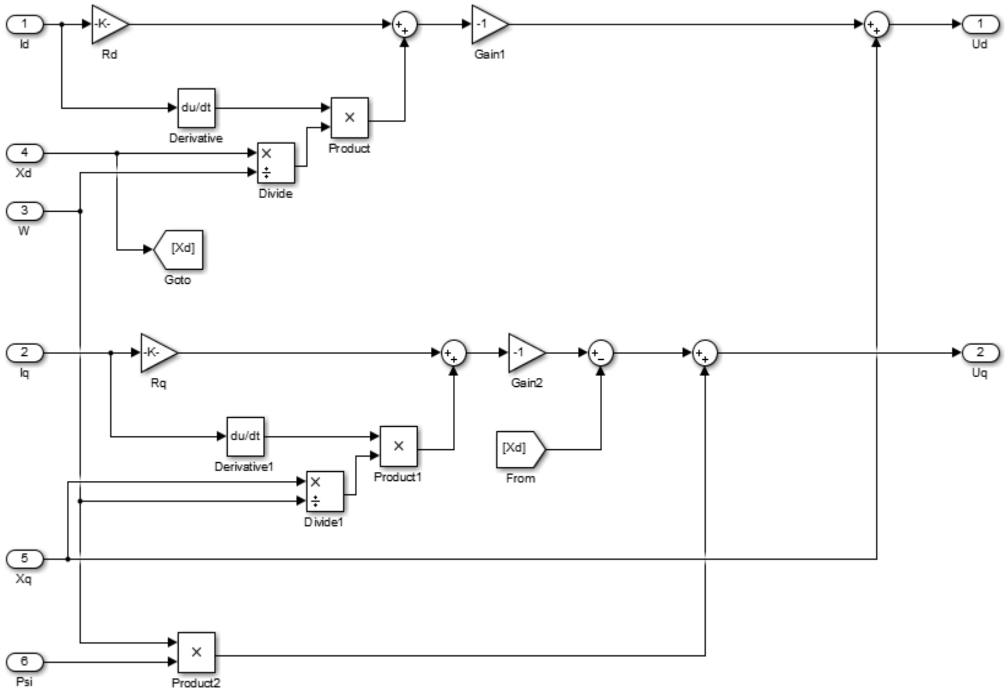

Fig. 3. Simulated model block describing the EMET with HCPM

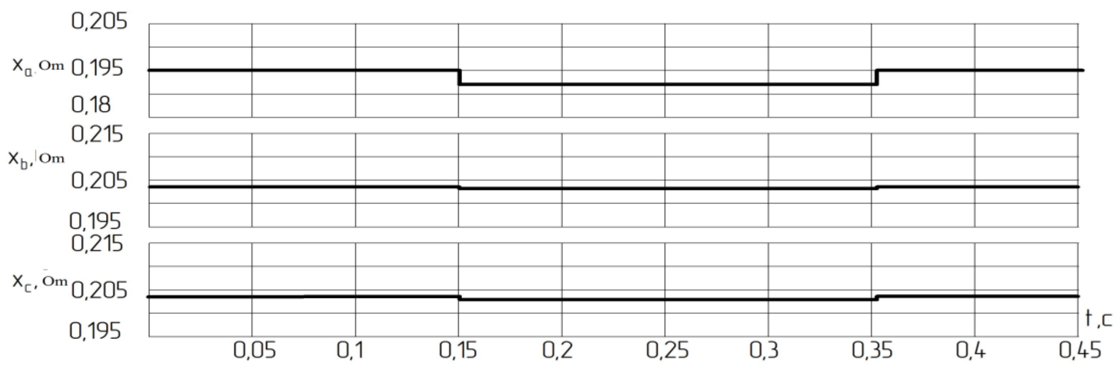

Fig. 4. Results of the simulation modeling process on the inductive reactance change in the EMET with HCPM during rotor displacement over the hybrid magnetic bearing and development of the hybrid magnetic bearing control system for rotor homing action.

As a result of the modeling the following results were obtained for the EMET investigated numerical parameters: when the rotor moves along the NCBS by 10% of the air-gap, so the inductive reactance increases by 0.5-1 % on the side of air-gap increasing. Herewith, the air-gap magnetomotive force changes by 3% on the side of air-gap increasing. These duration changes correspond to the whole time constant of the NCBS rotor position control system. Typical inconsiderable voltage failures appear in the voltage across the terminals of the EMET with HCPM as a result of rotor displacement and development of the NCBS control system for rotor homing action. Rate and duration of these failures are determined by NCBS stiffness and its control system time constant. Meaning that if the NCBS stiffness is rather low it is necessary to calculate the EMET stabilization system considering the NCBS parameters. It allows providing voltage constancy when using a NCBS in the EMET with HCPM. Moreover, the operation speed parameters and the system adjustability parameters must be consistent with the NCBS control system parameters.

Fig. 4 shows the changes of winding inductive reactance when the rotor displaces along the NCBS.

Moreover, the researches on the designed mathematical model have proved that if there is y- and x-axial displacement of the EMET rotor so air-gap conductivity changes. It leads to the magnetic field changes in the EMET air-gap, i.e. at the EMET rotor displacement on MBS some additional harmonic components of the air gap flux density appear in the air-gap:

– harmonics with a number of pairs of the poles $p \pm n$ (where n is 1,2,3...; p is a number of pairs of the poles) and the order $1 \pm p/n$, caused by magnetic conductivity components;

– higher harmonics with a number of pairs of the poles $p \pm n \pm kZ$ and the order $1 \pm n/p \pm kZ/p$ (where k is 1,2,3...; Z is a number of the stator teeth).

These harmonic components of magnetic field in the air-gap will appear in the electromotive force (EMF) and voltage generated by this magnetic field. In this case their presence and absolute value allow determining the EMET rotor displacement on NCBS and controlling this displacement with the help of either electromagnet or a nozzle. Herewith, these harmonic components are measured at the nominal operational mode of the EMET, i.e. when EMET winding temperature reaches its steady mode and its changing can happen only at emergency mode of the EMET operation, e.g. at the short-circuit condition, so the method suggested above is resistant to thermal processes in the EMET. If steady temperature of the EMET winding is 120 0C and winding resistance is 0.003 Ohm so the changing of this temperature by more than 10% means failure of the EMET. And if it changes by less than 10% there is no significant influence on harmonic components as the resistance will change only by 3-5%. Table 1 shows thorough criterial comparison of traditional (sensor) control method, sensorless ones developed by Simens and Ebara Corporation companies and the suggested method.

Table 1. Thorough criterial comparison of traditional (sensor) control method, sensorless ones developed by Simens and Ebara Corporation companies and the suggested method

Comparison criteria	Traditional control methods with a displacement transducer	Popular sensorless control methods on changing impedance or inductors of electromagnets	Developed sensorless control method
Weight-size parameters	High. Sensors reqire additional space for mounting them in the EMET shell	It is possible to decrease the EMET weight-size parameters	It is possible to decrease the EMET weight-size parameters
Price parameters	High. It is caused by sensors price	Low, as there are no rotor position sensors	Low, as there are no rotor position sensors
Integrability into the EMET functional systems	There are some difficulties caused by necessity to modernize and complicate functional test systems	There is difficulty with integrating them into the EMET functional systems as the control system is aimed to determine a rotor position	There are no unique challenges in integration as output currents and voltage can be gauged in functional systems of the EMET
Reliability	Low, which is caused both by sensors and necessity to pull an additional cable to them	There is possibility to increase their reliability	There is possibility to increase their reliability
Energy efficiency	Low, as in some cases sensors require additional power supply	Higher	Higher
Sensitivity to environments	High, as sensors are thermal- and vibro-resistant	Low, as parameters of automatic control system electromagnets depend on both inside temperature of the EMET shell and rotor vibrations	High, as parameters are connected directly with EMET parameters and can be corrected depending on EMET operability
Usage in different structural variations of hybrid magnetic bearings	Usable in all the structural variations	Usable only in variations with electromagnets	Usable in all the structural variations

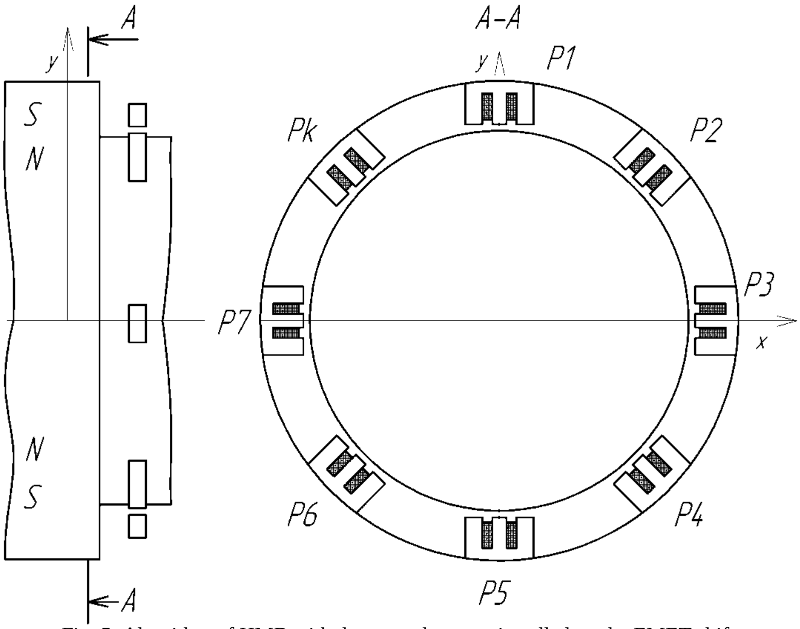

Fig. 5. Algorithm of HMB with the control system installed on the EMET shift

On the basis of the obtained results the authors of the paper have developed a unique algorithm of rotor position control in hybrid magnetic bearings (HMB) which allows avoiding the usage of rotor position sensors at high-speed EMET designing.

Description of the algorithm of HMB with a control system containing 8 electromagnets which are installed at equal distances along the EMET shift is given below in Pic. 5. Rotor motion angle is indicated by an encoder. For formalizing the algorithm the following notations are taken: P1, P2, P3, P4, P5, P6, P7, Pk are the control system electromagnets; ef is the encoder (it is not shown in Pic. 5).

1. At initial start-up of the EMET voltage sensors measure the effective value of EMF at the motion zero angle (under the electromagnet P1). EMF gain is resolved into components and it is possible to find relative deviation of EMF harmonics from the harmonics of the symmetrical mode at zero angle.

2. If the conditions $\Delta E_1 > 2\%$ and/or $\Delta E_3 > 2\%$ and/or $\Delta E_9 > 2\%$ and/or $\Delta E_{43} > 2\%$ are fulfilled then displacement under the electromagnet being analyzed can be found (31):

$$y_{P1} \approx \left[k_{21}\Delta E_3^3 + k_{22}\Delta E_3^2 + k_{23}\Delta E_3 + k_{24} \right]\delta_0 . \qquad (31)$$

3. If ΔE_1, ΔE_3, ΔE_9, ΔE_{43}>0 so through a power amplifier the electromagnet P1 gets a control signal which causes voltage increase in the electromagnet windings and pulling capacity, consequently. The propotional plus derivative control law is taken as a law of rotor position control on HMB. Its implementation is carried out by using a proportional-plus-derivative (PD) regulator:

$$I = \alpha \cdot y(P1) + \beta \cdot \frac{dy(P1)}{dt} . \qquad (32)$$

where α, β are control coefficients depending on the sum of forces which influence the rotor and structural features of HMB. Furthermore, control coefficients α, β must be chosen the way they provide steady rotor motion on HMB.

4. If ΔE_1, ΔE_3, ΔE_9, ΔE_{43}<0 so through a power amplifier the electromagnet P5 gets a control signal analogous the step 3.

5. The encoder ef readings allow determining rotor turning angle, and steps 1–3 are taken for the angles 45, 90, 135, 180, 225, 270 and 315 and set in the line of these angles (P2, P3, P4, P5, P6, P7, P8).

Results and conclusions.

– A generalized mathematical model of a high-speed EMET with HCPM on non-contact bearing supports has been developed and its numerical investigations have been carried out. The mathematical model allows realizing new algorithms of rotor control on different types of non-contact bearing supports providing by this higher speed. It has been shown that typical inconsiderable voltage failures appear in the voltage across the terminals of the EMET with HCPM as a result of rotor displacement and development of the NCBS control system for rotor homing action. Rate and duration of these failures are determined by NCBS stiffness and its control system time constant. That is their presence and absolute value allow determining the EMET rotor displacement on NCBS and controlling this displacement with the help of either an electromagnet or a nozzle. On the basis of the results obtained the authors have developed and investigated a new method of sensorless control of the rotor position on non-contact bearing supports. This means has a number of advantages over both famous sensorless control methods (developed by

Simens and Ebara Corparation) and traditional methods. So it has a chance of practical implementation.

– It has been established that for a high-speed EMET with HCPM on NCBS operating at a generator mode (with capacity of 120 kWt, rotor speed 35000 rpm, outlet linear stress of 200V, rotor outer diameter of 65 mm, effective length of 135 mm, stator bore diameter of 74 mm, stator outer diameter of 150 mm) when the rotor moves along the NCBS by 10% of the air-gap, so the inductive reactance increases by 0.5-1 % on the side of air-gap increasing. Herewith, the air-gap magnetomotive force changes by 3% on the side of air-gap increasing. These duration changes correspond to the whole time constant of the NCBS rotor position control system.

– It has been shown that typical inconsiderable voltage failures appear in the voltage across the terminals of the EMET with HCPM as a result of rotor displacement and development of the NCBS control system for rotor homing action. Rate and duration of these failures are determined by NCBS stiffness and its control system time constant.

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