Rendezvous of Non-Cooperative Spacecraft and Tug Using a Tether System

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Abstract— This paper focuses on a docking of two bodies by means of a tether in a force-free field which is studied in the framework of space debris removal problem. The two bodies are connected by the viscoelastic tether. One of the bodies (tug) is under constant thrust. In the initial time the bodies are at some distance from each other. Analysis of the tethered bodies dynamics is carried out assuming that the tether’s length can be changed according to a prescribed law. The goal is to find the control law that allows to realize impactless docking of two bodies. Analysis of the consistency of the proposed law with the requirements of safe rendezvous of the bodies is conducted. The obtained results can be applied to study the properties and possible configurations of the tethered spacecraft docking system.

Index Terms— Spacecraft rendezvous, tethered system, active space debris removal, space tug, tethered control

I. INTRODUCTION

In the recent years, the problems of space debris have come to the forefront of modern astronautics. According to the forecast made by Donald J. Kessler [1], space debris can put an end to further space exploration. There are more than 15,000 large objects on the orbits around the Earth. Only 7% of these are active spacecraft, 17% are nonfunctional spacecraft and 13% are orbital stages of the rockets. All these objects are tracked and an active spacecraft or a space station can avoid collision with such objects [2-4]. Collisions of the large space debris with other debris can significantly increase numbers of the small space junk on the Earth orbit. There are a large number of papers devoted to this problem [1-23]. There are various solutions to this problem [5-9]. Many modern spacecraft are equipped with de-orbiting facilities [24-27].

Various approaches have been offered to eliminate already running units. These approaches include powerful lasers to destroy satellites to reduce its orbital velocity [5], electrodynamics tether system to create a drag force [6], starting devices (e.g. chaser, debriter) to capture debris, and to lead away from the orbit [7-21]. Such methods are called the active debris removal [7-21].

In the papers [7-9] it were proposed to use a rocket upper stage as tethered tug for space debris removal. It is assumed that this upper stage have been completed its main mission and its secondary goal is to use the remaining fuel reserves to rendezvous with space debris with similar orbital parameters. Capturing of the space debris is performed by the tether released from the tug. The tug and debris are pulled together using the tether. The next stage is an orbital momentum changing which lowers the periapsis of both objects (tug and debris).

This study focuses on the stage of pulling space debris by the tether. The goal is to find the control law that allow one to realize the impactless docking of the tug and debris.

The paper is divided into three main parts. In the Section 2 a length control strategy for the two-body system with an inextensible tether is considered. In the Section 3 an analysis of the proposed tether length control laws is given in terms of possible implementation. Section 4 focuses on the study of behavior of the two-body with a viscoelastic tether.

II. LENGTH CONTROL STRATEGY FOR THE TWO-BODY SYSTEM WITH AN INEXTENSIBLE TETHER

The two-body system includes two points $m_1$ (the space tug) and $m_2$ (the space debris) connected by the tether which has the initial length $L_0$ as shown in Fig.1.

The position of the material points $m_1$ and $m_2$ can be defined by the coordinates $x_1$ and $x_2$ respectively. It is assumed that controlled motion is considered in a very short time interval compared to the orbital period. We suppose that all external forces significantly less than the thrust tug $F$ and this force coincides with the line connecting the two material points (Fig.1). Internal interaction of the bodies is determined by the tether tension force $T$. The dynamics of the system, taking into account

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the gravitational forces of the Earth, will be discussed in the following papers.

In addition, we assume that the tether is weightless. Taking into account the accepted assumptions, the motion equations of the two-body system with the inextensible tether can be written as [32]:

\[
\begin{align*}
\dot{x}_i &= \frac{1}{m_i}(F - T), \\
\dot{x}_2 &= \frac{1}{m_2}T.
\end{align*}
\]

(1)

Using the current length of the tether as a new variable

\[
l = x_1 - x_2
\]

(2)

and according to the equations (1) the equation of the relative motion of the two tethered bodies can be written as

\[
l = \frac{1}{m_1}(F - T) - \frac{1}{m_2}T.
\]

(3)

To ensure the safe rendezvous, the variable (2), must satisfy the following boundary conditions

\[
\begin{align*}
t = 0 & : l = 0, \\
t = t_k & : l = l_k,
\end{align*}
\]

(4)

Note that the relative velocity \(l\) of the two bodies at the end of pulling maneuver (4) must be zero. The conditions (4) are satisfied for the following series

\[
L = \frac{L_0}{2} \left(1 + \sum_{n=1}^{\infty} a_{2n+1} \cos \left[(2n+1)\omega t\right]\right),
\]

(5)

where

\[
\omega = \frac{\pi}{t_k}
\]

(6)

\(t_k\) is the duration of the maneuver, \(a_{2n+1}\) is the series coefficient that satisfy to following relationship

\[
\sum_{n=1}^{\infty} a_{2n+1} = 1.
\]

(7)

For the case when \(n = 0\), the series (5) has a very simple form

\[
L = \frac{L_0}{2} \left(1 + \cos \omega t\right).
\]

(8)

Fig.2 shows the function (5) in cases:

\[
\begin{align*}
n = 0, & \quad a_1 = 1 - \text{solid line}, \\
n = 2, & \quad a_1 = 0.5, \quad a_2 = 0.25 - \text{dotted line}, \\
n = 2, & \quad a_1 = a_2 = 0.2, \quad a_3 = 0.6 - \text{dashed line}.
\end{align*}
\]

Thus, the series (5) satisfies the boundary conditions (4).

Differentiating twice the equation (5) and equating the result to the equation (3), we get

\[
\frac{1}{m_1}(F - T) - \frac{1}{m_2}T = -\frac{L_0\omega^2}{2} \left(\sum_{n=1}^{\infty} a_{2n+1} (2n+1)^2 \cos \left[(2n+1)\omega t\right]\right).
\]

(9)

The expression for the tension force, in which the boundary conditions are satisfied (4), can be written as

\[
T = M \left(\frac{L_0\omega^2}{2} \sum_{n=1}^{\infty} a_{2n+1} (2n+1)^2 \cos \left[(2n+1)\omega t\right]\right) + \frac{F}{m_1},
\]

(10)

where \(M = m_1 m_2 / m\) is the reduced mass of the system, \(m = m_1 + m_2\) is the total mass of the two-body system.

Substituting the expression (10) in the equations (1) we obtain the equations of motion of the two-body system with the inextensible tether

\[
\dot{x}_1 = \frac{F}{m_{tot}} - \frac{M_1 L_0\omega^2}{2 m_{tot}} \sum_{n=1}^{\infty} a_{2n+1} (2n+1)^2 \cos \left[(2n+1)\omega t\right],
\]

(11)

\[
\dot{x}_2 = \frac{F}{m_{tot}} + \frac{M_1 L_0\omega^2}{2 m_{tot}} \sum_{n=1}^{\infty} a_{2n+1} (2n+1)^2 \cos \left[(2n+1)\omega t\right],
\]

where \(M_1 = m_1 / m\) and \(M_2 = m_2 / m\) are the non-dimensional masses of the tug and the space debris respectively.

For the case when \(n = 0\) the expression (10) has the following form:

\[
T = M \left(\frac{L_0\omega^2}{2} \cos \omega t + \frac{F}{m_1}\right),
\]

(12)

and the equations (11) can be written as:

\[
\dot{x}_1 = \frac{F}{m_{tot}} - \frac{M_1 L_0\omega^2}{2 m_{tot}} \cos \omega t,
\]

(13)

\[
\dot{x}_2 = \frac{F}{m_{tot}} + \frac{M_1 L_0\omega^2}{2 m_{tot}} \sin \omega t.
\]

III. TETHER LENGTH CONTROL LAWS

Let us examine the possibility of implementing of the tether length control laws (5). All numerical calculations will be performed for the two-body system with the parameters listed in Table 1.

We also use the following initial conditions for numerical simulation:

\[
\begin{align*}
x_1(0) &= L_0, & \dot{x}_1(0) &= 0, & \dot{\dot{x}}_1(0) &= 0, \\
x_2(0) &= 0, & \dot{x}_2(0) &= 0, & \dot{\dot{x}}_2(0) &= 0.
\end{align*}
\]

(14)

A. The requirements to the maneuver pulling

The rendezvous of the spacecrafts should occur without the collisions. This is unacceptable for the active debris removal missions, as this may lead to serious consequences,
for example, to damage of the tug, the formation of new debris, and increase of the debris environment. To prevent it we formulate the following conditions:

1. Any collisions should be avoided, i.e.
\[ \forall t < t_k : x_i \neq x_j. \] (15)

2. The magnitude of the tension force \( T \) should be as low as possible. The less the tension force \( T \), the smaller would be the mass of the mechanism that implements this force. That is why we are introducing this limitation.

3. A work of the tension force \( T \) on the time interval \( t \in [0, t_k] \) should be minimal:
\[ W = \int_0^{t_k} T \delta L \rightarrow \min. \] (16)

This requirement is due to the energy capabilities of the space tug.

4. The tether throughout the maneuver must be strained, i.e.
\[ T > 0. \] (17)

Otherwise, it can lead to negative consequences, such as tangling the tether.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>PARAMETERS OF THE TWO-BODY SYSTEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>Parameters</td>
</tr>
<tr>
<td>( L_0 )</td>
<td>initial length</td>
</tr>
<tr>
<td>( m_1 )</td>
<td>tug mass</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>space debris mass</td>
</tr>
<tr>
<td>( t_k )</td>
<td>duration of the maneuver</td>
</tr>
<tr>
<td>( EA )</td>
<td>stiffness of the tether</td>
</tr>
<tr>
<td>( C )</td>
<td>damping of the tether</td>
</tr>
<tr>
<td>( F )</td>
<td>the thrust tug</td>
</tr>
</tbody>
</table>

B. Choice of the thrust tug
According to the equation (10) and the condition (17), to the thrust tug \( F \) should satisfy the condition
\[ F > \frac{1}{2} m_1 L_0 \omega^2 \sum_{i=0}^{\infty} a_{2i+1} (2i+1)^2 \cos [(2i+1) \omega t]. \] (18)

As the case of constant thrust is considered in this paper, the tether remains strained during the maneuver if
\[ F > \frac{m_1 L_0 \omega^2}{2} \sum_{i=0}^{\infty} a_{2i+1} (2i+1)^2. \] (19)

For the case when \( n = 0 \), the condition (19) takes the form
\[ F > \frac{m_1 L_0 \omega^2}{2}. \] (20)

From (20) it follows that the decrease of the required thrust tug \( F \) can be achieved by increasing the maneuver time \( t_k \).

C. Influence of the coefficients \( a_{2i+1} \) on the tension force \( T \)
For example, we consider the effect of the coefficient \( a_{2i+1} \) on the value of the tension force \( T \) when \( n = 1 \).

Fig.3 shows that decreasing value of the coefficient \( a_i \) results in negative value of the tension force \( T \) (for instance \( a_i = 0.6 \), \( a_3 = 1 - a_i = 0.4 \)). Such cases do not correspond to the condition (17).

D. Work of the tension force \( T \)
To calculate the integral (16) we rewrite it in the form
\[ W = \int_0^{t_k} T \delta l = \delta l \frac{dl}{dt} = \int_0^{t_k} \frac{dl}{dt} dt. \] (21)

Substituting (10) and the derivative of the equation (5) in (21) we get
\[ W = -\frac{ML_0 \omega}{2} \left[ \sum_{i=0}^{n} a_{2i+1} (2i+1) \sin [(2i+1) \omega t] \right] dt + \]
\[ + \frac{L_0 \omega^2}{2} \left[ \sum_{i=0}^{n} a_{2i+1} (2i+1)^2 \cos [(2i+1) \omega t] \right]. \] (22)

The first integral in (22) can be easily computed using the conditions (7)
\[ \int_0^{t_k} \left[ \sum_{i=0}^{n} a_{2i+1} (2i+1) \sin [(2i+1) \omega t] \right] dt = \]
\[ - \frac{2}{\omega^2} \sum_{i=0}^{\infty} a_{2i+1} = - \frac{2}{\omega^2} a_i. \]

The subintegral function of the second integral in (22) is the odd periodic function of time \( t \) as the product of the sums of even and odd functions. Therefore, the second integral is zero. Finally, the work of the tension force \( T \) (22) takes the following form
\[ W = M_1 L_0 F = \frac{m_2}{m_1 + m_2} L_0 F. \] (23)

It follows that the work of the tension force \( T \) (23) depends only on the masses of the tug and of the debris, the total unstretched tether length and the thrust tug.

IV. EQUATIONS OF MOTION WITH A VISCOELASTIC TETHER
In this section we consider behavior of the two-body system under the length control law (8), taking into account the viscoelastic properties of the tether. The equations of motion can be written as:
\[
\dot{x}_1 = \frac{1}{m_1} \left[ F - H_x (T_s + T_d) \right],
\]
\[
\dot{x}_2 = \frac{1}{m_2} H_x (T_s + T_d),
\]
where
\[
H_x = \begin{cases} 
0, & \left| x_1 - x_2 \right| \leq L, \\
1, & \left| x_1 - x_2 \right| > L.
\end{cases}
\]

\(T_s\) is a tension force and \(T_d\) is a damping force:
\[
T_s = EA \varepsilon, \\
T_d = C \dot{\varepsilon},
\]
\[
\varepsilon = \frac{\left| x_1 - x_2 \right|}{L} - 1,
\]
\[
\dot{\varepsilon} = \frac{\left( x_1 - x_2 \right) (\dot{x}_1 - \dot{x}_2)}{\left| x_1 - x_2 \right|} \frac{\dot{L}}{L} (1 + \varepsilon),
\]
where \(EA\) is stiffness of the tether, \(C\) is a damping constant, \(L\) is the length control law (8). By substituting the expressions (25) - (28) into the equations (24), we get the equations of motion system with a viscoelastic tether.

In order to study the influence of the viscoelastic properties of the tether, several numerical techniques are used. The numerical results are based on the numerical integration of the equations (24) - (28) using an explicit fourth-order Runge-Kutta method. Table 1 presents parameters of the two-body system.

Fig.4 depicts the results of simulation for four the different initial conditions:
\[
(a) \quad x_1(0) = L_0, \quad x_1(0) = 0, \\
(b) \quad x_1(0) = 0.85L_0, \quad x_1(0) = 0, \\
(c) \quad x_1(0) = 1.15L_0, \quad x_1(0) = 0, \\
(d) \quad x_1(0) = 1.3L_0, \quad x_1(0) = 0.
\]

Fig.4b shows that the viscoelastic properties of the tether do not affect the behavior of the system in contrast to the cases (Figs. 4a, 4c, 4d).

In the Fig. 4d we have seen the collision between the tug and the debris at moments of time: \(t_1 = 13s, t_2 = 33s\). In order to prevent the collision the thrust tug should be increase (Fig. 5).

V. CONCLUSION

The tether length control law for the docking of two bodies by means of the viscoelastic tether in a force-free field is proposed. The general analysis shows that, the requirements of safe rendezvous of the two body using the proposed law can be implemented. It was also showed that one can realize a soft docking of the two bodies. We discovered that the elastic properties of the tether make it difficult to the soft docking. It is shown that a change in the thrust tug can compensate the action of the elastic properties of the tether. The obtained results can be applied to study the properties and possible configurations of the tethered spacecraft docking system.
REFERENCES


[27] SpaceX Falcon 9 description, access http://www.space.com/falcon9/.


