Availability Modeling of a Digital Electronic System with Intermittent Failures and Continuous Testing

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Abstract—**In this paper, we study a continuously tested digital electronic system subject to revealed, unrevealed, and intermittent failures. A new maintenance model is proposed for assessing the average availability of a digital electronic system by using a regenerative stochastic process with a finite number of states. Based on the properties of the regenerative process, general expressions are derived to calculate the mean times spent by the system in various operation and maintenance states. The average availability equation is derived under arbitrary and exponential failure distributions. Numerical examples illustrate the effect of fault coverage and failure rates on the average availability under exponential failure distributions. The proposed mathematical model considers the main failure types of digital telecommunications systems that allows more accurately assess the reliability of their operation.**

Index Terms—**Availability, intermittent failure, permanent failure, fault coverage, regenerative process.**

I. Introduction

CURRENTLY, most telecommunications systems use a digital information processing method. Failures of such systems usually include (1) sudden and complete permanent failures and (2) intermittent failures. The latter are usually detected by the results of periodic or continuous testing. Permanent failures are divided into revealed and unrevealed. Revealed failures are the failures that are covered by continuous testing. Conversely, unrevealed failures are not covered by continuous testing. However, hidden failures usually become revealed failures when the duration in the hidden state exceeds an upper time limit. As is well known [1], [2], the availability of telecommunications systems is one of the most important quality and reliability indicators.

In the case of periodic testing and exponential distribution of time to failures, mathematical modeling has been applied to assess the reliability maintenance of digital electronic systems with permanent and intermittent failures for decades. Nakagava [3] analyzed an inspection policy for intermittent faults where the test is planned at periodic times to detect the faults. Exponential distribution of time to an intermittent fault is assumed. Nakagava [4] considered a communication system subject to intermittent faults. Faults occur according to the exponential distribution and are hidden. Faults become permanent failures when the duration in the hidden state exceeds an upper time limit. Y. Hsu and C. Hsu [5] analyzed a three-state Markov model for fault-tolerant systems by taking both the effects of permanent and intermittent faults into consideration. Kranitis *et al*. [6] conducted a reliability analysis for optimal periodic testing of intermittent faults that minimizes the test cost. A Markov model is used for the probabilistic modeling of intermittent faults. The models for studying the reliability of digital systems subject to both permanent and intermittent faults were considered by Prasad [7], [8]. The models are based on a Markov model containing three states. Dharmaraja [9] developed an analytical model for the reliability and survivability analysis of UMTS network using Markov chains and a semi-Markov process with an exponential distribution of the random variables. However, unrevealed and intermittent failures are not considered in this model. Ulansky and Machalin [10] considered a maintenance model for a one-unit system with periodic testing subject to revealed and unrevealed failures under arbitrary failure distributions. Taghipour and Banjevic [11] considered hidden failures in the model with optimal periodic inspection for finite and infinite time horizons. Tai *et al.* [12] developed a maintenance model for maximizing the availability of the one-unit system assuming that maintenance is imperfect. Constantinescu [13] analyzed the hardware structure and fault occurrence using a continuous-time, discrete-state Markov model. Closed form solutions are derived for time-dependent and steady-state probabilities taking into consideration the weights of permanent and transient fault classes. Kim *et al.* [14] presented a semi-Markov model to assess and validate a hard real-time control system subject to permanent failure only. Analytical bounds are derived for exponential and Weibull failure distributions. Badia *et al.* [15], [16] developed maintenance models for a system subject to both revealed and unrevealed failures using a renewal process. Boonyathap and Jaturonnatee [17] formulated models of periodic preventive maintenance for used equipment under lease. Assumption of failures form is applied by the nonhomogeneous Poisson process, and failure distribution is appraised by Weibull. However, these models do not consider the effect of intermittent faults on a system, and may be useful only for determining optimal periodic inspection policy. Pham and Zhang [18] introduced a

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generalized model that incorporates a testing coverage measure into software reliability assessment. However, such testing coverage measures have not been considered for hardware of digital electronic systems. Malik and Kaiser [19] evaluated the reliability of a telecommunications system operating in harsh environments. The system's reliability was assessed by measuring shear strengths of solder joints of different electronic components at set intervals. Yasui *et al*. [20] formulated three typical stochastic models of data transmission and considered optimal policies to achieve high reliability of communication. Raza and Ulansky [21], [22] considered mathematical reliability models of a continuously tested avionic line replaceable unit subject to permanent and intermittent failures. However, these models cannot be applied to telecommunications systems because the systems have a continuous mode of operation, but the avionics systems are used in an interrupted mode of operation. Gil-Tomas *et al*. [23] considered a fault-tolerant microcomputer system against intermittent faults. Markov models for this system were generated, and some dependability functions, such as reliability and safety, were calculated. Li and Hao [24] proposed a novel system reliability assessment model with two dependent performance characteristics based on copula theory.

II. Problem Statement

The analysis of published studies showed the following features of the mathematical models developed to assess the reliability maintenance of digital systems with intermittent failures:

- The proposed mathematical models of reliability maintenance of hardware of digital electronic systems were developed for the case of periodical testing and 100% fault coverage.
- All models are based on the Markov or semi-Markov process.

There are no mathematical models that simultaneously consider unrevealed, revealed, and intermittent failures.

Therefore, the purpose of this study is to develop a mathematical model to calculate the average availability of a continuously tested telecommunications system subject to revealed, unrevealed, and intermittent failures under arbitrary failure distributions.

III. Notation

$Z(t)$	Random process with a limited number of states
T_0	Random regenerative cycle
T_i	Random time spent by the system in the state Z_i ($i = 1,\dots, 4$) per regenerative cycle
$E[T_i]$	Mean time of random variable T_i
Γ (≥ 0)	Random operating time to revealed failure
Ξ (≥ 0)	Random operating time to unrevealed failure
H (≥ 0)	Random operating time to intermittent failure
$\Phi(t)$	Distribution function of operating time to revealed failure for the part of the system that is covered by testing
$\phi(t)$	Revealed failure density function for the part of the system that is covered by testing
$F(t)$	Distribution function of operating time to unrevealed failure for the part of the system that is not covered by testing
$f(t)$	Unrevealed failure density function for the part of the system that is not covered by testing
$\Psi(t)$	Distribution function of operating time to intermittent failure for the part of the system that is covered by testing
$\psi(t)$	Intermittent failure density function for the part of the system that is covered by testing
μ	Revealed failure rate
λ	Unrevealed failure rate
θ	Intermittent failure rate

IV. Maintenance Model of a Continuously Tested Single-Unit Telecommunications System

Let us develop a maintenance model of a single-unit digital telecommunications system. The following assumptions are accepted:

- A revealed failure may occur only in the part of the system that is covered by continuous testing.
- An unrevealed failure may occur only in the part of the system that is not covered by continuous testing.
- The system leaves the state of an unrevealed failure only after the occurrence of a revealed or intermittent failure.
- Only one type of failure may occur at any time, and revealed and intermittent failures are detected immediately. The failed unit is perfectly repaired, renewing the system condition to be as-good-as-new.
- The time of replacement is negligible, and the number of spare parts is unlimited.

To determine the maintenance efficiency measures, we use a random regenerative process $Z(t)$ with a finite number of states

$$Z = \bigcup_{i=1}^{n} Z_i$$

on the infinite time horizon $(0, \infty)$. Further, we use a well-known property of regenerative stochastic processes [25], which states that the fraction of time for which the system is in the state Z_i ($i = 1, 2, \dots, n$) is equal to the ratio of the average time spent in the state Z_i per regeneration cycle to the average cycle duration. A possible realization of the stochastic process $Z(t)$ is shown in Fig. 1.

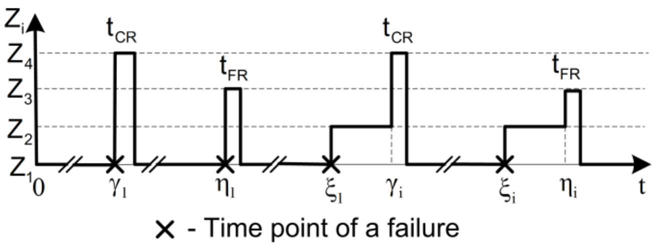

Fig. 1. Time diagram of realization of the regenerative process $Z(t)$.

Let us define the random process $Z(t)$. At any arbitrary time t, the system can be in one of the following states:

$Z(t) = Z_1$, if at time t, the system is in the operable state;

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$Z(t) = Z_2$, if at time t, the system is used as intended and is in an inoperable state because an unrevealed failure occurred in the part of the system that is not covered by the test;

$Z(t) = Z_3$, if at time t, the system is not used for its intended purpose because it is repaired after the occurrence of an intermittent failure;

$Z(t) = Z_4$, if at time t, the system is not used for its intended purpose because it is repaired after the occurrence of a revealed failure.

The system will be in the operable state from $t = 0$ to a time when one of the failures occurs as shown in Fig. 1.

We assume that a revealed failure occurs at time γ_i. The failure is detected by the testing equipment, and the system is judged as inoperable. Then, the system enters the state Z_4. We denote the average duration the system stays in the state Z_4 as t_{CR}. After completion of the repair, the system returns to the operable state Z_1.

We also assume that an intermittent failure occurs at an arbitrary time η_i. As a result, the system changes its state to the state Z_3 where it will stay for a time t_{FR}. After its repair, the system returns back to the operable state Z_1 being as-good-as-new.

We further assume that an unrevealed failure occurs at time ξ. Then, the system changes its state to Z_2, from which the system can pass to the state Z_3 or Z_4.

Let T_i be the time in the state Z_i ($i = 1,\dots, 4$). Obviously, T_i is a random variable with expected mean time $E[T_i]$. The average duration of the system regeneration cycle is determined by the following formula:

$$E\left[T_0\right] = \sum_{i=1}^{4} E\left[T_i\right]. \tag{1}$$

If the expected mean times $E[T_1]$, …, $E[T_4]$ are known, the average availability is determined as

$$A = E[T_1] \big/ E[T_0]. \tag{2}$$

A. The Mean Time the System Stays in the Operable State

An illustration of a possible duration of the system stay in the operable state is shown in Fig. 2

We then assume that an unrevealed failure occurs at an arbitrary time ξ ($0 < \xi \le \infty$) in the part of the system that is not covered by testing. Then, the system will be in the operable state in one of the three possible cases. The first case is when the system operates until the time γ ($\gamma < \xi$), when the revealed failure occurs as shown in Fig. 2(a). The second case is when the system operates until the time η ($\eta < \xi$), when the intermittent failure occurs as shown in Fig. 2(b).

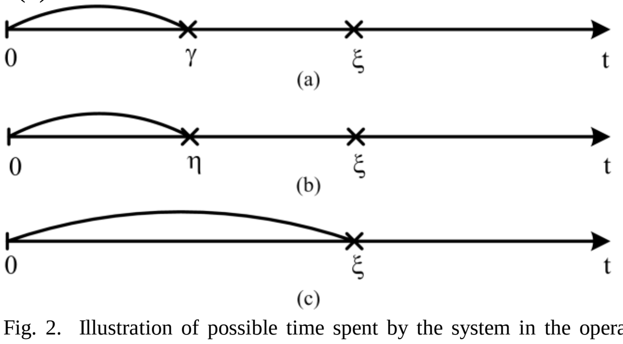

Fig. 2. Illustration of possible time spent by the system in the operable state: (a) The operable state of the system until the time when a revealed failure occurs (Case 1), (b) The operable state of the system until the time when an intermittent failure occurs (Case 2), and (c) The operable state of the system until the time when an unrevealed failure occurs (Case 3).

The third case is when the system operates until the time ξ, when the unrevealed failure occurs under the condition that no revealed or intermittent failure has occurred before it, as illustrated in Fig. 2(c).

The expected mean time the system stays in the operable state is determined by the law of total expectation:

$$E[T_1] = \int_0^\infty \left\{ E\left[T_1^\gamma \,|\, \Xi = \xi\right] + E\left[T_1^\eta \,|\, \Xi = \xi\right] + E\left[T_1^\xi \,|\, \Xi = \xi\right] \right\} dF(\xi), \tag{3}$$

where $E[T_1^\gamma \mid \Xi = \xi]$ is the conditional mathematical expectation of the random operating time to a revealed failure under the condition that $\Xi = \xi$, $E[T_1^\eta \mid \Xi = \xi]$ is the conditional mathematical expectation of the random operating time to an intermittent failure under the condition that $\Xi = \xi$, and $E[T_1^\xi \mid \Xi = \xi]$ is the conditional mathematical expectation of the random operating time to an unrevealed failure under the condition that $\Xi = \xi$.

Let us determine the conditional mathematical expectations in (3). From Fig. 2, we write the following expressions for the conditional mathematical expectations:

$$E\left[T_1^\gamma \,|\, \Xi = \xi \cap \Gamma = \gamma < \xi \cap \mathrm{H} > \gamma\right] = \gamma, \tag{4}$$

$$E\left[T_1^\eta \,|\, \Xi = \xi \cap \mathrm{H} = \eta < \xi \cap \Gamma > \eta\right] = \eta, \tag{5}$$

$$E\left[T_1^\xi \,|\, \Xi = \xi \cap \Gamma > \xi \cap \mathrm{H} > \xi\right] = \xi. \tag{6}$$

The random variable Γ varies from 0 to ξ due to the condition $\Gamma = \gamma < \xi$ in (4). The condition $\mathrm{H} > \gamma$ indicates that until the time $\Gamma = \gamma$, when the revealed failure occurs, the intermittent failure will not occur with the probability $P(\mathrm{H} > \gamma) = 1 - \Psi(\gamma)$. Therefore, based on other conditions in (4), we obtain the following formula:

$$E\left[T_1^\gamma \,|\, \Xi = \xi\right] = \int_0^\xi u\left[1 - \Psi(u)\right] d\Phi(u). \tag{7}$$

Similarly, the random variable H varies from 0 to ξ due to the condition $\mathrm{H} = \eta < \xi$ in (5). The condition $\Gamma > \eta$ indicates that until the time $\mathrm{H} = \eta$, when the intermittent failure occurs, the revealed failure does not occur with the probability $P(\Gamma > \eta) = 1 - \Phi(\eta)$. Therefore,

$$E\left[T_1^\eta \,|\, \Xi = \xi\right] = \int_0^\xi x\left[1 - \Phi(x)\right] d\Psi(x). \tag{8}$$

Due to the condition $\Gamma > \xi \cap \mathrm{H} > \xi$ in (6), the unrevealed failure may occur at time ξ only if the revealed and intermittent failures have not occurred until this time. The probability of this event is determined as $P(\Gamma > \xi \cap \mathrm{H} > \xi) = [1 - \Phi(\xi)][1 - \Psi(\xi)]$. Therefore,

$$E\left[T_1^\xi \,|\, \Xi = \xi\right] = \xi[1 - \Phi(\xi)][1 - \Psi(\xi)]. \tag{9}$$

By substituting (7)–(9) into (3), we obtain

$$E\left[T_1\right] = \int_0^\infty \left\{ \int_0^v [1 - \Psi(u)] u\, d\Phi(u) + \int_0^v [1 - \Phi(x)] x\, d\Psi(x) + v[1 - \Psi(v)][1 - \Phi(v)] \right\} dF(v). \tag{10}$$

B. The Mean Time the System Stays in the Inoperable State

An illustration of different variants for the duration of the system stay in the inoperable state is depicted in Fig. 3.

We first assume that the unrevealed failure occurs at an arbitrary time ξ ($0 < \xi \le \infty$). Then, the system will be in the inoperable state until the time γ ($\gamma < \xi$), when the revealed failure occurs, i.e., in the time interval (ξ, γ), as shown in Fig. 2(a), or until the time η ($\eta < \xi$), when the intermittent

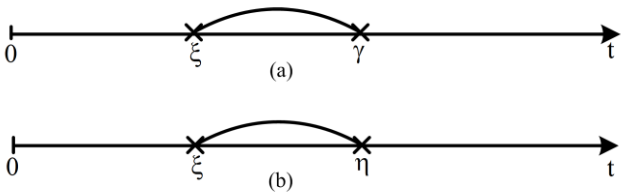

Fig. 3. Illustration of possible duration of the time spent by the system in the (a) inoperable state of the system in the time interval (ξ, γ) and (b) inoperable state of the system in the time interval (ξ, η).

failure occurs, i.e., in the time interval (ξ, η) as shown in Fig. 2(b).

The expected mean time the system stays in the inoperable state is determined by the law of total expectation

$$E\left[T_2\right]=\int_0^\infty\left\{E\left[T_2^\gamma\left|\Xi=\xi\right.\right]+E\left[T_2^\eta\left|\Xi=\xi\right.\right]\right\}dF\left(\xi\right), \tag{11}$$

where $E[T_2^\gamma \mid \Xi = \xi]$ is the conditional mean time the system stays in the inoperable state until the time γ, when the revealed failure occurs under the condition that Ξ = ξ, and $E[T_2^\eta \mid \Xi = \xi]$ is the conditional mean time the system stays in the inoperable state until the time η, when the intermittent failure occurs under the condition that Ξ = ξ.

Let us determine the conditional mean times included in (11). From Fig. 3, we write the following expressions:

$$E\left[T_2^\gamma\left|\Xi=\xi\cap\Gamma=\gamma>\xi\cap\mathrm{H}>\gamma\right.\right]=\gamma-\xi, \tag{12}$$

$$E\left[T_2^\eta\left|\Xi=\xi\cap\mathrm{H}=\eta>\xi\cap\Gamma>\eta\right.\right]=\eta-\xi. \tag{13}$$

Using (12)–(13), we determine the conditional mean times included in (11).

The random variable Γ takes values greater than ξ due to the condition Γ = γ > ξ in (12). The condition H > γ indicates that until the time Γ = γ, when the revealed failure occurs, the intermittent failure does not occur with the probability $P(\mathrm{H} > \gamma) = 1 - \Psi(\gamma)$. Therefore, based on other conditions in (12), we obtain

$$E\left[T_2^\gamma\left|\Xi=\xi\right.\right]=\int_\xi^\infty(u-\xi)\left[1-\Psi\left(u\right)\right]d\Phi\left(u\right). \tag{14}$$

Similarly, the random variable H takes values greater than ξ due to the condition H = η > ξ in (13). The condition Γ > η indicates that until the time H = η, when the intermittent failure occurs, the revealed failure does not occur with the probability $P(\Gamma > \eta) = 1 - \Phi(\eta)$. Therefore,

$$E\left[T_2^\eta\left|\Xi=\xi\right.\right]=\int_\xi^\infty(x-\xi)\left[1-\Phi\left(x\right)\right]d\Psi\left(x\right). \tag{15}$$

By substituting (14)–(15) into (11), we obtain

$$\begin{aligned}E[T_2]=\int_0^\infty\Big\{&\int_v^\infty(u-v)\left[1-\Psi(u)\right]d\Phi(u)+\\+&\int_v^\infty(x-v)\left[1-\Phi(x)\right]d\Psi(x)\Big\}dF(v).\end{aligned} \tag{16}$$

C. The Mean Time the System Stays in the Repair State Due to the Intermittent Failure

We assume that the intermittent failure occurs at an arbitrary time η under the condition that until this time, the revealed and unrevealed failures have not occurred. As a result, the false repair of the system will be performed for the time t_{FR} as illustrated in Fig. 4. After its repair, the system becomes as-good-as-new, and can be used for its intended purpose.

Let the intermittent failure occur in a time interval from η to η + *d*η (0 < η < ∞).

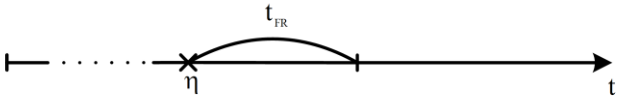

Fig. 4. Illustration of possible time spent by the system in the repair state Z_3.

Then, the conditional probability of such a random event is equal to the probability element ψ(η)*d*η (Fig. 5) if until the time η, the revealed and unrevealed failures have not occurred

$$P\left\{\eta<\mathrm{H}\le\eta+d\eta\left|\eta<\Gamma\cap\eta<\Xi\right.\right\}=\psi\left(\eta\right)d\eta. \tag{17}$$

The probability of the intermittent failure in the time interval (η, η + *d*η) is calculated as

$$P_{\xi>\eta}\left\{\eta<\mathrm{H}\le\eta+d\eta\right\}=\left[1-\Phi\left(\eta\right)\right]\left[1-F\left(\eta\right)\right]\psi\left(\eta\right)d\eta. \tag{18}$$

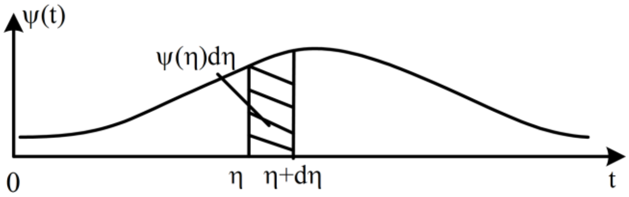

Fig. 5. The curve of the intermittent failure density function.

The probability of the intermittent failure in the time interval (0, ∞) is determined by the integration of (18) in the range of existence of random variable H, i.e.

$$P\left\{0<\mathrm{H}\le\infty\right\}=\int_0^\infty\left[1-\Phi\left(x\right)\right]\left[1-F\left(x\right)\right]\psi\left(x\right)dx. \tag{19}$$

The mean time the system stays in the repair state Z_3 is determined by multiplying the average repair time t_{FR} by the probability in (19)

$$\begin{aligned}&E[T_3]=t_{FR}P\left\{0<\mathrm{H}\le\infty\right\}=\\&=t_{FR}\int_0^\infty\left[1-\Phi(x)\right]\left[1-F(x)\right]\psi\left(x\right)dx.\end{aligned} \tag{20}$$

D. The Mean Time the System Stays in the Repair State Due to the Revealed Failure

An illustration of different variants for duration the system stays in the repair state Z_4 is depicted in Fig. 6.

We assume that the revealed failure occurs at an arbitrary time γ, and until that time, the intermittent failure has not occurred. As a result, the correct repair of the system will be performed during the time t_{CR} as illustrated in Fig. 6(a).

Further, we assume that the intermittent failure occurs at an arbitrary time η, and until that time, the unrevealed failure only occurred at time ξ (0 < ξ < η). Since the system is in the inoperable state due to the unrevealed failure that occurred in the part of the system not covered by testing, the repair of the system will be performed during the time t_{CR}, as illustrated in Fig. 6(b). After its repair, the system returns to the operable state being as-good-as-new.

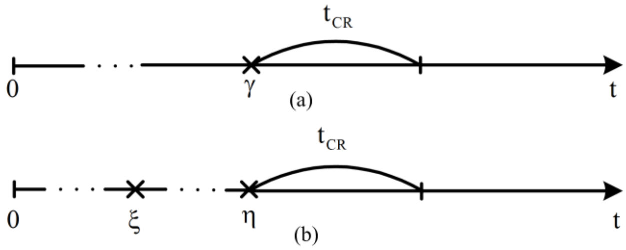

Fig. 6. Illustration of possible time spent by the system in the repair state Z_4.

Returning to Fig. 6(a), let the revealed failure occur in a time interval from γ to γ + *d*γ (0 < γ < ∞). Then, the conditional probability of such a random event is equal to the probability element ϕ(γ)*d*γ (Fig. 7):

$$P\{\gamma < \Gamma \le \gamma + d\gamma | \gamma < \mathrm{H}\} = \phi(\gamma)d\gamma. \tag{21}$$

The unconditional probability of the revealed failure in the time interval (γ, γ + *d*γ) is calculated as

$$P\{\gamma < \Gamma \le \gamma + d\gamma\} = [1 - \Psi(\gamma)]\phi(\gamma)d\gamma. \tag{22}$$

Let us now go back to Fig. 6(b), and assume that unrevealed failure occurs at the time ξ and the intermittent failure occurs in a time interval from η to η + *d*η (ξ < η < ∞). Then, the conditional probability of such a random event is equal to the probability element ψ(η)*d*η if until the time η, the unrevealed failure has not occurred:

$$P_{\xi<\eta}\{\eta < \mathrm{H} \le \eta + d\eta | \eta < \Gamma \cap \eta > \Xi\} = \psi(\eta)d\eta. \tag{23}$$

The unconditional probability of the occurrence of the intermittent failure in the time interval (η, η + *d*η) is calculated as

$$P_{\xi<\eta}\{\eta < \mathrm{H} \le \eta + d\eta\} = F(\eta)[1 - \Phi(\eta)]\psi(\eta)d\eta. \tag{24}$$

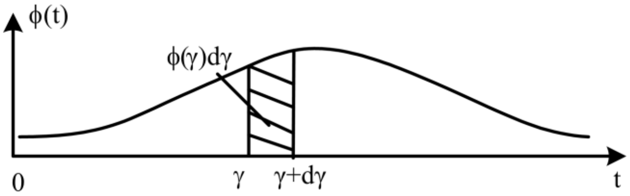

Fig. 7. The curve of the revealed failure density function

The probability of the occurrence of the revealed failure in the time interval (0, ∞) is determined by the integration of (22) in the range of existence of the random variable Γ, i.e.,

$$P\{0 < \Gamma \le \infty\} = \int_0^\infty [1 - \Psi(u)]\phi(u)du. \tag{25}$$

The probability of the occurrence of the intermittent failure in the time interval (0, ∞) is determined by the integration of (24) in the range of existence of the random variable Η, i.e.,

$$P_{\xi<\eta}\{0 < \mathrm{H} \le \infty\} = \int_0^\infty F(x)[1 - \Phi(x)]\psi(x)dx. \tag{26}$$

The mean time spent by the system in the repair state Z_4 is determined by the law of total expectation

$$\begin{aligned} E[T_4] &= t_{CR}P\{0 < \Gamma \le \infty\} + t_{CR}P_{\xi<\eta}\{0 < \mathrm{H} \le \infty\} = \\ & t_{CR}\int_0^\infty [1 - \Psi(u)]\phi(u)du + \\ & + t_{CR}\int_0^\infty F(x)[1 - \Phi(x)]\psi(x)dx. \end{aligned} \tag{27}$$

V. A Case of Exponential Distribution of Time to Failure

Let us determine the mean time spent by the system in the states Z_1, …, Z_4 and the average availability *A* for the case of an exponential distribution of time to failure. Distribution functions of time to failure for random variables Γ, Ξ, and Η are given by

$$\Phi(t) = 1 - \exp(-\mu t), \tag{28}$$

$$F(t) = 1 - \exp(-\lambda t), \text{ and} \tag{29}$$

$$\Psi(t) = 1 - \exp(-\theta t). \tag{30}$$

Failure density functions are determined respectively as

$$\phi(t) = \mu \exp(-\mu t), \tag{31}$$

$$f(t) = \lambda \exp(-\lambda t), \text{ and} \tag{32}$$

$$\psi(t) = \theta \exp(-\theta t). \tag{33}$$

Substituting (28)–(33) into (10), we obtain the formula for calculating the mean time the system stays in the operable state:

$$E[T_1] = \frac{1}{\mu + \lambda + \theta}. \tag{34}$$

Substituting (28)–(33) into (16), we obtain the formula for calculating the mean time the system stays in the inoperable state:

$$E[T_2] = \frac{\lambda}{(\mu + \theta)(\mu + \lambda + \theta)}. \tag{35}$$

Substituting (28)–(33) into (20) gives the expression for calculating the mean time the system stays in the repair state Z_3:

$$E[T_3] = \frac{t_{FR}\theta}{\mu + \lambda + \theta}. \tag{36}$$

Substituting (28)–(33) into (27) gives the expression for calculating the mean time the system stays in the repair state Z_4:

$$E[T_4] = t_{CR}\left(\frac{\mu}{\mu + \theta} + \frac{\theta\lambda}{(\mu + \theta)(\mu + \lambda + \theta)}\right). \tag{37}$$

And finally, substituting (35)–(37) into (2), we obtain the formula for calculating the system's average availability:

$$A = \frac{\mu + \theta}{(\mu + \lambda + \theta)(1 + t_{CR}\mu) + \theta[t_{FR}(\mu + \theta) + t_{CR}\lambda]}. \tag{38}$$

Let us determine relations between the fault coverage *a* and failure rates μ and λ. The fault coverage (*a*) can be determined as follows:

$$a = \frac{\lambda_{CBT}}{\lambda_{CBT} + \lambda_{NCBT}} = \frac{\mu}{\mu + \lambda}, \tag{39}$$

where λ_{CBT} is the failure rate for the part of the system that is covered by testing, and λ_{NCBT} is the failure rate for the part of the system that is not covered by testing.

From (39), we obtain the formula for determining the relationship between the unrevealed and revealed failure rates:

$$\lambda = \mu\frac{1 - a}{a}. \tag{40}$$

The revealed and unrevealed failure rates are linked by the following equation:

$$\Lambda = \lambda + \mu, \tag{41}$$

where Λ is the total permanent failure rate of the system.

VI. Numerical Examples

Let us assess the impact of the model parameters on the average availability of a non-redundant unit of the UA5000 (*Universal Access Unit*) system.

The UA5000 is a device for accessing both the narrowband and broadband services. The UA5000 provides subscribers quality voice and broadband access services and IP-based voice access and multimedia services [26]. To provide high level of QoS the availability of the UA5000 must be more than five nines after decimal point. The average availability of the UA5000's units affects the overall availability of the UA5000. The operation and maintenance process of any UA5000's unit corresponds to the time diagram shown in Fig. 1. Therefore, the proposed model can be used to assess the average availability of any UA5000's unit.

The hardware architecture of the UA5000 consists of cabinet, shelves, and boards. The rear-access UA5000 cabinet's shelf is called HABA. The HABA shelf is the master shelf of the UA5000.

The layout of the HABA shelf is shown in Fig. 8 [26], where PWX is the secondary power supply card, xPBM is the broadband control card (APMB/IPMB), PVx is the narrowband control card (PVU8/PVU4/PVM), RSUx is the remote subscriber unit (RSU8/RSU4), AIUB is the ATM interface card, xSL is the service line card (ASL/DSL/ADMB/VDLA/SDLB…), TSSB is the test card.

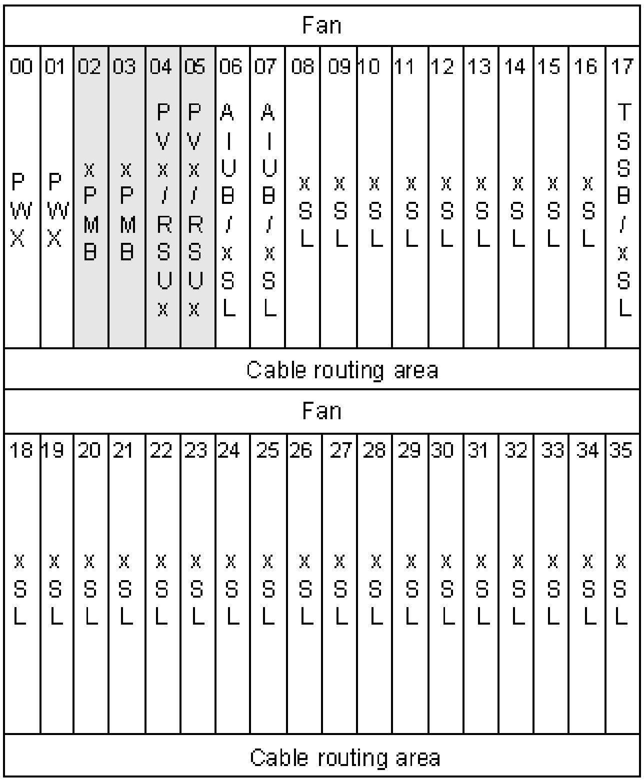

Fig. 8. Layout of the HABA shelf.

Example 1. Let us assess the average availability of the service board xSL as a non-redundant unit of the UA5000 and determine the dependence between the system's main parameters if $\theta = 1 \times 10^{-4}$ h^{-1} and $t_{CR} = t_{FR} = 1$ h.

The dependence of $E[T_1]$ on Λ when $a = 0.9$ is shown in Fig. 9. As seen, $E[T_1]$ decreases when Λ increases.

The dependence of $E[T_2]$ on the fault coverage a is shown in Fig. 10. The parameter of the family of curves is the rate of revealed failures μ. As seen in Fig. 10, $E[T_2]$ decreases when a increases.

The dependence of the average availability on the fault coverage at a different revealed failure rate is shown in Fig. 11. As it can be seen in the figure, the average availability increases when the fault coverage and revealed failure rate increase.

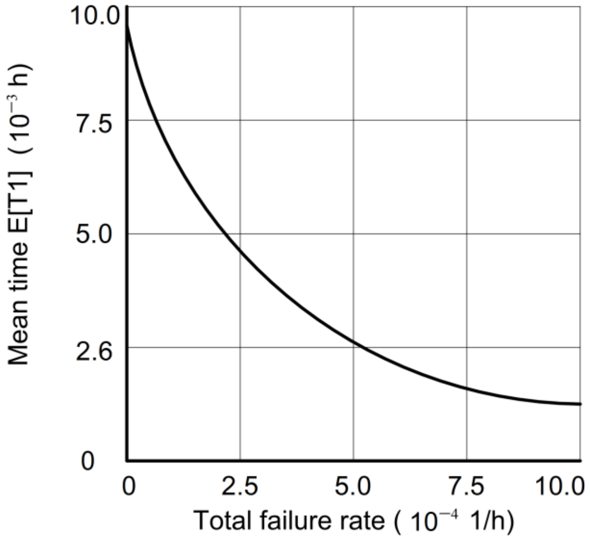

Fig. 9. Dependence of the mean time spent by the system in the operable state on the total failure rate.

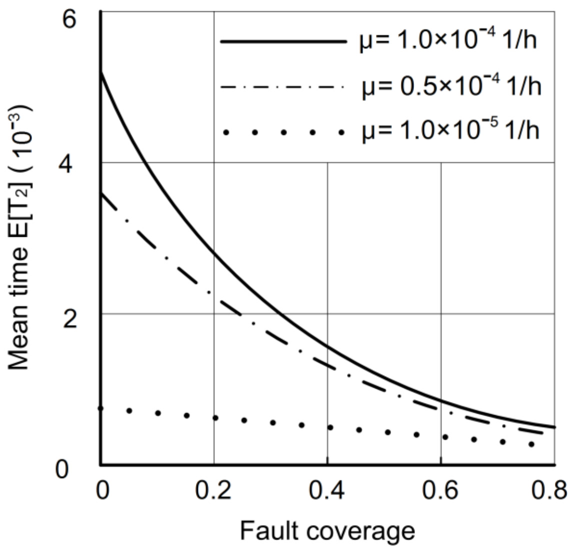

Fig. 10. Dependence of the mean time spent by the system in the inoperable state on the fault coverage.

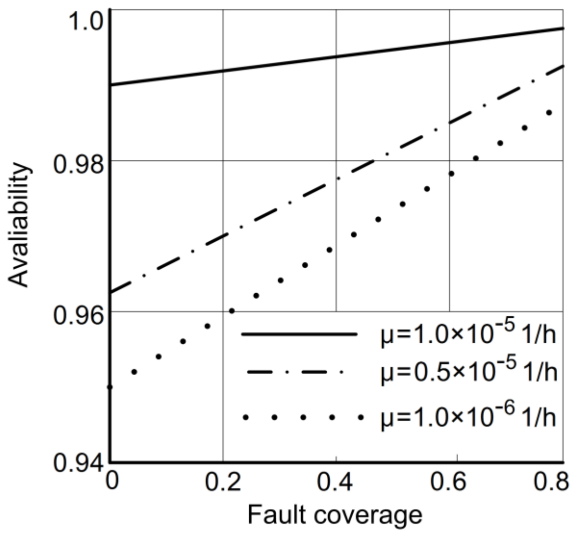

Fig. 11. Dependence of the average availability on the fault coverage.

Using (38), let us calculate the average availability of the non-redundant service board xSL at different values of the revealed failure rate when the fault coverage is unity. The results of the calculations are listed in Table 1.

As seen in Table 1, the average availability has weak dependence on the revealed failure rate when $a = 1$.

TABLE I
AVERAGE AVAILABILITY AS A FUNCTION OF THE REVEALED FAILURE RATE

μ	A
1×10^{-5}	0.99978
0.5×10^{-5}	0.99979
1×10^{-6}	0.99980

Example 2. Let us assess the impact of the intermittent failure rate of the service board xSL on the mean times $E[T_1]$, $E[T_2]$, and $E[T_3]$ for the initial data of Example 1.

The dependence of $E[T_1]$, $E[T_2]$, and $E[T_3]$ on the intermittent failure rate θ when $\mu = 1 \times 10^{-5}$ h^{-1} are shown in Fig. 12. The vertical axis is in log-10 scale.

As seen in Fig. 12, $E[T_1]$ decreases with increasing θ because any intermittent failure reduces the system's uptime. The dependence of $E[T_2]$ on θ is also decreasing because if there is an unrevealed failure in the part of the system not covered by testing and an intermittent failure occurs in the tested part, then the system is replaced by a new one.

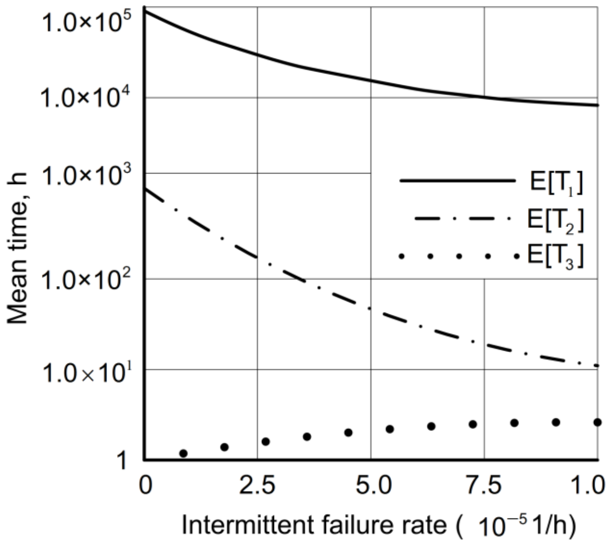

Fig. 12. Dependence of the mean times $E[T_1]$, $E[T_2]$, and $E[T_3]$ on the intermittent failure rate θ when $\mu = 1 \times 10^{-5}$ h^{-1}.

The shape of the curve of $E[T_3]$ versus θ shows that the average time spent by the system in the repair state Z_3 increases with the increasing intermittent failure rate. It should be also noted that $E[T_1]$ and $E[T_2]$ depend significantly on the intermittent failure rate.

Figure 13 shows the dependence of the average availability on the intermittent failure rate at different values of the revealed failure rate.

As seen in Fig. 13, the average availability of the service board xSL increases with increasing intermittent and revealed failure rates because the unrevealed failures are detected more quickly in such cases. It should also be noted that the impact of the revealed failure rate on the average availability decreases with the increasing intermittent failure rate.

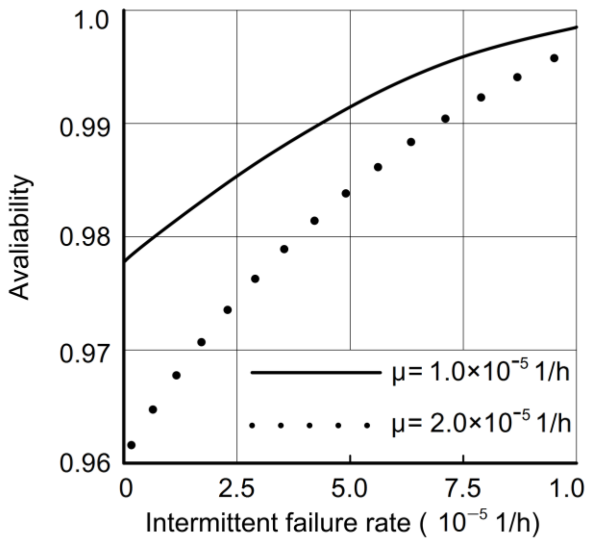

Fig. 13. Dependence of the average availability on the intermittent failure rate at different values of the revealed failure rate.

VII. CONCLUSION

In this study, we have developed a reliability maintenance model for a continuously tested digital telecommunications system subject to revealed, unrevealed, and intermittent failures under arbitrary failure distributions. The mathematical expressions for calculating the average availability of the telecommunications system have been derived. In the case of exponential failure distributions, it has been shown that the average availability largely depends on the fault coverage and intermittent failure rate. The obtained results are advisable to use in the design, operation, and maintenance phases of a telecommunications system.

Concerning the future work, we plan to generalize the developed mathematical model to a model in which an unrevealed failure becomes a revealed failure when the duration of the system stay in the hidden state exceeds the upper time limit. Such situations occur during the operation of some telecommunications systems.

REFERENCES

[1] M.L. Ayers, *Telecommunications System Reliability Engineering, Theory, and Practice*. New Jersey: Wiley, 2012.
[2] N. Olifer and V. Olifer, *Computer Networks: Principles, Technologies and Protocols for Network Design*. Chichester: Wiley, 2005.
[3] T. Nakagava, *Maintenance Theory of Reliability*. London: Springer, 2005.
[4] T. Nakagava, *Advanced Reliability Models and Maintenance Policies*. London: Springer, 2008.
[5] Y.T. Hsu and C.F. Hsu, "Novel model of intermittent faults for reliability and safety measures in long-life computer systems", *Int Journal of Electronics*, vol. 71, no. 6, pp. 917-937, 1991.
[6] N. Kranitis, A. Merentitis, N. Laoutaris, "Optimal periodic testing of intermittent faults in embedded pipeline processor applications", in *Proc. of 2006 IEEE Design, Automation and Test in Europe Conf.*, Munich, 2006, pp. 1-6.
[7] V.B. Prasad, "Computer networks reliability evaluations and intermittent faults", in *Proc. of 1990 IEEE 33th Midwest Symp on Circuits and Systems*, vol. 1 Calgary, Alta, 1990, pp. 327-330.
[8] V.B. Prasad, "Digital systems with intermittent faults and Markovian models," in *Proc. of 1992 IEEE 35th Midwest Symp on Circuits and Systems*, vol. 1, Washington, 1992, pp. 195-198.
[9] S. Dharmaraja, V. Jindal and U. Varshney, "Reliability and survivability analysis for UMTS network: an analytical approach,"

Network and service management, IEEE transactions, vol. 5, pp. 132-142, 2008.
[10] V.V. Ulansky and I.A. Machalin, "Maintenance policy of one-unit system with revealed and unrevealed failures," *Mathematical machines and systems*, vol. 3/4, pp. 245-256, 2007 (in Russian).
[11] S. Taghipour and D. Banjevic, "Periodic inspection optimization models for a repairable system subject to hidden failures," *Reliability, IEEE Trans.*, vol. 60(1), pp. 275-285, 2011.
[12] H.T. Allen, C. Ling-Yau, Z. Yongdao, L. Haitao, and E. A. Elsayed, "Condition based maintenance of periodically inspected systems," *Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering* 2009, WCE 2009, 1-3 July, 2009, London, U.K., pp1256-1261.
[13] C.A. Constantinescu, "Decomposition method for reliability analysis of real-time computing systems," in *Proc. of Reliability and maintainability Symposium*, Anaheim, CA, pp. 272-277, 1994.
[14] H. Kim, A.L. White and K.G. Shin, "Reliability modeling of hard real-time systems," in *Proc. of Fault-Tolerant Computing, Twenty-Eight Annual Int. Symposium*, Munich, 1998, pp. 304-313.
[15] F.G. Badia and M.D. Berrade, "Optimum maintenance of a system under two types of failure," *Int. Journal of Materials & Structural Reliability*, vol. 4(1), pp. 27-37, 2006.
[16] F.G. Badia, M.D. Berrade and C. A. Campos, "Optimal inspection and preventive maintenance of units with revealed and unrevealed failures," *Reliability Engineering and System Safety,* vol. 78, pp. 157-163, 2002.
[17] P. Boonyathap and J. Jaturonnatee, "Multiple periodic preventive maintenance for used equipment under lease", *Lecture Notes in Engineering and Computer Science: Proceedings of The International MultiConference of Engineers and Computer Scientists* 2010, IMECS 2010, 17-19 March, 2010, Hong Kong, pp1852-1857.
[18] H. Pham and X. Zhang, "NHPP software reliability and cost models with testing coverage," *European Journal of Operational Research*, vol. 145, pp. 443-454, 2003.
[19] S. Mallik and F. Kaizer, "A Study of the Reliability of Electronic Telecommunication Systems Working at Subsea Level," in *Transactions on Engineering Technologies* (WCE 2014), G. Yang, S. Ao and L. Gelman, Eds. Springer, pp. 35-48, 2014.
[20] K. Yasui, T. Nakagava and H. Sandoh, "Reliability models in data communication systems", in *Stochastic Models in Reliability and Maintenance,* S. Osaki Ed. Springer, pp. 281-306, 2002.
[21] A. Raza and V. Ulansky, "Minimizing total lifecycle expected costs of digital avionics' maintenance," *Procedia CIRP*, vol. 38, pp. 118-123, 2015.
[22] A. Raza and V. Ulansky, "Modelling of operational reliability and maintenance cost for avionics systems with permanent and intermittent failures," in *Proc. of the 9th IMA Int. Conf. on Modelling in Ind. Maintenance and Reliability*," 12-14 July, 2016, London, U.K., pp. 186-192.
[23] D. Gil-Tomás, J. Gracia-Morán, J. C. Baraza-Calvo, L. J. Saiz-Adalid and P.J. Gil-Vicente, "Injecting intermittent faults for the dependability assessment of a fault-tolerant microcomputer system," *IEEE Transactions on Reliability*, vol. 65, no. 2, pp. 648-661, 2016.
[24] C. Li and H. Hao, "A copula-based degradation modeling and reliability assessment", *Engineering Letters*, vol. 24, no. 3, pp295-300, 2016.
[25] W.L. Smith, "Regenerative stochastic processes", in *Proc. of the Royal Society. London. Series A. Mathematical, Physical and Engineering Sciences* 232, London, pp. 6–31, 1955.
[26] A.V. Roslyakov and P.S. Krylov, *Principles of Constructions, Application and Designing of Communication Platform U-SYS.* Moscow: Solon-Press, 2013 (in Russian).

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