Finite-Time Output Feedback Stabilization for a Class of Uncertain High Order Nonholonomic Systems

Yanling Shang, Deheng Hou and Fangzheng Gao

***Abstract*—In this paper, the problem of finite-time stabilization by output feedback is investigated for a class of high order nonholonomic systems in power chained form with uncertainties. By skilly using finite-time stability theorem and homogeneous domination approach, a constructive design procedure for output feedback control is given. Together with a novel switching control strategy, the designed controller renders that the states of closed-loop system are regulated to zero in a finite time. A simulation example is provided to illustrate the effectiveness of the proposed approach.**

***Index Terms*—high order nonholonomic systems, output feedback, homogeneous domination approach, sign function, finite-time stabilization.**

I. Introduction

IN the past decade, nonholonomic systems have attracted much attention because they can be used to model many real systems, such as mobile robots, car-like vehicle and under-actuated satellites, see, e.g., [1-3] and the references therein. However, due to the limitation imposed by Brockett's necessary condition [4], this class of nonlinear systems cannot be stabilized by stationary continuous state feedback, although it is controllable. As a consequence, the well-developed smooth nonlinear control theory and methodology cannot be directly used to such systems. To overcome this obstruction, with the effort of many researchers a number of intelligent approaches have been proposed,with the effort of many researchers a number of intelligent approaches have been proposed, which mainly include into discontinuous feedback [5,6], time-varying feedback [7,8] and hybrid stabilization [9]. Using there valid approaches, the robustness issue of nonholonomic systems with drift uncertainties has been extensively studied [10-15]. In particular, as the extension of the classical nonholonomic systems, the high order nonholonomic systems in power chained form have been recently achieved investigation [16-19]. Nevertheless, it should be noted that most of the existing works only consider the feedback stabilizer that makes the trajectories of the systems converge to the equilibrium as the time goes to infinity.

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Yanling Shang and Deheng Hou are with School of Software Engineering, Anyang Normal University, Anyang 455000, P. R. China hnnhsyl@126.com

Fangzheng Gao is with School of Mathematics and Statistics, Anyang Normal University, Anyang 455000, P. R. China gaofz@126.com

Compared to the asymptotic stabilization or exponential regulation, the finite-time stabilization, which renders the trajectories of the closed-loop systems convergent to the origin in a finite time, has many advantages such as fast response, high tracking precision, and disturbance-rejection properties [20]. Hence it is more meaningful to investigate the finite-time stabilization problem than the classical asymptotic stabilization. In recent years, the problem of finite-time stabilization of nonholonomic systems has been studied and some interesting results have been obtained [21-27]. Particularly, in the case when only parts of the states are measurable, [28] studied the finite-time stabilization by output feedback for a class of nonholonomic systems with nonlinearities. However, due to some intrinsic features of high order nonholonomic systems, such as its Jacobian linearization being neither controllable nor feedback linearizable, lead to the existing finite-time control methods highly difficult to this kind of systems or even inapplicable. To the best of the authors knowledge, there is no result referred to the finite-time stabilization of high order nonholonomic systems by output feedback.

Motivated the above discussion, by introducing a combined homogeneous domination and sign function approach, and overcoming some essential difficulties such as the weaker assumption on the system growth, the appearance of the sign function and the construction of a continuously differentiable, positive-definite and proper Lyapunov function, in this paper we will focus on providing a solution to the problem of finite-time output feedback stabilization for nonholonomic systems with uncertainties.

The rest of this paper is organized as follows. Section 2 introduces some preliminaries and formulates the control problem. Section 3 presents the control design procedure and the main results. Section 4 gives a simulation example to illustrate the theoretical finding of this paper. Finally, concluding remarks are proposed in Section 5.

II. Preliminaries and problem formulation

A. Preliminaries

The following preliminaries are to be used throughout the paper.

Notations. Throughout this paper, the following notations are adopted. R^+ denotes the set of all nonnegative real numbers and R^n denotes the real n-dimensional space. For any vector $x = (x_1, \cdots, x_n)^T \in R^n$ denotes $|x| = (\sum_{i=1}^n x_i^2)^{\frac{1}{2}}$. K denotes the set of all functions: $R^+ \to R^+$, which are continuous, strictly increasing and vanishing at zero; K_∞ denotes the set of all functions which are of class K and unbounded. A sign function $sign(x)$ is defined as follows: $sign(x) = 1$, if $x > 0$; $sign(x) = 0$, if $x = 0$ and $sign(x) = -1$, if $x < 0$. For any $a \in R^+$ and $x \in R$, the function $[x]^a$ is defined as $[x]^a = sign(x)|x|^a$. Besides, the arguments of the functions will be omitted or simplified, whenever no confusion can arise from the context. For instance, we sometimes denote a function $f(x(t))$ by simply $f(x)$, $f(\cdot)$ or f.

Definition 1. [29] Weighted Homogeneity: For fixed coordinates $(x_1, \cdots, x_n) \in R^n$ and real numbers $r_i > 0, i = 1, \cdots, n$.

- the dilation $\Delta_\varepsilon(x)$ is defined by $\Delta_\varepsilon(x) = (\varepsilon^{r_1}x_1, \cdots, \varepsilon^{r_n}x_n)$ for any $\varepsilon > 0$, where r_i is called the weights of the coordinates. For simplicity, we define dilation weight $\Delta = (r_1, \cdots, r_n)$.
- a function $V \in (R^n, R)$ is said to be homogeneous of degree τ if there is a real number $\tau \in R$ such that $V(\Delta_\varepsilon(x)) = \varepsilon^\tau V(x_1, \cdots, x_n)$ for any $x \in R^n \setminus \{0\}, \varepsilon > 0$.
- a vector field $f \in (R^n, R^n)$ is said to be homogeneous of degree τ if there is a real number $\tau \in R$ such that $f_i(\Delta_\varepsilon(x)) = \varepsilon^{\tau + r_i} f_i(x)$, for any $x \in R^n \setminus \{0\}, \varepsilon > 0$, $i = 1, \cdots, n$.
- a homogeneous p-norm is defined as $\|x\|_{\triangle,p} = (\sum_{i=1}^n |x_i|^{p/r_i})^{1/p}$ for all $x \in R^n$, for a constant $p \geq 1$. For simplicity, in this paper, we choose $p = 2$ and write $\|x\|_\triangle$ for $\|x\|_{\triangle,2}$.

Lemma 1. [29] Given a dilation weight $\Delta = (r_1, \cdots, r_n)$, suppose $V_1(x)$ and $V_2(x)$ are homogeneous functions of degree τ_1 and τ_2, respectively. Then $V_1(x)V_2(x)$ is also homogeneous with respect to the same dilation weight Δ. Moreover, the homogeneous degree of $V_1(x)V_2(x)$ is $\tau_1 + \tau_2$.

Lemma 2. [29] Suppose $V : R^n \to R$ is a homogeneous function of degree τ with respect to the dilation weight Δ. Then the following holds:

(i) $\partial V / \partial x_i$ is homogeneous of degree $\tau - r_i$ with r_i being the homogeneous weight of x_i.

(ii) There is a constant c such that $V(x) \leq c\|x\|_\triangle^\tau$. Moreover, if $V(x)$ is positive definite, then $\underline{c}\|x\|_\triangle^\tau \leq V(x)$, where $\underline{c}$ is a constant.

In the remainder of this section, we present the following lemmas which play an important role in the design process.

Lemma 3. [25] Consider the nonlinear system

$$\dot{x} = f(x,t) \ \ with\ f(0,t) = 0, \quad x \in R^n, \tag{1}$$

where $f : U_0 \times R^+ \to R^n$ is continuous with respect to x on an open neighborhood U_0 of the origin $x = 0$. Suppose there is a C^1 function $V(x,t)$ ($V(x,t) = 0$ if and only if $x = 0$) defined $\hat{U} \in R^n \times R$, where $\hat{U} \in U_0 \in R^n$ is a neighborhood of the origin, real numbers $c > 0$ and $0 < \alpha < 1$, such that (i)$V(x,t)$ is positive definite on $\hat{U}$; (ii)$\dot{V}(x,t) + cV^\alpha(x,t) \leq 0, \quad \forall x \in \hat{U}$. Then, the origin of (1) is finite-time stable with $T \leq \frac{V^{1-\alpha}(x(t_0),t_0)}{c(1-\alpha)}$ for initial condition $x(t_0)$ in some open neighborhood $U \in \hat{U}$ of the origin at initial time t_0. If $U = R^n$ and $V(x,t) \to +\infty$ as $|x| \to \infty$, the origin of system (1) is globally finite-time stable.

Lemma 4. [30] For $x \in R$, $y \in R$, $p \geq 1$ and $c > 0$ are constants, the following inequalities hold: (i) $|x+y|^p \leq 2^{p-1}|x^p+y^p|$, (ii) $(|x|+|y|)^{1/p} \leq |x|^{1/p}+|y|^{1/p} \leq 2^{(p-1)/p}(|x|+|y|)^{1/p}$, (iii) $||x|-|y||^p \leq ||x|^p - |y|^p|$, (iv) $|x|^p + |y|^p \leq (|x|+|y|)^p$, (v) $|[x]^{1/p} - [y]^{1/p}| \leq 2^{1-1/p}|x-y|^{1/p}$, (vi) $|[x]^p - [y]^p| \leq c|x-y||(x-y)^{p-1} + y^{p-1}|$.

Lemma 5. [30] Let x, y be real variables, then for any positive real numbers a, m and n, one has

$$\begin{aligned} a|x|^m|y|^n \leq{}& b|x|^{m+n} \\ &+\frac{n}{m+n}\Big(\frac{m+n}{m}\Big)^{-\frac{m}{n}} a^{\frac{m+n}{n}} b^{-\frac{m}{n}} |y|^{m+n} \end{aligned}$$

where $b > 0$ is any real number.

B. Problem formulation

In this paper, we consider the following high order nonholonomic systems:

$$\begin{array}{l} \dot{x}_0 = u_0^{p_0} + \phi_0^d(t, x_0) \\ \dot{x}_i = x_{i+1}^{p_i} u_0^{q_i} + \phi_i^d(t, x_0, x, u_0), \quad i = 1, \cdots, n-1 \\ \dot{x}_n = u_1^{p_n} + \phi_n^d(t, x_0, x, u_0) \\ y = (x_0, x_1)^T \end{array} \tag{2}$$

where $(x_0, x)^T = (x_0, x_1, \cdots, x_n)^T \in R^{n+1}$, $u = (u_0, u_1)^T \in R^2$, $y \in R^2$ are the system state, control input and system output, respectively; $p_i \geq 1$, $i = 0, 1, \cdots, n$ are odd integers; q_k, $k = 1, \cdots, n-1$ are integers,; and ϕ_i^d's are unknown continuous functions, referred as input and state-driven uncertainties.

The objective of this paper is to design an output feedback controller in the form:

$$\dot{\hat{x}} = \vartheta(\hat{x}, y), \quad u_0 = u_0(x_0), \quad u_1 = u_1(\hat{x}, y) \tag{3}$$

such that the finite-time regulation of the states are achieved; i.e., $\lim\limits_{t \to T}(|x_0(t)| + |x(t)|) = 0$ and $(x_0(t), x(t)) = (0, 0)$ for any $t \geq T$, where T is a finite settling time.

To this end, the following assumption regarding system (2) is imposed.

Assumption 1. For $i = 0, 1, \cdots, n$, there is constants $a, b > 0$ and $\tau \in (-\frac{1}{\sum_{l=1}^n p_1 \cdots p_{l-1}}, 0)$

$$|\phi_0(t, x_0)| \leq a|x_0|$$

$$|\phi_i(t, x_0, x, u_0)| \leq b(|x_1|^{(r_i+\tau)/r_1} + \cdots + |x_i|^{(r_i+\tau)/r_i}),$$

where $r_1 = 1$, $r_{i+1} = \frac{r_i + \tau}{p_i} > 0$, $i = 1, \cdots, n$ and $\sum_{l=1}^n p_1 \cdots p_{l-1} = 1$ for the case of $l = 1$.

Remark 1. It is worth pointing out that Assumption 1 encompasses the assumption in the closely related paper [28]. Specifically, when $p_i = 1$ and τ is some ratios of odd integers, it becomes the condition used in [28]. Therefore, an interesting problem is how to design a finite-time output feedback controller for high order nonholonomic system (2) under the weaker assumption of τ and r_i being arbitrary real numbers in some interval. In this paper, we will ingeniously combine homogeneous domination theory and sign function approach to solve this problem.

III. Finite-time output feedback controller design

In this section, we give a constructive procedure for the finite-time stabilizer of system (2) by output feedback. The design of finite-time output feedback controller is divided into the following two steps:

- We first stabilize the x-subsystem in a finite time by output feedback.
- Then we design a controller such that the x_0-subsystem is finite-time stable.

A. Finite-time output feedback stabilization of the x-subsystem

For the x_0-subsystem, we choose the control u_0 as

$$u_0 \equiv u_0^* \tag{4}$$

where u_0^* is a positive constant. In this case, the x_0-subsystem becomes

$$\dot{x}_0 = u_0^{*p_0} + \phi_0(t, x_0) \tag{5}$$

Noting that $\phi_0(t, x_0)$ satisfies the linear growth condition, it is easy to obtain that the solution of x_0-subsystem is bounded, for any given finite time $t_s > 0$. Hence, x_0 is well-defined on $[0, t_s]$. Under the control law (5), the x-subsystem can be written as

$$\begin{array}{l} \dot{x}_i = x_{i+1}^{p_i} u_0^{*q_i} + \phi_i^d(t, x_0, x, u_0), \quad i = 1, \cdots, n-1 \\ \dot{x}_n = u_1^{p_n} + \phi_n^d(t, x_0, x, u_0) \end{array} \tag{6}$$

Next we consider the finite-time output feedback stabilize for system (6). For convenience, we introduce an equivalent coordinates transformation:

$$\begin{array}{l} z_1 = x_1, \ \ z_i = \dfrac{\tilde{d}_i x_i}{L^{\kappa_i}}, \ \ i = 2, \cdots, n \\ \upsilon^{p_n} = \dfrac{u_1^{p_n}}{L^{\kappa_n + 1}} \end{array} \tag{7}$$

where $\tilde{d}_i = \prod_{j=1}^{i-1} u_0^{\frac{q_j}{p_j \cdots p_{i-1}}}$, $q_n = 0$, $\kappa_1 = 0$, $\kappa_{i+1} = \frac{\kappa_i + 1}{p_i}$, $i = 1, \cdots, n-1$ and $L > 1$ is a constant to be determined. Then, under (7), system (6) is transformed into:

$$\begin{array}{l} \dot{z}_i = L z_{i+1}^{p_i} + \dfrac{f_i}{L^{\kappa_i}}, \quad i = 1, \cdots, n-1 \\ \dot{z}_n = L \tilde{d}_n \upsilon^{p_n} + \dfrac{f_n}{L^{\kappa_n}} \end{array} \tag{8}$$

where $f_i = \tilde{d}_i \phi_i$ and the state $z_1 = x_1$ is measurable.

1) Homogeneous output feedback control of the nominal system : In this subsection, we will construct an output feedback stabilizer for the following nominal system

$$\begin{array}{l} \dot{z}_i = L z_{i+1}^{p_i}, \quad i = 1, \cdots, n-1 \\ \dot{z}_n = L \tilde{d}_n \upsilon^{p_n} \end{array} \tag{9}$$

The design of output feedback controller is divided into two steps. In Step A, we suppose that all the states are measurable, and develop a recursive design method to explicitly construct a state feedback control law for system (9). Then in step B, by constructing a nonsmooth reduced-order observer, we design an output feedback controller.

A. State feedback controller design

Step 1. Let $\xi_1 = [z_1]^{1/r_1}$ and choose the Lyapunov function

$$V_1 = W_1 = \int_{z_1^*}^{z_1} \Big[[s]^{1/r_1} - [z_1^*]^{1/r_1} \Big]^{2-\tau-r_1} ds \tag{10}$$

with $z_1^* = 0$. From (9), it follows that

$$\dot{V}_1 \leq -nL\xi_1^2 + L[\xi_1]^{2-\tau-r_1}(z_2^{p_1} - z_2^{*p_1}) \tag{11}$$

where the virtual controller is chosen as

$$z_2^* = -n^{1/p_1}[\xi_1]^{(r_1+\tau)/p_1} := -\beta_1^{r_2}[\xi_1]^{r_2} \tag{12}$$

Step i $(i = 2, \cdots, n)$**.** In this step, we can obtain the following property, whose similar proof can be found in [27] and hence is omitted here.

Proposition 1. Assume that at step $i-1$, there is a continuously differentiable, positive-definite and proper Lyapunov function V_{i-1}, and a set of virtual controllers $z_1^*, \cdots, z_i^*$ defined by

$$\begin{array}{lllllll} z_1^* & = & 0, & & \xi_1 & = & [z_1]^{1/r_1} - [z_1^*]^{1/r_1} \\ z_2^* & = & -\beta_1^{r_2}[\xi_1]^{r_2}, & & \xi_2 & = & [z_2]^{1/r_2} - [z_2^*]^{1/r_2} \\ \vdots & & & & \vdots & & \\ z_i^* & = & -\beta_{i-1}^{r_i}[\xi_{i-1}]^{r_i}, & & \xi_i & = & [z_i]^{1/r_i} - [z_i^*]^{1/r_i} \end{array} \tag{13}$$

with constants $\beta_1 > 0, \cdots, \beta_{i-1} > 0$ such that

$$\begin{array}{r} \dot{V}_{i-1} \leq -(n-i+2)L\displaystyle\sum_{j=1}^{i-1} \xi_j^2 \\ +[\xi_{i-1}]^{(2\sigma-\tau-r_{i-1})}(z_i^{p_{i-1}} - z_i^{*p_{i-1}}) \end{array} \tag{14}$$

Then the ith Lyapunov function defined by

$$V_i = V_{i-1} + \int_{z_i^*}^{z_i} \Big[[s]^{1/r_i} - [z_i^*]^{1/r_i} \Big]^{2-\tau-r_i} ds \tag{15}$$

is continuously differentiable, positive-definite and proper, and there is $z_{i+1}^* = -\beta_i^{r_{i+1}}[\xi_i]^{r_{i+1}}$ such that

$$\dot{V}_i \leq -(n-i+1)L\sum_{j=1}^{i} \xi_j^2 + L[\xi_i]^{2-\tau-r_i}(z_{i+1}^{p_i} - z_{i+1}^{*p_i}) \tag{16}$$

Hence at step n, choosing

$$\begin{array}{l} V_n = \displaystyle\sum_{i=1}^{n} \int_{z_i^*}^{z_i} \Big[[s]^{1/r_i} - [z_i^*]^{1/r_i} \Big]^{2-\tau-r_i} ds \\ z_{n+1}^* = -\beta_n^{r_{n+1}}[\xi_n]^{r_{n+1}} = -\Big[\displaystyle\sum_{i=1}^{n} \bar{\beta}_i [z_i]^{1/r_i} \Big]^{r_{n+1}} \end{array} \tag{17}$$

with $\bar{\beta}_i = \beta_n \cdots \beta_i$, from Proposition 1, we arrive at

$$\dot{V}_n \leq -L\tilde{d}_n \sum_{j=1}^{n} \xi_j^2 + L[\xi_n]^{2-\tau-r_n}(\upsilon^{p_n} - z_{n+1}^{*p_n}) \tag{18}$$

B. Output feedback controller design

Since $z_2, \cdots, z_n$ are unmeasurable, we construct a homogeneous observer

$$\begin{array}{l} \dot{\eta}_i = -Ll_{i-1}\hat{z}_i^{p_{i-1}}, \quad \hat{z}_i = [\eta_i + l_{i-1}\hat{z}_{i-1}]^{r_i/r_{i-1}} \\ \quad i = 2, \cdots, n \end{array} \tag{19}$$

where $\hat{z}_1 = z_1$ and $l_s > 0$; $s = 1, \cdots, n-1$ are the gains to be determined. By the certainty equivalence principle, we can replace z_i with $\hat{z}_i$ in (17) and obtain an output feedback controller

$$v(\hat{z}) = -\Big[\sum_{i=1}^{n} \bar{\beta}_i [\hat{z}_i]^{1/r_i} \Big]^{r_{n+1}} \tag{20}$$

where $\hat{z} = (\hat{z}_1, \hat{z}_2, \cdots, \hat{z}_n)$ and $\hat{z}_1 = z_1$. Considering

$$U_i = \int_{[\gamma_i]^{(2-\tau-r_{i-1})/r_{i-1}}}^{[z_i]^{(2-\tau-r_{i-1})/r_i}} ([s]^{r_{i-1}/(2-\tau-r_{i-1})} - \gamma_i) ds \tag{21}$$

where $\gamma_i = \eta_i + l_{i-1}z_{i-1}$ and setting the observation error $e_i = [z_i^{p_{i-1}} - \hat{z}_i^{p_{i-1}}]^{1/(r_i p_{i-1})}$, for $i = 2, \cdots, n$, from (9),

(19) and (21), it follows that

$$\begin{aligned}\dot{U}_i &= L\frac{\partial U_i}{\partial z_i}z_{i+1}^{p_i} + L\frac{\partial U_i}{\partial z_{i-1}}z_i^{p_{i-1}} - L\frac{\partial U_i}{\partial \eta_i}l_{i-1}\hat{z}_i^{p_{i-1}} \\ &= \frac{2-\tau-r_{i-1}}{r_i}L|z_i|^{(2-\tau-r_{i-1}-r_i)/r_i} \\ &\quad\times([z_i]^{r_{i-1}/r_i}-\gamma_i)z_{i+1}^{p_i} - Ll_{i-1}(z_i^{p_{i-1}}-\hat{z}_i^{p_{i-1}}) \\ &\quad\times\Big([z_i]^{(2-\tau-r_{i-1})/r_i}-[\hat{z}_i]^{(2-\tau-r_{i-1})/r_i}\Big) \\ &\quad -Ll_{i-1}(z_i^{p_{i-1}}-\hat{z}_i^{p_{i-1}}) \\ &\quad\times\Big([\hat{z}_i]^{(2-\tau-r_{i-1})/r_i}-[\gamma_i]^{(2-\tau-r_{i-1})/r_{i-1}}\Big)\end{aligned} \tag{22}$$

where $z_{n+1} = \upsilon(\hat{z})$.

Each term on the right-hand side of (22) can be estimated by the following propositions whose proofs are given in Appendix.

Proposition 2. There exists a positive constant λ_i such that

$$\begin{aligned}&-l_{i-1}(z_i^{p_{i-1}}-\hat{z}_i^{p_{i-1}}) \\ &\quad\times\Big([z_i]^{(2-\tau-r_{i-1})/r_i}-[\hat{z}_i]^{(2-\tau-r_{i-1})/r_i}\Big) \\ &\le -l_{i-1}\lambda_i e_i^2\end{aligned} \tag{23}$$

Proposition 3. For $i = 2,\cdots,n-1$,

$$\begin{aligned}&\frac{2-\tau-r_{i-1}}{r_i}|z_i|^{(2-\tau-r_{i-1}-r_i)/r_i}([z_i]^{r_{i-1}/r_i}-\gamma_i)z_{i+1}^{p_i} \\ &\le \frac{1}{12}\sum_{j=i-1}^{i+1}\xi_j^2+\alpha_i e_i^2+g_i(l_{i-1})e_{i-1}^2\end{aligned} \tag{24}$$

where g_i is a continuous function of l_{i-1}, $\alpha_i > 0$ is a constant, and $g_2 = 0$.

Proposition 4. For the controller $\upsilon(\hat{z})$, we obtain

$$\begin{aligned}&\frac{2-\tau-r_{n-1}}{r_n}|z_n|^{(2-\tau-r_{n-1}-r_n)/r_n} \\ &\quad\times([z_n]^{r_{n-1}/r_n}-\gamma_n)\upsilon^{p_n} \\ &\le \frac{1}{8}\sum_{j=1}^{n}\xi_j^2+\bar{\alpha}\sum_{i=2}^{n}e_i^2+g_n(l_{n-1})e_{n-1}^2\end{aligned} \tag{25}$$

where g_n is a continuous function of l_{n-1}, $\bar{\alpha} > 0$ is a constant.

Proposition 5. For $i = 3,\cdots,n$,

$$\begin{aligned}&-l_{i-1}(z_i^{p_{i-1}}-\hat{z}_i^{p_{i-1}}) \\ &\quad\times\Big([\hat{z}_i]^{(2-\tau-r_{i-1})/r_i}-[\gamma_i]^{(2-\tau-r_{i-1})/r_{i-1}}\Big) \\ &\le \frac{1}{16}(\xi_{i-1}^2+\xi_i^2)+e_i^2+\theta_i(l_{i-1})e_{i-1}^2\end{aligned} \tag{26}$$

where θ_i is a continuous function of l_{i-1}.

Choosing $U = \sum_{i=2}^{n}U_i$, by Propositions 2-5, we get

$$\begin{aligned}\dot{U} &= \frac{L}{2}\sum_{i=1}^{n}\xi_i^2 \\ &\quad+L\Big(-l_1\lambda_2+\alpha_2+\bar{\alpha}+g_3(l_2)+\theta_3(l_2)\Big)e_2^2 \\ &\quad+\sum_{i=3}^{n-1}\Big(-l_{i-1}\lambda_i+\alpha_i+1+\bar{\alpha}+g_{i+1}(l_i) \\ &\quad+\theta_{i+1}(l_i)\Big)e_i^2+(-l_{n-1}\lambda_n+1+\bar{\alpha})e_n^2\end{aligned} \tag{27}$$

By (17) and (20), we can estimate $\tilde{d}_n[\xi_n]^{(2-\tau-r_n)/\sigma}(\upsilon^{p_n} - z_{n+1}^{*p_n})$ in (18) by the following proposition, whose proof is given in Appendix.

Proposition 6. There exists a positive constant $\tilde{\alpha}$ such that

$$\tilde{d}_n[\xi_n]^{2-\tau-r_n}(\upsilon^{p_n}-z_{n+1}^{*p_n}) \le \frac{1}{4}\sum_{i=1}^{n}\xi_i^2+\tilde{\alpha}\sum_{i=2}^{n}e_i^2 \tag{28}$$

With the help of Proposition 6, defining $\Psi = V_n + U$, combining (18) and (27), and recursively choosing

$$\begin{aligned}&l_{n-1} \ge \lambda_n^{-1}\Big(\frac{1}{4}+1+\bar{\alpha}+\tilde{\alpha}\Big) \\ &l_{i-1} \ge \lambda_i^{-1}\Big(\frac{1}{4}+\alpha_i+1+\bar{\alpha}+\tilde{\alpha}+g_{i+1}(l_i)+\theta_{i+1}(l_i)\Big) \\ &\qquad i = n-1,\cdots,3 \\ &l_1 \ge \lambda_2^{-1}\Big(\frac{1}{4}+\alpha_2+\bar{\alpha}+\tilde{\alpha}+g_3(l_2)+\theta_3(l_2)\Big)\end{aligned} \tag{29}$$

we obtain

$$\dot{\Psi} = -\frac{L}{4}\sum_{i=1}^{n}\xi_i^2-\frac{L}{4}\sum_{i=2}^{n}e_i^2 \tag{30}$$

2) Homogeneous output feedback control of system (8) : Noting that from the construction of Ψ, it can be verified that Ψ is positive definite and proper with respect to $Z = (z_1,\cdots,z_n,\eta_2,\cdots,\eta_n)^T$. Denoting the dilation weight

$$\Delta = (\underbrace{r_1,\cdots,r_n}_{for\ z_1,\cdots,z_n},\underbrace{r_1,\cdots,r_{n-1}}_{for\ \eta_2\cdots,\eta_n}) \tag{31}$$

the closed-loop system can be rewritten as

$$\dot{Z} = LE(Z)+F(Z) \tag{32}$$

where $E(Z) = (z_2^{p_1},\cdots,z_n^{p_{n-1}},\upsilon^{p_n},\dot{\eta}_2\cdots,\dot{\eta}_n)^T$ and $F(Z) = (f_1,\frac{f_2}{L^{\kappa_2}},\cdots,\frac{f_n}{L^{\kappa_n}},0,\cdots,0)^T$. Furthermore, from Definition 1, it can be shown that $\Psi(Z)$ and $E(Z)$ are homogeneous of degree $2-\tau$ and τ with respect to Δ. By Lemmas 1 and 2, there is constants c_1, c_2 and c_3, such that

$$c_1||Z(t)||_\Delta^{2-\tau} \le \Psi(Z) \le c_2||Z(t)||_\Delta^{2-\tau} \tag{33}$$

$$\frac{\partial\Psi(Z)}{\partial Z}LE(Z) \le -c_3L||Z(t)||_\Delta^2 \tag{34}$$

By (7), Assumption 1 and $L > 1$, we can find constants $\delta_i > 0$ and $0 < \nu_i \le 1$ such that

$$\begin{aligned}\Big|\frac{f_i(\cdot)}{L^{\kappa_i}}\Big| &\le \frac{bu_0^{*q_1}\cdots u_0^{*q_i}}{L^{\kappa_i}}\sum_{j=1}^{i}|x_j(t)|^{(r_i+\tau)/r_j} \\ &\le \delta_i||Z(t)||_\Delta^{r_i+\tau}\end{aligned} \tag{35}$$

Noting that for $i = 1,\cdots,n$, $\partial\Psi(Z)/\partial Z_i$ is homogeneous of degree $2-\tau-r_i$, from Lemma 5, we obtain

$$\begin{aligned}\Big|\frac{\partial\Psi(Z)}{\partial Z}F(Z)\Big| &\le \sum_{i=1}^{n}\Big|\frac{\partial\Psi(Z)}{\partial Z_i}\Big|\Big|\frac{f_i(\cdot)}{L^{\kappa_i}}\Big| \\ &\le \rho_1||Z(t)||_\Delta^2\end{aligned} \tag{36}$$

where ρ_1 is a positive constant.

According to (30), (33), (34) and (36), we get

$$\dot{\Psi} \le -(c_3L-\rho_1)||Z(t)||_\Delta^2 \le -\frac{(c_3L-\rho_1)}{c_1^{2/(2-\tau)}}T^{2/(2-\tau)} \tag{37}$$

Hence, by choosing $L > max\{\rho_1/c_3, 1\}$ there exists a constant $\bar{c}_3$ such that

$$\dot{\Psi} \le -\bar{c}_3T^{2/(2-\tau)} \tag{38}$$

By Lemma 1, (38) leads to the conclusion that the closed-loop system (8), (19) and (20) is globally finite-time stable, which yields that system (6) can be globally finite-time stabilized by the output feedback. In addition, the settling time T_1 satisfying

$$T_1 \leq \frac{-(2-\tau)\Psi^{(-\tau)/(2-\tau)}(0)}{\bar{c}_3\tau} \tag{39}$$

B. Finite-time output feedback stabilization of the x_0-subsystem

From Section A, we know that $x(t) \equiv 0$ when $t \geq T_1$. Therefore, we just need to stabilize the x_0-subsystem in a finite time. When $t \geq T_1$, for the x_0-subsystem, we can take the following control law

$$u_0^{po} = -(k_0 + \phi_0(x_0))[x_0]^{\alpha_0}, \quad 0 < \alpha_0 < 1 \tag{40}$$

where α_0, k_0 are positive constants and $\phi_0(x_0) \geq a|x_0|^{1-\alpha_0} \geq 0$ is a smooth function. For instance, we can simply choose $\phi_0(x_0) = a(1 + x_0^2)$.

Taking the Lyapunov function $V_0 = x_0^2/2$, a simple computation gives

$$\dot{V}_0 \leq -k_0 x_0^{1+\alpha_0} \leq -k_0 V_0^{(1+\alpha_0)/2} \tag{41}$$

Thus by Lemma 1, x_0 tends to 0 within a settling time denoted by T_2 and

$$T_2 \leq \frac{2V_0^{(1-\alpha_0)/2}(0)}{k_0(1-\alpha_0)} \tag{42}$$

Up to now, we have finished the finite-time output feedback stabilizing controller design of the system (2). Consequently, the following theorem can be obtained to summarize the main results of the paper.

Theorem 1. Under Assumption 1, if the proposed control design procedure together with the above switching control strategy is applied to system (2), then the states of closed-loop system are regulated to zero in a finite time.

Remark 2. It should be pointed out that the $Z(0)$-dependent switching time T_1 leads to the proposed controller being a initial-value-dependent one and unavailable for the system with unknown initial values. However, for any given compact subset $U \in R^{2n-1}$, $Z(0) \in U \Rightarrow \Psi \leq \rho$, where ρ is a positive constant. This leads to

$$T_1 \leq \frac{-(2-\tau)\rho^{(-\tau)/(2-\tau)}(0)}{\bar{c}_3\tau} := T_1^{'}$$

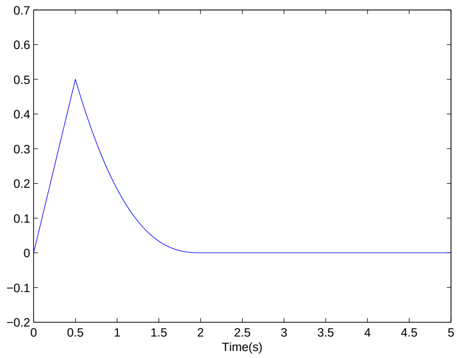
In this case, we can use $T_1^{'}$ to replace T_1 in the above controller design procedure and achieve the semi-global control objective.

IV. Simulation Example

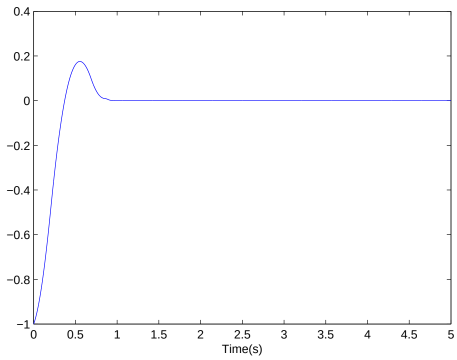
To illustrate the effectiveness of the proposed approach, we consider the following low-dimensional system

$$\begin{aligned} \dot{x}_0 &= u_0^5 + \tfrac{1}{4}\theta_0(t)x_0 \\ \dot{x}_1 &= x_2^3 u_0^3 + \tfrac{1}{8}ln(1 + (\theta_1(t)x_1)^2) \\ \dot{x}_2 &= u_1 + \tfrac{1}{8}\theta_2(t)x_2^{1/6}\sin x_2 \\ y &= (x_0, x_1)^T \end{aligned} \tag{43}$$

where $\theta_i(t)$, $i = 0, 1, 2$ are unknown functions satisfying $|\theta_i(t)| \leq 1$. Choose $\tau = -\frac{1}{13} \in (-\frac{1}{4}, +\infty)$, then $r_1 = 1$, $r_2 = \frac{r_1+\tau}{p_1} = \frac{4}{13}$ and $r_3 = \frac{r_2+\tau}{p_2} = \frac{3}{13}$. By Lemma 5, it can be verified that $|f_1| \leq \frac{1}{4}|x_0|$, $|f_1| \leq \frac{1}{8}|x_1|^{\frac{12}{13}}$ and $|f_2| \leq \frac{1}{10}(|x_1|^{3/13} + |x_2|^{3/4})$ satisfy Assumption 1 with $a = \frac{1}{4}$ and $b = \frac{1}{8}$.



(a) x_0



(b) x_1

Fig. 1. The responses of system states x_0 and x_1 .

Firstly, we define the control law $u_0 = 1$ and introduce the change of coordinates

$$z_1 = x_1, \quad z_2 = \frac{x_2}{L^{1/3}}, \quad \upsilon = \frac{u_1}{L^{4/3}} \tag{44}$$

under which, the x-subsystem of (43) is transformed into:

$$\begin{aligned} \dot{z}_1 &= L z_2^3 + f_1 \\ \dot{z}_n &= L\upsilon + \frac{f_2}{L^{1/3}} \end{aligned} \tag{45}$$

According to the design procedure shown in Section III, we can explicitly construct an output feedback controller for system (45). Specifically, we can choose

$$\begin{aligned} \dot{\eta}_2 &= -Ll_1[\eta_2 + l_1 z_1]^{12/13} \\ u &= -L^{4/3}\Big[\beta_2[\eta_2 + l_1 z_1]^{13/4} + \beta_2\beta_1[z_1]\Big]^{3/13} \end{aligned} \tag{46}$$

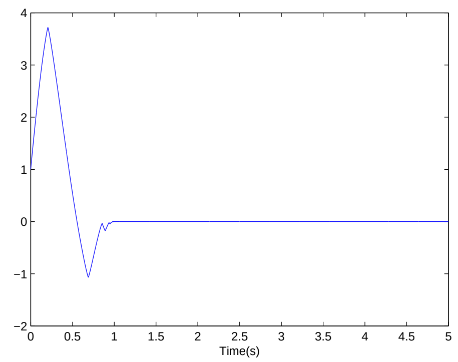
with appropriate positive constants l_1, β_1, β_2 and a large enough gain L such that output feedback controller (46) renders the system (45)(that is, the x-subsystem of (43) globally finite-time stable with a settling time T_1.

Then, when $t \geq T_1$, for the x_0-subsystem, we switch the control input u_0 to

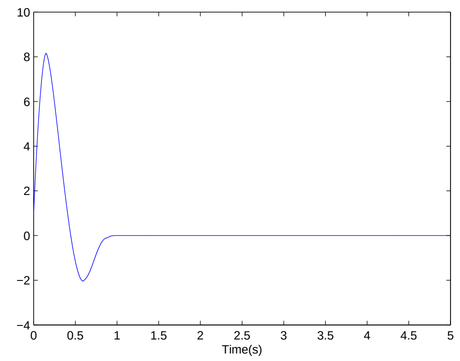
$$u_0 = -(k_0 + \tfrac{1}{4} + \tfrac{1}{4}x_0^2)^{1/5}[x_0]^{1/15} \tag{47}$$

where $k_0 > 0$ is a constant.

In the simulation, we assume $\theta_0(t) = \theta_1(t) = \theta_2(t) = sint$. When $(x_0(0), x_1(0),\ x_2(0),\ \eta_2(0))= (0, -1, 1, 1)$, by choosing the gains for the output laws as $L = 2$, $\beta_1 = 2.2$,

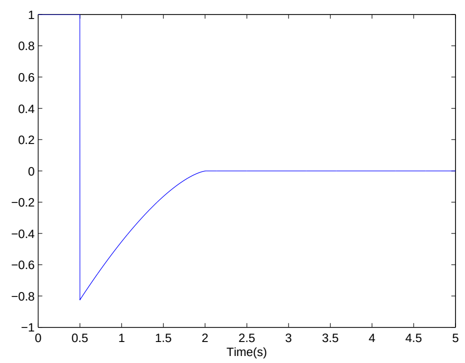


(a) x_2

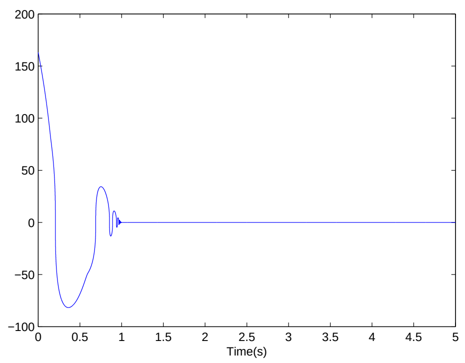


(b) η_2

Fig. 2. The responses of system states x_2 and η_2.



(a) u_0



(b) u_1

Fig. 3. The responses of control inputs u_0 and u_1.

$\beta_2 = 20$, $l_1 = 18$ and $k_0 = 1$, the simulation results are shown in Figs. 1-3, respectively. Fig. 1 shows the system states x_0 and x_1. Fig. 2 depicts the response of the system states x_2 and η_2. Fig. 3 illustrates the trajectories of control inputs. From Figs. 1 and 2, it can be seen that all the closed-loop system states indeed converge to zero in a finite time, which accords with the main results established in Theorem 1 and also demonstrates the effectiveness of the control method proposed in this paper.

V. Conclusion

In this paper, an output feedback stabilizing controller is presented for a class of high order nonholonomic systems in power chained form with uncertainties. The controller designed regulates the original system states to zero in a finite time. There are some related problems to investigate, e.g., for system (2) with unknown parameters, can an adaptive stabilizing controller be given under a similar assumption? In recent years, many results on stochastic nonholonomic systems have been achieved[31,32], however these works only consider the feedback stabilizer that makes the trajectories of the systems converge asymptotically to the equilibrium as the time goes to infinity. Therefore, how to design a finite time controller for stochastic nonholonomic systems is naturally regarded as an interesting research topic.

Appendix

Proof of Proposition 2. Noting that $(2-\tau-r_{i-1})/r_i p_{i-1} \geq 1$, by using Lemma 5 with $p=1$, $a=b=(2-\tau-r_{i-1})/r_i p_{i-1}$ and $e_i = [z_i^{p_{i-1}} - \hat{z}_i^{p_{i-1}}]^{1/r_i p_{i-1}}$, one leads to

$$\begin{aligned}
&-l_{i-1}(z_i^{p_{i-1}} - \hat{z}_i^{p_{i-1}}) \\
&\quad \times \left([z_i]^{(2-\tau-r_{i-1})/r_i} - [\hat{z}_i]^{(2-\tau-r_{i-1})/r_i}\right) \\
&\leq -l_{i-1}\lambda_i |e_i|^{1/r_i p_{i-1}} |e_i|^{(2-\tau-r_{i-1})/r_i p_{i-1}} \\
&= -l_{i-1}\lambda_i e_i^2
\end{aligned} \tag{A1}$$

where $\lambda_i = 2^{(2r_i p_{i-1}-2)/(2-\tau-r_{i-1})} > 0$ is a constant.

Proof of Proposition 3. Using $\gamma_i = \eta_i + l_{i-1}z_{i-1}$, (10), (16) and Lemmas 4-6, it follows that

$$\begin{aligned}
&\frac{2-\tau-r_{i-1}}{r_i}|z_i|^{(2-\tau-r_{i-1}-r_i)/r_i}\left([z_i]^{r_{i-1}/r_i} - \gamma_i\right)z_{i+1}^{p_i} \\
&= \frac{2-\tau-r_{i-1}}{r_i}|z_i|^{(2-\tau-r_{i-1}-r_i)/r_i} \\
&\quad \times \left([z_i]^{r_{i-1}/r_i} - [\hat{z}_i]^{r_{i-1}/r_i} + [\hat{z}_i]^{r_{i-1}/r_i} - \gamma_i\right) z_{i+1}^{p_i} \\
&\leq \frac{2-\tau-r_{i-1}}{r_i}|\xi_{i+1} - \beta_i\xi_i|^{r_{i+1}p_i} \\
&\quad \times |\xi_i - \beta_{i-1}\xi_{i-1}|^{(2-\tau-r_{i-1}-r_i)} \\
&\quad \times \left(|z_i^{p_{i-1}} - \hat{z}_i^{p_{i-1}}|^{r_{i-1}/p_{i-1}r_i} + l_{i-1}|z_{i-1} - \hat{z}_{i-1}|\right) \\
&\leq k_{i3}\left(|\xi_{i+1}|^{r_{i+1}p_i} + |\xi_i|^{r_{i+1}p_i}\right) \\
&\quad \times \left(|\xi_i|^{2-\tau-r_{i-1}-r_i} + |\xi_{i-1}|^{2-\tau-r_{i-1}-r_i}\right) \\
&\quad \times (|e_i|^{r_{i-1}} + l_{i-1}|e_{i-1}|^{r_{i-1}}) \\
&\leq \frac{1}{12}\sum_{j=i-1}^{i+1} \xi_j^2 + \alpha_i e_i^2 + g_i(l_{i-1})e_{i-1}^2
\end{aligned} \tag{A2}$$

where $k_{i3} > 0$, $\alpha_i > 0$ are constants and g_i is a continuous function of l_{i-1}.

Proof of Proposition 4. By (10), (17) and the definition of e_i, one gets

$$\begin{aligned}|v^{p_n}(\hat{z})| &= \Big|\sum_{i=1}^{n}\bar{\beta}_i[\hat{z}_i]^{\sigma/r_i}\Big|^{p_n r_{n+1}/\sigma}\\ &\le k_{i4}\Big(\sum_{i=1}^{n}|\xi_i|^{(r_n+\tau)/\sigma}+\sum_{i=1}^{n}|e_i|^{(r_n+\tau)/\sigma}\Big)\end{aligned} \tag{A3}$$

where k_{i4} is a positive constant.

Similar to (A2), with the use of Assumption 1, Lemmas 4-6 and (A3), (22) holds immediately.

Proof of Proposition 5. According to $\gamma_i = \eta_i + l_{i-1}z_{i-1}$, (10), Lemmas 3-5 and the definition of e_i, one obtains

$$\begin{aligned}& l_{i-1}(z_i^{p_{i-1}}-\hat{z}_i^{p_{i-1}})\\ &\quad\times\Big([\hat{z}_i]^{(2-\tau-r_{i-1})/r_i}-[\gamma_i]^{(2-\tau-r_{i-1})/r_{i-1}}\Big)\\ &\le k_{n5}|e_i|^{r_i p_{i-1}}|e_{i-1}|^{r_{i-1}}\Big(|e_{i-1}|^{2-\tau-2r_{i-1}}\\ &\quad+|\xi_{i-1}|^{2-\tau-2r_{i-1}}+|\xi_i|^{2-\tau-2r_{i-1}}+|e_i|^{2-\tau-2r_{i-1}}\Big)\\ &\le \frac{1}{16}(\xi_{i-1}^2+\xi_i^2)+e_i^2+\theta_i(l_{i-1})e_{i-1}^2\end{aligned} \tag{A4}$$

where k_{n5} is a positive constant and θ_i is a continuous function of l_{i-1}.

Proof of Proposition 6. By (10), (22) and Lemmas 4-6, it follows that

$$\begin{aligned}&\tilde{d}_n[\xi_n]^{(2\sigma-\tau-r_n)/\sigma}(v^{p_n}-z_{n+1}^{*p_n})\\ &\le |\xi_n|^{2-\tau-r_n}\Big|\sum_{i=2}^{n}\bar{\beta}_i([z_i]^{1/r_i}-[\hat{z}_i]^{1/r_i})\Big|^{r_n+\tau}\\ &\le k_{n6}|\xi_n|^{2-\tau-r_n}\Big(\sum_{i=2}^{n}|z_i-\hat{z}_i|\\ &\quad\times(|z_i-\hat{z}_i|^{(1-r_i)/r_i}+|z_i|^{(1-r_i)/r_i})\Big)^{r_n+\tau}\\ &\le \bar{k}_{n6}|\xi_n|^{2-\tau-r_n}\Big(\sum_{i=2}^{n}|e_i|^{r_i}\\ &\quad\times(|e_i|^{1-r_i}+|\xi_{i-1}|^{1-r_i}+|\xi_i|^{1-r_i})\Big)^{r_n+\tau}\\ &\le \frac{1}{4}\sum_{i=1}^{n}\xi_i^2+\tilde{\alpha}\sum_{i=2}^{n}e_i^2\end{aligned} \tag{A5}$$

where k_{n6}, $\bar{k}_{n6}$ and $\tilde{\alpha}$ are positive constants.

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